Radioactive Decay

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Abstract

Radioactive decay is the process of emitting radiation by an unstable atom. In nuclear science, the decay chain refers to a series of radioactive decays of different radioactive decay products as a sequential series of transformations.[1] In this experiment, we examined decay of Radon gas (^{220}Rn), which is one of the elements produced in the radioactive decay chain starting from ^{232}Th and ending at ^{208}Pb, called "thorium series" or "thorium cascade".(see Figure 1) The half-life of Radon gas is 55.6 seconds and we tried to find this value in the experiment.

1 Introduction and Theoretical Motivation

Radioactive decay is a stochastic (i.e. random) process at the level of single atoms. According to quantum theory, it is impossible to predict when a particular atom will decay, regardless of how long the atom has existed. However, for a collection of atoms, the collection's expected decay rate is characterized in terms of their measured decay constants or half-lives. This is the basis of radiometric dating. The half-lives of radioactive atoms have no known upper limit, spanning a time range of over 55 orders of magnitude, from nearly instantaneous to far longer than the age of the universe. The decaying nucleus is called the parent radionuclide (or parent radioisotope), and the process produces at least one daughter nuclide. Except for gamma decay or internal conversion from a nuclear excited state, the decay is a nuclear transmutation resulting in a daughter containing a different number of protons or neutrons (or both). When the number of protons changes, an atom of a different chemical element is created.[2]

Unstable isotopes may decay through various processes but they all follow the same decay law:

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

where λ is the decay constant and N_0 is the initial number of the unstable isotope nuclei.

The half-life is expressed in terms of the decay constant as

$$t_{1/2} = \frac{ln2}{\lambda} \tag{2}$$

[1] https://en.wikipedia.org/wiki/Decay_chain#Thorium_series

[2] https://en.wikipedia.org/wiki/Radioactive_decay

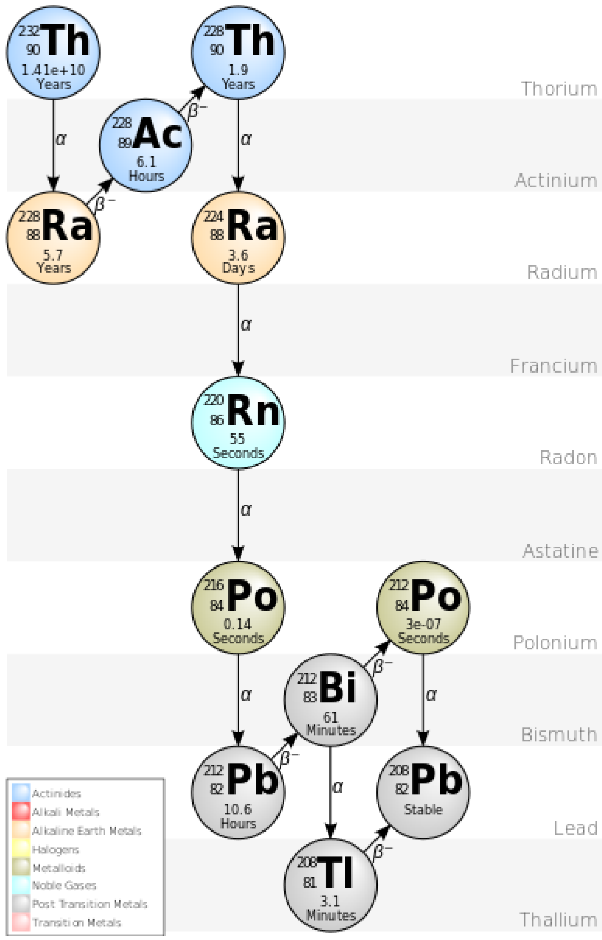

Figure 1: Thorium cascade (image taken from the link at footnote 1.)

