Matter Waves

Berat Gönültaş

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Abstract

In this experiment we used an electron diffraction tube to find Planck's constant and the two distinct crystal spacing distance of the graphite. We diffracted the electrons at the graphite and observe the circles they made on the fluorescent screen. We measured the radii of the circles and used equations to get our results. After my analysis, I found $d_{small} = 0.25 \pm 0.20 \times 10^{-9} m$ and $d_{big} = 0.43 \pm 0.14 \times 10^{-9} m$. Accepted values are 0.123 nm and 0.213 nm, respectively. So , the result for first one is one sigma away from the accepted value while the second is two sigma away. These are relatively good results though our uncertainties are not small. I also found $h = 0.12 \pm 3.9 \times 10^{-34} Js$, which is 2 sigma away from the accepted value, 6.626×10^{-34} Js.

1 Introduction

In 1923, de Broglie postulated that matter can behave like a wave and its wavelength is given by

$$\lambda = \frac{h}{p} \tag{1}$$

where h is the Planck's constant and p is the momentum of the particle.[1]

The relationship is now known to hold for all types of matter: all matter exhibits properties of both particles and waves. In 1926, Erwin Schrödinger published an equation describing how a matter wave should evolve—the matter wave analogue of Maxwell's equations—and used it to derive the energy spectrum of hydrogen.[2]

We have the following equation for an electron accelerated in a potential.

$$q_e V = \frac{p^2}{2m_e} \tag{2}$$

where m_e and q_e are the mass and the charge of an electron and V is the potential difference between cathode and anode. Combining Equation 1 and Equation 2, we get,

[1]Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

[2]`https://en.wikipedia.org/wiki/Matter_wave`

$$\lambda = \frac{h}{\sqrt{2q_e m_e V}} \tag{3}$$

If we send electrons to a crystal, they scatter according to the Bragg's Law.[3]

$$2d\sin\theta = m\lambda \tag{4}$$

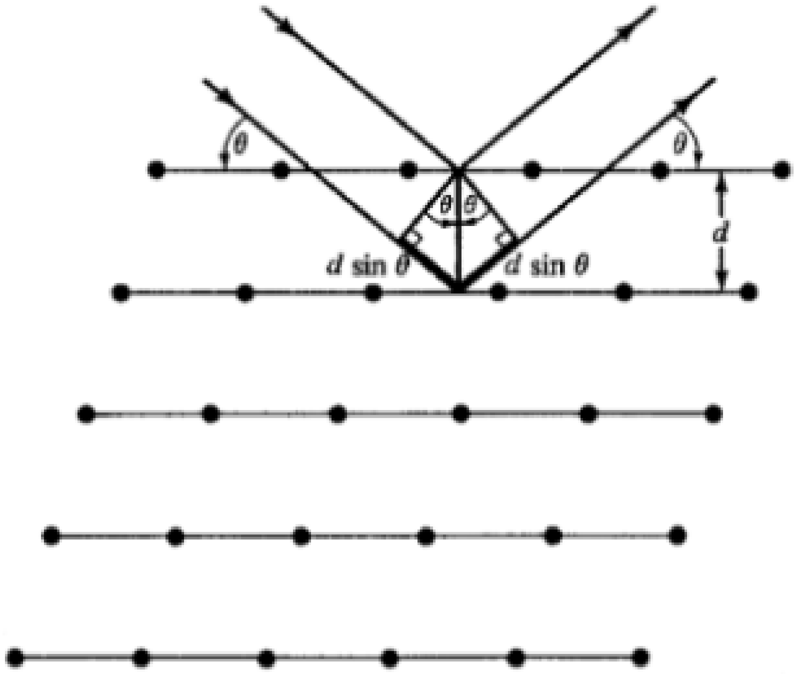

Figure 1: Bragg's Law

where θ is the angle corresponding to the mth maxima in the diffraction pattern; d is the crystal spacing.

