Adaptive Speckle Reduction Filter for Log-Compressed B-Scan Images

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***Abstract*— A good statistical model of speckle formation is useful for designing a good adaptive filter for speckle reduction in ultrasound B-scan images. Previously, statistical models have been used, but they failed to account for the log compression of the echo envelope employed by clinical ultrasound systems. Log-compression helps in reducing the dynamic range of the B-scan images for display on a monitor as well as enhancing weak backscatterers. In this article, statistics of log-compressed echo images, using the K-distribution statistical model for the echo envelope, are used to derive a parameter that can be used to quantify the extent of speckle formation. This speckle quantification can be used with an unsharp masking filter to adaptively reduce speckle. The effectiveness of the filter is demonstrated on images of contrast detail phantoms and on *in-vivo* abdominal images obtained by a clinical ultrasound system with log-compression.**

I. Introduction

Due to coherence of the backscattered echo signals, images obtained from echo ultrasound imaging systems have interference patterns called speckle. Speckle tends to degrade the resolution and the object detectability (contrast). Many methods have been proposed using frequency and spatial compounding to reduce this speckle [1]–[5]. But these methods do not utilize the information about the speckle contained in the images. Recently, a method has been proposed which uses the statistics of the image to adaptively filter out speckle [6]–[9]. In this method, local statistics of a point in the image is used to quantify the extent of the speckle formation. This statistic is then used to selectively filter the image using a modified version of the unsharp masking filter. One drawback of this filter is that it is designed to work with uncompressed images, whereas clinical imaging systems normally employ logarithmic compression to reduce the dynamic range of the envelope. Thus, previous methods are not useful unless the images are decompressed first.

In this article, a statistic which quantifies the extent of the speckle formation for log-compressed envelope images will be derived. This statistic will be used in unsharp mask filtering of log-compressed B-scan images without having to uncompress images first. The statistical analysis is based on the K-distribution model for the echo envelope signal. This model has been proposed previously for speckle statistics of microwave sea echoes [10], laser speckle in atmospheric transmission [11], and ultrasound echo speckle [12]–[16], and provides the statistics of scattering media with arbitrary scatterer densities.

In the following section, a statistic for log-compressed images based on the K-distribution model will be used to design an unsharp masking filter for speckle reduction. In the penultimate section, the results of using this filter on images from a clinical imaging system will be presented, and in the final section we will state our conclusions.

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II. Statistics of the Envelope Image

The echo signal from a scattering medium is a coherent sum of individual scattered signals which results in the formation of speckle. The resolution cell, defined as the volume of medium which contributes to the echo signal at a given instant, determines the number of scatterers contributing to the echo signal. The statistics of the echo envelope for the case of many scatterers per resolution cell (>10) is Rayleigh distributed [17], [18]. This distribution is given as

$$p(A) = \frac{A}{\sigma^2} \exp\left(-\frac{A^2}{2\sigma^2}\right); \qquad A \geq 0 \tag{1}$$

where A is the envelope amplitude and σ^2 is the variance of the random backscatter amplitude of the individual scatterers. When the scatterer density (the number of scatterers per resolution cell) is small, or when the effective scatterer density is reduced due to correlation in scatterers, the probability density function deviates from the Rayleigh distribution. A generalized version of the Rayleigh density function, the K distribution, can be used to model the statistics of the echo envelope when the scatterer densities are smaller than the Rayleigh limit [10], [19]. This K distribution is given as

$$p(A) = \frac{2b}{\Gamma(\alpha)} \left(\frac{bA}{2}\right)^{\alpha} K_{\alpha-1}(bA);$$
$$b = 2\sqrt{\frac{\alpha}{E\{A^2\}}}; \qquad A \geq 0; \qquad \alpha > 0 \tag{2}$$

where α is a parameter of the K distribution that characterizes the scatterer density, $K_\beta(x)$ is the modified Bessel function of second kind of order β, $\Gamma(x)$ is the Gamma function, and $E\{A^2\}$ is the second moment of the envelope amplitude, E denoting the expectation operator. Parameter α has been previously shown [10] to be related to the number of scatterers

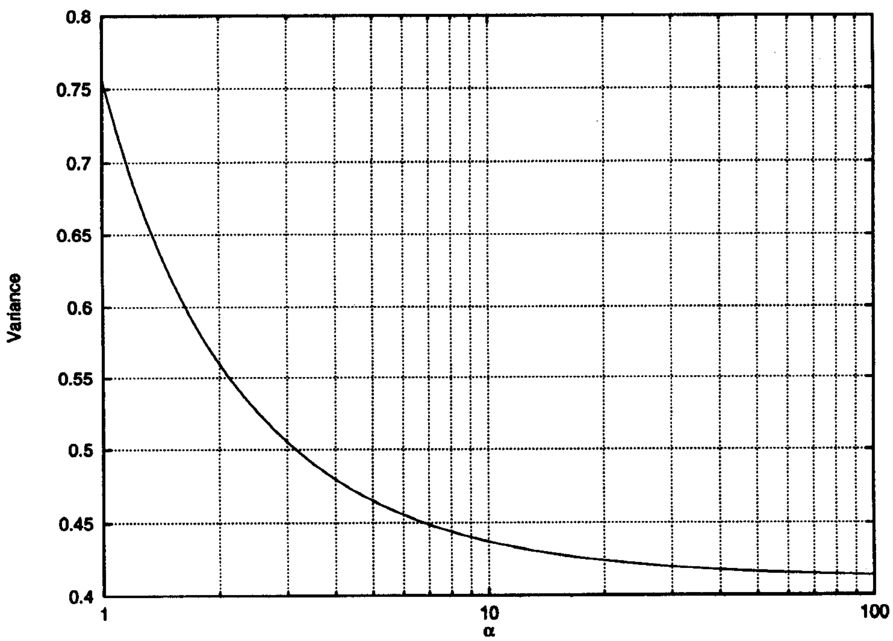


Fig. 1. Variance of log-compressed K distributed data with the K parameter α.

per resolution cell, N_s

$$\alpha = (\nu + 1)N_s; \qquad \nu > -1$$

where ν is a function of the scanning geometry and the backscatter coefficient statistics. This relation shows that α can be defined as the "effective" number of scatterers in the resolution cell, the number of scatterers that can be "resolved" with the K-distribution model.

The various moments of the K distribution can be shown to be [15]

$$E\{A^\eta\} = \frac{(2\sigma^2)^{\eta/2}\Gamma\left(1+\frac{\eta}{2}\right)\Gamma\left(\alpha+\frac{\eta}{2}\right)}{\alpha^{\eta/2}\Gamma(\alpha)} \tag{3}$$

where η is the moment order.

III. Envelope Compression in B-Scan Imaging

Clinical ultrasound imaging systems employ nonlinear signal processing to reduce the dynamic range of the input echo signal to match the smaller dynamic range of the display device and to emphasize objects with weak backscatter. Typically, the input image could have dynamic ranges of the order of 50–70 dB whereas a display device would have dynamic range of the order of 20–30 dB. This reduction in dynamic range is normally achieved through a logarithmic compression which selectively compresses large input signals. Logarithmic compression is also useful for homomorphic filtering if the noise is multiplicative.

