Interference & Noise Robust Image Reconstruction for Magnetic Particle Imaging

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Background

- MPI
- X-Space Reconstruction
- Partial field-of-view Center Imaging (PCI)
    - PCI
    - Lumped PCI

X-Space Reconstruction

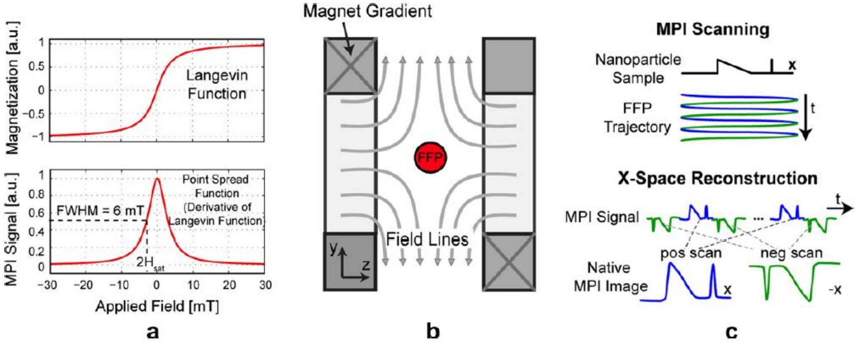

Fig. 1: Principles of x-space MPI imaging. (a) SPIO magnetization characterized by a Langevin function, and Point Spread Function (PSF). (b) Selection fields created by permanent magnets, and Field Free Point (FFP). (c) FFP is moved rapidly with excitation field. In x-space reconstruction, MPI signal is gridded to the instantaneous position of the FFP. [1]

- **Speed Compensation**

$$\hat{\rho}(x_s(t)) = \rho(x) * h(x)|_{x = x_s(t)}$$

$$s(t) = \gamma \dot{x_s}(t)\hat{\rho}(x_s(t))$$

$$\hat{\rho}(x_s(t)) = \frac{s(t)}{\gamma \dot{x_s}(t)}$$

P. W. Goodwill and S. M. Conolly, “The x-space formulation of the magnetic particle imaging process: 1-D signal, resolution, bandwidth,SNR, SAR, and magnetostimulation,” *IEEE Transactions on Medical Imaging*, vol. 29, no. 11, pp. 1851– 1859, 2010.

- **Fourier Series of MPI Signal**

$$s(t) = \sum_{n=1}^{\infty} S_n \sin(2\pi n f_0 t)$$

- **Image Basis of MPI Image**

$$\hat{\rho}(x) = \left.\frac{s(t)}{\gamma \dot{x}_s(t)}\right|_{t=\frac{1}{(2\pi f_0)}\arccos\left(\frac{2x}{W}\right)}$$

$$= \sum_{n=1}^{\infty} S_n \left.\frac{\sin(2\pi n f_0 t)}{\sin(2\pi f_0 t)}\right| t = \frac{1}{(2\pi f_0)}\arccos\left(\frac{2x}{W}\right)$$

$$= \sum_{n=1}^{\infty} S_n \frac{\sin\left(\pi \arccos\left(\frac{2x}{W}\right)\right)}{\sin\left(\arccos\left(\frac{2x}{W}\right)\right)}$$

$$= \sum_{n=1}^{\infty} S_n U_{n-1}\left(\frac{2x}{W}\right)$$

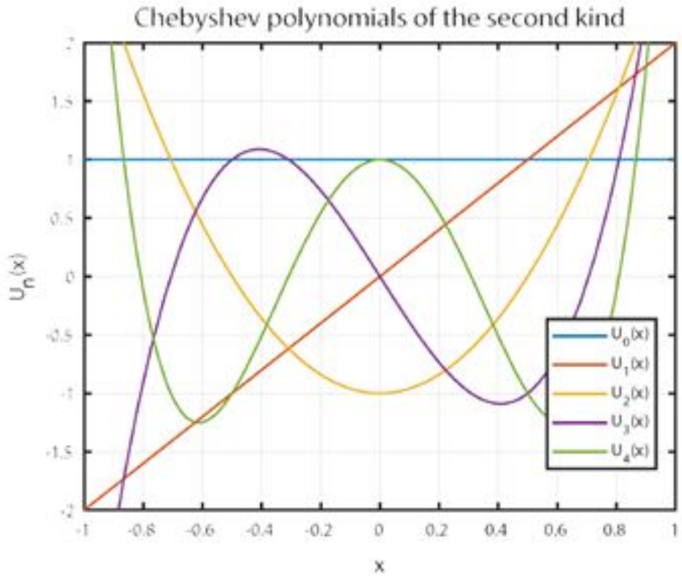

Fig. 2: Chebyshev Polynomials of the Second Kind. [2]

K. Lu, P. W. Goodwill, E. U. Saritas, B. Zheng, and S. M. Conolly, “Linearity and shift invariance for quantitative magnetic particle imaging,” *IEEE Transactions on Medical Imaging,* vol. 32, no. 9, pp. 1565–1575, 2013.

