CS 554
Computer Vision

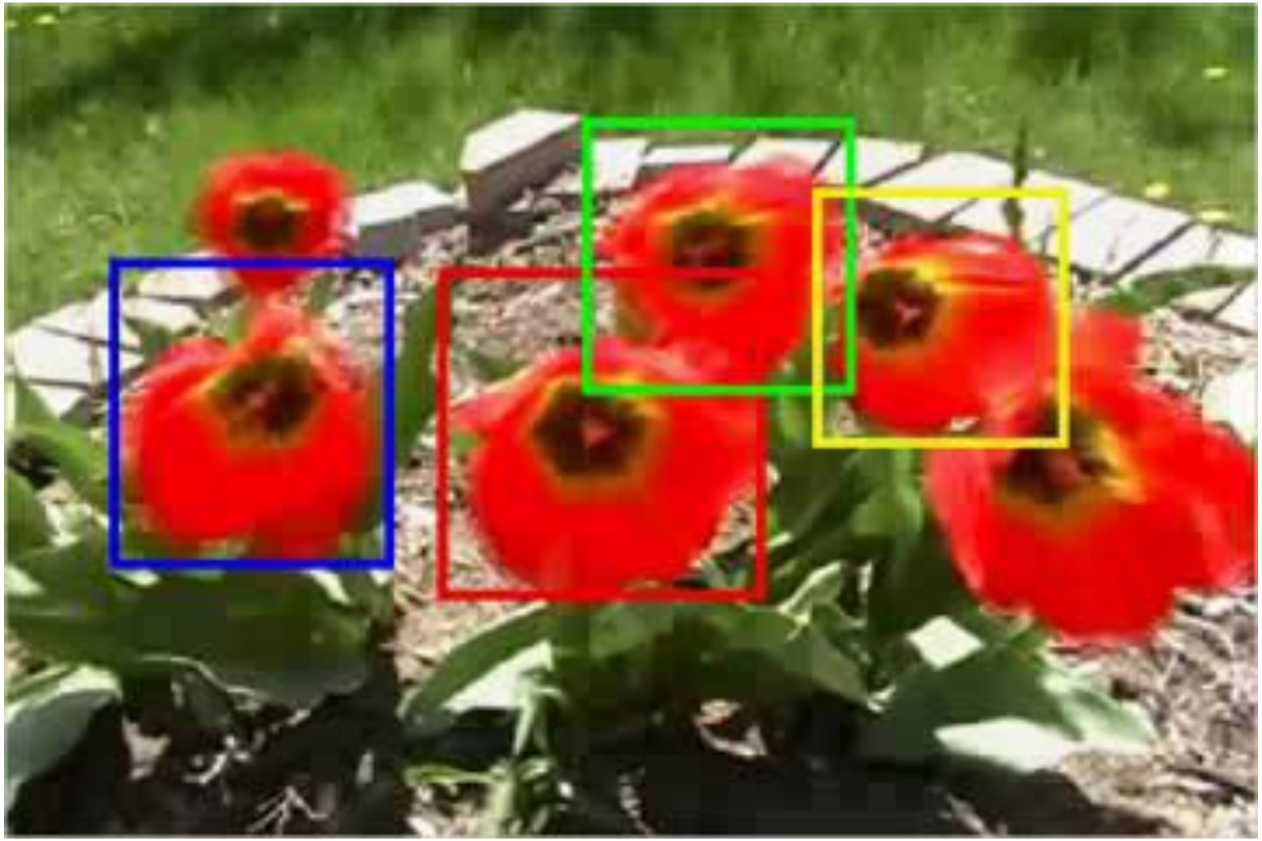
Tracking

Hamdi Dibeklioğlu

Slide Credits: L. van der Maaten

Tracking

Tracking aims to follow an object or object configuration in a video sequence:



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Recall *Bayes' rule:*

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Search strategy can be one of the following four approaches:

- Kalman (only for linear-Gaussian likelihood and prior) or particle filtering
- Lucas-Kanade algorithm (only for template matching using squared errors)
- Mean-shift tracking
- Sliding-window search (essentially brute-force search)

Kalman filter

Kalman filter

- We aim to infer the location of the object, $\mathbf{x}_t$, at time t

- A simple *motion prior* could be defined as:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}) \propto \exp\left(-\frac{1}{2\sigma^2}\|\mathbf{x}_t - \mathbf{A}\mathbf{x}_{t-1}\|^2\right)$$

- A simple *likelihood* model could be defined as:

$$p(\mathbf{I}_t|\mathbf{x}_t) \propto \exp\left(-\frac{1}{2\tau^2}\|\mathbf{B}\mathbf{x}_t - \phi(\mathbf{I}_t)\|^2\right)$$