In graphite, there are two different lattice constants because of the crystal structure of the graphite. The conditions in the setup we are using in this experiment allows only the first order diffraction to be seen. However, since there are two different lattice spacings, two first order diffraction maxima are observed.[4]

Keeping in mind $m = 1$, if we combine Equation 3 and 4, we get

$$\frac{1}{\sqrt{V}} = d\frac{\sqrt{8q_e m_e}}{h}\sin\theta \tag{5}$$

[3]Figure 1 and 2 were taken from `http://instructor.physics.lsa.umich.edu/adv-labs/Electron_Diffraction/electron_diffraction2.pdf`

[4]Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

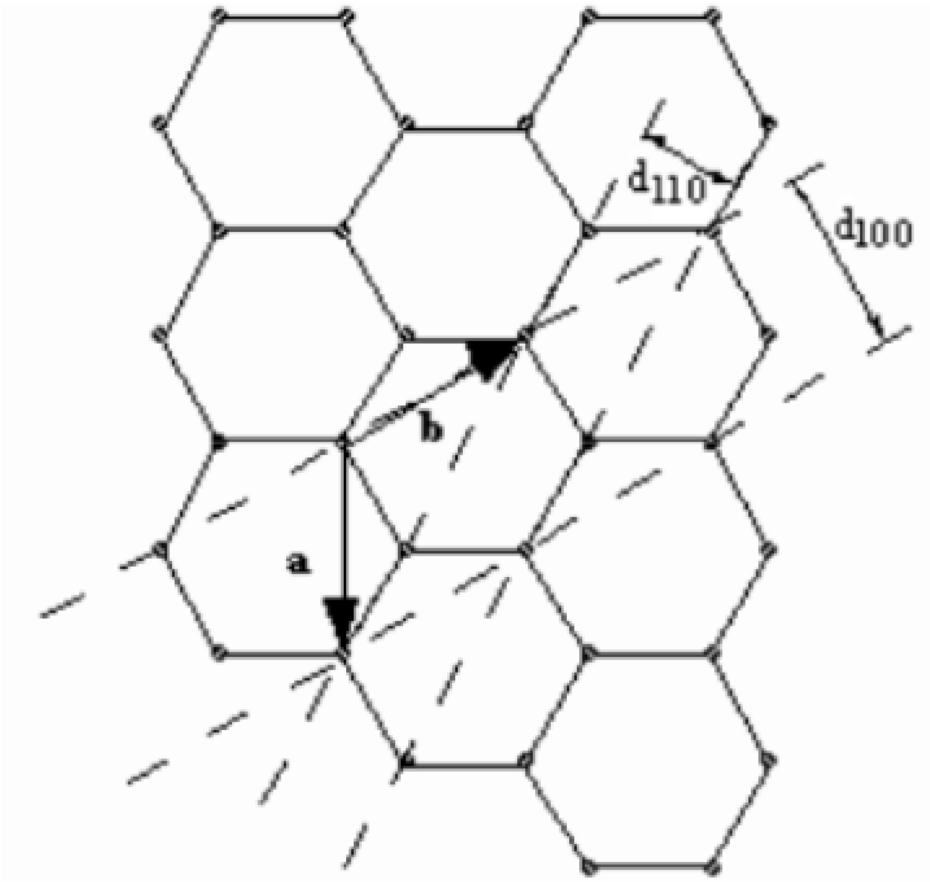

Figure 2: Lattice spacings for graphite

$\sin\theta$ can be calculated by founding θ as

$$\theta = \frac{\arctan \frac{r}{L}}{2} \tag{6}$$

2 Apparatus and Procedure

In this experiment, we used electron diffraction tube, 0-50 kV DC power supply, 5kV DC power supply, connecting leads, microammeter 0-50 μA, and rheostat.[5]

The electron diffraction tube is a small cathode ray tube (CRT) similar to that in a small (old-fashioned) TV set.[6] The electron diffraction tube consists of an electron gun that

[5]Figure 3 was taken from the Erhan Gülmez's book. There is 2θ instead of θ, since the incoming ray is horizontal so that one imagine that the Figure 1 was rotated θ degrees to the left.