This kind of nonlinear compression totally changes the statistics of the input envelope signal. The K-distribution model cannot be used for compressed B-scan image statistics. A closed form expression for the density function of the log transformed K distributed data is yet to be derived. However, it is possible to obtain statistics for the limiting case of large scatterer densities, i.e., the statistics of log-compressed, Rayleigh distributed data [20]. One way to solve the problem would be to obtain an approximation to the K-distribution function which would enable derivation of statistics of log-compressed K distributed data. The approximate density function could then be used to derive a statistic quantifying the extent of speckle formation which can be used for filtering speckle. This is the approach taken in this article.

A. Logarithmic Compression Model

The logarithmic compression transfer function can be written as

$$X = D \ln A + G \tag{4}$$

where A is the input to the compression block and X is the output of the compression block. D is a parameter of the compressor which represents the dynamic range of input, and G is the linear gain of the compressor. Here it is assumed that the input is nonzero.

The linear gain parameter, G, does not affect the statistics of the output signal because it just changes the mean of the distribution function. But the dynamic range parameter, D, scales the output signal and is thus important to estimate if one has to invert this logarithmic transfer function.

If the minimum and maximum input values $A_{\min}$ and $A_{\max}$ are mapped to minimum and maximum output values $X_{\min}$ and $X_{\max}$ by this logarithmic compression, then the relationship between them can be written as

$$X_{\max} - X_{\min} = D \ln\left(\frac{A_{\max}}{A_{\min}}\right). \tag{5}$$

The right-hand side of (5) represents the dynamic range of the input and the left hand side is the range of the output. The input dynamic range, R, can be written as

$$R = 20 \ln\left(\frac{A_{\max}}{A_{\min}}\right). \tag{6}$$

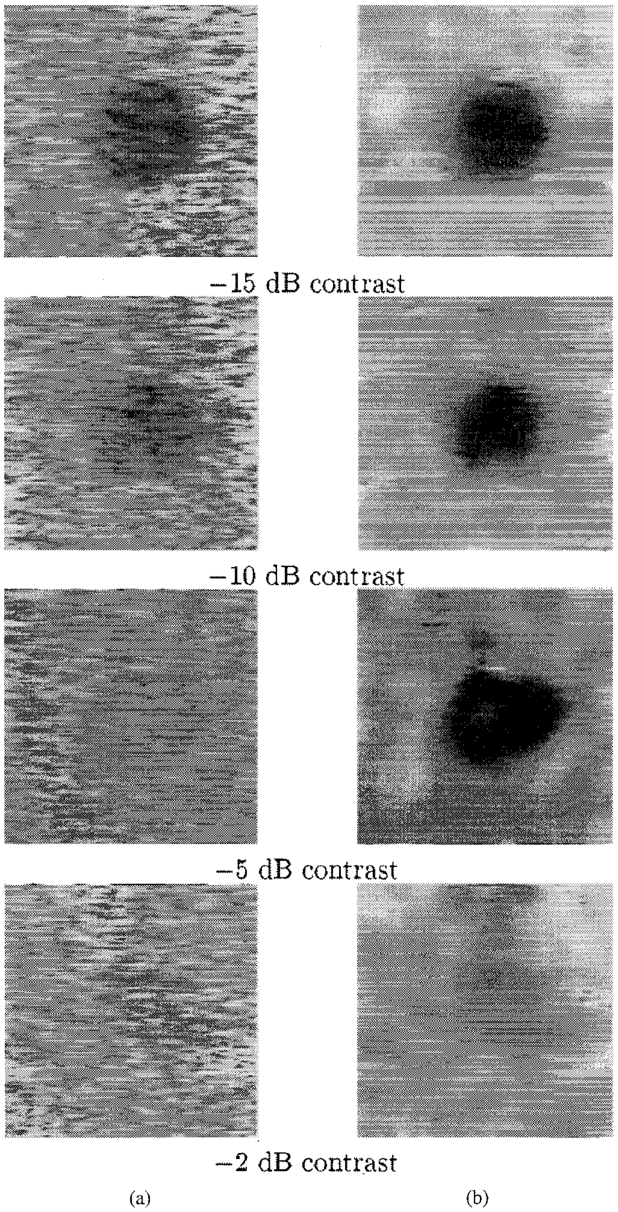


Fig. 2. Comparison of log-compressed envelope images obtained by log transforming images from a linear scanner with its filtered images: (a) log-compressed envelope images and (b) filtered images. These images show negative contrast.

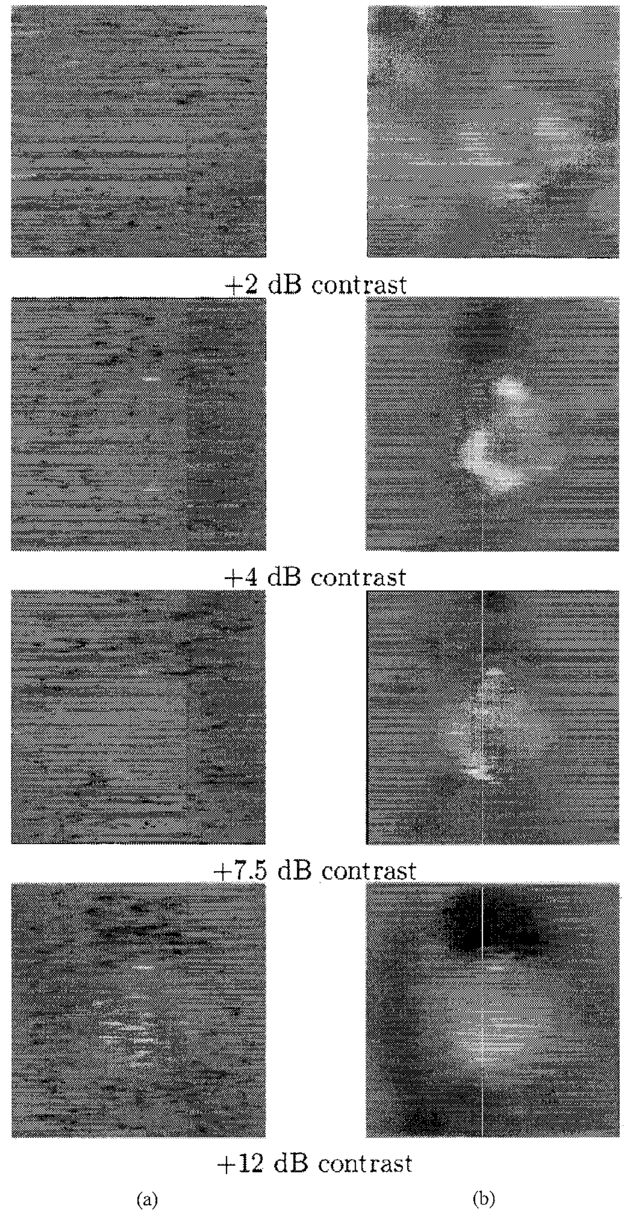


Fig. 3. Comparison of log-compressed envelope images obtained by log transforming images from a linear scanner with its filtered images: (a) log-compressed envelope images and (b) filtered images. These images show positive contrast.

Therefore

$$D = \frac{20}{R}\,(X_{\max} - X_{\min}). \tag{7}$$

Thus if the input dynamic range and the output range are known, then the logarithmic compression parameter D could be estimated. Also, the above equation shows why the parameter D is denoted as the dynamic range parameter.