- DC loss in MPI Image

$$\hat{\rho}_{lost}(x) = \alpha S_1 U_0 \left(\frac{2x}{W}\right) = \alpha S_1$$

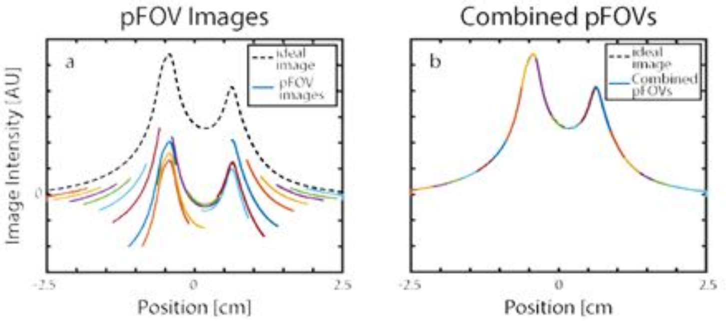

Fig. 3: Standard x-space reconstruction. (a) pFOV images having different DC losses due to high-pass filtering (b) Combining pFOV images by enforcing the smoothness and non-negativity. [2]

➔ FFP is moved to scan field-of-view (FOV). Due to safety limitations, FOV is divided into partial FOVs (pFOV).

➔ pFOVs are combined to obtain FOV after speed compensation in x-space reconstruction.

➔ First harmonic signal is lost due to high-pass filtering MPI signal to get rid of direct feedthrough signal.

K. Lu, P. W. Goodwill, E. U. Saritas, B. Zheng, and S. M. Conolly, “Linearity and shift invariance for quantitative magnetic particle imaging,” *IEEE Transactions on Medical Imaging,* vol. 32, no. 9, pp. 1565–1575, 2013.

Partial FOV Center Imaging (PCI)

- Partial FOV Centering Imaging (PCI) is a novel robust x-space image reconstruction for MPI developed with the motivation of eliminating non-ideal effects on signal. There are two proposed methods: PCI and Lumped PCI.

- PCI and Lumped-PCI methods offer a trade-off between interference and noise robustness. PCI is robust against interference and sensitive to noise, while Lumped PCI is robust against noise and sensitive to interference.

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

PCI

➔ PCI samples MPI signal at the centers of pFOVs and locates them closely by overlapping pFOVs.

$$x_s(t)|_{t=t_{oj}} = x_{0j}, \text{ for } j = 1, \dots, N$$

$$\begin{aligned}\hat{\rho}_0(x_{oj}) = s(t)|_{t = t_{oj}} \\ = \alpha \dot{x_s}(t_{0j})\hat{\rho}(x_{0j}) \\ = \beta_0\hat{\rho}(x_{0j})\ for\ j = 1, \dots, N\end{aligned}$$

➔ Then, applies deconvolution to recover DC loss.

$$\tilde{\rho}_0(x) = \hat{\rho}(x) * h_0(x)$$

$$h_0(x) = \beta_0\left(\delta(x) - \frac{4}{\pi W}\sqrt{1 - \left(\frac{2x}{W}\right)^2}\right)$$

$$\hat{\rho}(x) = \tilde{\rho}_0(x) *^{-1} h_0(x)$$

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

Lumped PCI

➔ Samples at each position on pFOVs are assigned to their centers.

$$x_s(t)\big|_{t\,=\,t_{kj}} = x_{kj},\, for\ j = 1, \ldots, N\ and\ k = \ -K, \ldots, K$$

$$\hat{\rho}_k(x_{oj}) = \beta_k \hat{\rho}(x_{kj})$$

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

Lumped PCI

➔ MPI image can be obtained from each raw image.

$$h_k(x) = \beta_k \left(\delta\left(x - (x_{0j} - x_{kj})\right) - \frac{4}{\pi W}\sqrt{1 - \left(\frac{2x}{W}\right)^2} \right)$$

$$\hat{\rho}(x) = \tilde{\rho}_k(x) *^{-1} h_k(x)$$

➔ Raw Lumped-PCI image is the summation of all raw images for each position.

$$\tilde{\rho}_{lum}(x) = \sum_{k=-K}^{K} \tilde{\rho}_k(x)$$

$$h_{lum}(x) = \sum_{k=-K}^{K} h_k(x)$$

$$\hat{\rho}(x) = \tilde{\rho}_{lum}(x) *^{-1} h_{lum}(x)$$

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

Deconvolution Kernels

- **Modified Lumped PCI Equations**

$$\tilde{\rho}_{lum}(x) = \sum_{k=-K}^{K} \alpha_k \tilde{\rho}_k(x)$$

$$\begin{aligned} h_{lum}(x) &= \sum_{k=-K}^{K} \alpha_k h_k(x) \\ &= \sum_{k=-K}^{K} \alpha_k \beta_k \delta\left(x - \left(x_{0j} - x_{kj}\right)\right) \end{aligned}$$

