Homework 3, Final Report
Finite Elements

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Index

GitHub Repository

The code and data for this project are available on GitHub at the following link:

https://github.com/berckanala/01-Finite-Element

1 Stress Analysis (Part B)

1.1 Introduction

This study uses the finite element method to analyze maximum principal stresses in a part with a stress concentration. Four mesh sizes, two refinement strategies (global and local), and two element types (Quad4 and Quad9) are compared, resulting in 16 simulations. The goal is to evaluate how these factors affect stress distribution, particularly near the critical region, and to observe convergence with mesh refinement.

1.2 Results

1.2.1 Quad4 Elements

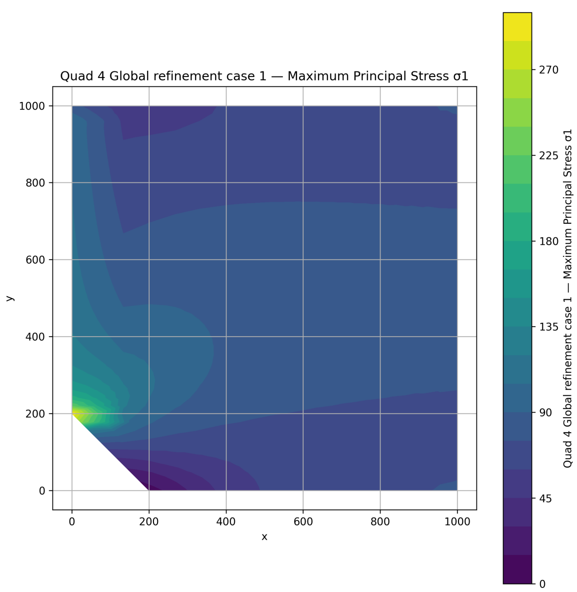