- Combining the two via Bayes' rule: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{I}_t) \propto p(\mathbf{I}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{x}_{t-1})$

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- Because both terms are Gaussian, the result is also Gaussian:

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$$p(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t \mid \boldsymbol{A}\mathbf{x}_{t-1}, \sigma^2)$$

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$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{I}_t) = \mathcal{N}(\mathbf{x}_t|\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

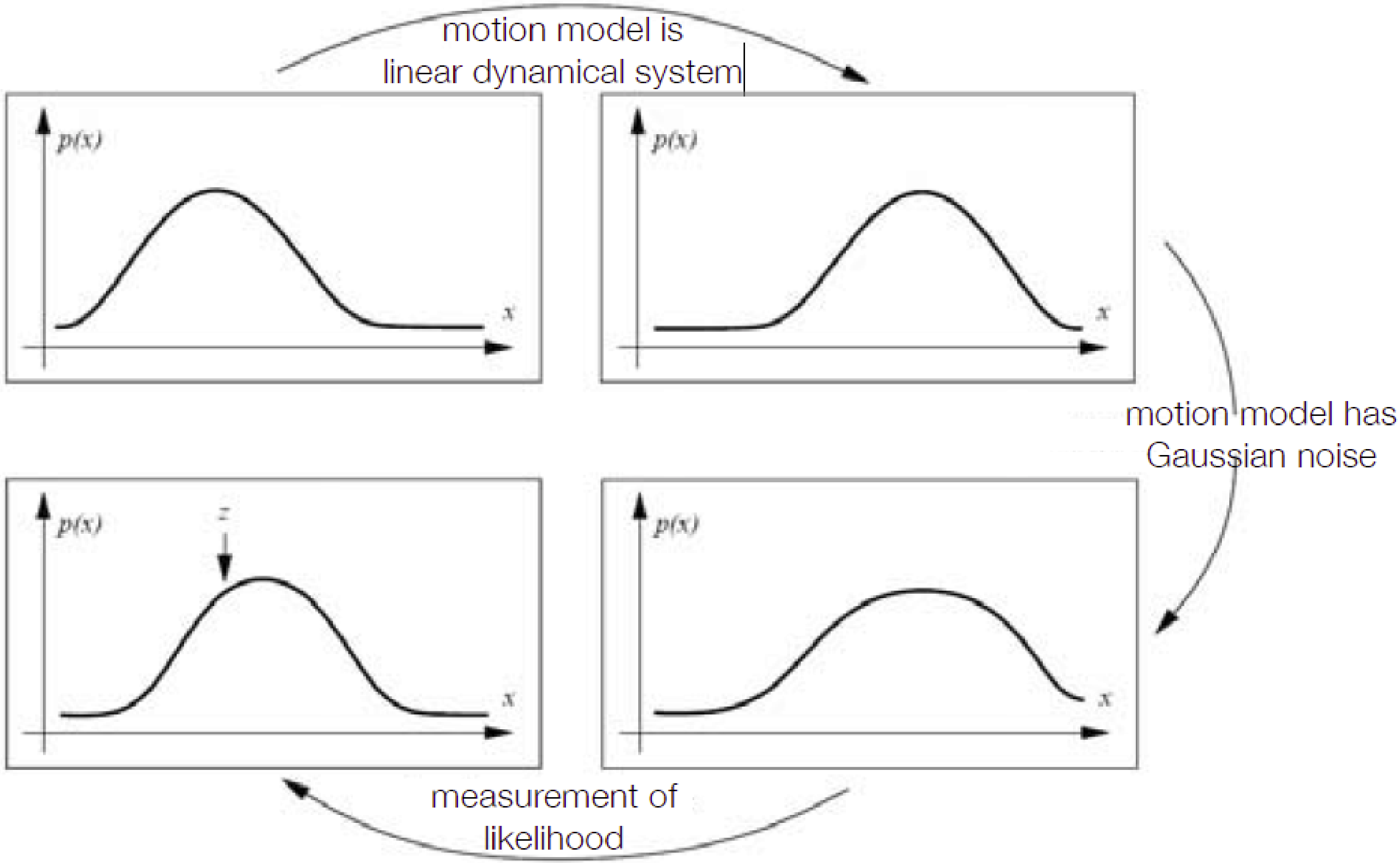
$$\boldsymbol{\mu}_t = \mathbf{A}\boldsymbol{\mu}_{t-1} + \mathbf{K}_t(\phi(\mathbf{I}_t) - \mathbf{B}\mathbf{A}\boldsymbol{\mu}_{t-1})$$

