BILKENT UNIVERSITY ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EEE - 321: Signals and Systems
Lab Assignment 1

Please carefully study this assingment before coming to the laboratory. You may begin working on it or even complete it if you wish, but you do not have to. There will be short quizzes both at the beginning and end of the lab session; these may contain conceptual, analytical, and Matlab-based questions. Within one week, complete the assignment in the form of a report and turn it in to the assistant. Some of the exercises will be performed by hand and others by using Matlab. What you should include in your report is indicated within the exercises. Provide codes for all parts except for Part 1 in the lab report.

Part 1

This part contains several basic Matlab exercises. The goal is to briefly remind you about the vector-based programming logic of Matlab, so that you can use it in the most efficient manner during our labs. Try all the exercises below in **Matlab command window**, and answer the related questions. Write down your answers in your report. You do not need to provide any code for the exercises in this part.

- a) First type **a=[2 -3.7 5*rand exp(1i*2*pi*3)]**, then type **a=[2; -3.7 ;5*rand; exp(1i*2*pi*3)]**. What is the difference?
- b) In order to do this item, you need to learn how to measure a computation time of a Matlab code. Learn how *tic* and *toc* commands work. You can use *help* command in Matlab for this purpose. Then measure the time difference between generation of the variables **a=[2 -3.7 5*rand exp(1i*2*pi*3)]** and **a=[2 -3.7 5*rand exp(1i*2*pi*3)];**. When is it useful to put ";" at the end of a command line?

- c)Type **a=[2 -3.7 5*rand exp(1i*2*pi*3)];** and **b=[5 6 7 8];**. Then type **c=a*b**. What message do you receive? Why do you receive this message?

- d)Repeat part c by typing **c=a.*b**. What is the result? What is the effect of adding the dot in front of the multiplication symbol? If you type **c=b.*a**, does the result change?

- e) Repeat part c but type **b=[5; 6; 7; 8];**. Then type **c=a*b**. What is the result? What has Matlab done now?

- f) Type **a=[2; -3.7 ;5*rand; exp(1i*2pi*3)]** and **b=[5 6 7 8];**. Then type **c=a*b**. What is the result? What has Matlab done now?

- g) Type **a=[0:0.05:5]**. What does such a command do? What is the purpose of 0.05 in this comment? Can we also use the colon operator '**:**' with two numbers only e.g., **a=[1:5]**? If yes, what has Matlab done now?

- h)Then type **a=[0:0.05:5];**. Measure the spent time while generating this variable.

- i) Now generate the variable **a** in part h by using a *for* loop without allocating any memory for **a** before the loop. Again measure the spent time.

- j) Again generate the variable **a** in part i by using a *for* loop, but, in this case, allocate memory for **a** before the loop. You can allocate memory by using *zeros* or *ones* commands. (You can do a web search to learn how to use these commands for memory allocation). Measure the spent time to generate the variable after memory allocation. Now compare the measured time in parts g,h and i. Which method is the most efficient one?

Now you have learned how to generate basic Matlab variables and also you have compared the efficiency of different variable generation methods. From now on, you are expected to write your Matlab codes not only correct but also efficient. Besides these methods, there might be some other cases that make your Matlab code more efficient in this lab and further labs. You can always do a web search or get help from the TAs on how to make your code more efficient.

- k) Type **a=[0:pi/12:2*pi]**;. Then type **b=sin(a);**. Examine the resulting **b**. Notice that as the input argument for the **sin** function, we used the vector **a**. This does not make sense mathematically, because we have to insert a single real valued number to the $\sin(x)$ function. For instance, $\sin(\frac{\pi}{5})$ has a meaning but $\sin([\frac{\pi}{3}\,\frac{\pi}{5}])$ does not in the ordinary sense. But Matlab still returns a result when we insert **a** into the **sin** function. What does Matlab do?

- l) Type **t=[0:0.03:10];**, and type **x=3*cos(2*pi*t);**. Now first type **plot(x)**, then type **plot(x,t)** and **plot(t,x)**. What is the difference? You do not need to include the plots to the report, just provide your answer to this question.

- m) For the above part, type **plot(t,x,'-+')**. What do you observe? Type **plot(t,x,'+')**. What happens?

Type **help plot** in the Matlab command window, and see what else you can do with the **plot** command, which is one of the vital commands of Matlab. Also study the Matlab commands **xlabel**, **ylabel**, **title**, **xlim**, **ylim** and **grid**. These commands are essential for producing professional looking plots in Matlab. Make sure that you can confidently use these commands. As a final exercise to further understand the **plot** command, we will graph the function $x(t) = 2\cos(2\pi t - 3\pi/25)$. For this part, provide your answers to the questions that are asked below. You will also provide a graph. Again, no codes are necessary.