[6]University of Michigan, Advanced Physics Laboratories, Electron Diffraction and Crystal Structure, `http://instructor.physics.lsa.umich.edu/adv-labs/Electron_Diffraction/electron_`

accelerates electrons towards a graphite foil. In contrast to the cathode ray tube and the fine beam tube a much higher voltage is used. The source of the electron beam is the electron gun, which produces a stream of electrons through thermionic emission at the heated cathode and focuses it into a thin beam by the control grid. A strong electric field between cathode and anode accelerates the electrons, before they leave the electron gun through the spaces of the anode-grid. Afterwards the accelerated electrons hit a thin graphite foil. The electrons are diffracted there so they fly towards the screen in different directions. When electrons strike the fluorescent screen, light is emitted so that the diffraction picture gets visible. The whole configuration is placed in a vacuum tube to avoid collisions between electrons and gas molecules of the air, which would attenuate the beam.[7]

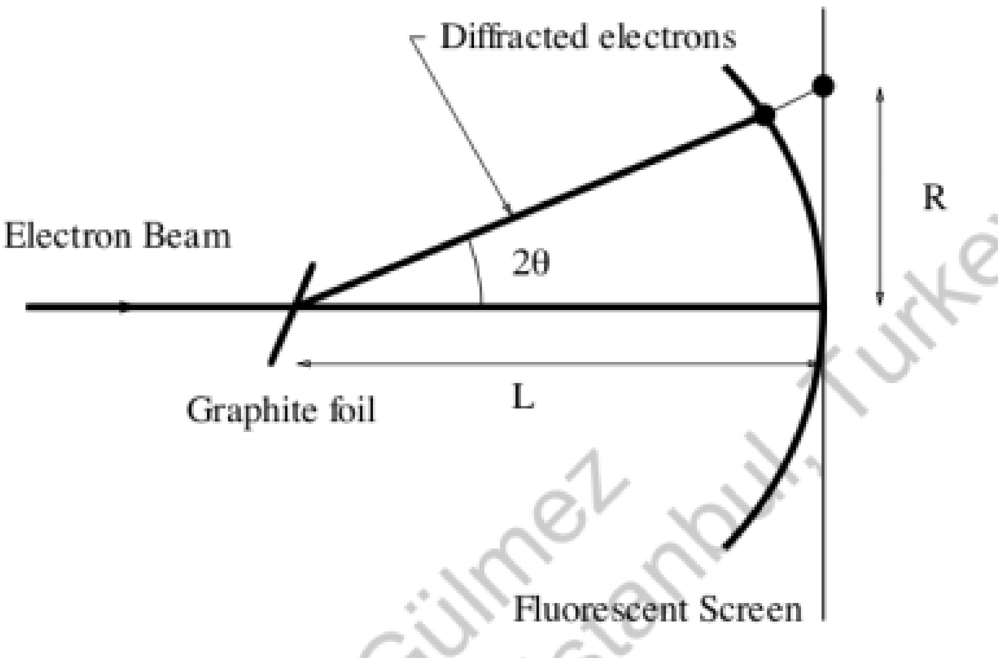

Figure 3: Geometry of the electron diffraction tube

The cathode is internally connected to the filament through a large resistor. As the tube current increases, the current passing through this large resistor increases also. The increased current causes the indirectly heated cathode element to be more negative hence reducing the number of electrons reaching the cathode-anode region where they are accelerated and passing through the target. This arrangement to limit the current passing through the tube is useful to protect the thin graphite target to be damaged because of accidental high currents or user abuse.At high current values, the thin graphite target can be overheated and damaged easily.[8]

diffraction2.pdf

[7]CERN Teachers Lab, https://project-physicsteaching.web.cern.ch/project-physicsteaching/english/experiments/electron-diffraction-tube.pdf

[8]Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

3 Data and Analysis

Our raw data is given below.(Figure 5)[10]

V-high(kV)	V-reverse(V)	I-f(microA)	r-small(cm)	r-big(cm)
			1.27 +- 0.10	2.04 +- 0.10
4.6 +- 0.1	8 +- 1	0 +- 20	1.35 +- 0.10	2.19 +- 0.10
			1.25 +- 0.10	2.48 +- 0.10
			1.32 +- 0.10	2.51 +- 0.10
4.8 +- 0.1	6 +- 1	20 +- 20	1.49 +- 0.10	2.58 +- 0.10
			1.42 +- 0.10	2.38 +- 0.10
			1.42 +- 0.10	2.64 +- 0.10
4.4 +- 0.1	6 +- 1	20 +- 20	1.48 +- 0.10	2.51 +- 0.10
			1.51 +- 0.10	2.28 +- 0.10
			1.22 +- 0.10	2.35 +- 0.10
4.2 +- 0.1	9 +- 1	0 +- 20	1.49 +- 0.10	2.71 +- 0.10
			1.39 +- 0.10	2.60 +- 0.10
			1.35 +- 0.10	2.35 +- 0.10
4.7 +- 0.1	7 +- 1	20 +- 20	1.35 +- 0.10	2.37 +- 0.10
			1.22 +- 0.10	2.37 +- 0.10