$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{B})\left(\mathbf{A}\boldsymbol{\Sigma}_{t-1}\mathbf{A}^{\mathrm{T}} + \sigma^2\mathbf{I}\right)$$

$$\mathbf{K}_t = \left(\mathbf{A}\boldsymbol{\Sigma}_{t-1}\mathbf{A}^{\mathrm{T}} + \sigma^2\mathbf{I}\right)\mathbf{B}^{\mathrm{T}}\left[\mathbf{B}\left(\mathbf{A}\boldsymbol{\Sigma}_{t-1}\mathbf{A}^{\mathrm{T}} + \sigma^2\mathbf{I}\right)\mathbf{B}^{\mathrm{T}} + \tau^2\mathbf{I}\right]^{-1}$$

"Kalman gain matrix"

Kalman filter



Example: Kalman filter

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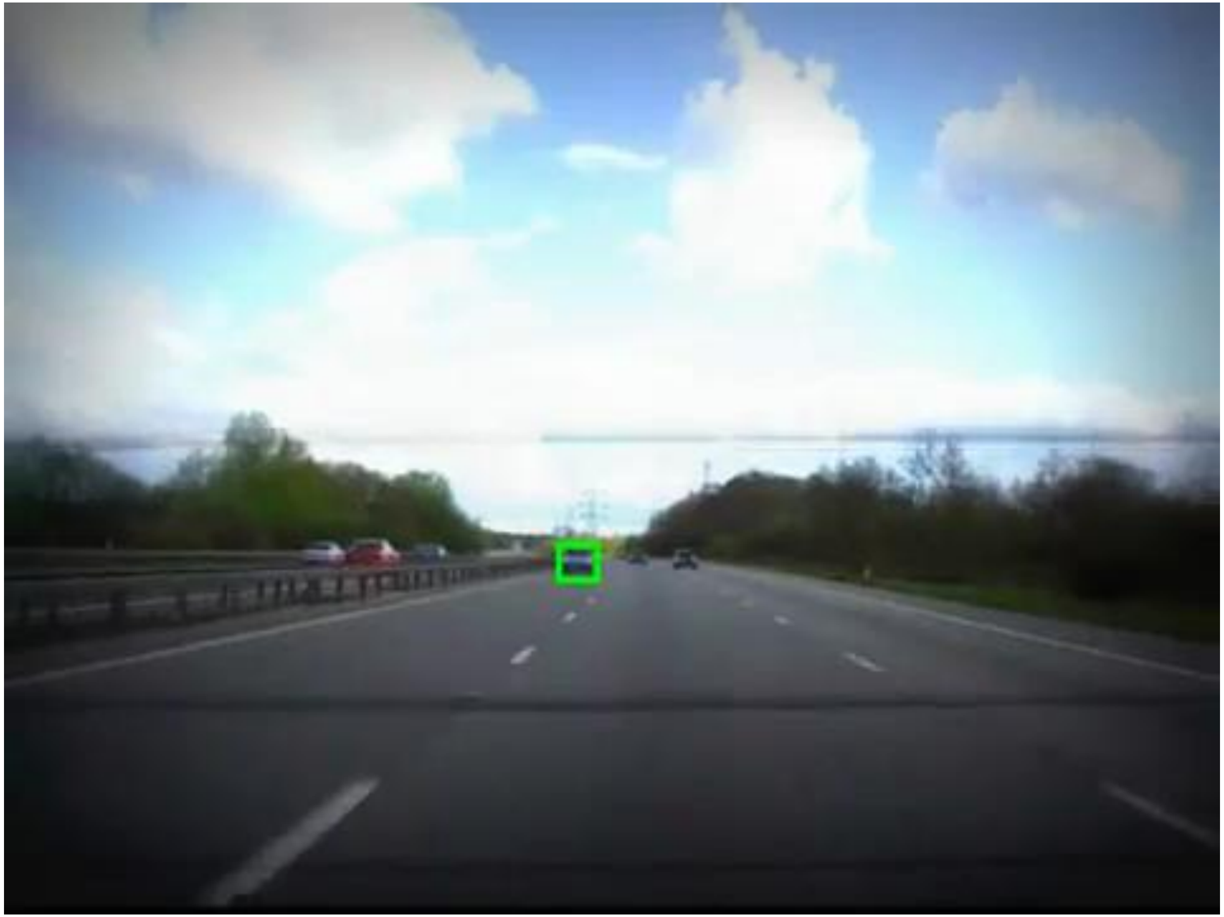

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In practical settings, the Kalman filter does often not work very well:

