Cavendish Experiment

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Abstract

In this experiment, we found the Universal Gravitational Constant using a setup similar to the one which was used by Cavendish in 1798. The setup, known as torsion balance, consists of a rod having an appropriate torsion constant and two pairs of lead balls having a gravitational attraction in between. At the end of our analysis, we found the constant as $2300 \pm 89 \times 10^{-14} m^3 kg^{-1} s^{-2}$.

1 Theoretical Motivation

Henry Cavendish's experiments determining the density of the Earth were published in the Philosophical Transactions of the Royal Society in 1798. His method, following a procedure obtained from his friend John Michell, consisted of using a torsional spring to find the gravitational force between lead spheres smaller than 1 foot in diameter. In doing so he not only found the mass of the Earth, which then yielded masses for other celestial objects such as the Sun and Moon, but also verified the universal nature of Newton's Law of Gravitation. At that time Newton's law had proven to be of ample use in predicting the motion of the planets (Falconer), but as Cavendish notes in the opening of his paper, an important merit of Michell's procedure was its ability to "[render] sensible the attraction of small quantities of matter" (Cavendish), thereby bringing the inverse square law down from celestial orbits and into the laboratory. Famously, we can use Cavendish's results to calculate the gravitational constant G; the Cavendish experiment is, in fact, commonly described as the first determination of this constant, though the man himself entirely avoids it.[1]

The geometry of the system is given at Figure 1[2]. M's represents the big masses where m's are representing the small masses which rotates with the wire and results in a torsion. Laser light is fallen on the mirror which also rotates with the wire. The reflected laser light on the ruler permits us to calculate the rotation angle.

In general we can write gravitational force as follows.

$$F_G = G\frac{Mm}{r^2} \tag{1}$$

And we can write the torque due to a torsion on a wire as follows.

[1] `http://large.stanford.edu/courses/2007/ph210/chang1/`

[2] Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

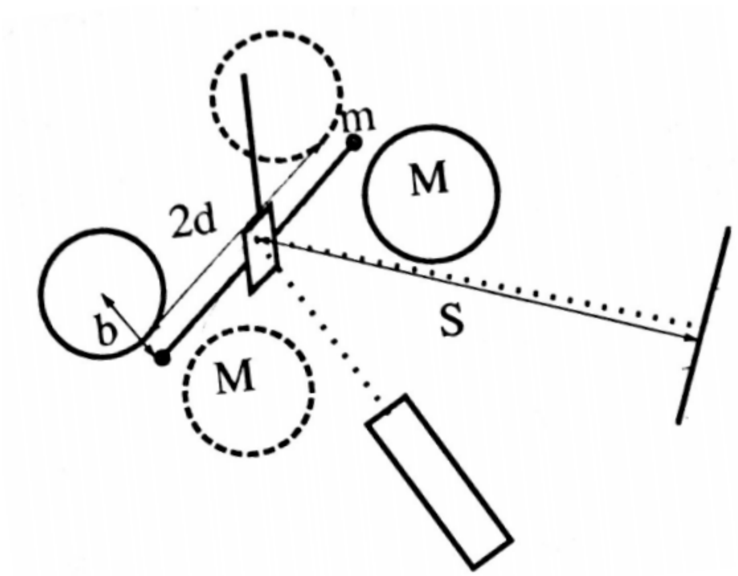

Figure 1: Geometry of the setup

$$\tau = -k\theta \tag{2}$$

From 1 and 2, we can write G as using torque equation as follows.

$$G = \frac{k\theta_{eqb}r^2}{MmL} \tag{3}$$

where L is the perpendicular distance of gravitational force to the rotation axis and θ_{eqb} is the resonance angle.

The period of torsional system is given as

$$T = 2\pi\sqrt{\frac{I}{k}} \tag{4}$$

The same system has an angular frequency of

$$\omega_0 = \frac{2\pi}{T} = \sqrt{\frac{k}{I}} \tag{5}$$

If the system is not ideal and has a damping parameter β, then our damping frequency is

$$\omega_d = \sqrt{\omega_0^2 - \beta^2} \tag{6}$$

2 Apparatus, Procedure and Data

In this experiment, we used a ruler, a laser and a computer and a torsion balance setup with large masses.