V-high(kV)	V-reverse(V)	I-f(microA)	r-small(cm)	r-big(cm)
			1.37 +- 0.10	2.37 +- 0.10
4.9 +- 0.1	9 +- 1	0 +- 20	1.37 +- 0.10	2.37 +- 0.10
			1.25 +- 0.10	2.39 +- 0.10
			1.52 +- 0.10	2.55 +- 0.10
4.6 +- 0.1	6 +- 1	20 +- 20	1.47 +- 0.10	2.56 +- 0.10
			1.18 +- 0.10	2.71 +- 0.10
			1.26 +- 0.10	2.48 +- 0.10
4.6 +- 0.1	11 +- 1	0 +- 20	1.20 +- 0.10	2.35 +- 0.10
			1.20 +- 0.10	2.30 +- 0.10
			1.27 +- 0.10	2.39 +- 0.10
3.9 +- 0.1	9 +- 1	0 +- 20	1.27 +- 0.10	2.39 +- 0.10
			1.28 +- 0.10	2.39 +- 0.10
			1.35 +- 0.10	2.53 +- 0.10
4.4 +- 0.1	9 +- 1	0 +- 20	1.35 +- 0.10	2.35 +- 0.10
			1.35 +- 0.10	2.35 +- 0.10

Figure 5: Data table

I first calculated the wavelength corresponding to each HV value.(Figure 6)[11]

To utilize Equation 5 to find d, we plotted $1/\sqrt{V}$ versus $\sin\theta$ graphs for both big radius and small radius.[12] Figure 7 shows the data for the small radius. When I tried to fit a first order polynomial to this plot, it failed. All clicks gave different p0 and p1 values, which are the y intersection and the slope, respectively. And I never saw any fit on the screen. To get at least a result, even a bad one, I threw the upper left data point away, which is the most outlying one. Then what I got is the Figure 8.

Then, I did the same plot for the big radius values.(Figure 9)

The slope values we get are equals to $d\frac{\sqrt{8q_e m_e}}{h}$ by Equation 5.

[10] I put 0.10 cm uncertainty on radii values although vernier is sensible enough to measure the second value after decimal point. I did so, because the rings had a thickness and it was an uncertainty to add the analysis.

[11] Calculations can be found in the spreadsheet called "calc.odt" in the link at the Appendix.

[12] Calculations can be found in the spreadsheet called "calc.odt" in the link at the Appendix.

HV (kV)	Corresponding Wavelength (m)
4.6 ± 0.1	$5.7 \pm 0.062 \times 10^{-10}$
4.8 ± 0.1	$5.6 \pm 0.058 \times 10^{-10}$
4.4 ± 0.1	$5.8 \pm 0.066 \times 10^{-10}$
4.2 ± 0.1	$6.0 \pm 0.071 \times 10^{-10}$
4.7 ± 0.1	$5.7 \pm 0.060 \times 10^{-10}$
4.9 ± 0.1	$5.5 \pm 0.057 \times 10^{-10}$
4.6 ± 0.1	$5.7 \pm 0.062 \times 10^{-10}$
4.6 ± 0.1	$5.7 \pm 0.062 \times 10^{-10}$
3.9 ± 0.1	$6.2 \pm 0.080 \times 10^{-10}$
4.4 ± 0.1	$5.8 \pm 0.066 \times 10^{-10}$

