Scattering in Two Dimensions

Berat Gönültaş

March 2019

Abstract

Scattering is a general physical process where some forms of radiation, such as light, sound, or moving particles, are forced to deviate from a straight trajectory by one or more paths due to localized non-uniformities in the medium through which they pass.[1] In this experiment, we studied scattering in two dimensions with a very simple model to find the diameter of a vertically placed cylindrical target. Though our model is a modest one, the essence of the process is the same with the brilliant applications of scattering in particle physics and other fields.

1 Theoretical Motivation

One of the best ways to understand the structure of particles and the forces between them is to scatter them off each other. This is particularly true at the quantum level where the systems cannot be seen in the literal sense and must be probed by indirect means. The scattering process gives us information about the projectile, the target, and the forces between them.[2] CERN teams, who conduct the largest scientific experiments in the world, work on scattering phenomena, as well. For example, here is a quotation from an old news from the CERN internet site: "Physicists from the ATLAS experiment at CERN have found the first direct evidence of high energy light-by-light scattering, a very rare process in which two photons – particles of light – interact and change direction. The result, published today in Nature Physics, confirms one of the oldest predictions of quantum electrodynamics (QED)."[3]

Consider a 2D scattering model as in Figure 1. A projectile (a steel ball) shot towards the target with an impact parameter b will scatter at an angle θ. Assuming that the distance between the target and the detector is large, the scattering angle θ is given by[4]

$$\frac{b}{r} = \cos\frac{\theta}{2} \tag{1}$$

[1] `https://en.wikipedia.org/wiki/Scattering`

[2] R.Shankar, Principle of Quantum Mechanics, Second Edition

[3] `https://home.cern/news/news/experiments/atlas-observes-direct-evidence-light-light-scattering`

[4] Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

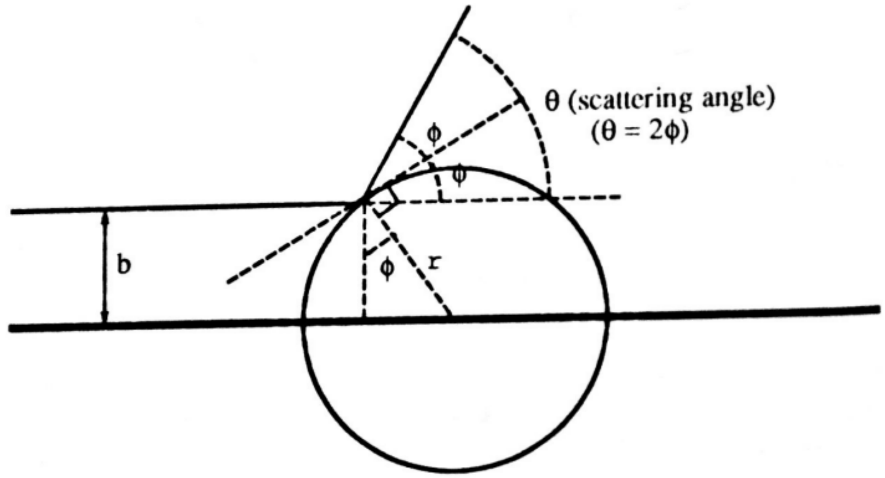

Figure 1: Geometry of the Scattering in two dimensions

The number of particles scattered at a given angle is

$$dN = -Idb = \frac{Ir}{2}\sin\frac{\theta}{2}d\theta \tag{2}$$

where I is the incident flux (shots/cm)

There is a minus sign in front of Idb in order to get plus sign with the additional minus coming from the derivative of cosine function.

The differential cross section is defined as

$$\frac{d\sigma}{d\Omega} = \frac{Y}{Id\Omega} \tag{3}$$

where Y is the yield in $d\Omega$, $d\Omega$ is the solid angle, and I is the incident flux.

Considering our model in Figure 1, Equation 3 becomes,

$$\frac{d\sigma}{d\theta} = \frac{dN}{Id\theta} \tag{4}$$

If we simplify Equation 4, it becomes,

$$d\sigma = \frac{dN}{I} \tag{5}$$

2 Apparatus and Experimental Procedure

In the experiment, we used the system in Figure 2, which has a gun shooting steel balls by vacuuming. There are pressure sensitive papers on the inner surface of the cylindrical rim at the edge of the circular tray. The scattered balls leave a point-like trace on the paper.