Figure 1: Quad4 Case 1 – Global Refinement

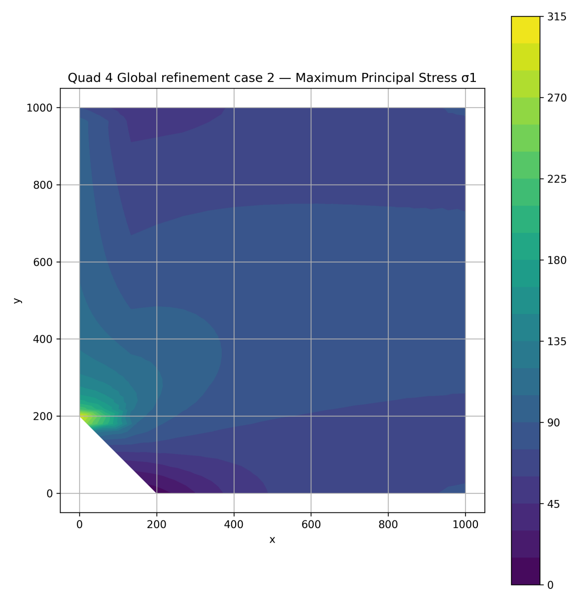

Figure 2: Quad4 Case 2 – Global Refinement

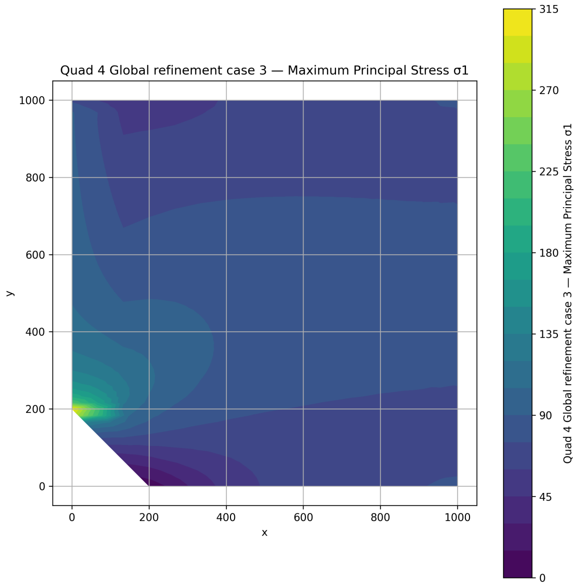

Figure 3: Quad4 Case 3 – Global Refinement

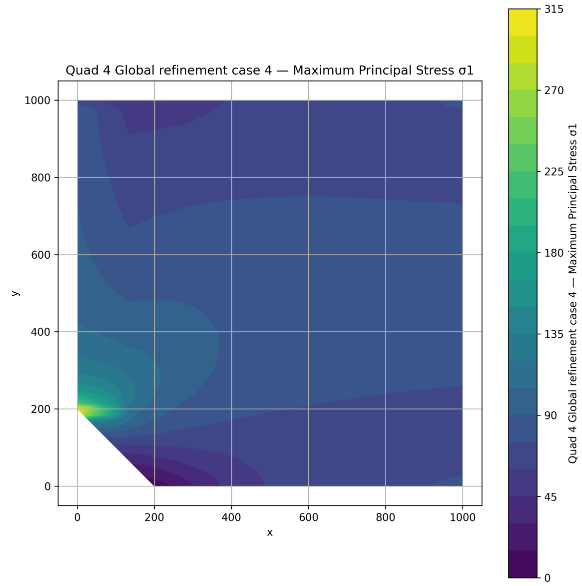

Figure 4: Quad4 Case 4 – Global Refinement

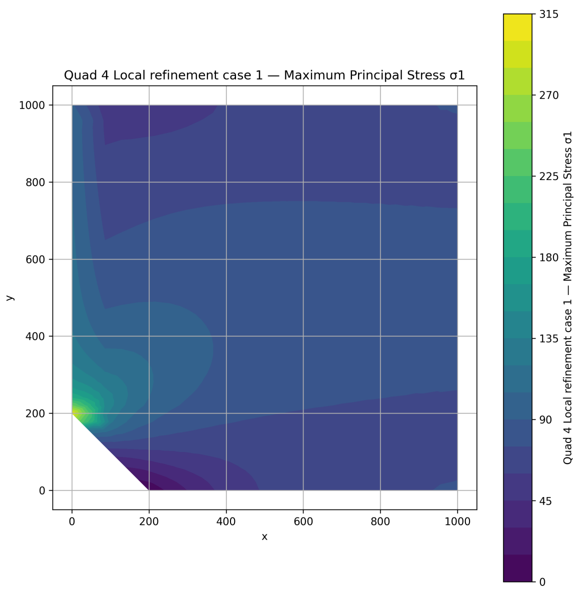

Figure 5: Quad4 Case 1 – Local Refinement

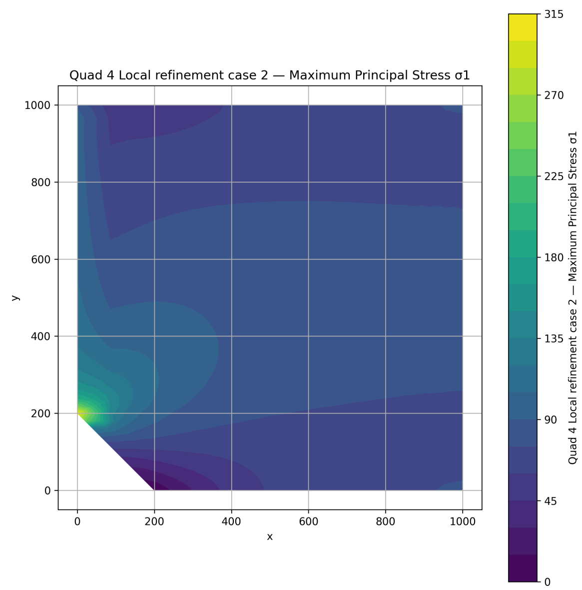

Figure 6: Quad4 Case 2 – Local Refinement

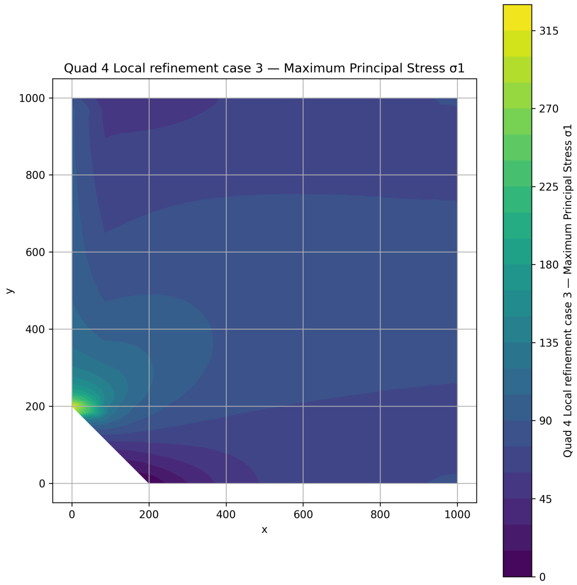

Figure 7: Quad4 Case 3 – Local Refinement

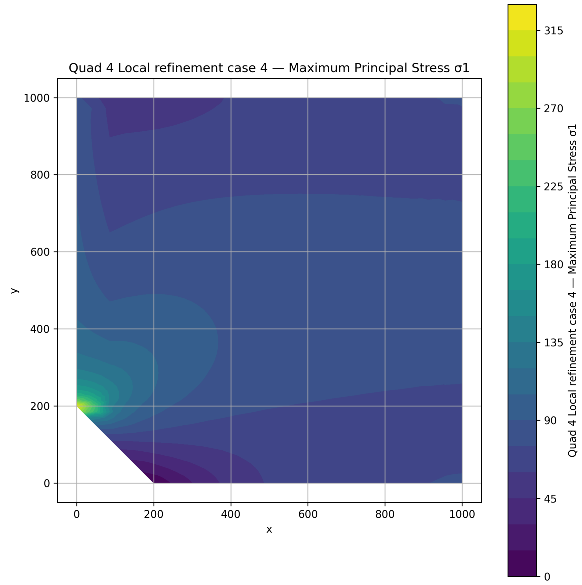

Figure 8: Quad4 Case 4 – Local Refinement

1.2.2 Quad9 Elements

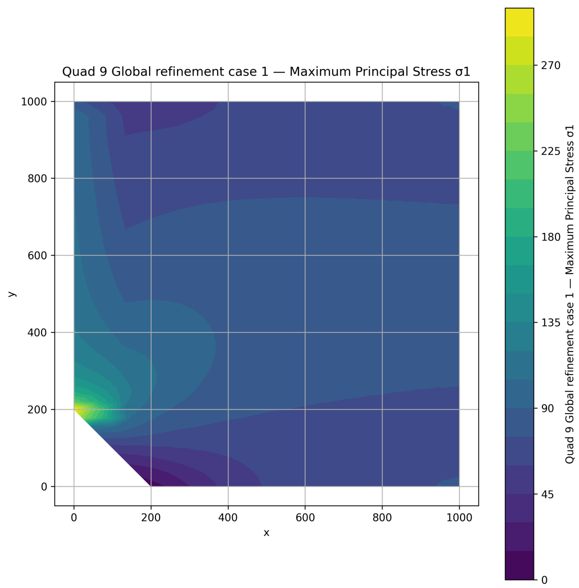

Figure 9: Quad9 Case 1 – Global Refinement

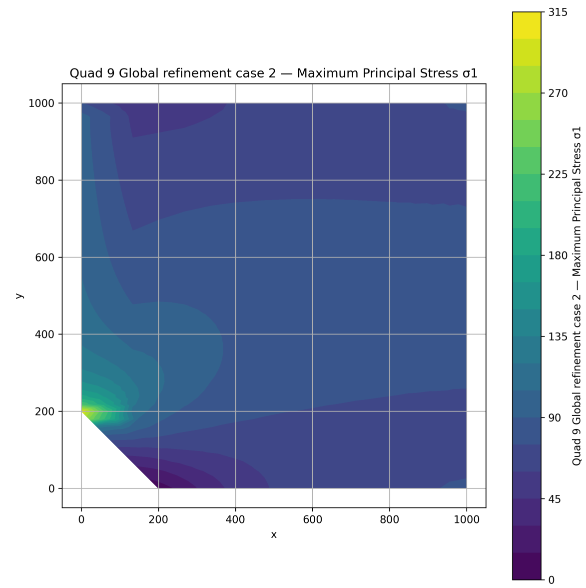