Figure 6: HV-Wavelenght

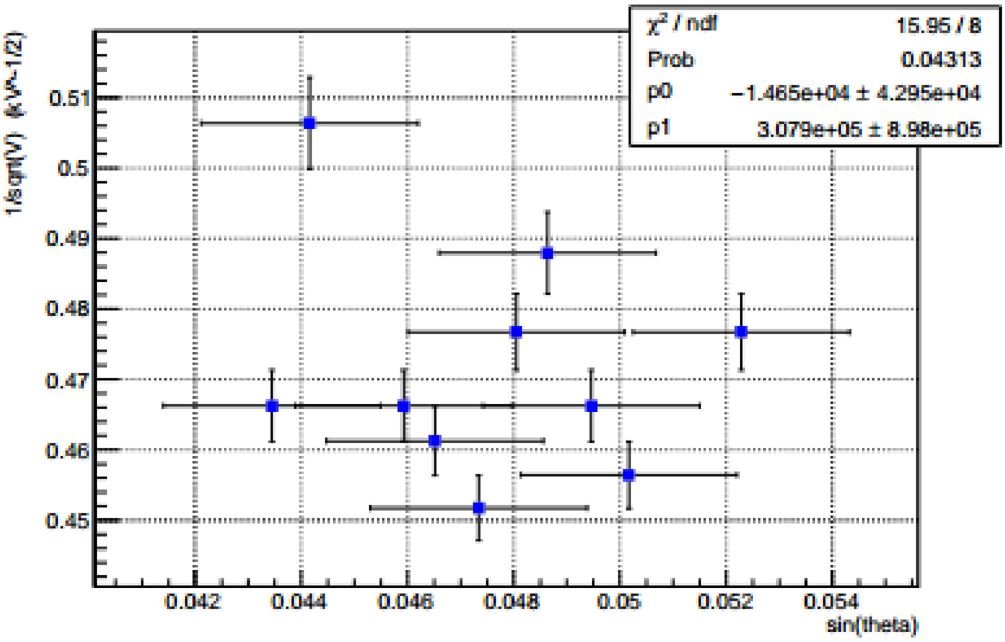

Figure 7: Small radius plot which didn't fit

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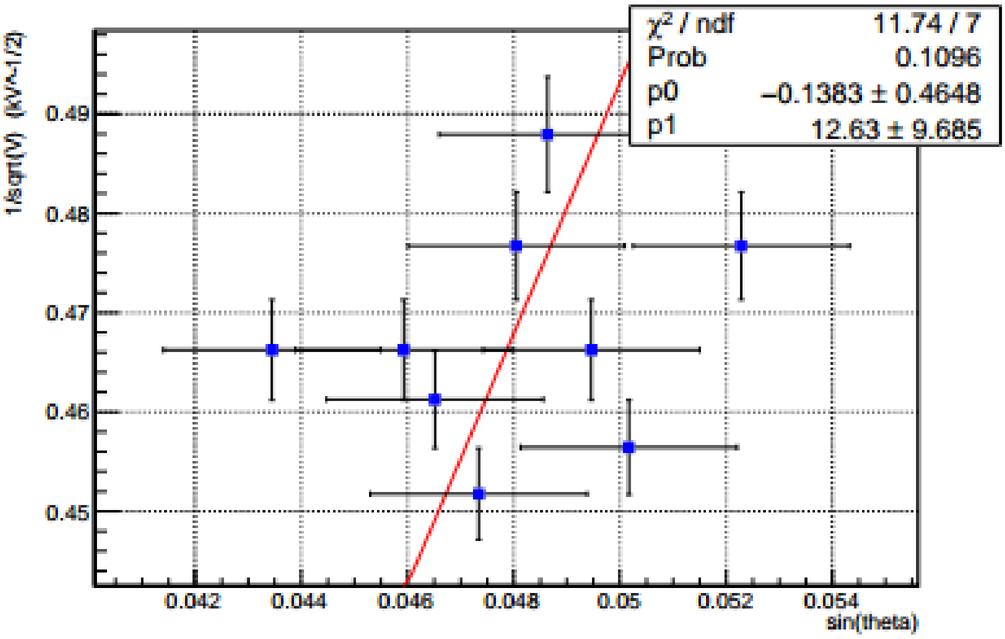