Figure 2: General view of the apparatus

First, we lined the rim with the special paper very carefully. In order to get better traces, it was important to line the paper not too loose or too tight. Then we marked the angles on the paper with 20 degree of intervals. We started shooting from the right edge of the gun range and we slowly take the gun to the left till the balls start to scatter. Then we marked that position of the gun and shot 20 balls. Then we moved the gun to the left by turning the screw one full turn. We continued shooting 20 balls and then turning the screw. We tried to shoot the balls with equal pressure. At the end, we ended shooting when the balls started not to scatter. We measured the distance between that final position of the gun and the initial position by ruler. Then, we measured the diameter of the target by vernier. The measured value of diameter is $2r = 5.64 \pm 0.01 cm$. Lastly, we took the paper off the system and read the angles. While taking data we omitted the interval of 350-10 degrees since a portion of it lies behind the target and the other portion is a place where we theoretically concluded that no scattered balls can hit but only those which are not scattered and directly go can hit. For other parts of the paper, we took the average of the angles for each interval. For example, the counts hit to the interval of 10-30 degrees is noted down as 20 degrees in data.

3 Data and Analysis

At the end of the experiment, we got a distribution of number of traces on the paper like

Angle	dN
20	25
40	19
60	27
80	40
100	49
120	49
140	62
160	109
180	94
200	67
220	69
240	58
260	70
280	30
300	21
320	24
340	6

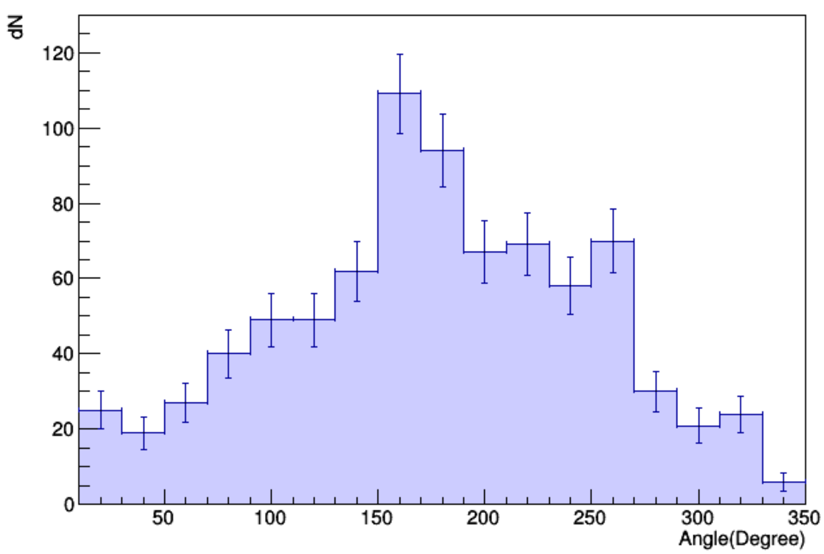

Figure 3: Histogram of the counts for each interval of angle

Since, the system is symmetric, to increase data, we added counts of 20 degrees and 340 degrees, 40 degrees and 320 degrees, 60 degrees and 300 degrees and so on. 180 degrees has no pair, so we multiplied it by 2. Then we got

Angle	dN
20	31
40	43
60	48
80	70
100	119
120	107
140	131
160	176
180	188

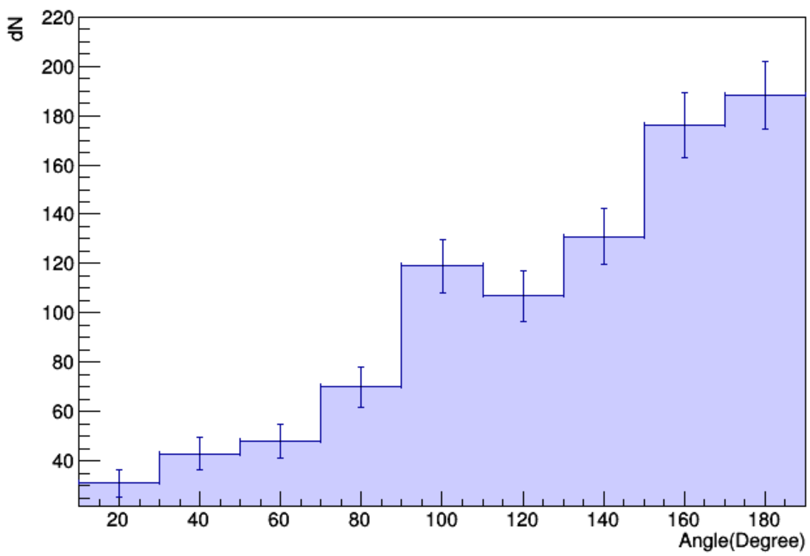

Figure 4: Histogram of the symmetrically added counts for each pair of intervals of angle

Then we plotted $sin\frac{\theta}{2}$ vs dN graph and we fit a line to the graph.[5]