2 Apparatus and Experimental Procedure

In the experiment, we used Wulf's electroscope, thorium salt, an ionization chamber, a HV(high voltage) power supply, a stopwatch and a tea tray. (Figure 2)

Wulf's electroscope is a design also used by Victor Hess, the physicist who discovered cosmic rays. Wulf's electroscope has an additional electrode opposite to the moving electrode of a conventional electroscope. This additional electrode is connected to the ground. When the moving electrode reaches and comes into contact with this additional electrode, it discharges and begins to charge again.[3]

In the experiment, by opening the clamp shutting the air flow between the Thorium vessel and the ionization chamber and by squeezing the bottle several times, a sample of radioactive material was placed in an enclosure and a HV was applied between the two terminals in the same closure. A current was produced in the HV circuit because of ionized air. Measuring the current continuously would result in a quantity proportional to the amount of decay as a function of time. However, the currents involved were too small to be

[3]Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

Figure 2: General view of the apparatus.

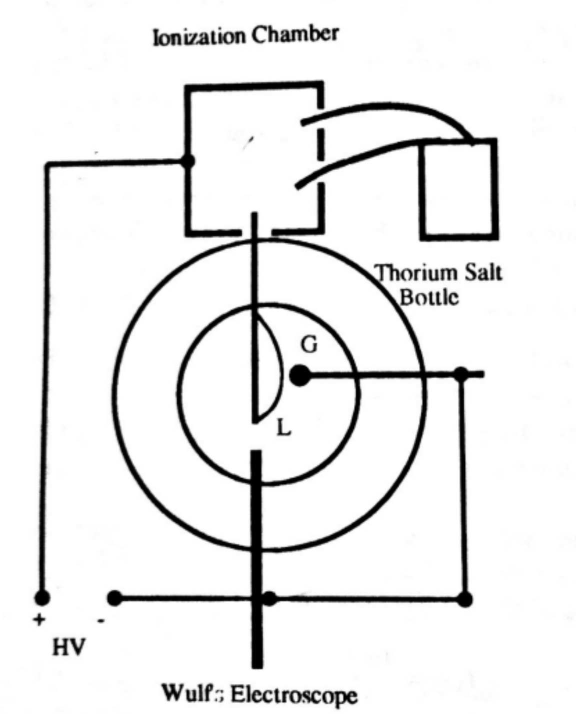

Figure 3: Schematic diagram of the Wulf's electroscope and the ionization chamber.

measured with a simple ammeter. A current amplifier was needed. On the other hand, since the actual quantity in absolute terms was not required to verify the exponential decay and to extract the decay constant, a simpler approach would be to use an electroscope. Wulf's electroscope was more convenient since it discharges automatically after reaching a preset charged state.

Measuring the very low currents with the help of Wulf's electroscope was achieved by measuring the time between successive discharges. Since the amount of charge required before Wulf electroscope discharges is fixed, the current was proportional to the inverse of the time elapsed to accumulate this full charge, assuming that the charging is linear with respect to time:

$$Q = Is \implies I = \frac{Q}{s} \implies I \propto \frac{1}{s} \tag{3}$$

where s is the time elapsed to reach the full charge. Hence, the radioactive decay can be studied simply by measuring the time duration it takes to charge Wulf's electroscope connected to an ionization chamber.[4]

We took 15 different data sets by applying 5 different HV values, i.e. 2500 V, 3000V, 3500 V, 4000 V, 4500 V. We observed and measured the elapsed time intervals(s_i's) for charging the electroscope for 3 different cases for each voltage value. For each HV value, we measured time after squeezing the bottle 5 times, 10 times and 15 times, respectively. Each time, we opened the ionization chamber to the air and fanned it with the tea tray before proceeding and taking a new data.

Then we calculated the times of discharges and average times for all the sets.

$$s_i = t_{i+1} - t_i \tag{4}$$

and

$$T_i = \frac{t_{i+1} + t_i}{2} \tag{5}$$

where t_i is the time corresponding to the instants we observed a discharge.

Then using Equation 1 and 3, and keeping in mind that the quantity of sample is proportional to the decay rate (and implicitly, to the current), we can write

$$ln\frac{1}{s(t)} = ln\frac{1}{s_0} - \lambda t \tag{6}$$

Here we can approximately use T_i's instead of t, and rewrite Equation 6 as

$$ln(s) = constant + \lambda T \tag{7}$$

so that λ is the slope of the T versus $ln(s)$ graph.

After finding weighted average value of λ, we calculated $t_{1/2}$ by Equation 2.