- n) Let **t=[0 0.02 0.04 ... 0.96 0.98 1]**. By now, you know that we can prepare the array **t** with the single-line command **t=[0:0.02:1];**. How many time points are included in **t**?

- o) It is possible to generate **t** in other ways. How would you generate the variable **t** in part o using **linspace** command?

- p) Now compute $x(t)$ over the time grid specified by **t**, and denote the resulting array with **x**. By now, you know that we can obtain **x** with the single-line command **x=2*cos(2*pi*t - 3*pi/25);**.

- q) Type **figure;**. You will see that an empty figure window will be opened. Then type **plot(t,x,'b');**. You will see that your function is plotted, where the color of the curve is blue. Then type **hold on;**. This command will enable you to make further plots within the same figure window while preserving all the old plots.

- r) Now let **t=[0 0.05 0.1 0.15 ... 0.90 0.95 1]** and compute **x=2*cos(2*pi*t - 3*pi/25)** similar to previous part. How many time points do we take this time? Type **plot(t,x,'r');** this time. You will notice that a red curve is added to the old figure window.

- s) Add the same sine signal definition **x=2*cos(2*pi*t - 3*pi/25)** , and plot with the time index **t=[0 0.1 ... 0.9 1]** with a different color.

- t) Add the same sine signal plot with the time index **t=[0 0.2 0.4 0.6 0.8 1]** with a different color. Then, label the x-axis as **time (in seconds)** and y-axis as **Amplitude**. Display the major grid lines by typing **grid on**. Finally, introduce a title for your figure as **Sinusoidal Signal**.

- u) Closely examine the figure that you obtained, perhaps zooming in or out. Which choice of **t** produces the plot which is most likely for the continuous $x(t)$? Why?

- v) How does the **plot** command "fill" the space between data points?

- w) Suppose **t=[1 2 3 4]** and **x=[6 pi -2 2*pi]**. Roughly sketch with your hand the plot that Matlab would generate if we issued the command **plot(t,x)**.

- x) After closing the figure window you used during the previous items, repeat the exercise in item s by using the **stem** command instead of the **plot** command. Do not provide any graphs, but just answer the following question: What is the difference between the **plot** command and **stem** command?

- y) Repeat the exercise in item r, s and t but instead of plotting on the same figure, use different figures using **subplot** command(type **help subplot** in the command window to learn it). Decorate the figure as described in item t but make different titles. What is the benefit of using **subplot** command?

Part 2

In this part, we will experience how different signal waveforms sound. First consider a signal of the form

$$x_1(t) = A_c \cos(2\pi f_0 t + \phi). \tag{1}$$

where A_c is the amplitude, f_0 is the frequency, t is the time,and ϕ is the phase. For this part, assume $A_c = 2$, ϕ =0 and **t=[0:1/8192:1]** .

- a) First examine **sound** and **soundsc** commands. Are both appropriate to listen the discrete version of above signal in Matlab?

- b) Take $f_0 = 220$Hz. Compute $x_1(t)$ and store it in an array named **x1**. Plot **x1** versus **t**. Clearly label the plot and put an appropriate title. Turn on the speakers of your computer. Then type **sound(x1)** or **soundsc(x1)**. Listen to the sound.

- c) Repeat a for $f_0 = 440$Hz, but do not produce any plot.

- d) Repeat a for $f_0 = 880$Hz, but do not produce any plot.

What happens to the pitch of the sound as the frequency increases?

Now, generate a new time vector t, for $\phi = 0$ and $f_0 = 1000$Hz so that you can listen thus tone to 10 seconds. Type **t = [0 : δt : 10]**, where δt is the increment needed to be computed such that at each period of cosine signal will have 200 sample points. What is the total number of time samples in 10 seconds for a 1000Hz tone signal? Name the signal for the new vector as **x2** and listen to **x2** with **sound** command.

- e) For $f_0 = 1000$Hz, compute **x2** with $\phi = \pi/4$. Listen to the sound using **sound** command, but do not produce any plot.

- f) Repeat d for $\phi = \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 2\pi$, but do not produce any plot.

What is the effect of the phase on the sound that hear. Plot three periods of **x2** for all phase values described in items e and f into a single figure. Clearly label the plot and put an appropriate title. Display the major grid lines by typing **grid on**. Use the **legend** command to distinguish the signals.

Now consider a second signal defined as

$$x_3(t) = e^{-at} \cos(2\pi f_0 t). \tag{2}$$

Take $f_0 = 880$ and $a = 7$. Compute $x_3(t)$ and store it in an array named **x3**. Write a single line code for computing **x2**(no **for** loops). Provide this code to your report. Make a plot of **x3** versus **t**, and listen to **x3** by the **sound** command.Clearly label the plot and put an appropriate title. Compare your plot and what you hear to the results you obtained for $x_1(t)$ when f_0 is 880. What is the effect of adding the e^{-at} term to the sound that you hear? Which one of $x_1(t)$ and $x_3(t)$ resembles the sound produced by a piano more, which one resembles that of a flute more? Now take $a = 1$, and recompute **x3** (do not change **t** and f_0). Compare the sound you hear with that of the $a = 5$ case. Repeat for $a = 20$. How does the duration of the sound that you hear change as a increases?