B. Statistical Analysis of Log-Compressed Fully Formed Speckle

Even though the density function of the log-compressed K distribution would be quite complicated, the density function of the compressed signal from fully formed speckle, which has Rayleigh density function for uncompressed data, is easy to derive. In fact, as shown by Kaplan and Ma [20], [21], the statistics of log-compressed fully developed speckle can be used to estimate the log transfer function.

For any arbitrary compression transfer function, the density function of the compressed output, which is used to produce the B-scan images, is given by

$$p_X(X) = \frac{p_X(X^{-1})}{\left|\dfrac{dX}{dA}\right|}$$

where A is the input and X is the output of the compressor, and X^{-1} is the input value which is mapped to output X. This density function for the case when the input A is Rayleigh distributed (1), can be evaluated as [21], [22]

$$p_X(X) = \frac{1}{\lambda} \exp\{-g \exp(-g)\} \tag{8}$$

where

$$g = \frac{\rho - X}{lambda}$$
$$\rho = D\frac{\ln(2\sigma^2)}{2} + G$$

and

$$\lambda = \frac{D}{2}.$$

The density function defined in (8) is the double exponential or Fisher–Tippet density function. The mean and variance of the double exponential density function can be written as [22, p. 930]

$$E\{X\} = \rho - \gamma\lambda \tag{9}$$

and

$$\begin{aligned}\text{Var}\{X\} &= E\{(X - E\{X\})^2\} \\ &= \frac{\pi\lambda^2}{6}\end{aligned} \tag{10}$$

where γ is the Euler constant ($\gamma \approx 0.5772$). Thus the mean and the variance of a log-compressed envelope signal can be written as

$$E\{X\} = D\left[\frac{\ln(2)}{2} + \ln(\sigma) - \frac{\gamma}{2}\right] + G \tag{11}$$

and

$$\text{Var}\{X\} = \frac{\pi^2}{24}D^2. \tag{12}$$

Thus from (12) it can be seen that the variance of the log-compressed envelope is independent of the backscattered energy of the echo and only depends on the compression dynamic range parameter, D. Thus the parameter, D, could be estimated from the variance of the log-compressed envelope in a region of the image containing fully formed speckle.

C. Statistics of the Log-Compressed Partially Formed Speckle

The density function for normalized intensity, $I_n = X^2/2\sigma^2$ of Rayleigh distributed data, given by density function (1), can be written as

$$p_I(I_n) = \exp(-I_n), \qquad I_n \geq 0$$

which is an exponential distribution. Similarly, the density function for normalized intensity, when the amplitude is K distributed, can be written as

$$p_I(I_n) = \frac{2\alpha^{(\alpha+1)/2}}{\Gamma(\alpha)} I_n^{(\alpha-1)/2} K_{\alpha-1}\left(\sqrt{4\alpha I_n}\right), \qquad I_n \geq 0.$$

Because the density function involves Bessel's function, it is difficult to derive a closed form expression for the density function of the log-compressed K distributed data. One possibility to overcome this problem would be to find an approximation to the K-distribution function which would avoid using the Bessel function. As the K distribution is a generalization of the Rayleigh distribution, it would be useful to come up with an approximation which models the K distribution as a modulated version of the Rayleigh distribution. This could be done by expressing the K distribution in terms of the Laguerre orthogonal polynomials [23], [24]. The Laguerre polynomials' weighting function is negative exponential which is the distribution of the normalized intensity for the Rayleigh distributed amplitude. Thus the Laguerre polynomial expansion for the normalized intensity would be

$$p_I(I_n) = \sum_{m=0}^{\infty} c_m \phi_m(I_n) W(I_n) \tag{13}$$

where $\phi_m(x)$ is the Laguerre polynomial of mth order, and $W(x)$ is the weighting function for the Laguerre polynomials [which is exponential, $\exp(-x)$], and, c_m are the coefficients of the Laguerre series expansion for the K distribution.

Approximating the K distribution with its first three Laguerre polynomial terms as derived in Appendix A, the density function can be written as

$$p_I(I_n) \approx \left[1 + \frac{1}{\alpha}\left(1 - 2I_n + \frac{I_n^2}{2}\right)\right] \exp(-I_n).$$

Now if the intensity is log transformed through function

$$U = D_I \ln I_n + G_I \tag{14}$$

where D_I and G_I are dynamic range and gain parameters for the log transfer function, then the density function of log transformed data, U, is given by

$$\begin{aligned}p_U(U) &= \frac{p_I(I_n^{-1})}{\left|\frac{dU}{dI_n}\right|} \\ &= \frac{\exp\left(\frac{U - G_I}{D_I}\right)}{D_I} p_I\left(\exp\left[\frac{U - G_I}{D_I}\right]\right).\end{aligned}$$

The density function for the log transformed normalized intensity, using the Laguerre polynomial approximation to the K-distribution function (14), can be written as

$$\begin{aligned}P_U(U) = \frac{1}{D_I}&\left[\left(1 + \frac{1}{\bar{\alpha}}\right) - \frac{2}{\alpha}\exp\left(\frac{U - G_I}{D_I}\right)\right. \\ &\left.+ \frac{1}{2\alpha}\exp\left(2\frac{U - G_I}{D_I}\right)\right] \\ &\cdot \exp\left(\frac{U - G_I}{D_I}\right) \exp\left[-\exp\left(\frac{U - G_I}{D_I}\right)\right].\end{aligned} \tag{15}$$

The above equation is a modified form of double exponential distribution.

The mean and the variance of the log transformation can be obtained using the first two moments. Thus, from the moments derived in Appendix B, the mean, Mean$\{U\}$, and the variance,

