Numerical optimization: Assignment 5

Deadline: the lab on 2024.04.29

In this assignment, we are going to look into uniform approximation, ie. approximation with norm $\|\cdot\|_\infty$. You will notice that changing the norm from the least squares makes it somewhat harder to solve. To properly do the tasks, I recommend looking into §4.5.7 (Approximation in the Maximum Norm) in *G. Dahlquist, Å. Björck, Numerical Methods in Scientific Computing, Vol. I, SIAM 2008*.

Task 3 from Assignment 4 can be handed in for full points during the lab on April 22.

1. [1 point] Formulate a uniform approximation problem for the data in Assignment 3, Task 3.

2. [2 points] Implement the Remez exchange algorithm to solve (with a reasonable margin of error) the previous task. Try to fit the polynomial of degree 4.

3. [2 points] Formulate a problem of approximating a circular arc (a piece of a circle corresponding to $(1, [-\alpha, \alpha])$ in polar coordinates; usually $\alpha \leq \pi/2$) with a Bézier curve with 2D control points. An easy starter on Bézier curves is in, eg., §2.1-2.2 in *H. Prautzsch, W. Boehm, M. Paluszny, Bézier and B-Spline Techniques, Springer 2002*. Formulate it both for the least squares and the uniform norm.

4. [1 point] Solve the problem formulated in Task 3 for the least squares norm, for degrees $2, 3, 4$.

5. [4 points] Solve (with a reasonable margin of error) the problem formulated in Task 3 for the uniform norm, for degrees $2, 3, 4$.