[4] Advanced Physics Experiments by Erhan Gülmez (Boğaziçi University Publications, 1999, ISBN 975-518-129-6)

3 Data and Analysis

The data and the graph for 3000 V and 10 squeezes are given below.[5]

$t_i(seconds)$	$s_i(seconds)$	$T_i(seconds)$	ln s_i
9.20	9.20	18.36	2.22
27.51	18.31	39.60	2.907
51.69	24.18	69.46	3.186
87.23	35.54	119.33	3.571
151.43	64.20	-	4.162

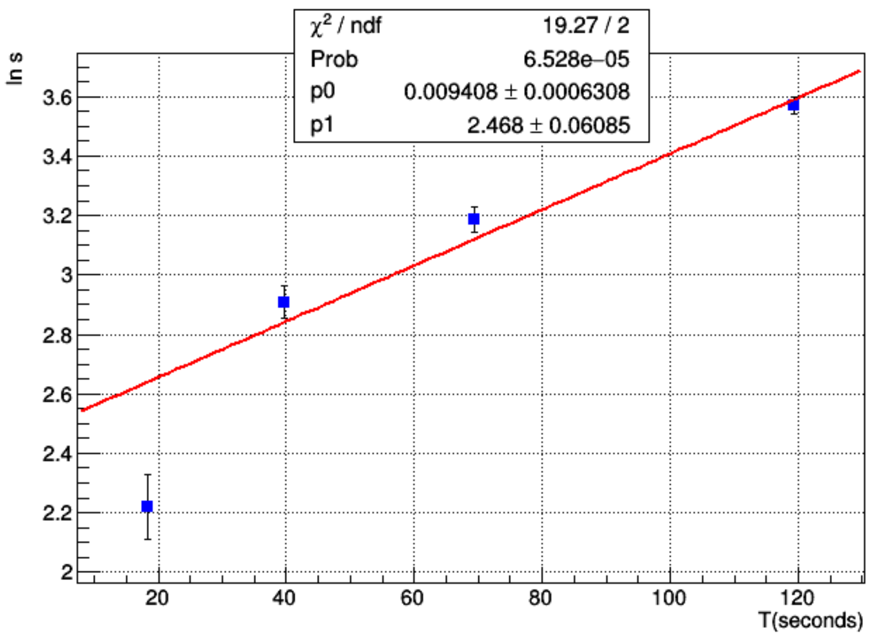

Figure 4: T versus $ln(s)$ graph for 3000 V and 10 squeezes.

Here, p0 represents the slope which is λ (by Equation 7), and p1 is the lns_0 which is an irrelevant constant. The error bars for x values of the data points were also put. However, they are not visible since the error values of x values are very small compared to the x values themselves.

Thus, for 3000 V and 10 squeezes, $\lambda = 0.009408 \pm 0.000631$ by Figure 4.

[5] For all data sets and graphs, see appendix.

With a similar approach, the λ value for each data set has been found. Then we calculated weighted average of λ's by

$$\lambda_{w.ave} = \frac{\sum_i w_i \lambda_i}{\sum_i w_i} \tag{8}$$

where

$$w_i = \frac{1}{\sigma^2}$$

At the end, we found

$$\lambda_{w.ave} = 0.007580 \pm 0.000122 \tag{9}$$

Then, by Equation 2, $t_{1/2}$ can be found as

$$t_{1/2} = \frac{ln2}{\lambda_{w.ave}} = 91.44 \pm 1.47 \; seconds \tag{10}$$

However, this result was too much and because of the reasons mentioned at "Conclusion" section in next pages, we thought that there is no source for such a big error. Then, we went back to our data and observed it. Looking at the graphs we have drawn, we saw that the first data point on the graph was almost neglected for most of the graphs because of the big y error they have. However, the result would be much more near to the our expectation if those first points were also taken into account in fitting. So, we decided to put less error values on those points than the propagated error values and we drew the graphs again to find new slopes. Chosen value for the error on the first data points was 0.06, which is a reasonable number.

The new graph with new error settings for the 3000 V and 10 squeezes is given below.