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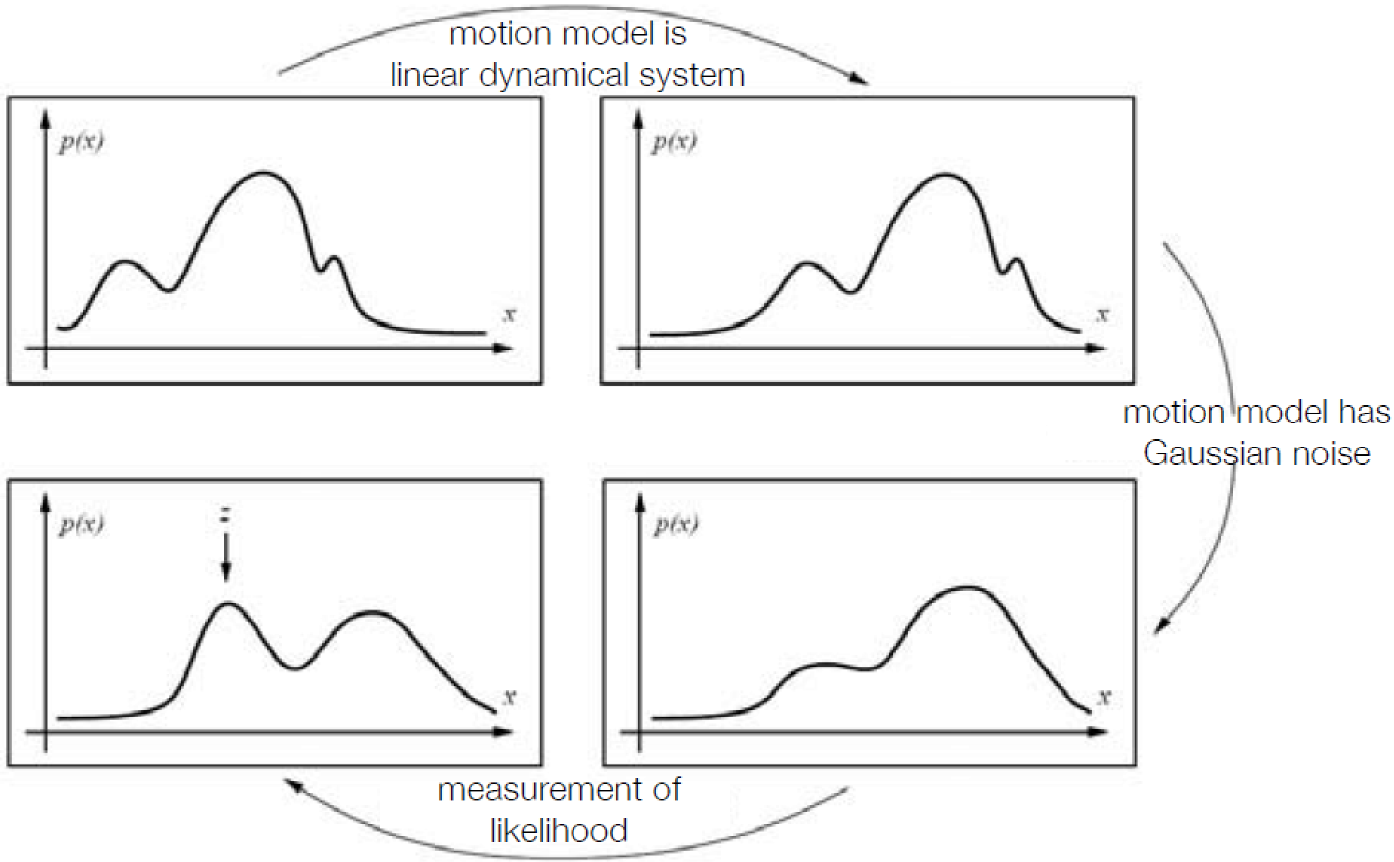
Changing the likelihood to something non-Gaussian complicates inference:

- Particle filters do this: solve inference problem by *importance sampling*

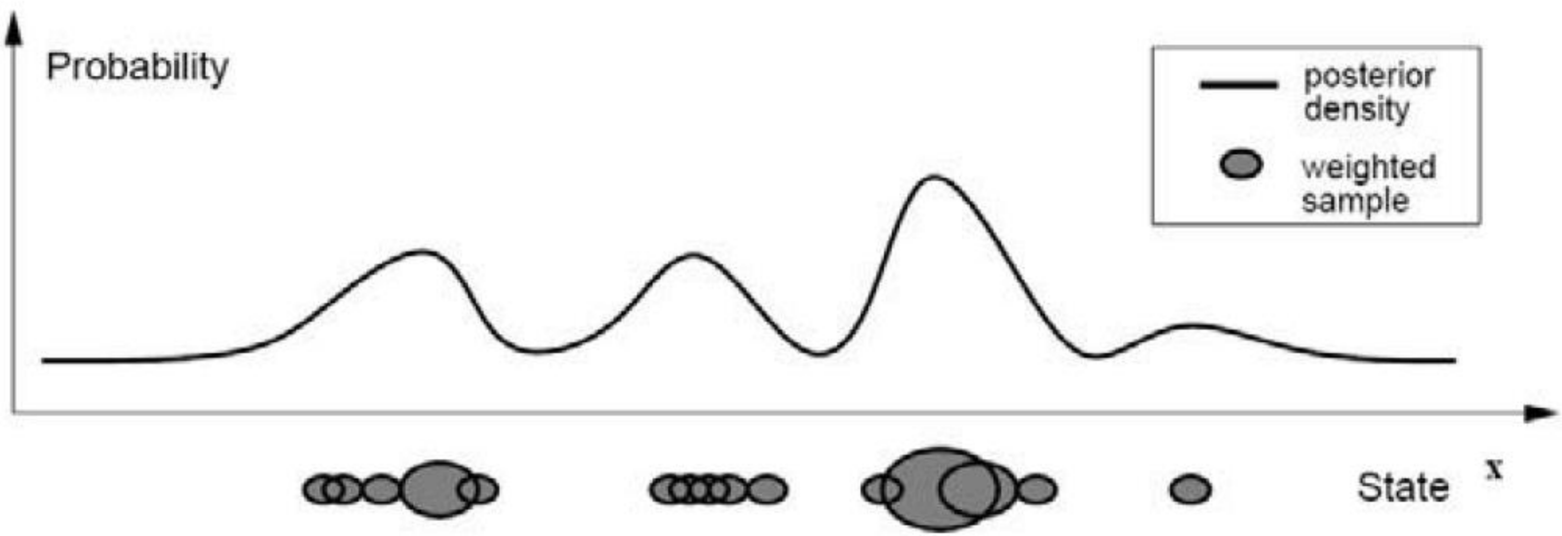
Particle filter

Particle filter



Particle filter (Condensation)

- For complex appearance likelihood functions, exact inferences are impossible
- Particle filters represent $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{I}_t)$ by a set of weighted samples ("particles"):



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  - Prediction may be weighted average of particles, or most likely particle

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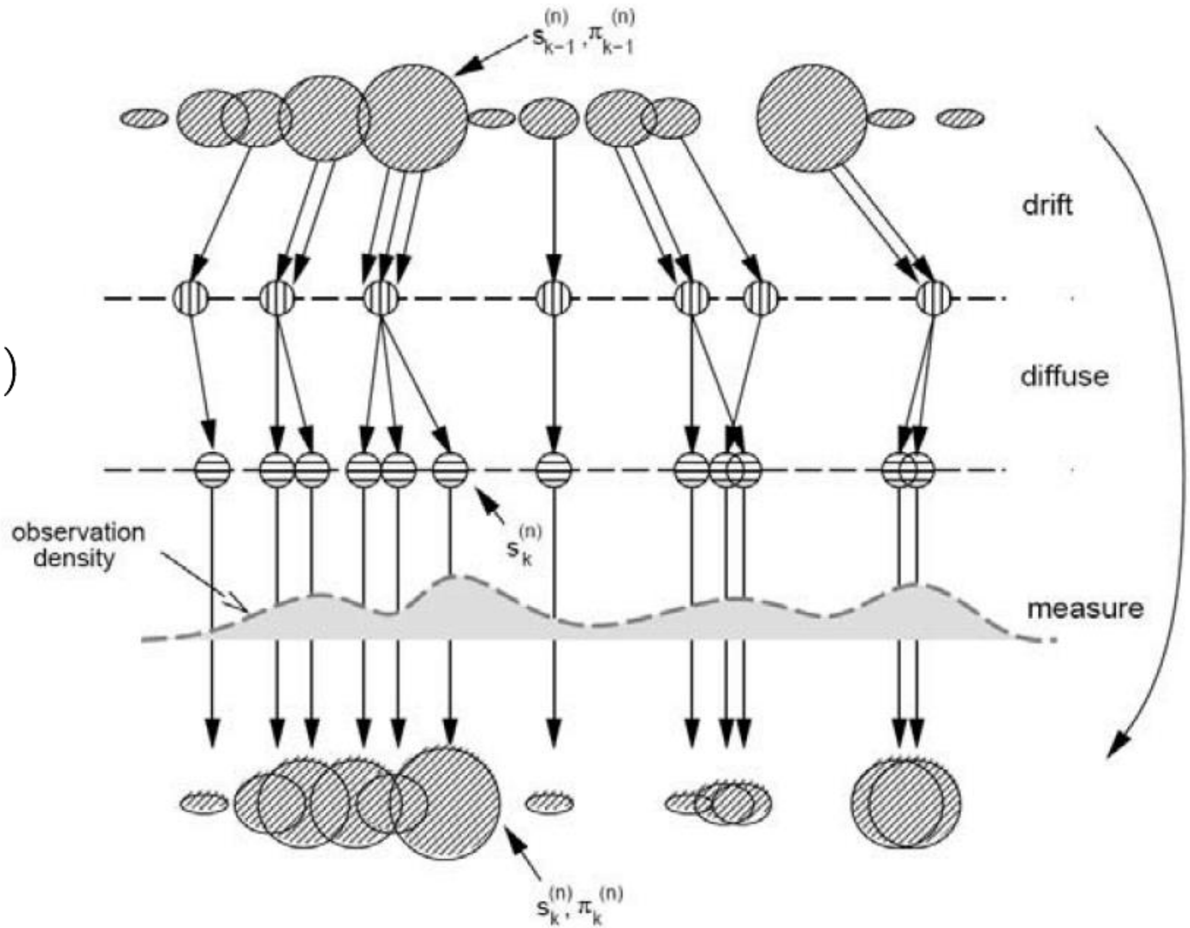
Pick sample $\mathbf{s}_{t-1}^{(n)}$ according to weights