Figure 10: Quad9 Case 2 – Global Refinement

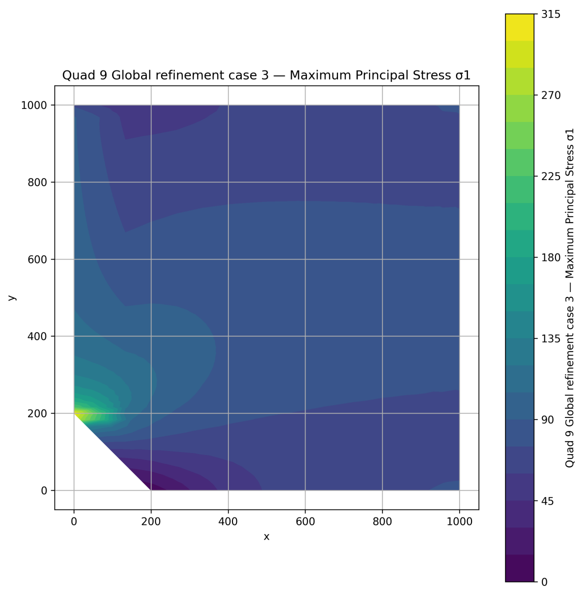

Figure 11: Quad9 Case 3 – Global Refinement

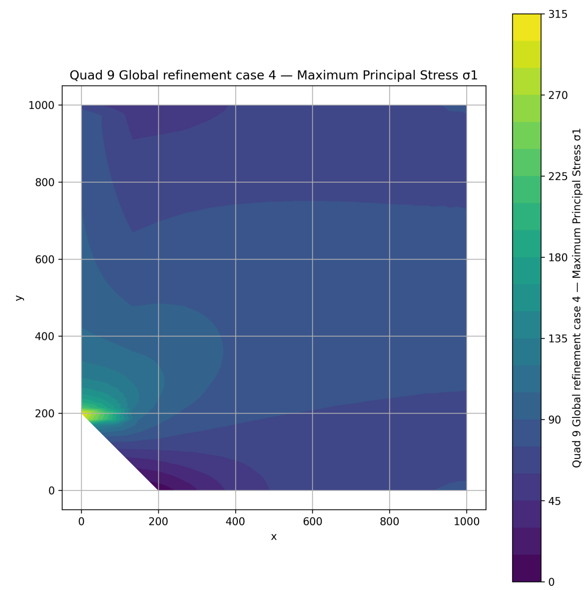

Figure 12: Quad9 Case 4 – Global Refinement

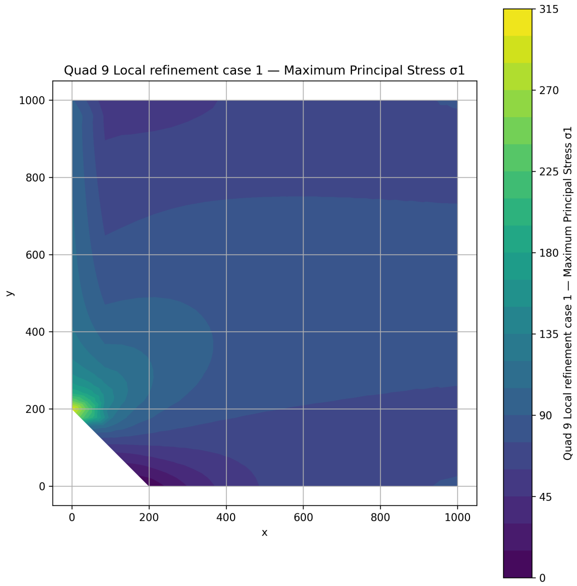

Figure 13: Quad9 Case 1 – Local Refinement

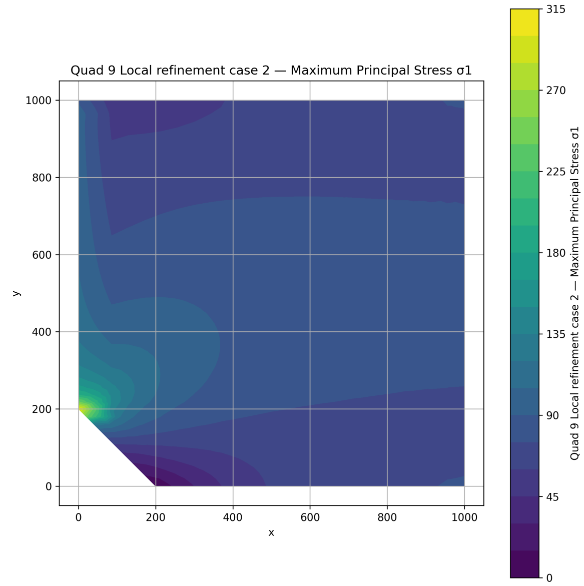

Figure 14: Quad9 Case 2 – Local Refinement

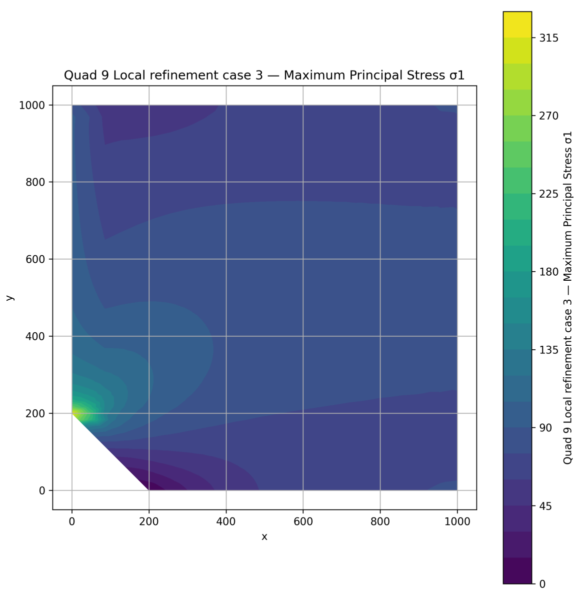

Figure 15: Quad9 Case 3 – Local Refinement

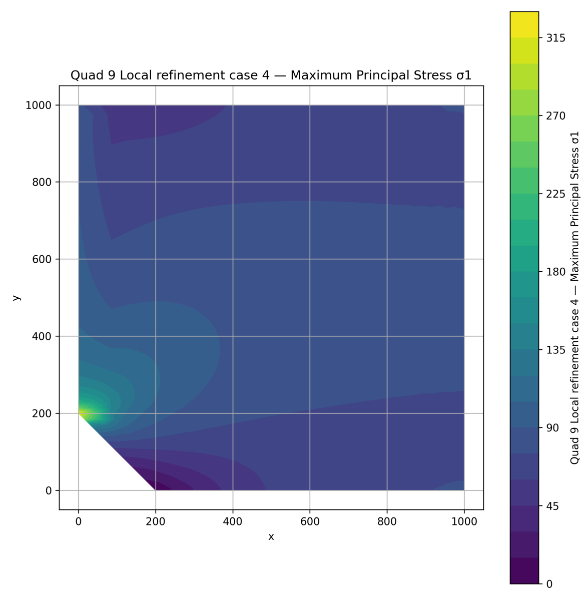

Figure 16: Quad9 Case 4 – Local Refinement

1.3 Results Summary and Discussion

The following tables summarize the maximum principal stress values obtained from the finite element simulations using Quad4 and Quad9 elements. For each element type, results are