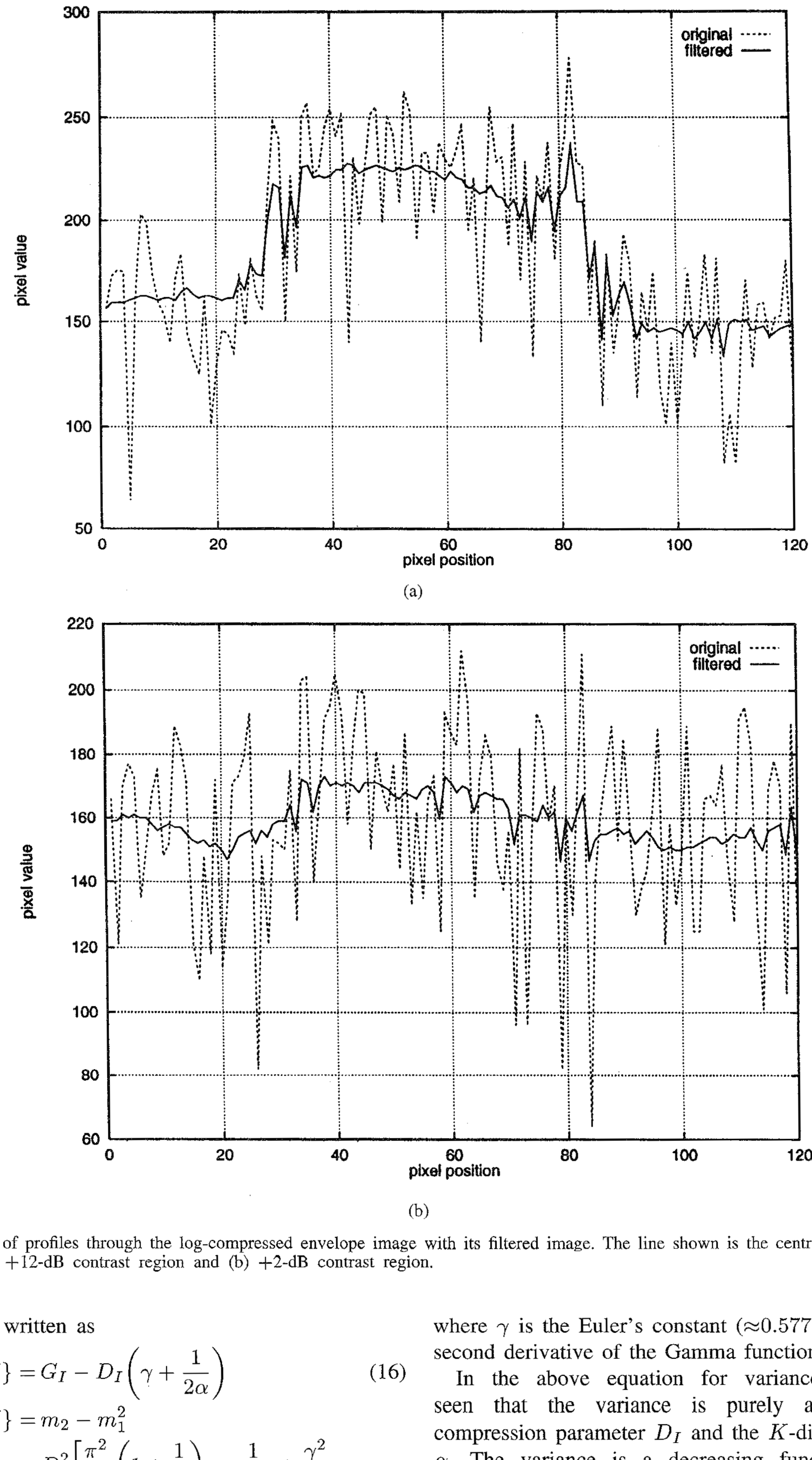


Fig. 4. Comparison of profiles through the log-compressed envelope image with its filtered image. The line shown is the central column in the images corresponding to (a) +12-dB contrast region and (b) +2-dB contrast region.

$\mathrm{Var}\{U\}$, can be written as

$$\mathrm{Mean}\{U\} = G_I - D_I\left(\gamma + \frac{1}{2\alpha}\right) \tag{16}$$

$$\begin{aligned}\mathrm{Var}\{U\} &= m_2 - m_1^2 \\ &= D_I^2\left[\frac{\pi^2}{6}\left(1+\frac{1}{\alpha}\right) - \frac{1}{4\alpha^2} + \frac{\gamma^2}{\alpha}\right. \\ &\qquad \left. -\frac{\gamma}{\alpha} + \frac{\Gamma''(3)}{2\alpha} - \frac{2\Gamma''(2)}{\alpha}\right]\end{aligned} \tag{17}$$

where γ is the Euler's constant ($\approx$0.5772) and $\Gamma''(x)$ is the second derivative of the Gamma function.

In the above equation for variance (17), it can be seen that the variance is purely a function of log compression parameter D_I and the K-distribution parameter α. The variance is a decreasing function of α and is scaled by D. For the limiting case of large α (large scatterer density), the variance reaches the value of $\pi^2/6D_I^2$.

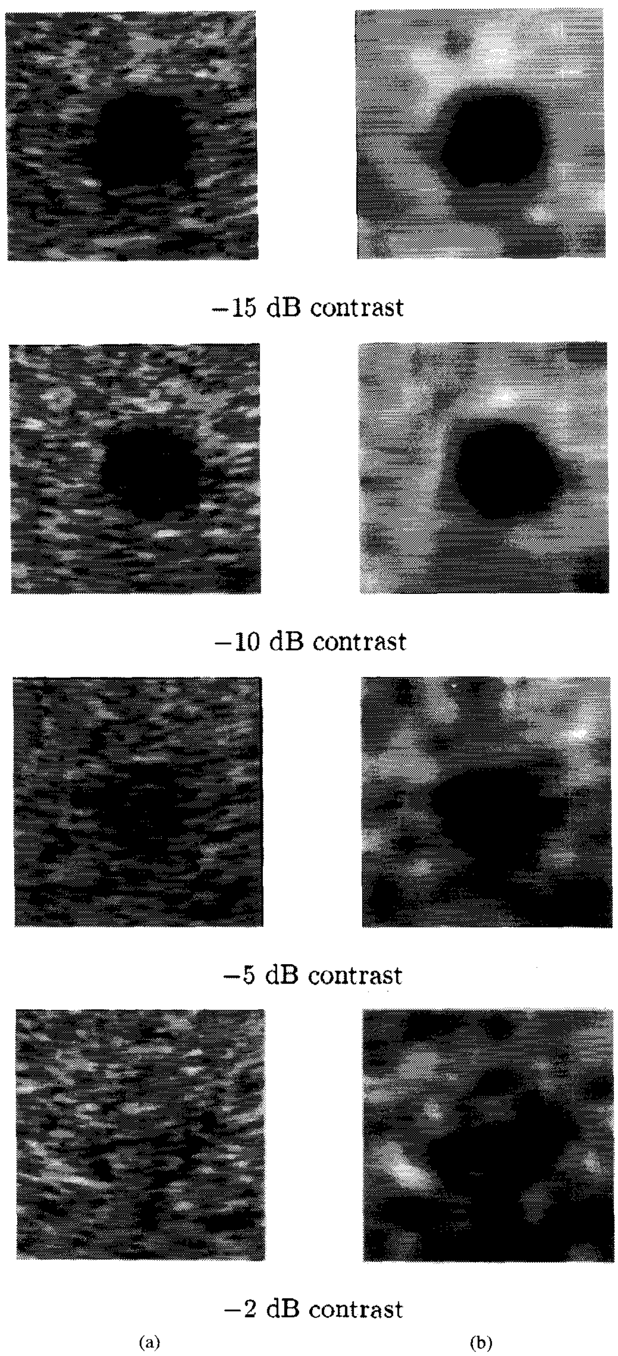


Fig. 5. Comparison of B-scan envelope image from HP clinical scanner with its filtered images: (a) original envelope images and (b) filtered images. These images show negative contrast.