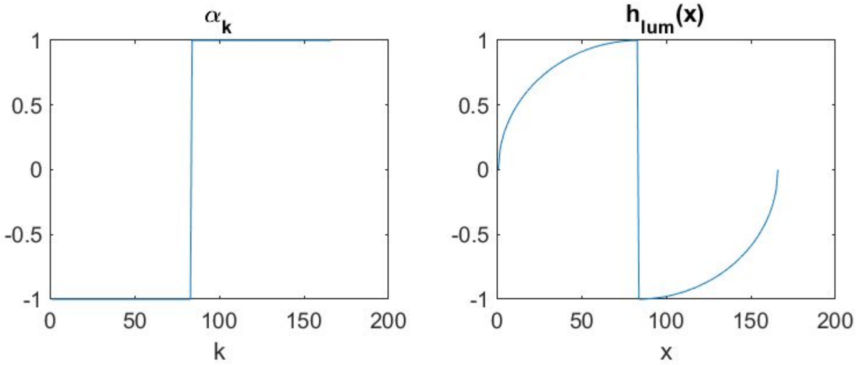

Fig.4 : Deconvolution Kernels for Lumped-PCI

Results

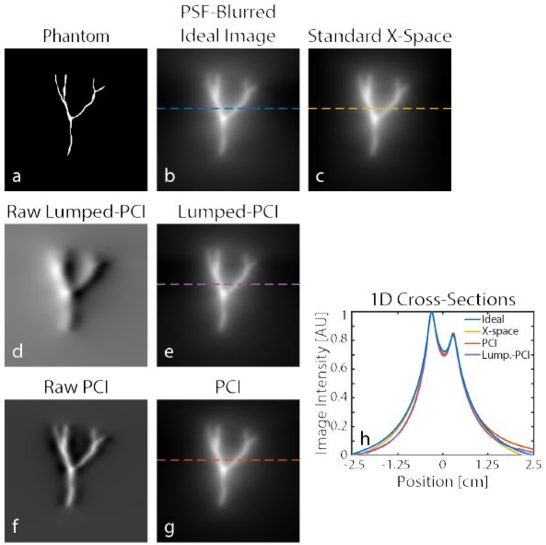

Fig. 5: Image Results for Standard X-Space Reconstruction, Lumped PCI and PCI. [3]

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

Noise

Noise

- **Signal-to-Noise Ratio (SNR) Definition in Simulations**

$$SNR = 20\log_{10}\left(\frac{\max_t |s(t)|}{\sigma}\right)$$

➔ White Gaussian noise

➔ Noise robustness difference between PCI and Lumped-PCI

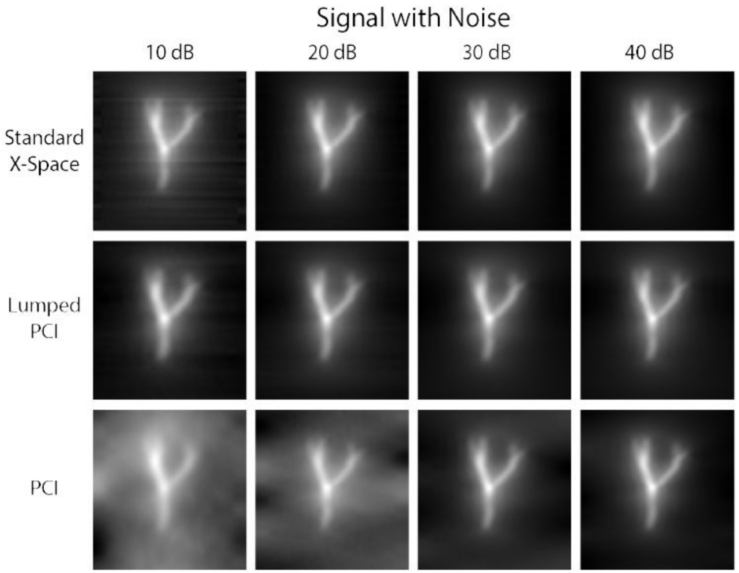

Fig. 6: Comparison of Standard X-Space Reconstruction, Lumped PCI and PCI Results at Different SNR Levels. [3]

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

Noise Analysis

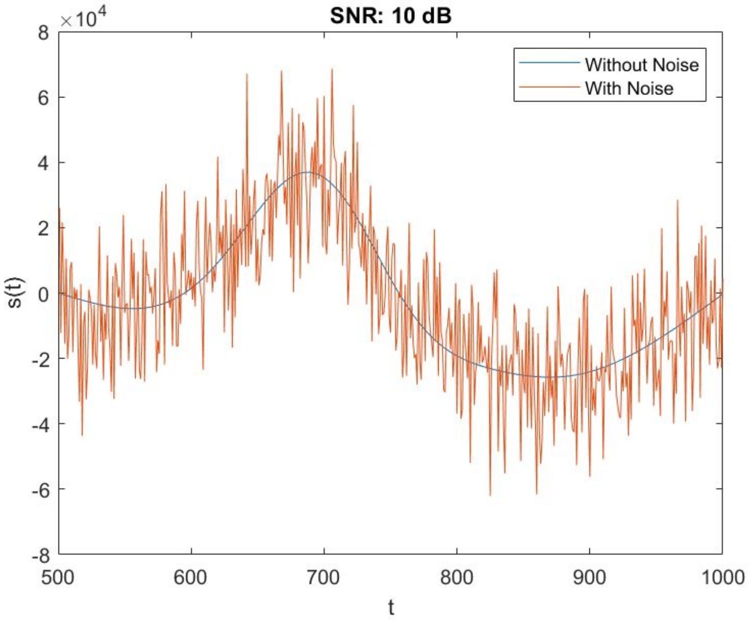

Fig. 7: Noise Profile in Middle Line (#26) and Middle pFOV (#291)