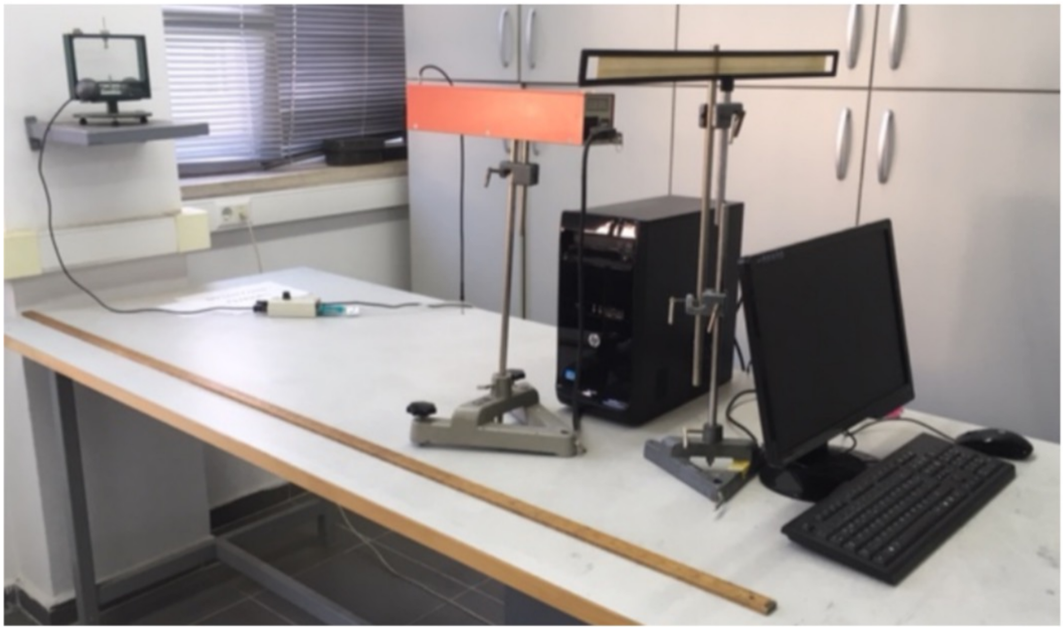

Figure 2: A general view of the apparatus

The details of torsion balance system was given at previous section. The setup was so that when the wire gets bend, we read a voltage value proportional to the angle of bending. The computer was giving us the voltage values corresponding to angles values. However, we needed a conversion between the voltage and angle. So, we used the ruler to find angles at resonance. Then we found our conversion equation by using the voltage value of the resonance.

The way we take the setup into resonance was simply turning the big masses to always made forces act so that they support the motion, i.e. the forces were always arranged to be in the same direction with the motion. When the voltage values (angles) are not increasing after some point, we concluded that we reached to the resonance. We took some pair of data at resonance with corresponding reads on ruler; however, at the last turning of the big masses, we were a bit careless so that we disturbed the whole system. Normally we were supposed to let the system to damp after that last turn but our data went absurd. We waited to see if it would damp around the same voltage value. But it did not. That is to say we wouldn't be able to determine V_{mean} values, and consequently V_{eqb} to find θ_{eqb}. The useless data we had is given below. (Figure 3)

Then we got the data of the other team doing the same experiment on that same day. The resonance and damping graphs of the data are given below. (Figure 4 and 5)

The values for the readings on ruler are given below.[3] (Figure 6)

[3] The value 11 in the second and third row are actually 11.0. All values are in centimeters and having an uncertainty of 0.1 cm