presented under two mesh refinement strategies (global and local) and across four mesh sizes. These values help evaluate the convergence behavior and the influence of element type and mesh refinement on the accuracy of stress predictions, particularly near regions of stress concentration.

Element Type	**Case**	**Max σ_1 [MPa]**
Quad4	1	353.84
Quad4	2	359.28
Quad4	3	375.95
Quad4	4	375.71

Table 1: Quad4 Elements – Global Refinement

Element Type	**Case**	**Max σ_1 [MPa]**
Quad4	1	330.87
Quad4	2	338.38
Quad4	3	355.20
Quad4	4	358.17

Table 2: Quad4 Elements – Local Refinement

In the case of Quad4 elements, the maximum principal stress values exhibit a clear convergence trend as the mesh is refined. The use of local refinement leads to slightly lower and more stable stress values compared to global refinement, especially in finer meshes. This suggests that concentrating mesh density in regions of high stress is more effective than refining the entire domain uniformly.

Element Type	**Case**	**Max σ_1 [MPa]**
Quad9	1	353.83
Quad9	2	359.28
Quad9	3	375.94
Quad9	4	375.71

Table 3: Quad9 Elements – Global Refinement

Element Type	**Case**	**Max σ_1 [MPa]**
Quad9	1	330.86
Quad9	2	338.38
Quad9	3	355.20
Quad9	4	358.17

Table 4: Quad9 Elements – Local Refinement

In the case of Quad9 elements, the maximum principal stress values demonstrate consistent convergence trend as the mesh is refined. Local refinement leads to slightly lower stress values than global refinement, particularly in the finer mesh cases, indicating improved accuracy around stress concentration zones.

