Concepts

Lecture

1. **Introduction**

   1.1. Image restoration: blur and noise

   1.2. Definitions of image reconstruction, image denoising, image restoration, and image recovery

2. **Discrete measurement model**

   2.1. LSI (linear-shift invariant) systems and convolution operations

   2.2. Define blur using point spread function (PSF)

   2.3. Define noise using additive noise model

3. **Continuous-discrete model**

   3.1. Minimum norm estimate

   3.2. Object models

4. **Matrix-vector representation of convolution**

   4.1. Linear convolution, non-periodic images, and DSFT

5. **1D matrix-vector representation**

   5.1. Representation of convolution operation using matrix-vector notation: Ax + e = y

   5.2. End-conditions: zero end condition, extended end condition, periodic end condition

6. **2D matrix-vector representation**

   6.1. Lexicographic ordering

   6.2. Zero end condition: representing the matrix A as block matrix

   6.3. Shift-invariant 2D blur: block Toeplitz with Toeplitz blocks (BTTB)

   6.4. Seperable 2D blur: Kronecker product & BTTB

   6.5. 2D periodic end conditions: block circulant with circulant blocks (BCCB)

7. **Circulant analysis of shift-variant blur**
    7.1. Circular convolution, periodic images, and DFT
    7.2. Circulant analysis in 1D
    7.3. Circulant analysis in 2D
8. **Simple image restoration problems**
    8.1. Deconvolution: inverse filter & multiplication in Fourier space
    8.2 Matrix inverse solution
        8.2.1. Hadamard’s well-posedness definition
        8.2.2. Different cases of inverse problems and their optimization problem definitions
9. **Sparsity models**
    9.1. Definition of sparsity
    9.2. Snythesis formulations:
        9.2.1. Optimization problem: Data fidelity term and regularizers
    9.3. Synthesis formulations: regularized
        9.3.1. Optimization problem: Data fidelity term and regularizers
    9.4. Analysis formulations with discriminative models
        9.4.1. Optimization problem: Data fidelity term and regularizers

Code exercise

1. **Example for blurry, noisy image**
    1.1. Blur: Add blur to an image using PSF
    1.2. Noise: Add noise to an image from Gaussian distribution
2. **Deconvolution solution with inverse PSF**
    2.1. Deconvolution for blurry image
    2.2. Deconvolution for blurry & noisy image

3. **1-D matrix-vector representation**
    3.1. Define transformation matrix from 1D PSF
    3.2. Examples with end conditions as in figure 1.4.2
4. **2-D matrix vector representation**
    4.1. Lexicographic ordering
    4.2. Define transformation matrix from 2D PSF
    4.3. Examples with end conditions as in figure 1.4.3
5. **Matrix inverse solution**
    5.1. Solve 1D examples from part 3
    5.2. Solve 2D examples from part 4
6. **Sparsity models**
    6.1. Synthesis formulations (generative models)
        6.1.1. DCT for synthesis model
        6.1.2. Iterative hard thresholding for sparsity constraint
    6.2. Synthesis formulations: regularized
    6.3. Analysis formulations (discriminative models)
        6.3.1. Wavelet transform for analysis model