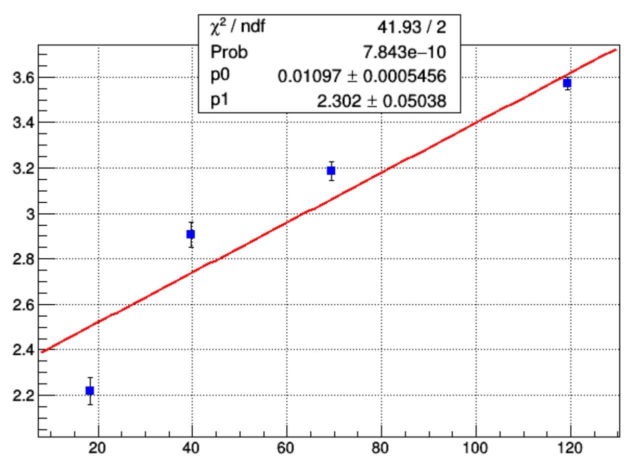

Figure 5: T versus $ln(s)$ graph for 3000 V and 10 squeezes with less error on first data point.

It is obvious that the fit in Figure 5 much more better than the Figure 4 at representing the data.

Thus, we followed the same procedure with a new error setting for some of the graphs(those in which the fit was very far away from the first data point) hoping that we will get a better result.

After calculations, we found the new $\lambda_{w.ave}$ and half-life as

$$\lambda_{w.ave} = 0.009642 \pm 0.000130 \tag{11}$$

$$t_{1/2} = 71.89 \pm 1.47 \ seconds \tag{12}$$

Here, the error of the half-life is assumed as same as previous one since the new setting gave a less error for half-life and we can't take it as if it is the true value. Still, we think that it is better to note down the calculated error value for new half-life. It was found as 0.97

4 Conclusion

With first error setting, we found the half-life of Radon gas significantly more, almost twice as large than the accepted value, 55.6 seconds. The calculations have been checked several times so that it can be said that there is no mistake at calculations. The data taking procedure was very simple. We couldn't relate all this error to the error in data taking procedure.

The Thorium cascade doesn't end with Radon gas, and there is Polonium in the chain after Radon. Polonium has half-life of 0.14 seconds.(See Figure 1) So, we were gathering the charge resulting from Polonium decay as well as Radon decay. After Polonium, there is Lead in the decay chain, which has half-life of 10.6 hours so that we mustn't consider the effects of Lead decay and the decay of its daughters. Besides, the effect of the decay of the daughters of Thorium till Radon are also insignificant because their half-lives are very long. Back to the effect of Polonium, Radon has 86 protons while Polonium has 84. And also Lead, the daughter of Polonium, has 82 protons. Thus, the effect of Radon decay and Polonium decay will be the same, i.e, they cause same amount of charge accumulation at electroscope. However, this effect of Polonium is not something more than putting a factor of 2 in front of I ,and consequently of Q, at the Equation 3. However, still I will be proportional to $\frac{1}{s}$ and no effect will occur in our slope calculation. In other words, Polonium's effect is just doubling the charge accumulated; it is not changing the decay rate of charge accumulation.

While using T instead of t_i we were approximating. However, considering Equation 1, taking the midpoint does not correspond to the mid value of charge accumulated. $e^{-\lambda t}$ must be made equal to the 1/2 to get the point where half of the charge accumulated in the interval of t_i and t_{i+1}. After calculation we see that, it is not the mid point but it corresponds to the 0.693/1 of the interval. Thus, our T values are "underestimated". If we would take the 0.693/1 of the interval as T, we would have a smaller slope since the rise in T would be bigger in big values of t_i since the intervals get longer. Smaller the slope(λ), bigger the $t_{1/2}$. Thus, this correction would make the deviation of our result from the accepted value much more dramatic.

Thus, neither effect of other daughters in Thorium cascade nor the approximation of T was not a justification for the deviation at first setting. So, after a long thinking period, we figured out that the errors on the first data points for some graphs were the source of the deviation at the end. Thus, we used second setting. Probably, 0.6 was not the best value to assign as the errors of first data points, and this also has effects on result for sure.

The experiment can be improved by excluding the human factor from the experiment. If we could have less errors for time by using a machine to take data, these would make error for lns much less, which could give us better data so that we wouldn't need small touches as we did in errors for some first data points.