Figure 8: Small radius plot with a fit

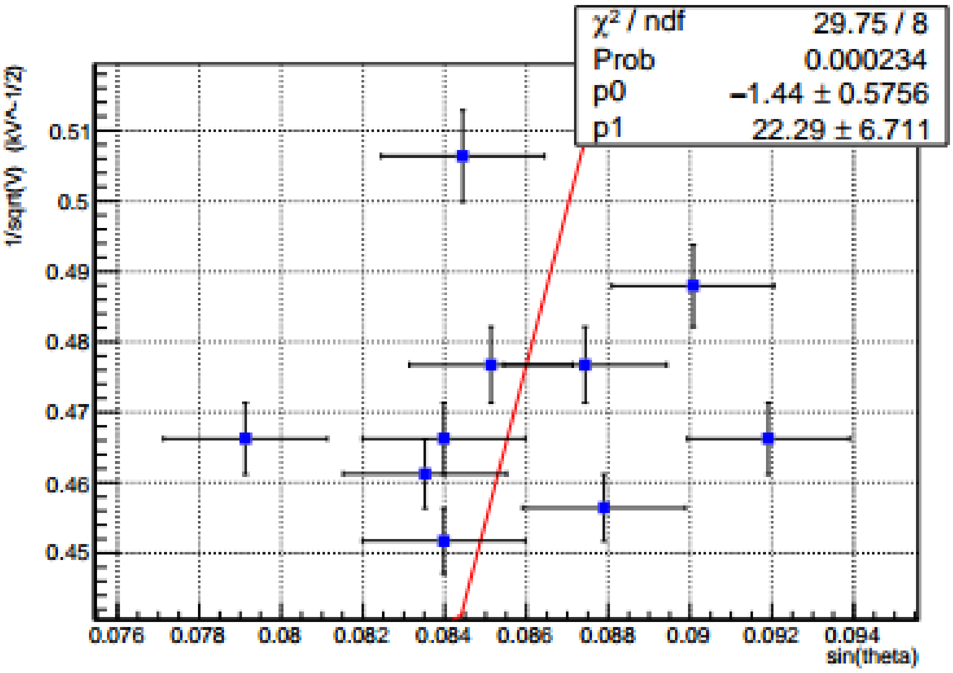

Figure 9: Big radius plot

The slope values are

HV (kV)	Weighted Average of the Wavelength(m)
4.6 ± 0.1	$2.00 \pm 0.0447 \times 10^{-11}$
4.8 ± 0.1	$2.17 \pm 0.0446 \times 10^{-11}$
4.4 ± 0.1	$2.19 \pm 0.0445 \times 10^{-11}$
4.2 ± 0.1	$2.18 \pm 0.0446 \times 10^{-11}$
4.7 ± 0.1	$2.04 \pm 0.0447 \times 10^{-11}$
4.9 ± 0.1	$2.06 \pm 0.0446 \times 10^{-11}$
4.6 ± 0.1	$2.22 \pm 0.0445 \times 10^{-11}$
4.6 ± 0.1	$2.00 \pm 0.0447 \times 10^{-11}$
3.9 ± 0.1	$2.02 \pm 0.0447 \times 10^{-11}$
4.4 ± 0.1	$2.09 \pm 0.0446 \times 10^{-11}$

Figure 10: HV-Weighted average of wavelength for corresponding radii

$$m_{small} = 13 \pm 10 \frac{1}{\sqrt{kV}} = 0.41 \pm 0.32 \frac{1}{\sqrt{V}} \tag{7}$$

$$m_{big} = 22 \pm 7 \frac{1}{\sqrt{kV}} = 0.70 \pm 0.22 \frac{1}{\sqrt{V}} \tag{8}$$

so that

$$d_{small} = 0.25 \pm 0.20 \times 10^{-9} m \tag{9}$$

and

$$d_{big} = 0.43 \pm 0.14 \times 10^{-9} m \tag{10}$$

Then, I calculated the wavelenght values using Equation 4. As it instructed in the handbook, I used known d values given in the handbook. I calculated d for both inner and outer ring for each case and calculated weighted average of them.(Figure 10)[13]

Then, I plotted the graph of the data given in Figure 10 to observe the linear relation between them. (Equation 3) Unfortunately again, the root couldn't make a fit to these data points.(Figure 11)

Then I, again, took away the upper left data point since it is the most outlying one. The result is in Figure 12.

Our slope here, $p1$ equals to

$$p1 = 14 \pm 4.6 \times 10^{9} \frac{1}{\sqrt{kV}m} = 0.44 \pm 0.14 \times 10^{9} \frac{1}{\sqrt{V}m} = \frac{\sqrt{2q_e m_e}}{h} \tag{11}$$

by Equation 3. Thus we found h as

$$h = 0.12 \pm 3.9 \times 10^{-34} Js \tag{12}$$

[13]Calculations are in the spreadsheet.

