Problem List 4
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We are considering graphs with a single binary relation $E(s, t)$. In this list, we are interested in paths of non-zero length.

1 (1 pt) Let's consider the following query in Datalog:

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T(X, Y) :- E(X,Y).
T(X, Y) :- T(X,Z), T(Z,Y).
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Recall the definition of Datalog semantics, and then show that for each $i \in \mathbb{N}_+$ it holds that $T^i = \{(a, b) |$ there is a path from a to b of length $\leq 2^{i-1}\}$.

Write the following Datalog queries. Use constants n and m where needed.

2 (0.5 pts) Return the vertices that can be reached by a path from n or by a path from m.

3 (0.5 pts) Return the vertices that can be reached by a path from both n and m.

4 (1 pt) Return pairs of vertices that can be reached from vertex n by paths of the same length.

5 (1 pt) Return pairs of vertices x, y such that there exist paths from n to x and to y with different lengths.

We consider graphs with a ternary relation $E(s, t, a)$. In the third column, we store the color of the edge (from a palette of 16 million colors). Write each of the following Datalog queries, or prove that it does not exist.

6 (0.5 points) Return pairs of vertices x, y such that there is no path from x to y.

7 (0.5 points) Return pairs of vertices x, y such that there exists a monochromatic path from x to y.

8 (0.5 points) Return pairs of vertices x, y such that every path from x to y is composed of edges of at least two colors.

9 (1 point) Return pairs of vertices x, y such that there exists a path from x to y composed of edges of at most two colors.

10* (0 points, bonus up to 2 points) We consider graphs with a binary relation $E(s, t)$. Find out what Ehrenfeucht-Fraïssé games are, and use them to show that first-order logic (DRC/TRC) cannot express the query $P_*(x, y)$ which is satisfied when there exists a path from x to y of any length.