5 Appendix

All data sets and graphs as well as the sample code used in analysis can be found in the following github link: `https://github.com/beratgonultas/phys442/tree/master/radioactive`

Photos of used Wulf's electroscope are given below.

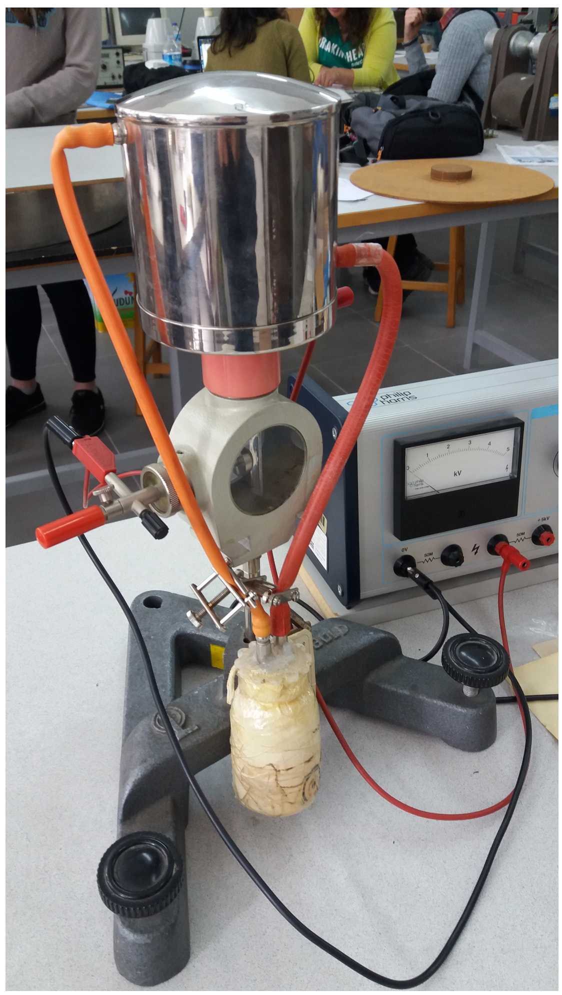

Figure 6: Wulf's electroscope

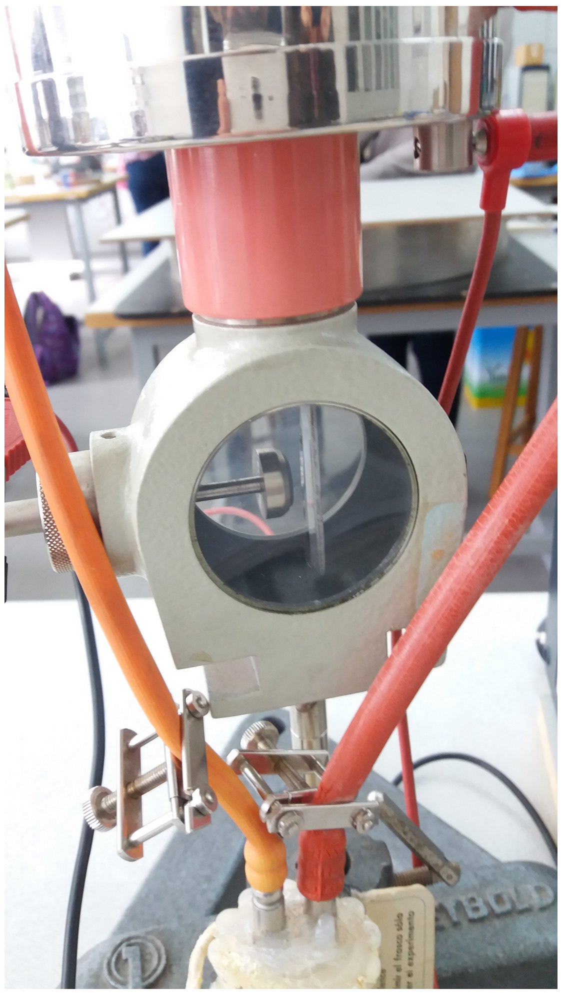

Figure 7: Wulf's electroscope (near)