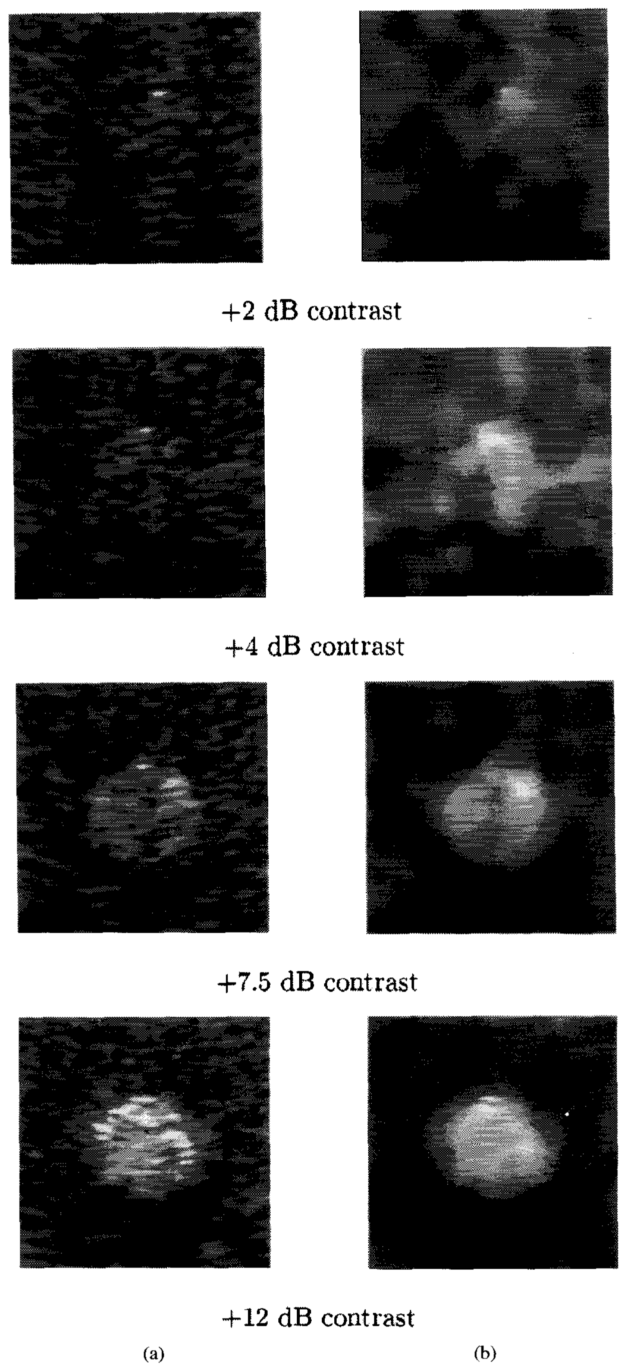


Fig. 6. Comparison of B-scan envelope image from HP clinical scanner with its filtered images: (a) original envelope images and (b) filtered images. These images show positive contrast.

Now if the log compression was done for amplitude rather than the intensity

$$X = D \ln A + G \tag{18}$$

where D and G are dynamic range and gain parameters for the log compression of the amplitude, then the parameter D and G can be written as

$$D = 2D_I \tag{19}$$

$$G = G_I - D_I \ln(2\sigma^2). \tag{20}$$

Hence, the mean and the variance for the log-compressed amplitude can be written as

$$\text{Mean}\{X\} = G + \frac{D}{2}\left[\ln(2\sigma^2) - \left(\gamma + \frac{1}{2\alpha}\right)\right] \tag{21}$$

$$\begin{aligned}\operatorname{Var}\{X\} &= m_2 - m_1^2 \\ &= \frac{D^2}{4}\left[\frac{\pi^2}{6}\left(1+\frac{1}{\alpha}\right) - \frac{1}{4\alpha^2}\right. \\ &\qquad \left. + \frac{\gamma^2}{\alpha} - \frac{\gamma}{\alpha} + \frac{\Gamma''(3)}{2\alpha} - \frac{2\Gamma''(2)}{\alpha}\right]. \end{aligned} \tag{22}$$

Define the α dependent part of $\operatorname{Var}\{Y\}$ as

$$\begin{aligned} r(\alpha) &= \frac{1}{\alpha} - \frac{6\gamma(1-\gamma)}{\alpha\pi^2} + 3\frac{\Gamma''(3) - 4\Gamma''(2)}{\alpha\pi^2} - \frac{3}{2\alpha^2\pi^2} \\ &= \frac{0.608}{\alpha} - \frac{0.152}{\alpha^2} \end{aligned} \tag{23}$$

where limiting value of $r(\alpha)$ is $\lim_{\alpha\to\infty} r(\alpha) = 0$. Now the variance of the log-compressed amplitude can be written as

$$\operatorname{Var}\{X\} = \frac{\pi^2}{24} D^2[1 + r(\alpha)]. \tag{24}$$

It should be noted here that the above equation is an approximation to the variance because we have only used the first three Laguerre polynomials in the Laguerre polynomial expansion for the K distribution. Higher order polynomials could be added to obtain higher order $(1/\alpha)^n$ terms in $R(\alpha)$ to improve the approximation. If for example, the third-order Laguerre polynomial is added to improve the approximation to the K distribution, the expression for $r(\alpha)$ up to the second order of α improves to

$$r(\alpha) = \frac{0.608}{\alpha} + \frac{0.231}{\alpha^2}. \tag{25}$$

Equation (24) shows that the variance of the log-compressed envelope will be the smallest at large scatterer densities (approaching the limit of $\pi^2/24 \approx 0.4112$) and will be large for small scatterer densities.

The normalized variance (with respect to the compression dynamic range parameter, D), V_N, defined as

$$\begin{aligned} V_N &= \frac{24}{\pi^2}\frac{\operatorname{Var}\{X\}}{D^2} \\ &= 1 + r(\alpha) \end{aligned} \tag{26}$$

is a function of the scatterer density, α, which has a large value for small densities and reaches a value of unity for larger densities. Fig. 1 shows the normalized variance of log-compressed K distributed amplitude data as given by (24).

Thus this statistical analysis shows that the normalized variance of log-compressed envelope images can be used as a statistic to parameterize scatterer densities, or the extent of speckle formation as low densities would result in fully resolved scatterers and large densities result in fully formed speckle. The interesting finding is that such a simple statistic like variance gives the extent of speckle formation; the larger the variance, the less the extent of speckle formation.

D. Filter Statistic

Defining

$$\begin{aligned} f(\alpha) &= \frac{1}{V_N} \\ &= \frac{\pi^2}{24}\frac{D^2}{E\{(X - E\{X\})^2\}} \end{aligned} \tag{27}$$

we observe that $f(\alpha)$ tends to zero for small scatterer densities and to unity for large scatterer densities. Thus this statistic can be used as the filtering parameter for an unsharp masking filter to selectively filter the image when $f(\alpha)$ is close to unity.

The sample variance of the log-compressed images can be estimated as

$$V = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2 \tag{28}$$

where the sample mean $\overline{X}$ is given as

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \tag{29}$$

where X_i are the N samples of the echo envelope. So the estimate for $f(\alpha)$ can now be written as

$$\widehat{f(\alpha)} = \frac{\pi^2}{24}\frac{\hat{D}^2}{V} \tag{30}$$

where $\hat{D}$ is an estimate of the logarithmic compression parameter from the dynamic range and the output image range as given by (7).

IV. Speckle Reduction Filter

Using the estimated statistic, $\widehat{f(\alpha)}$, one can design an unsharp masking filter similar to the filter proposed by Bamber, Daft, and Crawford *et al.* [6]–[9]. The unsharp masking filter smoothes the image based on some local statistic. The output Y of an unsharp masking filter for input X is given by

$$Y = \overline{X} + c(X - \overline{X}) \tag{31}$$

where c is the local statistic and $\overline{X}$ is the local mean. If the statistic is limited to range [0, 1], then the filter output will range from maximal smoothing (mean) to no filtering. If c were allowed to be larger than one, then it will enhance the edges, in which we are not interested.