Sample from $p(\mathbf{s}_t^{(n)}|\mathbf{s}_{t-1}^{(n)})$

Update weights $\pi_t^{(n)}$



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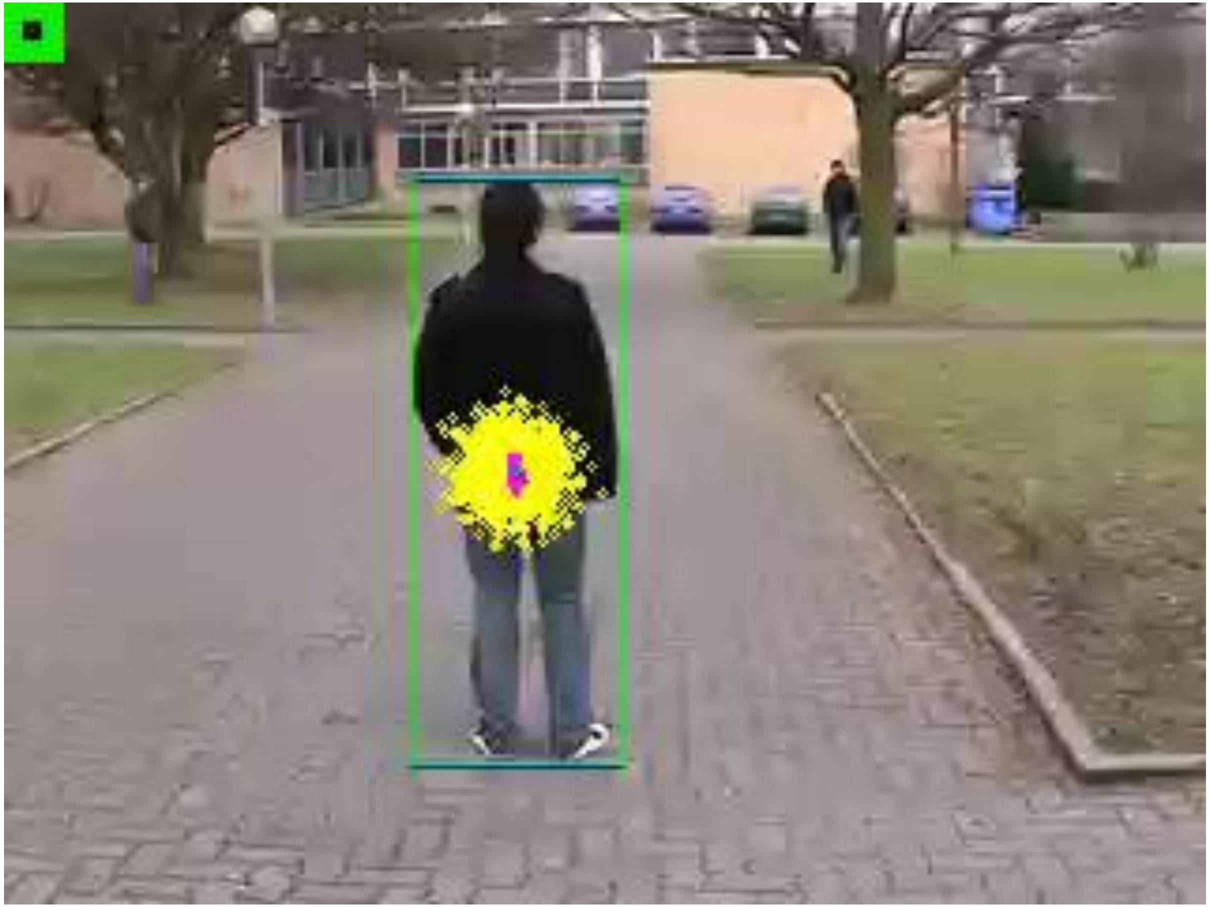

