École Nationale Supérieure d'Informatique pour l'Industrie et l'Entreprise

ENSIIE - Évry

SABR Model

Group Project

BERETE Salifou, CHHEANG Vinha

Nom Professeur: Stephane Kamta

Interest Rate

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Contents

1 Swaptions

A swaption is an option to enter into an interest rate swap. The time distance between ”today” and the option’s exercise date is called expiry, while the time distance between the exercise date and the swap’s maturity is called tenor

2 The SABR Model

The SABR (Stochastic Alpha Beta Rho) model (Hagan et al. 2002) is one of the simplest possible generalizations of the single-FRA rate Black model with stochastic volatility. Choosing the simplest reasonable process for α now yields the “stochastic $\alpha\beta\rho$ model,” which has become known as the SABR model. In this model, the forward price and volatility are:

$$dF = \alpha F^{\beta} dW_1 \quad F(0) = f \tag{1}$$

$$d\alpha = \nu\alpha dW_2 \quad \alpha(0) = \alpha \tag{2}$$

under the forward measure, where the two processes are correlated by:

$$dW_1 dW_2 = \rho dt \tag{3}$$

2.1 The volatility

To compute the SABR implicit volatilities $\sigma_B(f, K)$:

$$\begin{aligned}\sigma_B(f,K) = &\frac{\alpha}{(fK)^{(1-\beta)/2}[1 + \frac{(1-\beta)^2}{24} log^2(f/K) + \frac{(1-\beta)^4}{1920} log^4(f/K)]}(\frac{z}{x(z)})\\ &(1 + [\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(fK)^{1-\beta}} + \frac{1}{4}\frac{\rho\beta\alpha\nu}{(fK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24}\nu^2]t_{ex})\end{aligned} \tag{4}$$

Where :

$$z = \frac{\nu}{\alpha}(fK)^{(1-\beta)/2} log(f/K) \tag{5}$$

And x(z): is defined by

$$x(z) = log(\frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho}) \tag{6}$$

For the special case of at-the-money (AMT) options, options struck at $K = f$, this formula reduces to

$$\sigma_{ATM} = \frac{\alpha}{f^{(1-\beta)}}[1 + \frac{(1-\beta)^2}{24}\frac{\alpha^2}{f^{2(1-\beta)}} + \frac{1}{4}\frac{\rho\beta\alpha\nu}{f^{(1-\beta)}} + \frac{2-3\rho^2}{24}\nu^2]t_{ex} \tag{7}$$

2.2 Calibration of parameters

To calibrate the model with respect to market data the following function is minimized:

$$min\sqrt{\sum_{i=-x}^{x}(\sigma_{SABR,i}-\sigma_{MKT,i})^2}$$

$$s.t: \alpha > 0,\ 0 \leq \beta \leq 1,\ -1 \leq \rho \leq 1,\ \nu > 0$$

i is the strike index, so the sum is computed along the swaption's "smile".

3 Application

3.1 Loading Data set

We will representation data by using "pd.read_excel('market_data.xlsx',skiprows=0)", we get the data frame:

	Unnamed: 0	Unnamed: 1	Unnamed: 2	Market Volatilities for strike spreads in bps:	Unnamed: 4	Unnamed: 5	Unnamed: 6	Unnamed: 7	Unnamed: 8	Unnamed: 9	Unnamed: 10	Unnamed: 11
0	Tenor	Expiry	Fwd	-150.0000	-100.0000	-50.0000	-25.0000	0.0000	25.0000	50.0000	100.0000	150.0000
1	2	0.25	0.010764	0.0000	1.0470	0.4812	0.4327	0.4268	0.4148	0.4253	0.4322	0.4495
2	2	0.5	0.011099	0.0000	0.9647	0.5079	0.4637	0.4477	0.4390	0.4377	0.4452	0.4576
3	2	0.75	0.011602	0.0000	0.8253	0.5033	0.4648	0.4494	0.4387	0.4348	0.4375	0.4463
4	2	1	0.012194	0.0000	0.6796	0.4788	0.4474	0.4501	0.4435	0.4478	0.4611	0.4754
5	2	2	0.016196	0.0000	0.9119	0.5417	0.4628	0.4529	0.4461	0.4386	0.4387	0.4442
6	2	5	0.028436	0.4040	0.3541	0.3218	0.3107	0.3048	0.2975	0.2923	0.2873	0.2870
7	2	10	0.033873	0.3026	0.2725	0.2510	0.2422	0.2343	0.2279	0.2228	0.2161	0.2128
8	5	0.25	0.016017	1.1870	0.6027	0.4655	0.4278	0.4030	0.3879	0.3789	0.3710	0.3725
9	5	0.5	0.016802	0.9568	0.5800	0.4661	0.4339	0.4125	0.3969	0.3888	0.3801	0.3785
10	5	0.75	0.017682	0.8325	0.5562	0.4578	0.4288	0.4078	0.3914	0.3821	0.3719	0.3692

Figure 1: Loading data set

We will apply the SABR model into the Data Set "Market_data". To be find the volatility that respect to:

- α, β, ρ, ν are the parameters
- F : is the forward rate
- K : is the strike of the option
- expiry
- MKT: is the market volatility
- i: is the index for the tenor/expiry swaption
- j: is the index for the moneyness of the option

3.2 Values of variables

By using Python to be loading data set,we get the data of some variables to be performance in SABR model such that F, K, expity, and MKT.

3.3 Parameter α, β, ρ, ν

3.3.1 Function

We will created the function name: "objfunc(par,F,K,time,MKT)" in python file. This function will be contain the values of :

$$\sqrt{\sum_{i=-x}^{x} (\sigma_{SABR,i} - \sigma_{MKT,i})^2}$$

After that, we was created the function name "calibration(starting_par,F,K,time,MKT)" that will be able to minimized the function "objfunc".