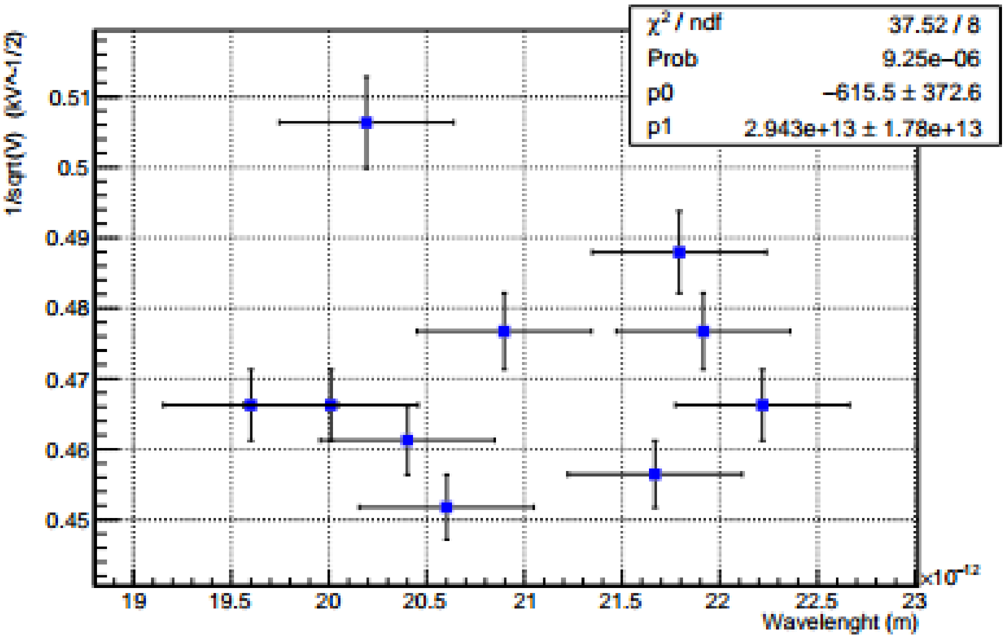

Figure 11: First try for the data in Figure 10

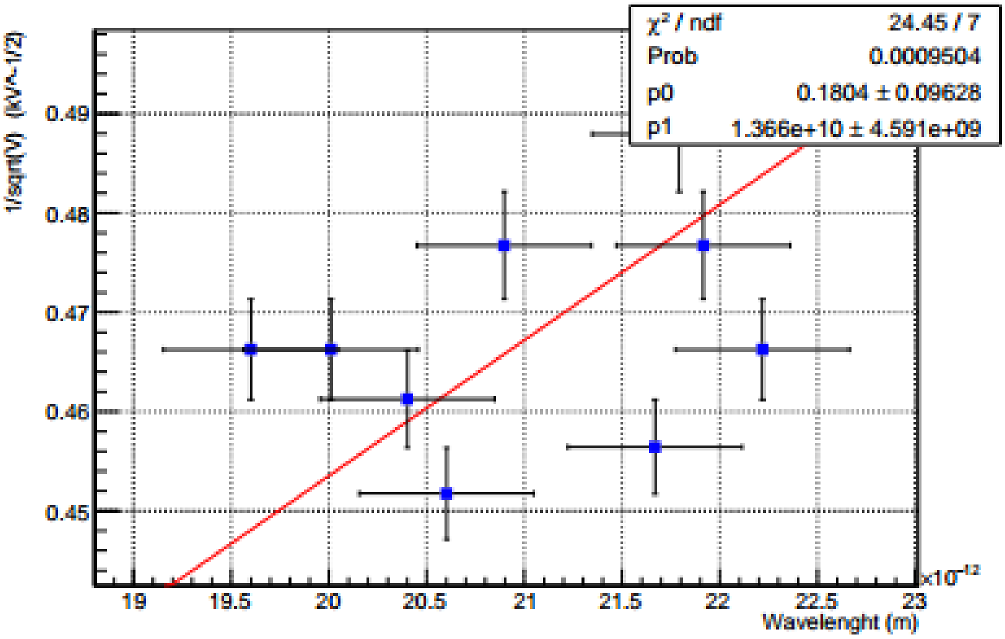

Figure 12: Plot of the data in Figure 10 with a linear fit

4 Conclusion

As mentioned in Abstract, the results are one or two sigma away from the accepted values. However, especially for the Planck's constant, the uncertainty is very high. In addition, we were expecting to get a linear distribution at Figure 12, but they are not so if one looks at the chi2 value. The another thing to keep in mind is that we get some fits after throwing away a data point. So, these fits are bad, but they are the best thing -more than the best- we can do with this data. So, problem must be at our data.