In our design we use $\widehat{f(\alpha)}$ as the statistic

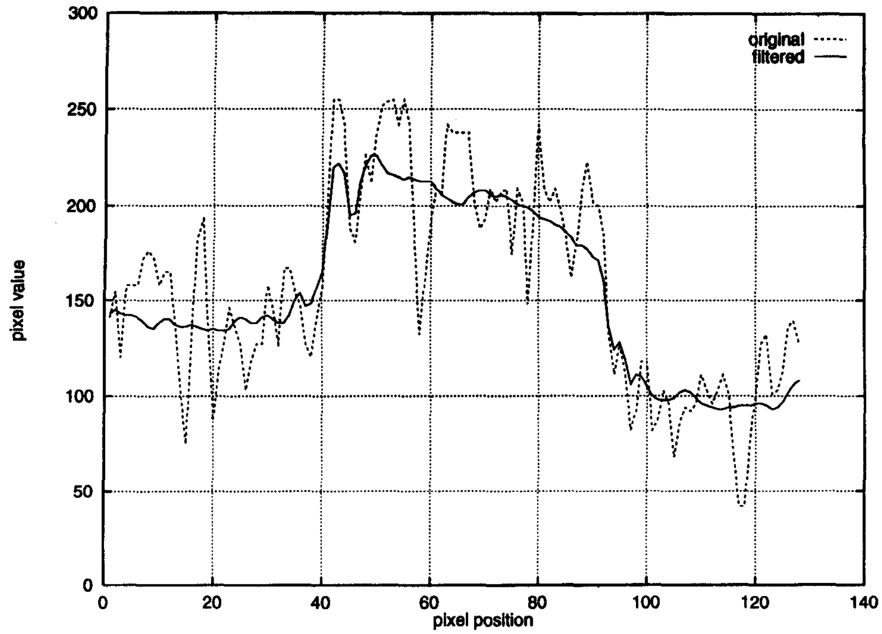
$$\begin{aligned} c &= 1 - \widehat{f(\alpha)} \\ &= 1 - \frac{\pi^2}{24}\frac{\hat{D}^2}{V}. \end{aligned} \tag{32}$$

So an unsharp masking filter which smoothes when $\widehat{f(\alpha)}$ is large could be designed as

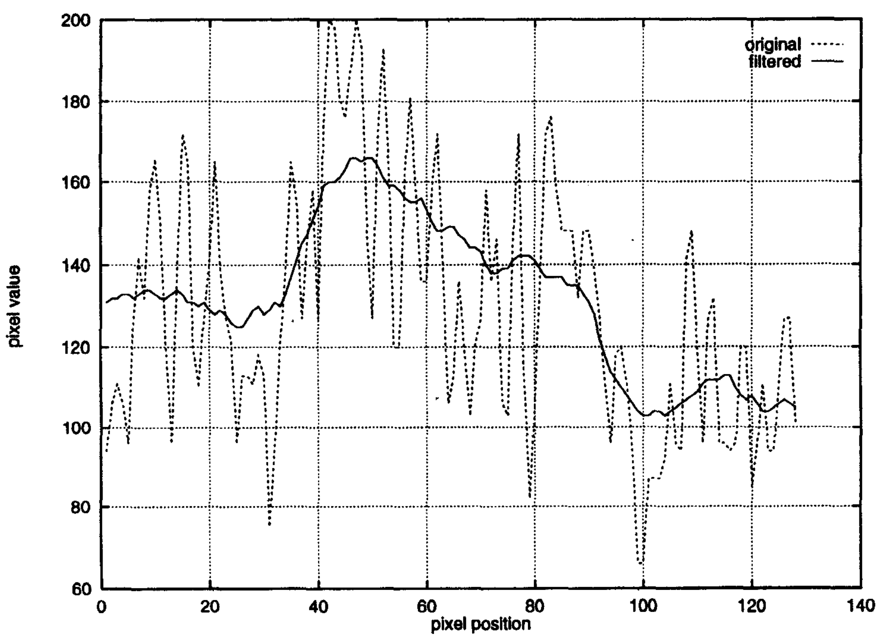
$$Y = \overline{X} + [1 - \widehat{f(\alpha)}](X - \overline{X}) \tag{33}$$

where the range of $\widehat{f(\alpha)}$ is limited to [0, 1]. Any larger estimates of $\widehat{f(\alpha)}$ will be truncated to one. Larger values are possible when there is coherent signal present in the data. In such cases, we still would like to do maximum smoothing to reduce the speckle to better show the coherent component.

Clinical imaging systems often employ some sort of low-pass filtering after the log compression of the echo envelope to smooth the images. The low-pass filtering will reduce the variance of the signal and thus affect the adaptive speckle filter because it is based on the variance of the signal. We add a parameter, $\phi \in (0, 1]$, to compensate the reduction in the variance due to low-pass filtering. $\phi = 1$ corresponds to no low-pass filtering, and $\phi \to 0$ corresponds to highly smoothed



(a)



(b)

Fig. 7. Comparison of profiles through the log-compressed envelope image with its filtered image. The line shown is the central column in the images corresponding to (a) +12-dB contrast region and (b) +4-dB contrast region.

images

$$Y = \overline{X} + [1 - \phi \widehat{f(\alpha)}](X - \overline{X}). \quad (34)$$

V. Experiments

A. Filtering Images of a Contrast Detail Phantom

A contrast detail phantom with circular regions of differing contrast levels was imaged with a water tank based rectilinear scanning imaging system. In this system, the phantom to be imaged is placed in a water tank opposite to the echo imaging transducer. This system acquires the rf echo A-lines in a linear scan which is envelope detected to produce a B-scan image that does not have any envelope compression. The images were acquired with a weakly focused 3.5-MHz transducer, with 19-mm diameter and 60–120-mm focused range, at a depth of about 80 mm. The images were logarithmically compressed in the post-processing stage to simulate compressed B-scan