- **Noise**

$$s(t) = s_{ideal}(t) + n(t)$$

SNR Optimization for Lumped PCI

- Weighting each sample before raw Lumped-PCI creation such that SNR of raw Lumped-PCI image will be optimized.

$$\tilde{\rho}_{lum,ideal} = \sum_{k=-K}^{K} \beta_k \tilde{\rho}_k \text{, where } k = -K, \ldots, K$$

$$\begin{aligned}
\tilde{\rho}_{lum} &= \sum_{k=-K}^{K} (\beta_k \tilde{\rho}_k + n_k) \\
&= \sum_{k=-K}^{K} c_k (\beta_k \tilde{\rho}_k + n_k) \\
&= \sum_{k=-K}^{K} c_k \beta_k \tilde{\rho}_k + \sum_{k=-K}^{K} c_k n_k
\end{aligned}$$

$$SNR = 20 \log_{10} \left(\frac{\left| \sum_{k=-K}^{K} c_k \beta_k \tilde{\rho}_k \right|}{\sigma'} \right)$$

$$\sigma' = \sqrt{\sum_{k=-K}^{K} {c_k}^2 \sigma^2} = \sigma \sqrt{\sum_{k=-K}^{K} {c_k}^2}$$

SNR Optimization for Lumped PCI

$$\frac{\partial}{\partial c_j}\left(\frac{\sum_{k=-K}^{K} c_k\beta_k\tilde{\rho}_k}{\sigma\sqrt{\sum_{k=-K}^{K} {c_k}^2}}\right) = \frac{\sigma\beta_j\tilde{\rho}_j\sqrt{\sum_{k=-K}^{K} {c_k}^2} - \sum_{k=-K}^{K} c_k\beta_k\tilde{\rho}_k \cdot \left(\frac{1}{2\sqrt{\sum_{k=-K}^{K} {c_k}^2}}\, 2\, c_j\, \sigma\right)}{\sigma^2 \sum_{k=-K}^{K} {c_k}^2}$$

$$= \beta_j\tilde{\rho}_j\sqrt{\sum_{k=-K}^{K} {c_k}^2} - \sum_{k=-K}^{K} c_k\beta_k\tilde{\rho}_k \cdot \frac{c_j}{\sqrt{\sum_{k=-K}^{K} {c_k}^2}}$$

$$= \beta_j\tilde{\rho}_j \sum_{k=-K}^{K} {c_k}^2 - \left(\sum_{k=-K}^{K} c_k\beta_k\tilde{\rho}_k\right) . c_j = 0$$

SNR Optimization for Lumped PCI

- Signal samples at each gridding position should be weighted by itself.
- For conserving Linear Shift-Invariance (LSI) property of Lumped-PCI, PSF blurred image is assumed to be uniform.

$$c_j = \beta_j \tilde{\rho}_j \frac{\sum_{k=-K}^{K} {c_k}^2}{\sum_{k=-K}^{K} c_k \beta_k \tilde{\rho}_k}$$

$$\mu = \frac{\sum_{k=-K}^{K} {c_k}^2}{\sum_{k=-K}^{K} c_k \beta_k \tilde{\rho}_k}$$

$$c_j = \mu \beta_j \tilde{\rho}_j$$

Results

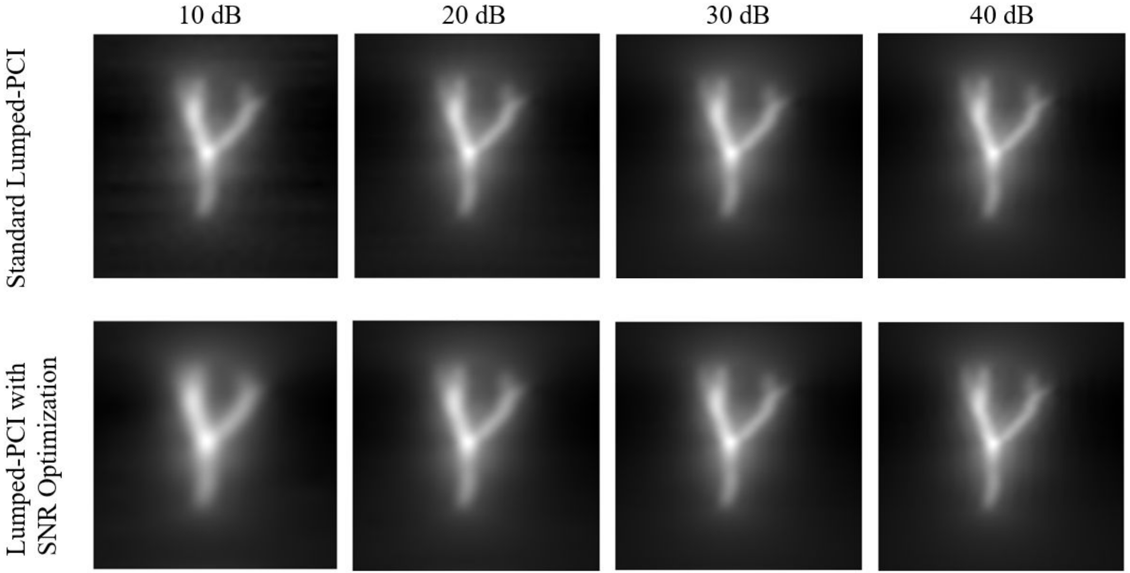

Fig. 8: Results of Lumped-PCI with SNR Optimization

Interference

Interference

- **Signal-to-Interference Ratio (SIR) Definition in Simulations**

$$SIR = 20\log_{10}\left(\frac{\max_f |S_n(f)|}{\gamma_n}\right)$$

$$S_n(f) = \begin{cases} S(f), & \left(n-\frac{1}{2}\right) f_0 < f < \left(n+\frac{1}{2}\right) f_0 \\ 0, & otherwise \end{cases}$$