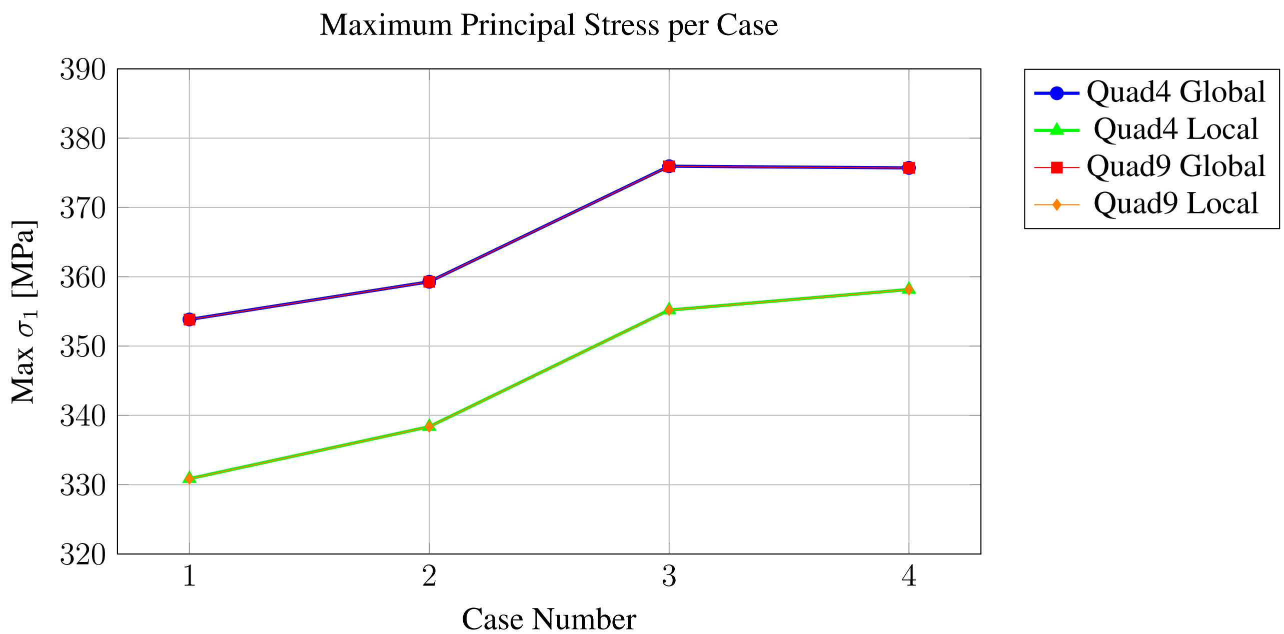

Figure 17: Maximum principal stress for each refinement case and element type

When comparing the results from Quad4 and Quad9 elements, it is evident that both types are capable of capturing the overall behavior of the principal stress distribution. However, Quad9 elements, being higher-order, exhibit smoother and more accurate convergence, especially in regions of stress concentration. They can deliver more precise results with fewer elements, making them computationally efficient when combined with properly applied local mesh refinement.
On the other hand, Quad4 elements are simpler and require less computational effort per element, making them suitable for preliminary analyses or when resources are limited. However, their accuracy relies more heavily on mesh density, and they benefit significantly from local refinement near critical areas.

2 Design Modification and Stress Redistribution (Part C)

The following shows the stresses obtained from the 9L4 model (Quad9, local refinement, case 4), which was used for the profile modification. This includes both the original and the modified cases.