Tracking-by-detection

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- Appearance likelihood: Your favorite object detector (V&J, D-T, *etc.*)
  - Now, we are using a *conditional likelihood*:

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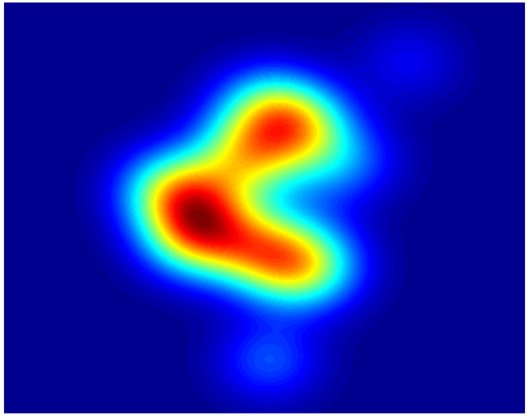
- Potential problem: Object appearance may change over time

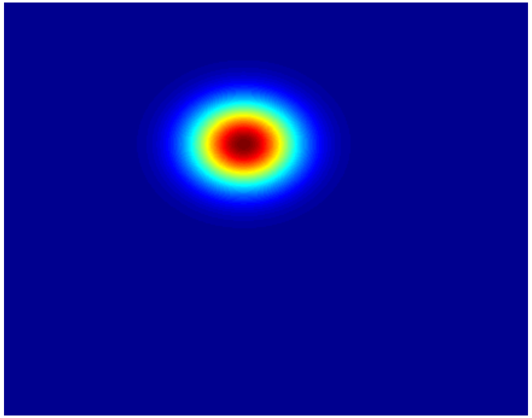
Track-learn-detect

Can we gather positive and negative examples to update appearance model?

- Positive example: Assume the track in the previous frame is correct
- Negative example: Example with high detection score but low motion prior



appearance likelihood



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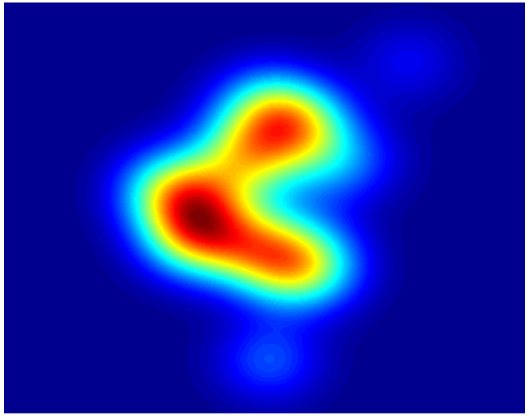

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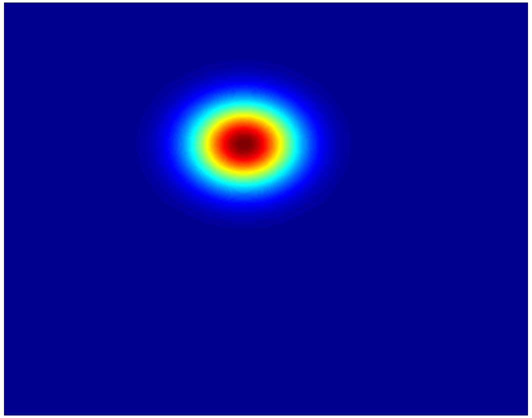

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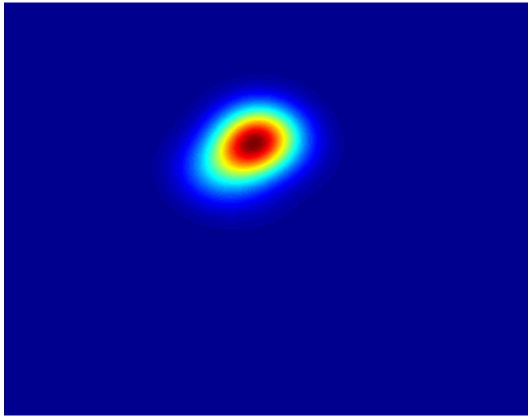

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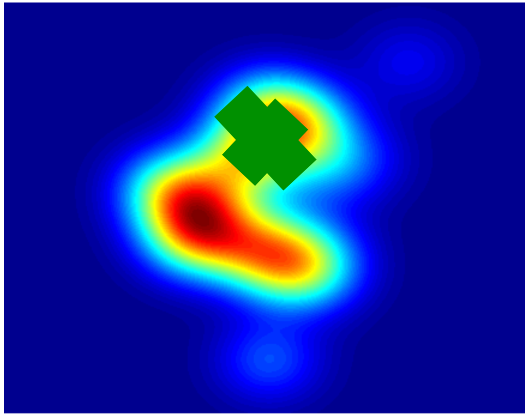

