EECE 3207 - Recitation Class #7
Problems #7
Signals and Systems, EECE 3203 Contents
Oct 4, Spring 2022, University of Memphis

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Exam Oct 5. 2022 during class hour

Signal operation : inner product

Given two signals $x(t)$, $\phi(t)$

$$(x(t), \phi(t)) = \int x(t)\, \phi^*(t)\, dt$$

A. Solve the following inner products on the given interval

Given: $x(t) = 2 + 0.1\sin(2\pi t) - 0.2\sin(4\pi t)$, $\phi_0(t) = 1$, $\phi_1(t) = \sin(2\pi t)$, and $\phi_2(t) = \sin(4\pi t)$.

1. Calculate the innerproduct $(x(t), \phi_0(t))$ in the interval $t = 0$ to 1
2. Calculate the innerproduct $(x(t), \phi_1(t))$ in the interval $t = 0$ to 1
3. Calculate the innerproduct $(x(t), \phi_2(t))$ in the interval $t = 0$ to 1.

B.

1. Given a signal x(t) = t for t = 0 to 1, calculate the following inner products (a,b, d, and e require integration by parts):
   a. (x(t), $e^{-i4\pi t}$)
   b. (x(t), $e^{-i2\pi t}$)
   c. (x(t), 1)
   d. (x(t), $e^{i2\pi t}$)
   e. (x(t), $e^{i4\pi t}$)

C. Solve problems 1.16 (a), (c), (e) (Alkin, 2014)

Hint: Euler's formula is given as

$$e^{ja} = \cos(a) + j\sin(a)$$

$$e^{-ja} = \cos(a) - j\sin(a)$$

1.16. Using Euler's formula, prove the following identities:

a. $\cos(a) = \frac{1}{2}e^{ja} + \frac{1}{2}e^{-ja}$

b. $\sin(a) = \frac{1}{2j}e^{ja} - \frac{1}{2j}e^{-ja}$

c. $\frac{d}{da}[\cos(a)] = -\sin(a)$

d. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

e. $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

f. $\cos^2(a) = \frac{1}{2} + \frac{1}{2}\cos(2a)$

D. What is the period of $x(t) = \sin \pi t + \sin 2\pi t/5$? [1]

Solution: If both terms are periodic, their common period is LCM(2, 5) = 10

E. Solve problems 1.25 (Alkin, 2014)

1.25. Identify which of the signals in Fig. P.1.25 are even, which ones are odd, and which signals are neither even nor odd.

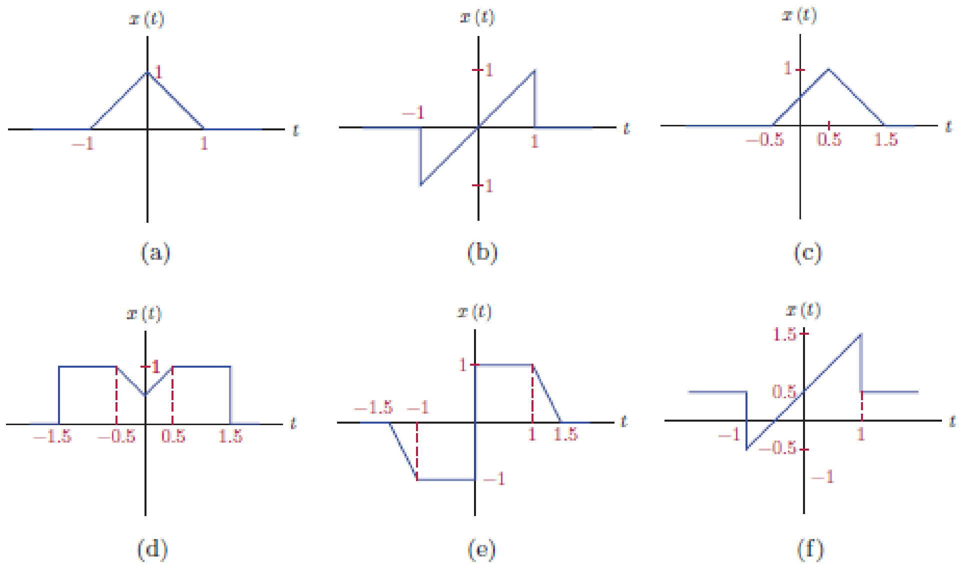

Figure P. 1.25

F. Solve problems 1.17 (Alkin, 2014)

1.17. Using the definition of periodicity, determine if each signal below is periodic or not. If the signal is periodic, determine the fundamental period and the fundamental frequency.

a. $x(t) = 3\cos(2t + \pi/10)$

b. $x(t) = 2\sin\left(\sqrt{20}t\right)$

c. $x(t) = 3\cos(2t + \pi/10)\, u(t)$

d. $x(t) = \cos^2(3t - \pi/3)$

e. $x(t) = e^{-|t|}\cos(2t)$

f. $x(t) = \sum_{k=-\infty}^{\infty} e^{-(t-kT_s)}\, u(t - kT_s)$

g. $x(t) = e^{j(2t+\pi/10)}$

h. $x(t) = e^{(-1+j2)t}$

References

[1] Stanford University, Signals and Systems Fall 2021, Lect 03

[2] Alkin, Oktay - Signals and Systems (2014)