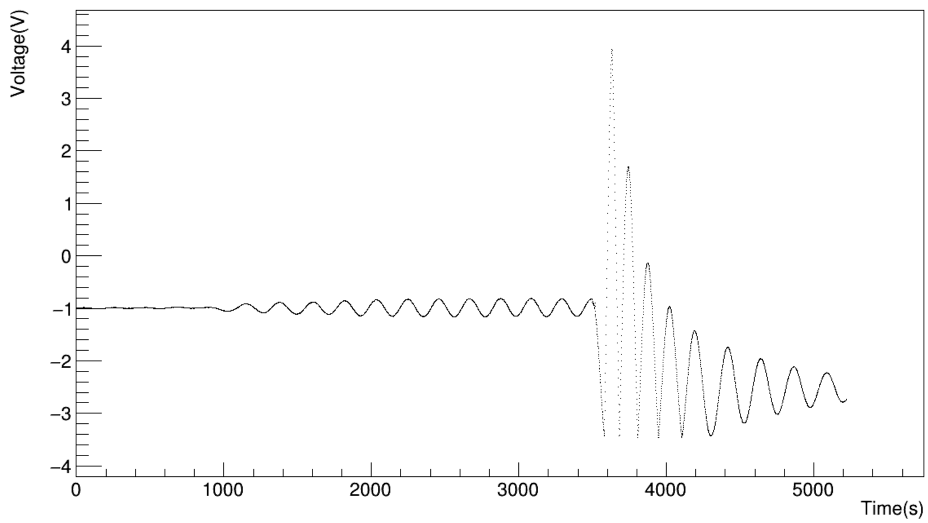

Figure 3: The broken data (it happens)

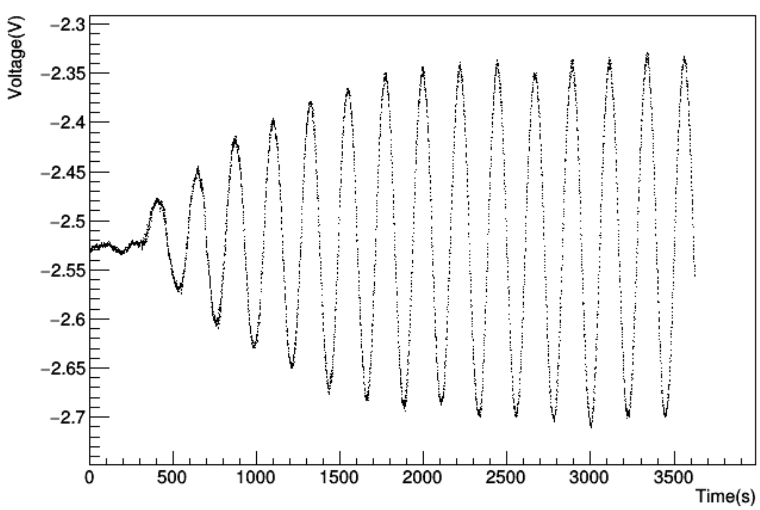

Figure 4: The resonance phase

3 Analysis

We performed fits of the following functions to the graphs.

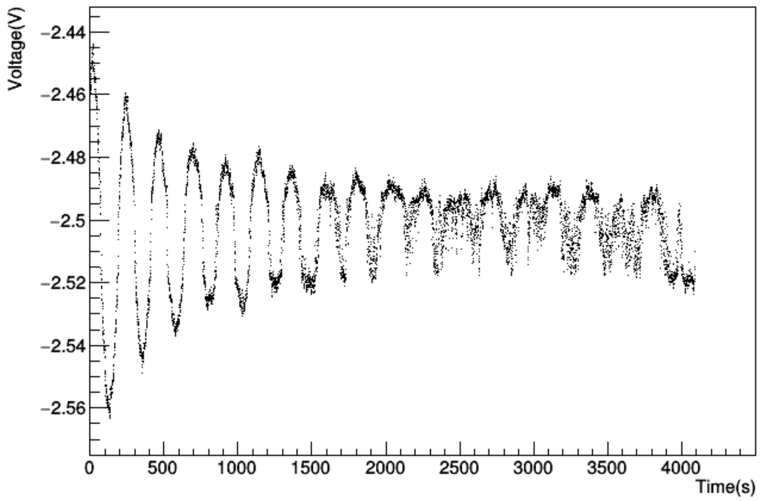

Figure 5: The damping phase

11.2	7.1
11	7.1
11	7.2
11.2	7.2
11.2	7.1
11.2	7.1

Figure 6: The readings on ruler

For the resonance part:

$$Acos(\omega t + \phi) + C \tag{7}$$

For the damping part:

$$A'cos(\omega_d t + \phi') + C' \tag{8}$$

Then we got the following fits.

