EECE 3207 - Recitation Class # 12
Problems # 11 - April 12, 2022
Signals and Systems, EECE 3203 Contents
Spring 2022, University of Memphis

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Code:
https://github.com/bereketeshete/Signals-and-Systems-EECE-3207/tree/main/Spring%202022/Week%2011%20-%20Spectrum%20Plots

Hints,

Trigonometric Fourier series (TFS)

Synthesis equation:

$$\tilde{x}(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

Analysis equations:

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \cos(k\omega_0 t)\, dt\,, \quad \text{for} \quad k = 1, \ldots, \infty$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \sin(k\omega_0 t)\, dt\,, \quad \text{for} \quad k = 1, \ldots, \infty$$

$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t)\, dt \qquad \text{(dc component)}$$

Exponential Fourier series (EFS)

Synthesis equation:

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t}$$

Analysis equation:

$$c_k = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t)\, e^{-jk\omega_0 t}\, dt\,, \quad k = -\infty, \ldots, \infty$$

4.43. Consider the periodic pulse train shown in Fig. 4.15 on page 289. Its EFS coefficients were determined in Example 4.7 and given in Eqn. (4.77). Write a script to compute and graph the EFS line spectrum for duty cycle values $d = 0.4$, 0.6 , 0.8 and 1. Comment on the changes in the spectrum as d is increased.

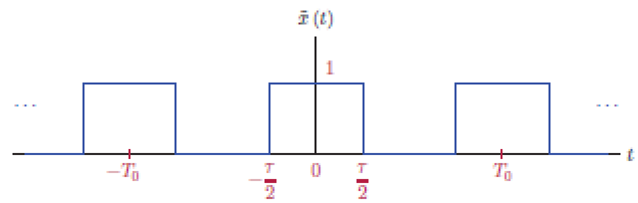

Figure 4.15 – Periodic pulse train for Example 4.7.

$$c_k = \frac{1}{T_0}\left[\int_{-\tau/2}^{\tau/2} (1)\, e^{-j2\pi kt/T_0}\, dt\right] = \frac{\sin(\pi kd)}{\pi k}$$

$$c_k = d\ \mathrm{sinc}\,(kd)$$

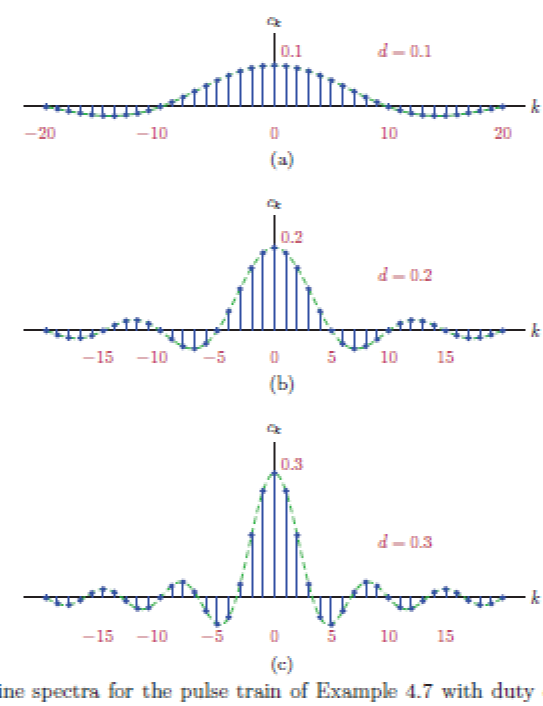

Figure 4.16 – Line spectra for the pulse train of Example 4.7 with duty cycles (a) $d = 0.1$, (b) $d = 0.2$, and (c) $d = 0.3$.

4.41. Compute and graph finite-harmonic approximations to the signal $\tilde{x}(t)$ shown in Fig. P.4.7 using 3, 4, and 5 harmonics. Also graph the approximation error for each case.

4.7. Determine the TFS coefficients for the periodic signal shown in Fig. P.4.7.

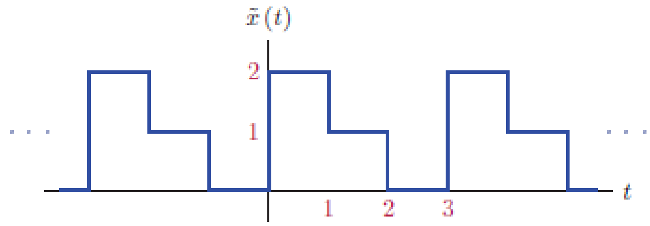

Figure P. 4.7

Solution:

The fundamental period is $T_0 = 3$ seconds which corresponds to a fundamental frequency of $f_0 = 1/3$ Hz or $\omega_0 = 2\pi/3$ rad/s.

$$a_0 = \frac{1}{3}\int_0^3 \tilde{x}(t)\,dt = \frac{1}{3}\left[\int_0^1 (2)\,dt \int_1^2 (1)\,dt\right] = 1$$

$$\begin{aligned} a_k &= \frac{2}{3}\int_0^3 \tilde{x}(t)\cos\left(\frac{2\pi k t}{3}\right)dt \\ &= \frac{2}{3}\left[\int_0^1 (2)\cos\left(\frac{2\pi k t}{3}\right)dt + \int_1^2 (1)\cos\left(\frac{2\pi k t}{3}\right)dt\right] \\ &= \frac{1}{\pi k}\left[\sin\left(\frac{2\pi k}{3}\right) + \sin\left(\frac{4\pi k}{3}\right)\right] = 0 \end{aligned}$$

$$\begin{aligned} b_k &= \frac{2}{3}\int_0^3 \tilde{x}(t)\sin\left(\frac{2\pi k t}{3}\right)dt \\ &= \frac{2}{3}\left[\int_0^1 (2)\sin\left(\frac{2\pi k t}{3}\right)dt + \int_1^2 (1)\sin\left(\frac{2\pi k t}{3}\right)dt\right] \\ &= \frac{1}{\pi k}\left[2 - \cos\left(\frac{2\pi k}{3}\right) - \cos\left(\frac{4\pi k}{3}\right)\right] \end{aligned}$$

$$\tilde{x}^{(m)}(t) = a_0 + \sum_{k=1}^{m} a_k \cos(k\omega_0 t) + \sum_{k=1}^{m} b_k \sin(k\omega_0 t) \qquad (4.45)$$