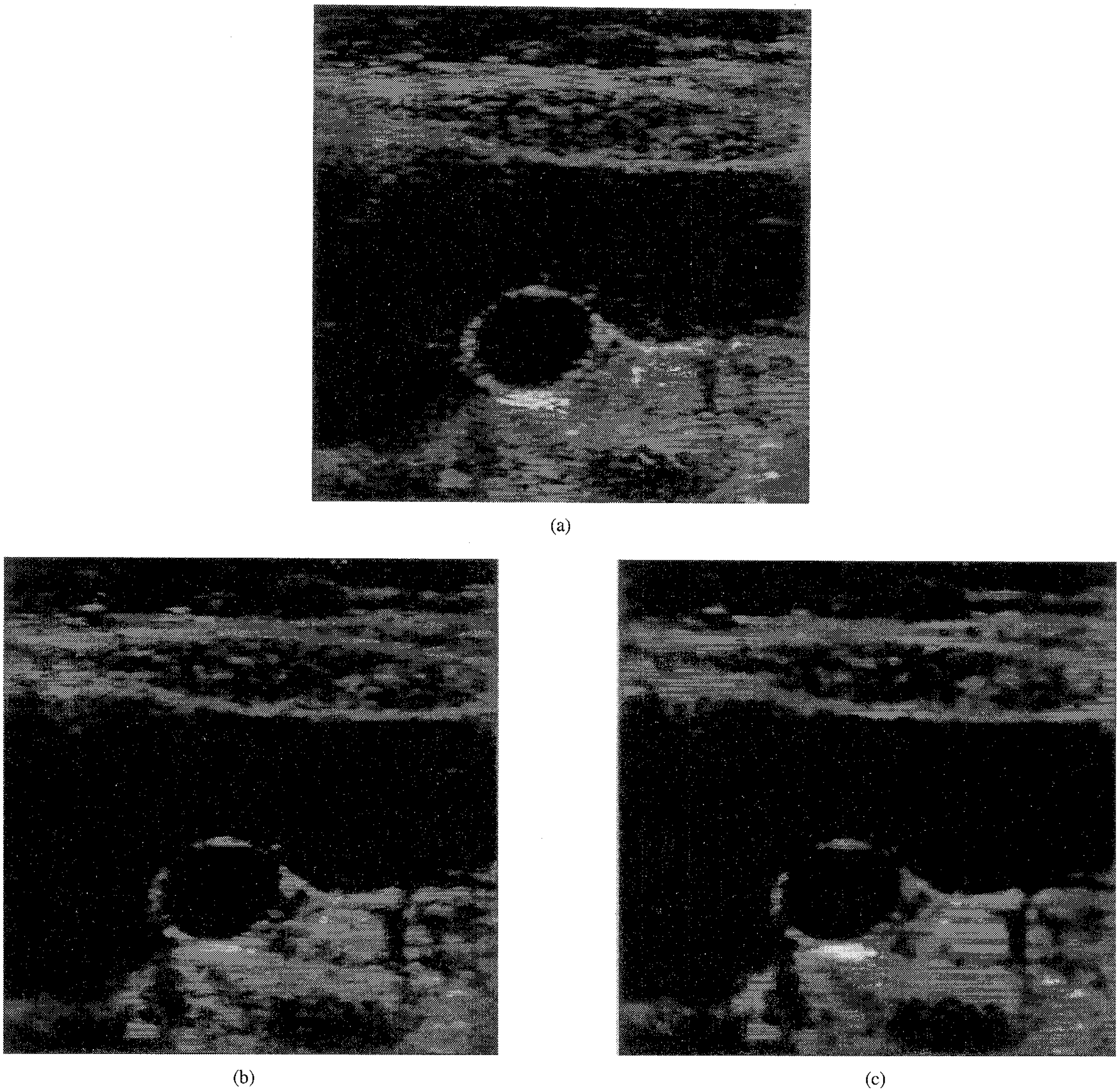
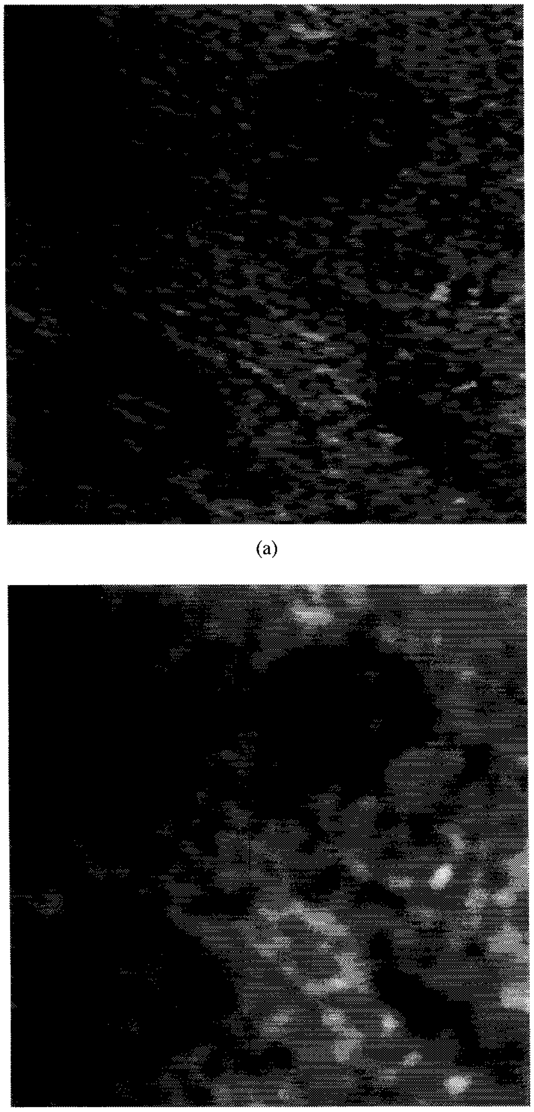


Fig. 8. Comparison of B-scan image of abdomen from Acuson echo imaging system with its filtered versions (a) original image and adaptive filtered images, (b) with $\phi = 0.75$, and (c) with $\phi = 1.00$.

images. Then the images were filtered with the unsharp masking filter given in (33). The statistic was computed from a region of 16×16 pixels centered around each pixel of these images. The results of filtering are shown in Figs. 2 and 3. The first figure shows the negative contrast regions and the second shows the positive contrast regions. In these filtered images, even the low contrast regions (-2 and $+2$ dB) can be seen, whereas they can not be distinguished in the compressed envelope images.

In Fig. 4, a line through the log-compressed image and its filtered image is shown for the $+12$-dB contrast region and the $+2$-dB contrast region. As can be seen in the figure, the variance in the contrast region and its background has been reduced while preserving the edges. Also for the case of the $+2$-dB region, the contrast between the central region and its background is clearly visible in the filtered image which is not the case for the compressed B-scan image.

The same phantom was also scanned with an Hewlett-Packard clinical imaging system (HP 77 020A) with a 32-element phased array transducer of 3.5-MHz central frequency at a depth of about 80 mm. The logarithmic compression transfer function for this imaging system was obtained by using the statistics of the images of the phantom and the statistics of images obtained earlier from the water tank based linear scanning system. The mean of images from the contrast region obtained by the two systems was used to obtain the log transfer function relation between the two. The least squares fitted log transfer function was used as the log compression model, and the dynamic range parameter of this fitted log transfer function was used to estimate the local normalized variance of the compressed image from the HP system. The images were filtered with the unsharp masking filter. The parameter D was evaluated to be 52.6 by the least squares method outlined earlier. The statistic was obtain using image



(a)

(b)

Fig. 9. Comparison of B-scan image of liver from Acuson echo imaging system with its filtered versions (a) original image and (b) adaptive filtered image with $\phi = 1.00$.

samples in a region of 16×16 around each pixel in the image. The results are shown in Figs. 5 and 6. These results also show that the filter is effective in reducing the speckle.

In Fig. 7, a line through the image obtained from the HP system and its filtered image is shown for +12-dB contrast region and +4-dB contrast region. The results obtained are similar to the results obtained for speckle reduction for log-compressed images from watertank-based imaging system. The filtering shows reduction in the variance with edge preservation. Thus these results from the contrast detail phantom show that the filter is effective in speckle reduction as expected from theoretical analysis.

B. Filtering Abdominal Images from Clinical Imaging System

The filter was used to reduce speckle in images acquired in a clinical setting using a Acuson imaging system, 128XP. This ultrasound system provides the logarithmic compression input dynamic range, R, which can be used to estimate the compression parameter, D. The parameter D is required for estimating the normalized variance of the log-compressed images.

Figs. 8 and 9 show the results of filtering clinical images of liver and abdominal region acquired from the Acuson ultrasound system. The transducer frequency was 3.5 and 2.5 MHz, and images obtained at 80 and 120 mm respectively. The input dynamic range for these images was 35 and 51 dB, respectively. The patch size for statistic estimation was 7×7 pixels. The filtering was done assuming no smoothing of images [Figs. 8(c) and 9(c)] and some smoothing of images [$\phi = 0.75$, Figs. 8(b) and 9(b)]. With $\phi = 0.75$, the edges are sharper for these images because they have been smoothed by the echo imaging system which is compensated by taking a nonunity ϕ for adaptive filtering. In the liver image, Fig. 9, the tumor region boundaries are much clearer in the filtered images as compared to the original image.