In Figure 7, p0 represents A, p1 represents ω, p2 represents ϕ and p3 represents C. In Figure 8, p0 represents A', p1 represents ω_d, p2 represents ϕ, p3 represents C' and p4 represents the damping parameter β.

From Equation 3, we see that what we need to know to find G are k and θ_{eqb} since we know the other values, which are m, M, L and r.

From Equation 6, we found that

$$\omega_0^2 = \omega_d^2 + \beta^2 = 28000 \pm 14 \times 10^{-6} s^{-1} \tag{9}$$

We also know that $k = \omega_0^2 I$. So we found k as

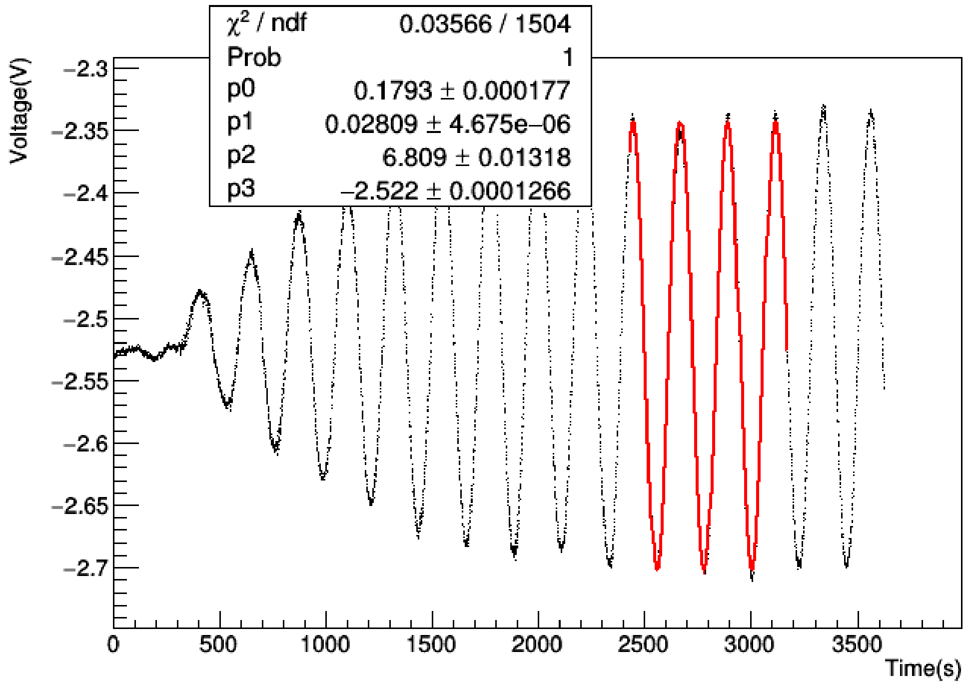

Figure 7: The fit of the resonance phase

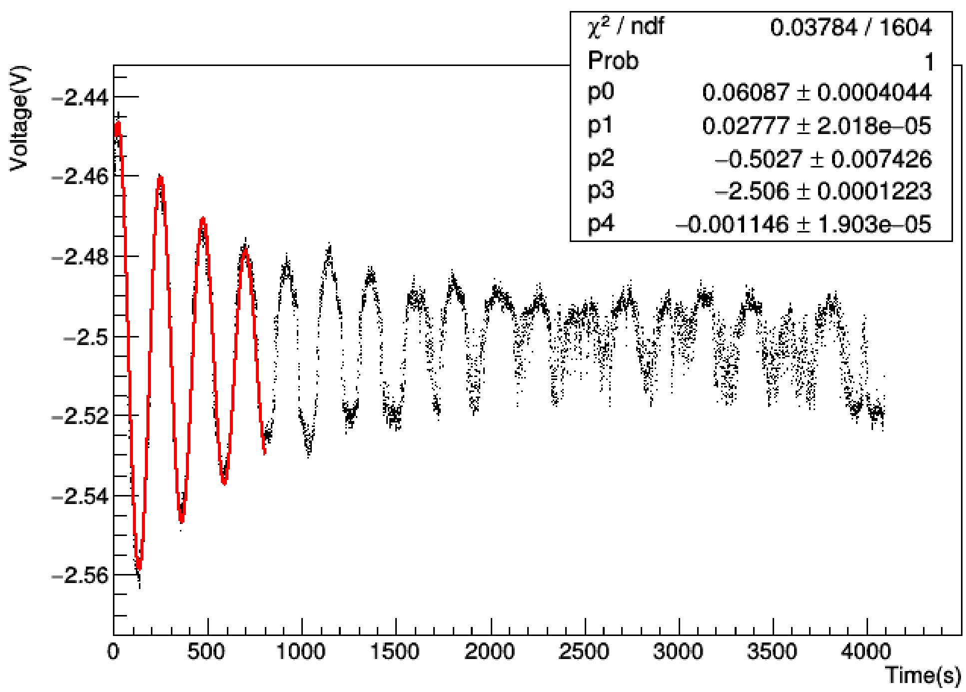

Figure 8: The fit of the damping phase

$$k = 11000 \pm 69 \times 10^{-11} kgm^2 s^{-2} \tag{10}$$

We found average ΔS from Figure 6 as 4.0 ± 0.1 cm.
$\frac{\Delta S}{L}$ gives us the corresponding $\Delta\theta$. Thus, we found

$$\Delta\theta = 2500 \pm 62 \times 10^{-5} radians \tag{11}$$

We are looking for a relation like $\Delta\theta = a\Delta S$. So, dividing them to find a, we get

$$a = 630 \pm 22 \times 10^{-5} radians/meters \tag{12}$$

We read $V_{mean,damping}$ and $V_{mean,resonance}$ as C and C' from the fits. Thus,

$$V_{equilibrium} = V_{mean,resonance} - V_{mean,damping} = 1600 \pm 18 \times 10^{-5} Volts \tag{13}$$

Since we know the conversion constant a, we can find the corresponding $\theta_{equilibrium}$ as follows.

$$\theta_{equilibrium} = a \times V_{equilibrium} = 1010 \pm 37 \times 10^{-7} radians \tag{14}$$