$$f = nf_0$$

➔ Interference robustness difference between PCI and Lumped-PCI

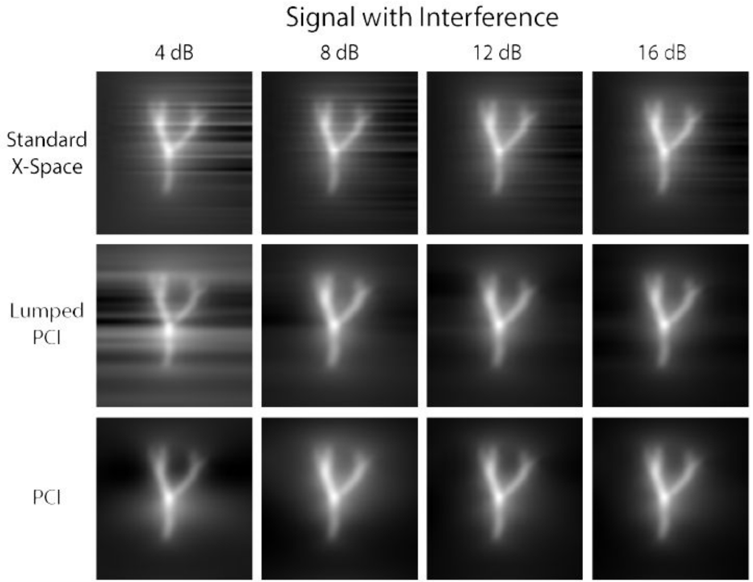

Fig. 9: Comparison of Standard X-Space Reconstruction, Lumped PCI and PCI Results at Different SIR Levels. [3]

S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

Interference Analysis

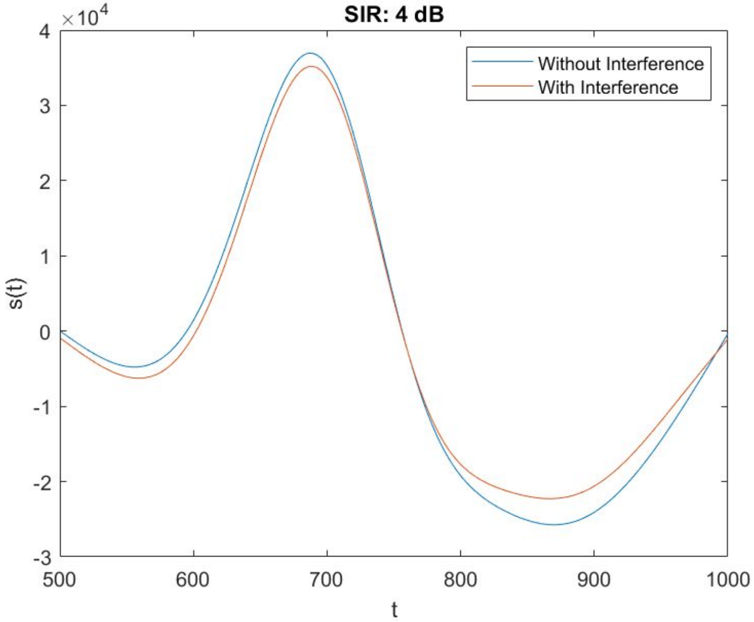

Fig. 10: Interference Profile in Middle Line (#26) and Middle pFOV (#291)

- **Harmonic Interference**

$$s(t) = s_{ideal}(t) + i(t)$$

$$s(t) = \sum_n S_n \sin(2\pi f_n t) + \sum_n I_n \sin(2\pi f_n t + \phi_n)$$

$$where\ I_n \sim U(0, \gamma_n)\ and\ \phi_n \sim N\left(0, \left(\frac{\pi}{10}\right)^2\right)$$