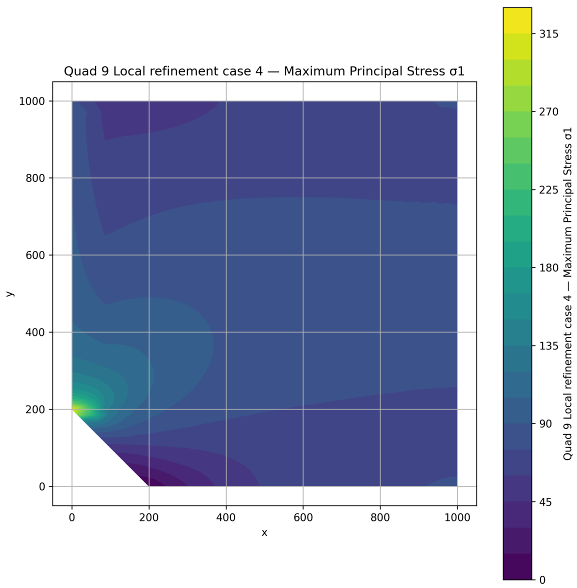

Figure 18: Original case – Quad9 Local Case 4

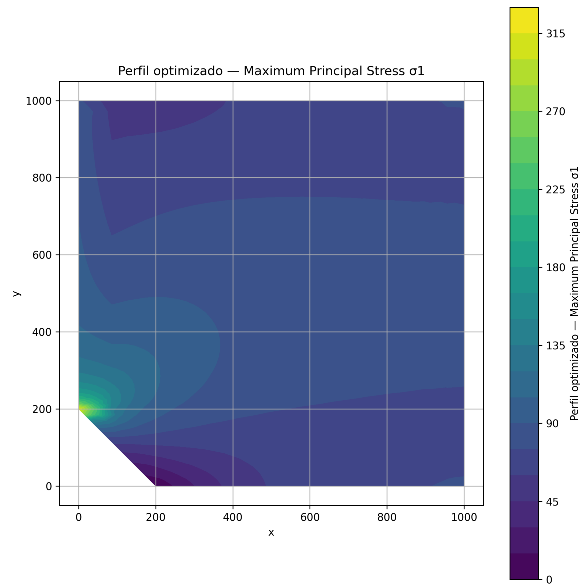

Figure 19: Modified case – Quad9 Local Case 4

In this case, the maximum principal stress in the original configuration is 358.17 MPa, while in the modified configuration it is 355.41 MPa, showing a reduction of approximately 3 MPa.
The profile is directly supported at its ends. To improve its performance, we modified the cross section by reducing 5% of the material from the region located directly above the supports and redistributing it to the area farther away from them. Although this approach may initially seem counterintuitive, it increases the stiffness of the outer region, generating a greater moment due to the self-weight, which pushes the profile downward.
Since the applied force tends to lift the profile upward, this redistribution acts as a counterbalance, reducing the upward deflection and stabilizing the system. As a result, the internal stresses are reduced. In the model, Steel 2 refers to the portion farther from the supports, while Steel 1 refers to the portion closer to them. This modification was analyzed using the 9L4 model to ensure accurate results.

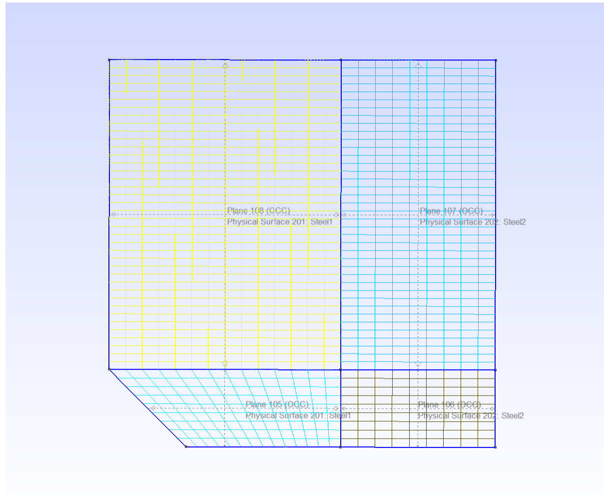

Figure 20: Gmsh of the profile

3 Summary Essay (Part D)

This assignment provided insights into how mesh refinement and local geometry modifications influence the accuracy and interpretation of stress analysis using the finite element method (FEM). One of the most important lessons was understanding the trade-off between mesh density and computational efficiency. As the mesh becomes finer, especially in critical regions where stress concentrations occur, the predicted stress values become more accurate and reliable. Local mesh refinement proved particularly effective, offering improved resolution where needed without excessively increasing the global element count.

Through comparative simulations using both Quad4 and Quad9 elements under global and local refinement strategies, we observed that higher-order elements such as Quad9 deliver smoother and more precise results with fewer elements, especially when combined with local refinement. This underscored the importance of selecting appropriate element types and refinement techniques based on the goals of the analysis and available computational resources.

A key part of the assignment involved applying a local geometric modification: redistributing material from the region directly over the supports to the area farther away. Although initially counterintuitive, this modification effectively reduced the maximum principal stress. By increasing the moment generated by self-weight, the profile counteracted the upward bending caused by external loads. This highlighted how even small geometric changes can significantly influence stress distribution and structural behavior.

Finally, the process emphasized the iterative nature of FEM modeling. From mesh generation and boundary condition definition to result interpretation, careful attention to detail is essential. Errors in geometry, meshing, or assumptions can lead to misleading conclusions. Overall, this assignment deepened my understanding of how thoughtful modeling strategies and refinement choices directly impact the fidelity and usefulness of FEM analyses in structural engineering contexts.