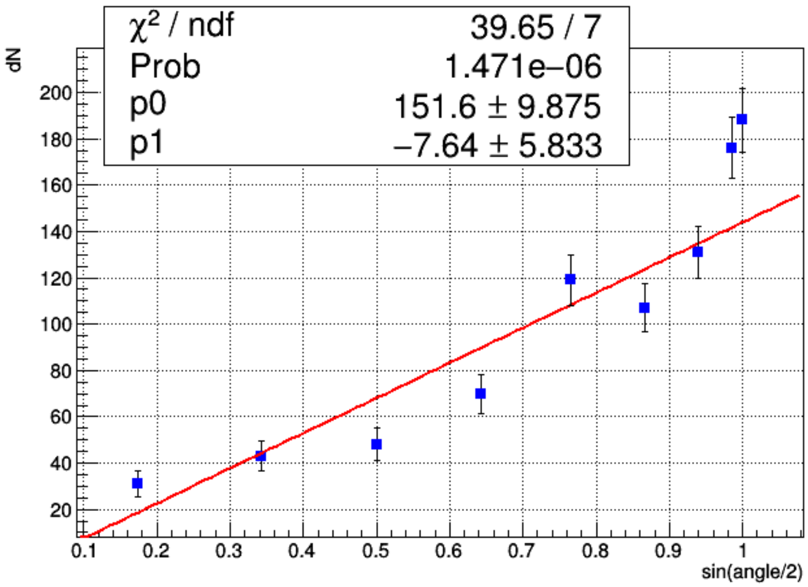

Figure 5: $sin\frac{\theta}{2}$ vs dN graph

Here, from Equation 2, the slope of the line can be expressed as

$$m = I\frac{r}{2}d\theta \tag{6}$$

Thus,

$$r = \frac{2m}{Id\theta} \tag{7}$$

where

$$I = \frac{20}{5.7/41}cm^{-1} \tag{8}$$

where 20 is the number of balls shot at each position, 5.7 (cm) is the distance between the initial and the final position of the gun, and 41 is the total number of turnings of screw we did.

Plugging in the numbers inside Equation 7, we get

$$r = 6.040 \pm 0.393cm \tag{9}$$

and

$$2r = 12.08 \pm 0.79cm \tag{10}$$

[5]p0 represents the slope and p1 represents the y intercept.

Summing over Equation 5, we find that

$$\sigma = \sum \frac{dN}{I} = \frac{25 + 19 + .. + 24 + 6}{I} \tag{11}$$

Thus we find that

$$2r = \sigma = 5.70 \pm 0.20 \tag{12}$$

4 Conclusion

We found $2r$ by two different methods. The first method gave a result which doesn't agree with the measured value. However, the second method gave a result which fits the real value. It is not surprising because the second method was much more straightforward and by being so, it excludes additional source of errors. The first method got effected by both the error in total N and the error in its distribution to each angle while the second method suffered only the error in total N. The fit at first method(Figure 5) has a too big chi2/ndf value. The uncertainty in angles is 10 degrees, which is not so small. We tried to decrease chi2/ndf value by adding the uncertainties of $sin(\frac{\theta}{2})$ to the graph using error propogation, but chi2/ndf value decreased so much that it gave a worser result than previous setting (0 error for $sin(\frac{\theta}{2})$ in graph). Thus, we left it like this.

We started shooting from the right. Finding the point where balls start to hit the target took a while and although we assumed these "insignificant"(not scattered) shootings are going to hit the range of 350-10 degrees, it is possible that some of the balls hit the neighbour interval, i.e., the interval of 10-30 degrees, which shows up as 20 degrees in data.

And also, for sure, this long experiment made the shooter tired and he couldn't shoot the balls with the same pressure after a point. Thus, the traces might have become not visible for some shoots.

The number of counts of traces between 280 degrees and 340 degrees is considerably less than its symmetric "partner", between 20 degrees and 80 degrees. The pressure sensitive paper was a bit loose around the region of 300 degrees so that we were probably not able to count some balls hit there since the some balls didn't leave any trace at that loose region. On the other hand, the paper had some other traces than the ball hits. Our each touch to the paper was a candidate for a fake count.

The fit in Figure 5 doesn't pass over the origin. It is expected since we don't add the interval of 350-10 to the data, the dN counts don't start where sin is 0. In other words, at the beginning, there is a tiny range where dN is zero.

The results can be improved by taking more data, i.e., shooting more balls. Using a device to shoot balls instead of manpower, and using alternative and much more "readable" detectors will make the results better.

5 Appendix

The codes used for analysis can be found at the following github link:

`https://github.com/beratgonultas/phys442/tree/master/2D_scattering/`