Interferencing Signal Decomposition

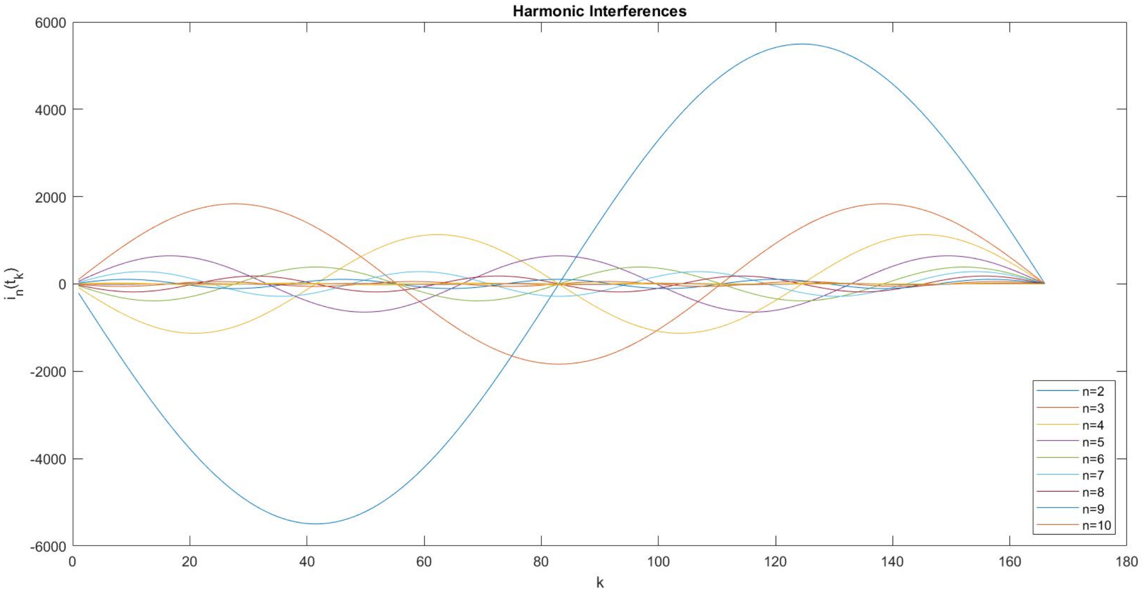

Fig. 11: Expectances of Harmonic Interferences for Middle Line.