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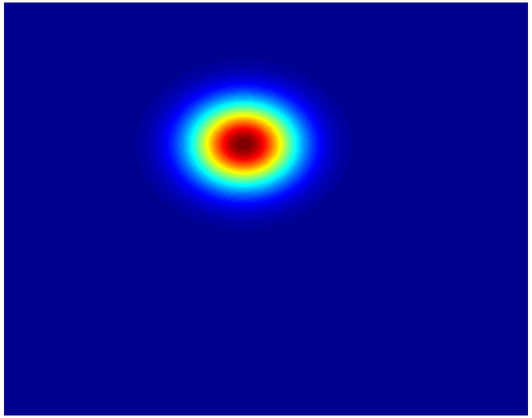

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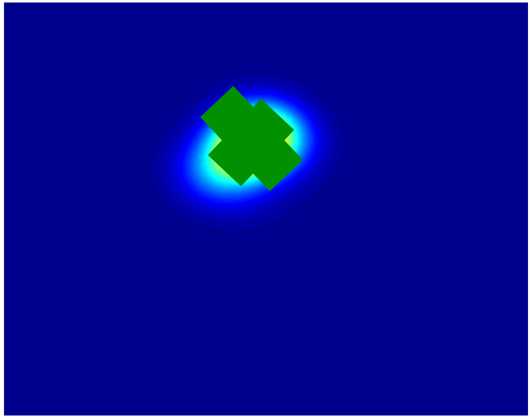

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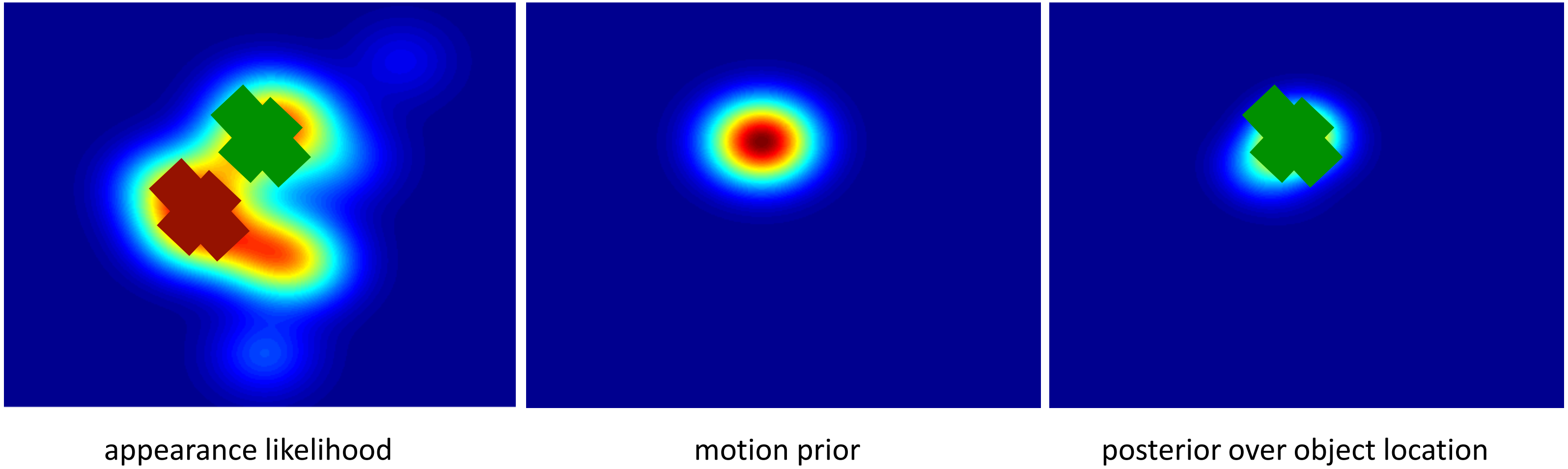

Example: Track-learn-detect

Kanade-Lucas-Tomasi tracker

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- Detect feature points in object using *Shi-Tomasi corner detector* (like Harris)

- Track feature points in the next frame by minimizing the squared error:

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- Note that this is yet another (non)linear least squares minimization problem!

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- Note that this is yet another (non)linear least squares minimization problem!
    - The minimization can be performing using the Lucas-Kanade algorithm

Example: Kanade-Lucas-Tomasi Tracker

Reading material:

- M. Isard and A. Blake. "Condensation—Conditional Density Propagation for Visual Tracking." International Journal of Computer Vision 29(1), 5-28, 1998.
- Section 1 and 2 of “Lucas-Kanade 20 Years On: A Unifying Framework,” International Journal of Computer Vision 56(3), 221–255, 2004.