During the data taking process, we always took data in the HV region near 5kV, which was the maximum value for our setup. We did so because the rings were much clearer in high potentials since the electrons were much more energetic. However, it was a bad idea for statistical reasons. We had to look wider to see the picture, i.e., the linear behavior. Having a better linear behavior would give us better fits which have smaller uncertainties. Another stupid thing we did was taking data at same HV values. At that very moment in the lab, I thought still it is a different data since the reverse voltage is different, but it is nonsense. The reverse voltage is very small to effect the results. Actually, it is an agent we put to get better picture, it wouldn't give and didn't gave us any different picture. So, besides taking data in a small range, we also repeated some points. Still we measured different radii values but I had not to put them on the graph as different data points, I could use them to get a better average value.

As I mentioned previously, we took filament current datato see whether or not there is a pattern related to it. It seems there is not.

There are negative non-zero intercepts at Figure 8 and 9. They are nonsense. What I expect is a positive intercept. That would mean for some low HV values, the radii is 0 (there is no circle). This is logical since there is threshold value for voltage so that for the values below it, there is no electron hitting the screen. This unexpected negative intercept value is a flase result due to our bad data. We saw this expected positive intercept at Figure 12. It is significantly big to consider.

Taking data on a spherical surface like the one of tube was hard and some error must be due to this data taking process.

The velocity of the electrons speed up in 5kV is approximately 1/3 of the speed of light. It is enough to treat relativistically but the effect is not too big since the factor would be $\sqrt{1 - v^2/c^2} = 8/9$

5 Appendix

All references are given as footnotes.

The spreadsheet including the calculations and the code used for the plots are given in the link below.

`https://github.com/beratgonultas/443/tree/master/matter%20waves`

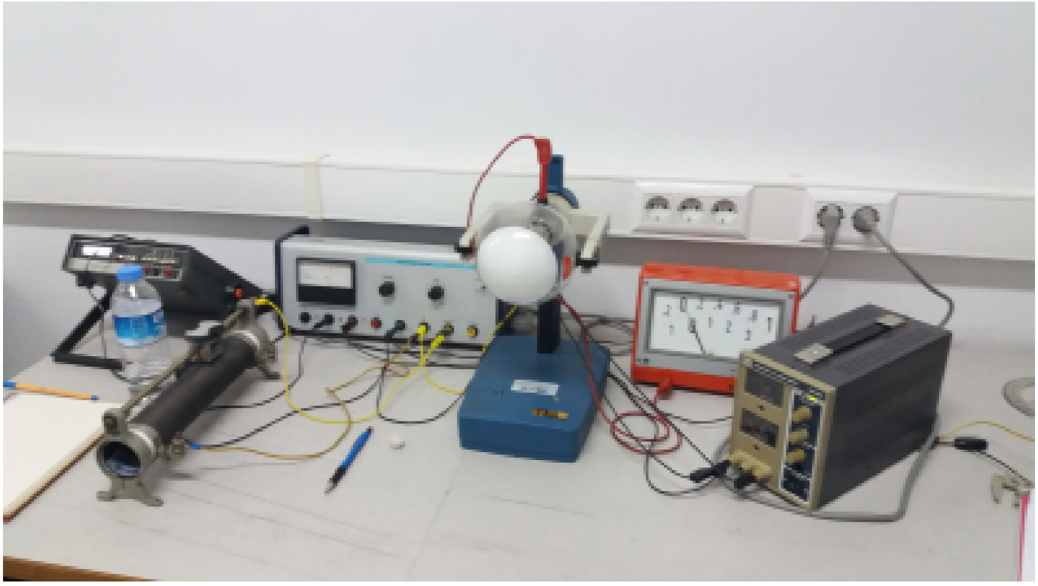

Figure 13: The setup