3.3.2 methodology

- Through the calibration we want to find the parameters (for each smile) that make model volatilities as close as possible to market volatilities.
- The objective function computes, with arbitrary parameters, the volatilities for a single smile, then sums the squares of all market differences. Its square root is the final quantity that we want to minimize.
- Notice that volatility differences for missing market data are set as zero, so that the minimization is not affected.
- For a specification without the shift, negative strikes can be neglected setting the differences zero as before.
- we used the method "SLSQP" for constrained minimization of multivariate scalar functions, where the objective function is minimized with starting values for the parameters. The bounds for the parameters are those specified before in the model.

3.3.3 Result

	tenor	expiry	alpha	beta	rho	nu	Forward
0	2y	0.25	0.005869	0.000000	-0.617576	1.430.250	0.010764
1	2y	0.5	0.005797	0.000000	-0.441660	1.171.850	0.011099
2	2y	0.75	0.005580	0.000000	-0.182584	0.899599	0.011602
3	2y	1	0.005923	0.024513	0.180814	0.810425	0.012194
4	2y	2	0.006057	0.000000	0.009768	0.997566	0.016196

Figure 2: Loading data result

3.4 Volatility

After we get the values of each variable such that F,K,expiries,MKT from data set and the parameters α, β, ρ, ν from minimized the function:

$$min\sqrt{\sum_{i=-x}^{x}(\sigma_{SABR,i} - \sigma_{MKT,i})^2}$$

$$s.t : \alpha > 0,\ 0 \leq \beta \leq 1,\ -1 \leq \rho \leq 1,\ \nu > 0$$

Therefore, we will be able to calculated the volatility of each time and the strike: K by apply on the function name: "SABR_vol_matrix(alpha,beta,rho,nu,F,K,expiries,MKT)" in Python file.

3.5 Representation the result by graphic

On this section, we plot the graph of the volatily with stike and forward rate when for differente values of beta.

3.5.1 Volatility and Strike: K

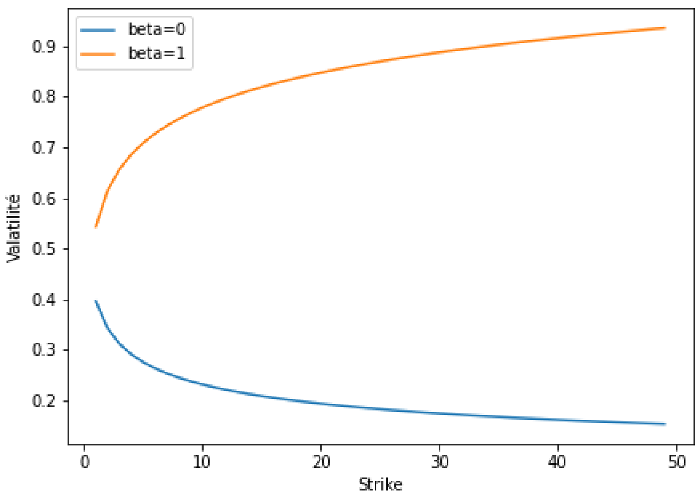

Figure 3: Volatility-K

3.5.2 Volatility and Forward Rate: F

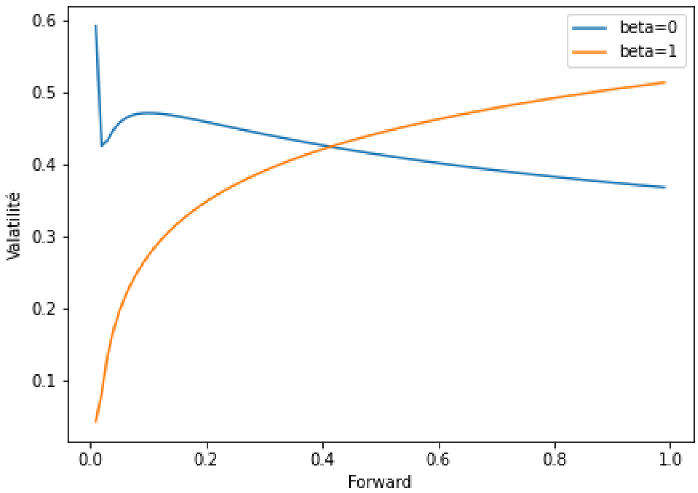

Figure 4: Volatility-F

3.5.3 Volatility and Forward Rate: F

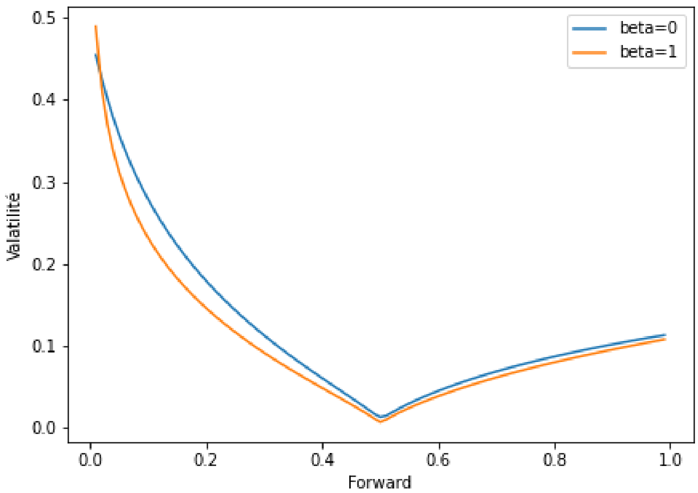

Figure 5: Volatility-F

At the end we show that we get approximatively the same graph if we are in at the money condition for differente values of beta.