Fig. 10 shows a line through the B-scan image obtained with the Acuson system shown earlier in Fig. 8. The line shown was a vertical line which passes approximately through the middle of the vessel at the bottom of the image. As can be seen in the image, the filter clearly preserves the edges of the vessel, while reducing the speckle in the middle of the image as well as inside the vessel.

VI. Discussions

The statistical analysis of log compression described in this article has shown that the speckle reduction of B-scan images does require decompression of the images. In fact, this statistical analysis shows that a simple statistical metric like the variance can be used to perform such filtering. Also, previous adaptive methods [6], [8], [25] involved filters in which the filtering parameters were required to be manually optimized. Such methods require decompression of the compressed B-scan images before filtering which makes them dependent on optimality of the employed decompression methods. The statistical modeling approach using the K-distribution analysis resulted in a filter which does not require manual optimization [26, ch. 6]. Also, because the algorithm works with compressed images directly, the method presented here avoids problems associated with suboptimal decompression.

The low-pass filtering of log-compressed images has not been directly incorporated into the statistical method presented here. The low-pass filter will reduce the local variance of the compressed echo envelope. Thus, to counter the reduction in variance introduced by low-pass filtering, an additional parameter, ϕ, was introduced. If the low-pass filter characteristics are known, then it should be possible to determine the optimal ϕ and the need for manual optimization for this parameter will not be needed.

The interesting observation for the design of speckle reduction filters is that the local variance of the image quantifies the extent of speckle formation. Thus, any optimal filter design that uses local variance for speckle quantification, in addition to the unsharp masking filter used here, can be tried for speckle reduction.

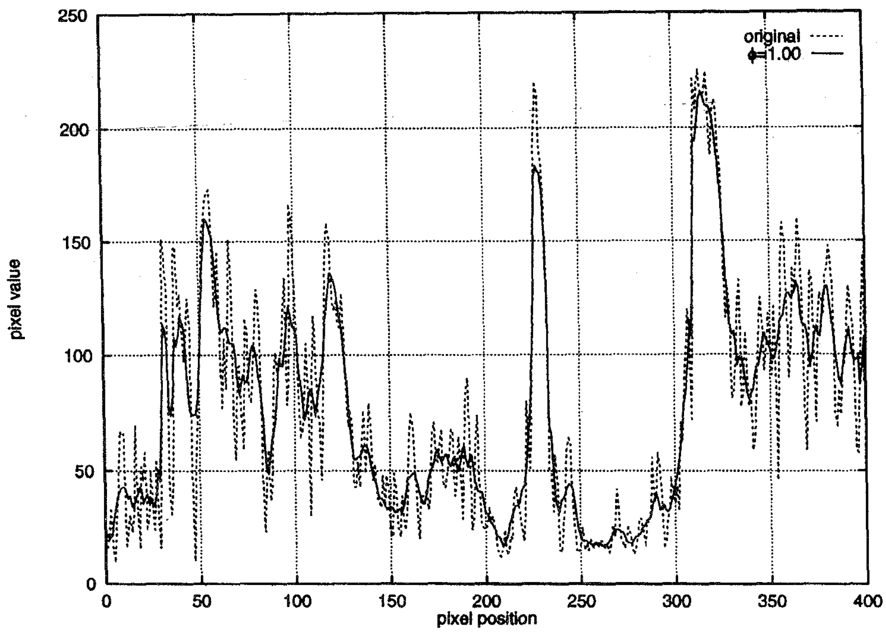


Fig. 10. Comparison of profiles through the abdominal image from Acuson system with its filtered image obtained with $\phi = 1.00$. The line shown is the vertical line passing through the middle of the vessel at the bottom of the image as seen in Fig. 8.

VII. Conclusions

A new statistical model of log-compressed B-scan images allows us to derive a statistic which parameterizes the extent of speckle formation. This parameterization is useful for designing an adaptive speckle reduction filter. The interesting result was that a simple statistic like variance can be used to parameterize the extent of speckle formation. The adaptive filter was then used on numerically log-compressed images of a contrast detail phantom and on images from clinical systems which produce logarithmically compressed B-scan images. The results show that the filter selectively reduces speckle while retaining object boundaries. Thus this filter shows a potential for speckle suppression in clinical ultrasound imaging systems.

Appendix A
Laguerre Polynomial Basis

Any function $f(x)$ defined on positive real basis, $x \in [0, \infty)$, can be written in terms of the Laguerre basis functions as

$$f(x) = \sum_{m=0}^{\infty} c_m \phi_m(x) W(x) \tag{A1}$$

where $\phi_m(x)$ is the Laguerre polynomial of mth order, and $W(x)$ is the weighting function for Laguerre polynomial $[W(x) = \exp(-x)]$, and c_m are the coefficients of the Laguerre series expansion for the function $f(x)$.

Using the orthogonality of the Laguerre polynomials, the coefficients c_m can be written as

$$c_m = \int_0^{\infty} p_I(I_n)\phi_m(I_n)\, dI_n. \tag{A2}$$

The first three Laguerre polynomials are

$$\phi_0(x) = 1 \tag{A3}$$

$$\phi_1(x) = 1 - x \tag{A4}$$

$$\phi_2(x) = 1 - 2x + \frac{x^2}{2}. \tag{A5}$$

Therefore, using the expression for the moments for the K distribution, (3), the first three coefficients can be evaluated as

$$c_0 = 1 \tag{A6}$$

$$c_1 = 0 \tag{A7}$$

$$c_2 = 2\left(1 + \frac{1}{\alpha}\right). \tag{A8}$$

Appendix B
Moments of the Log Compressed K Distributed Data

A νth order moment is given by

$$m_\nu = \int_{-\infty}^{\infty} U^\nu p_U(U)\, dU. \tag{B1}$$

Therefore, using $V = \exp[(U - G_I)/D_I]$, the first two moments can be written as

$$m_1 = \int_0^{\infty} (D_I \ln V + G_I)e^{-V} \cdot \left[\left(1 + \frac{1}{\alpha}\right) - \frac{2}{\alpha}V + \frac{1}{2\alpha}V^2\right] dV \tag{B2}$$

$$m_2 = \int_0^{\infty} (D_I \ln V + G_I)^2 e^{-V} \cdot \left[\left(1 + \frac{1}{\alpha}\right) - \frac{2}{\alpha}V + \frac{1}{2\alpha}V^2\right] dV. \tag{B3}$$

Using the definite integrals for combinations of exponentials and logarithms [27], these two moments can be derived to be

$$m_1 = G_I - D_I\left(\gamma + \frac{1}{2\alpha}\right) \tag{B4}$$

$$m_2 = D_I^2\left[\left(\frac{\pi^2}{6} + \gamma^2\right)\left(1 + \frac{1}{\alpha}\right) + \frac{\Gamma''(3)}{2\alpha} - \frac{2}{\alpha}\Gamma''(2)\right] + G_I^2 - D_I G_I\left(2\gamma + \frac{1}{\alpha}\right) \tag{B5}$$

where γ is the Euler's constant ($\approx$0.5772), and, $\Gamma''(x)$ is the second derivative of the Gamma function.

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