Interferencing Signal

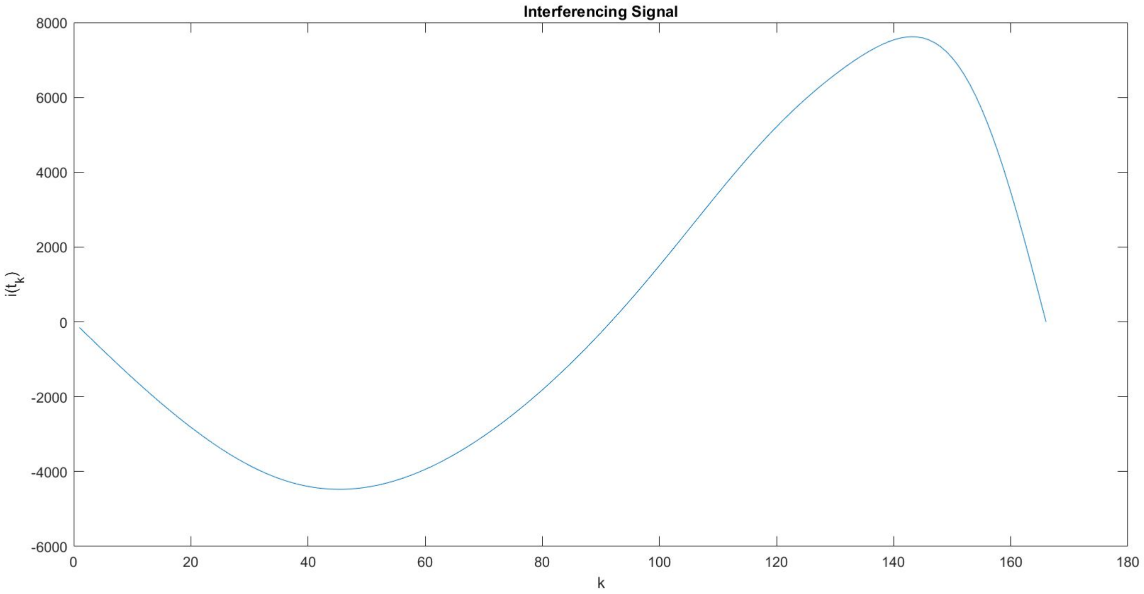

Fig. 12: Total Expectance of Harmonic Interferences for Middle Line

Interferencing Signal Decomposition

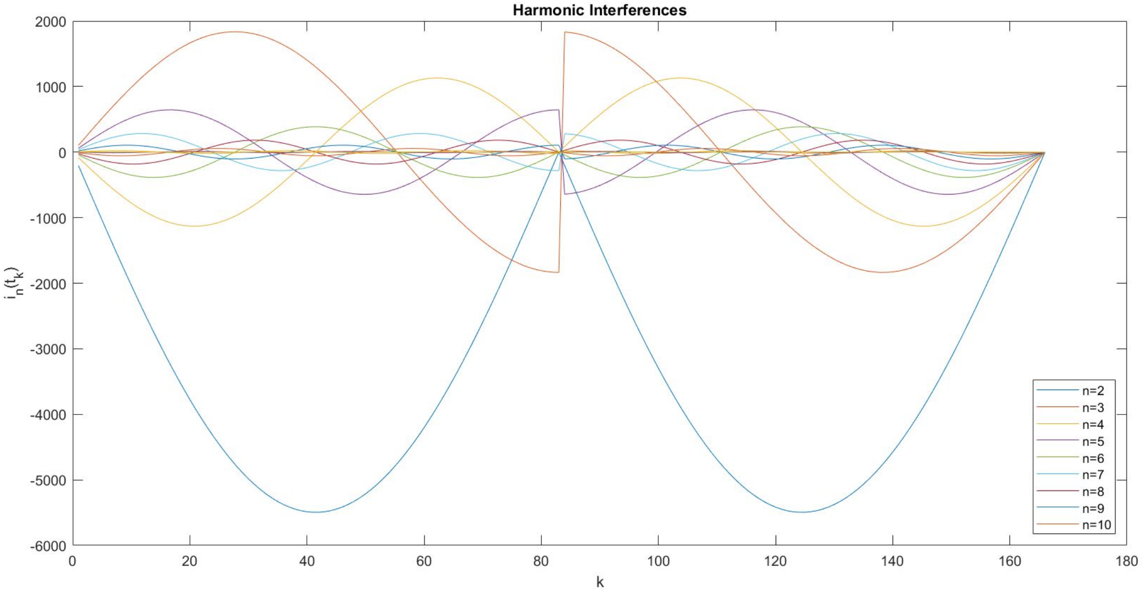

Fig. 13: Expectances of Harmonic Interferences for Middle line
(Kernel signs are included.)

Interferencing Signal

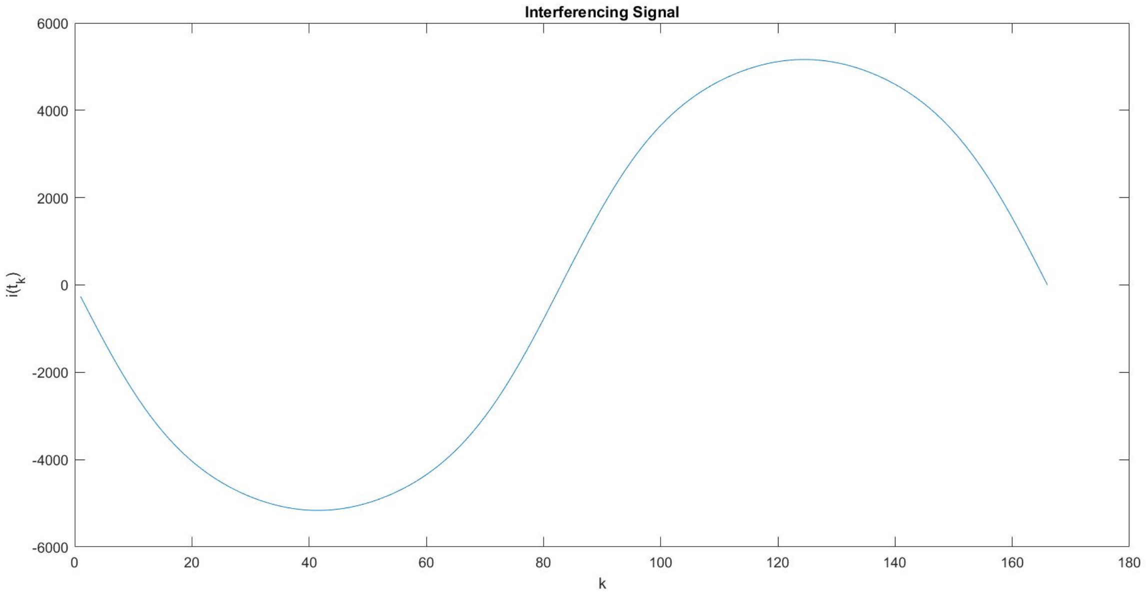

Fig. 14: Total Expectance of Harmonic Interferences for Middle Line. (Kernel signs are included.)

Results

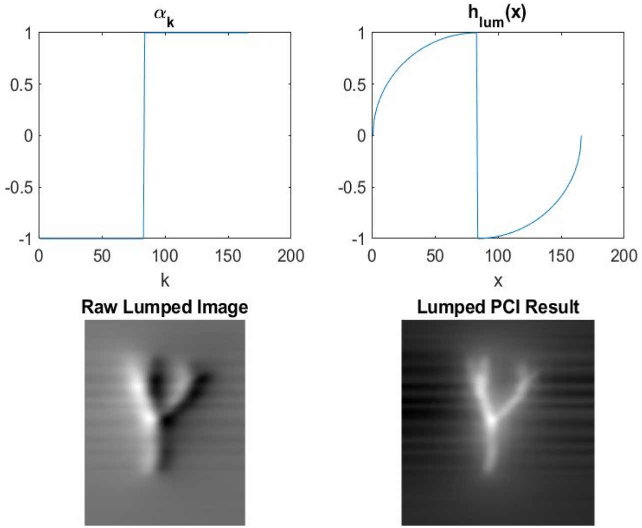

Fig. 15: Lumped PCI result at 4 dB SIR level. Kernel is the standard kernel.

