Let’s find

Let’s find the eigenvectors

For

For

We end up with two orthogonal vectors which point in the and directions. Rotating the system by radians to obtain the normalized eigenvectors:

The eigenvectors are unchanged by a rotation; that is:

Solve for the “motion” of a point as it is fed back to the system:

Solve for

Solve for

We will now search for a family of curves that will pass through the origin (unstable) and reach at the boundary of the square. These curves will define the sizes of the regions. All points in a region are classified together, whereas points in different regions are classified apart.

Eliminating and isolating , we get:

A point with initial coordinates will follow the curve expressed by:

With:

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We must now consider the behaviour of these curves at the boundary of the square. A point will travel the curve given by the equation above, until it reaches the boundary. Only the component that is tangent to the boundary will contribute to the motion.

We are interested in finding the curve represented by the motion equation, for which a point will reach the boundary and stop. This will occur when the above equation is set to zero. Thus we wish to find the point at which:

Considering only the positive quadrant, with a normalized tangent vector:

Given

We get

Since on the boundary, we have:

Therefore:

With:

The lines, and determine the intersection with the boundary four (unstable) equilibrium points. The curves representing all interior points of the square will move so as to intersect the boundary at one of these equilibrium points.

The regions of the two dimensional system are thus determined.