Numerical optimization: Assignment 4

Deadline: the lab on 2024.04.15

In this assignment, we are going to do some linear programming. The recommended Python library is CVXPY: https://www.cvxpy.org/. In each assignment, apart from writing the code, you should also formulate a problem in LaTeX. You can also hand in Task 5 from Assignment 3 during the labs dedicated to Assignment 4 for 100% points.

1. 2 points You and your friends play an economic board game. It is your last turn and need to get more than 70 points to win. You earn points by selling certain resources. Your town produces 10 wood, 10 stone and 10 iron per turn. Additionally, you can produce:

   - 1 unit of tools by spending 1 wood and 1 iron;
   - 1 unit of housing by spending 2 wood and 3 stone.

   At the end of the turn, you sell all resources and produced wares. The price of the goods is as follows:

   - wood: 1 point;
   - stone: 2 points;
   - iron: 3 points;
   - tools: 5 points;
   - housing: 10 points.

   You can produce non-integer amounts of tools and housing, ie. producing 5.2 units of tools and 1.3 units of housing, with 2.2 wood, 4.8 iron and 6.1 stone left (amounting to 68.8 points) is perfectly legitimate.

   (a) Formulate a linear optimization problem. Precisely specify what the variables are.
   (b) Solve the linear optimization problem using the CVXPY library. Can you win?

2. 3 points One of your friends has an expansion pack for the game, which emulates human labour. This time, you have 100 labour units in your town. The cost of producing resources is now:

   - 1 unit of wood requires spending 1 unit of labour;
   - 1 unit of stone requires spending 3 units of labour;
   - 1 unit of iron requires spending 2 units of labour and at most 10 units can be extracted per turn;
   - 1 unit of tools requires spending 1 wood, 1 iron, 1 labour;
   - 1 unit of housing requires spending 2 wood, 3 stone, 2 labour.

   Like in the previous task, all amounts can be non-integer.

   For the sake of simplicity, you can use the wood which you extracted in the same turn to produce e.g. tools.

   The point values of each resource are the same as in the previous task. Labor units cannot be exchanged for points.

   (a) Formulate a linear optimization problem. Precisely specify what the variables are.

(b) Solve the linear optimization problem using the CVXPY library. How many points are you able to get?

3. **5 points** There is a solo mode for the board game and you have decided to practice a bit. In the solo mode, the game takes 5 turns. This time, even more moves are to be considered! This time, you have a stockpile which can store 30 units, which can store wood, stone, iron, tools (ie. you can store 5 units of tools, 10 units of wood, 12 units of iron, and have space for 3 more units of any kind). All resources which do not fit into the stockpile at the end of the turn have to be exchanged for points (and you can choose not to have a full stockpile, if you want). Additionally, the housing which you produced can be exchanged for more labour units per turn. You begin the game with 50 units of labour per turn, and the iron mine allows you to extract at most 5 units of iron per turn. The initial contents of the stockpile are: 5 units of tools, 5 units of iron, 5 units of wood, and 5 units of stone.

This time, to produce the resources, there are the following requirements:

- 1 unit of wood requires spending 1 unit of labour and 0.1 units of tools;
- 1 unit of stone requires spending 3 units of labour and 0.1 units of tools;
- 1 unit of iron requires spending 2 units of labour and 0.1 units of tools; at the beginning, the mining limit is 5 units of iron per turn;
- 1 unit of tools by spending 1 wood, 1 iron, 1 labour, 0.1 tools;
- 1 unit of housing by spending 2 wood, 3 stone, 2 labour, 0.2 tools.

This time, you can only use the resources which were put into the stockpile in the previous turn, ie. you cannot extract wood and at the same turn use it to manufacture tools. After production, you have to decide what to do with the resources:

- wood, stone, iron and tools can either be exchanged for points or stored in the stockpile (max capacity: 30);
- the mining limit can be increased; increasing it by 1 unit requires spending 1 unit of labour and 0.2 units of tools;
- housing can either be exchanged for points or spent to increase the number of labour units (1 housing gives 10 labour units per turn).

The increased values of labour units per turn and mining limits start having any effect at the next turn. It applies until the end of the game. For example, if you increase the number of labour units by 15 in the first turn and later by 20 in your third turn, the total number of labour units in each of the five turns is: 50, 65, 65, 85, 85. Similarly, if you increase the mining limit in the second turn by 1.2 and in the fourth one by 3.7, your mining limits in each of the five turns are: 5, 5, 6.2, 6.2, 9.9.

The items which were put in the stockpile cannot be exchanged for points at later turns. All items put in the stockpile at one turn, if they are not used in the following turn, are wasted. The point values are the same as in the previous task.

(a) Formulate a linear optimization problem. Precisely specify what the variables are.

(b) Solve the linear optimization problem using the CVXPY library. How many points are you able to get?