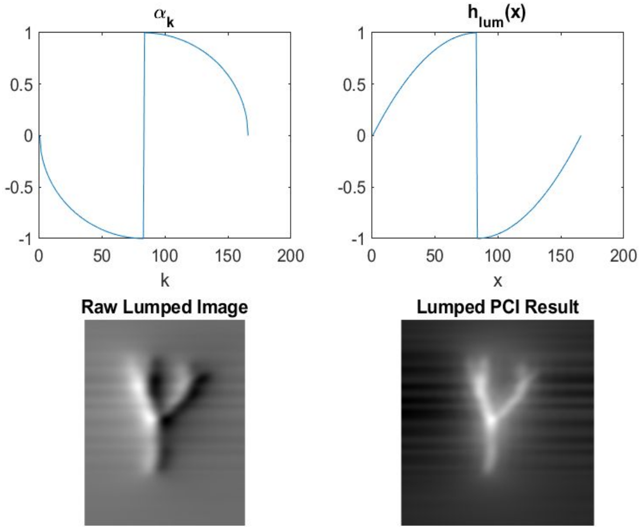

Fig. 16: Lumped PCI result at 4 dB SIR level. Kernel is the kernel used in SNR optimization for Lumped PCI.

Results

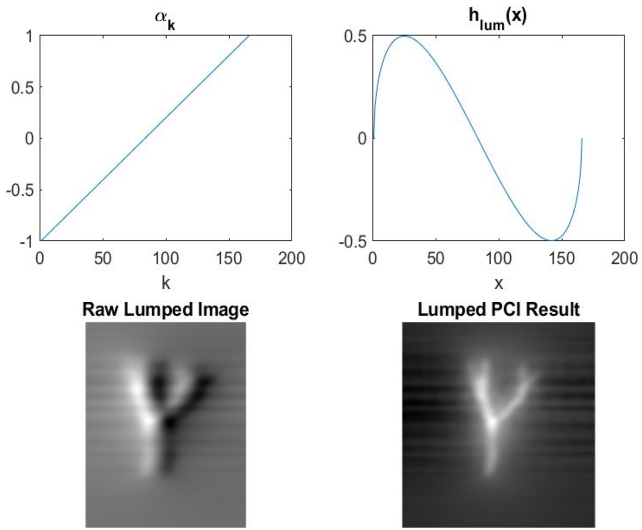

Fig. 17: Lumped PCI result at 4 dB SIR level. Kernel is used for emphasizing edges.

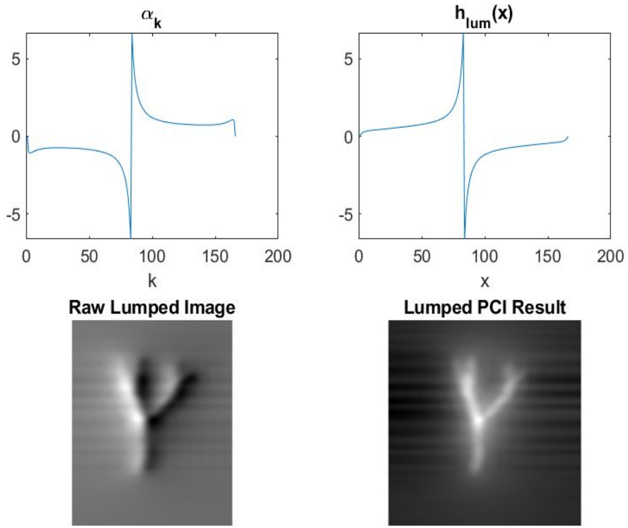

Fig. 18: Lumped PCI result at 4 dB SIR level. Kernel is used for suppressing samples with maximum interference.

Discussion

Discussion

- ➔ Noise robustness and interference robustness were studied separately.
- ➔ Noise robustness of Lumped PCI was increased by SNR optimization technique.
- ➔ Interference profile for Lumped PCI was analysed.
- ➔ Interference robustness of Lumped PCI could not be increased yet.

Future Work

Future Work

New Approaches for Interference Robustness of Lumped-PCI

- ➔ Weighting strategy for summing harmonic interference expectations to be zero. Kernel has the property of odd function. Interference at certain harmonics are already summed up to be zero.
- ➔ Another approach may be using PCI image for Lumped-PCI to boost interference robustness.

References

[1] P.W. Goodwill, E.U. Saritas, L.R. Croft, T.N. Kim, K.M. Krishnan, D.V. Schaffer, S.M. Conolly, X-Space MPI: magnetic nanoparticles for safe medical imaging, Adv. Mater. 24 (2012) 3870–3877.

[2] S. Kurt, “Rapid and Robust Image Reconstruction for magnetic Particle imaging,” M.S. Thesis, Grad. Sch. of Engineering and Science, Bilkent University, 2020.

[3] S. Kurt, Y. Muslu and E. U. Saritas, "Partial FOV Center Imaging (PCI): A Robust X- Space Image Reconstruction for Magnetic Particle Imaging," *in IEEE Transactions on Medical Imaging,* vol. 39, no. 11, pp. 3441-3450, Nov. 2020. doi: 10.1109/TMI.2020.2995410

[4] P. W. Goodwill and S. M. Conolly, “The x-space formulation of the magnetic particle imaging process: 1-D signal, resolution, bandwidth, SNR, SAR, and magnetostimulation,” *IEEE Transactions on Medical Imaging*, vol. 29, no. 11, pp. 1851– 1859, 2010.

[5] K. Lu, P. W. Goodwill, E. U. Saritas, B. Zheng, and S. M. Conolly, “Linearity and shift invariance for quantitative magnetic particle imaging,” *IEEE Transactions on Medical Imaging,* vol. 32, no. 9, pp. 1565–1575, 2013.