Finally, we have the values of k and $\theta_{equilibrium}$, which were necessary to find G. After plugging in the values in Equation 3, we found G as follows.

$$G = 2300 \pm 89 \times 10^{-14} m^3 kg^{-1} s^{-2} \tag{15}$$

4 Conclusion

Our result is not convincing since it is much sigma away from the known value. Our sigma value is very small. Investigating the source of such a low sigma value to conclude if it is a result of a mistake in calculation or not, we see that many values are taken from the sheet on the slide which errors were very small. However, we are not suspecting that those values can be wrong. Focusing on the values we found, i.e., k and θ, we see that our fit results have also given us values having very small sigma values. Especially, the data of the damping phase is very suspicious. We didn't take the right side of the graph into fit to keep that bad fluctuations away; however, we might still suffering from that phenomena.

Our system was very sensitive so that any motion around it might ended up with a distortion in our data. But still, our result is too bad to explain by that.

The result can be improved by waiting much more for resonance and by conducting the experiment a more isolated room.

For the interested reader, we suggest the paper of Cavendish. (H. Cavendish, Phil. Trans. Roy. Soc. 88, 469; 1798)

5 Appendix

References are given as footnotes. A sample of the codes used in analysis can be found in the following github link:

`https://github.com/beratgonultas/phys442/tree/master/cavendish`