Next, consider the signal

$$x_4(t) = \cos(2\pi f_1 t)\cos(2\pi f_0 t), \tag{3}$$

where $f_1 \ll f_0$. Take $f_0 = 1000$ and $f_1 = 6$. Again using a single line command, compute **x4** (provide this code to your report) and plot and listen to it. Compare you results with that of $x_2(t)$. What is the effect of the low frequency cosine term $\cos(2\pi f_1 t)$ on the sound you hear? Recompute **x4** for $f_1 = 3$ and $f_1 = 9$. What is the change in the sound that you hear? By using a trigonometric identity, write $x_4(t)$ as a sum of two different frequency cosine signals and also interpret what you hear in this part using this identity.

Part 3

The instantaneous frequency of a signal of the form

$$x(t) = \cos(2\pi\phi(t)) \tag{4}$$

is defined as

$$f_{ins}(t) = \frac{d\phi(t)}{dt}. \tag{5}$$

Show that the instantaneous frequency of the signal $x_1(t)$ given in Eq. 1 is given as $f_{ins}(t) = f_0$ for all t. Next, consider a signal of the following form

$$x_5(t) = \cos(\pi\alpha e^t). \tag{6}$$

Show that the instantaneous frequency of the signal $x_5(t)$ is given as $f_{ins}(t) = \dfrac{\alpha e^t}{2}$ for all t. Thus, instantaneous frequency changes exponentially with time. What is the frequency at $t = 0$? What is the frequency at some $t = t_0$? To get a feeling about the physical implication of the exponentially changing instantaneous frequency, let us compute $x_5(t)$ and listen to it. Take **t=[0:1/8192:1]**. Then go to the website www.random.org and generate a random integer between 300 and 800(and write the randomly selected number on the report). You will use this generated number as α. With these selections, what are the values between which the frequency will change? Now compute **x5** again with a single line command and provide this command in your report. Then, listen to **x5**. Now, comment on the physical implication of the linearly changing instantaneous frequency. Keep **t** the same, and repeat the experiment with $\alpha_1 = \alpha/2$ and $\alpha_2 = 2\alpha e^{\alpha t}$. Comment on the changes. Now consider the following signal:

$$x_6(t) = \sin(2\pi(-500t^4 - 300t^2 + 1600t)). \tag{7}$$

Take **t=[0:1/8192:2]**. Prepare **x6** with a single line command and provide your code. Then listen to **x6**. How does the frequency of the signal change as time goes on? Find the instantaneous frequency for $x_g(t)$. What is the frequency at $t = 0$? What is the frequency at $t = 1$? What is the frequency at $t = 2$?

Part 4

Let $x_1(t) = A_1 \cos(2\pi f_0 t + \phi_1)$ and $x_2(t) = A_2 \cos(2\pi f_0 t + \phi_2)$ where $A_1 \geq A_2 \geq 0$. Let $x_3(t) = x_1(t)x_2(t)$. I claim that $x_3(t)$ has the form $x_3(t) = A_3 \cos(2\pi f_3 t + \phi_3)$ where $A_3 \geq 0$. Find A_3, f_3 and ϕ_3 in terms of A_1, A_2, ϕ_1, ϕ_2, f_0 and t by providing full derivation. Note that, A_3 may be time dependent. Given A_1, A_2 and f_0, find a condition on ϕ_1 and ϕ_2 for a fixed t such that

- a) A_3 is minimum.
- b) A_3 is maximum.

What are the maximum and minimum possible values for A_3?

Part 5

Consider a discrete-time cosine signal,

$$x[n] = A\cos(\omega n + \phi). \tag{8}$$

where $n \in Z$. A is the amplitude, ω is the normalized frequency in radians and ϕ is the phase shift in radians. First consider the following discrete-time signal:

$$x_7[n] = \cos(0.05\pi n). \tag{9}$$

- a) Generate a segment of $x_7[n]$ for $n \in (0, 255)$ and name it as **x7** in Matlab.
- b) Plot **x7** in Matlab, considering that the signal has a discrete nature. Clearly label the plot and put an appropriate title.

A discrete-time signal $x[n]$ is said to be periodic if there exists a **positive integer** N that satisfies

$$x[n] = x[n+N], \forall n \tag{10}$$

The smallest N that satisfies the periodicity condition is again called as the fundamental period.

- c) Consider the general discrete-time cosine signal. Is that signal always periodic for all values of ω and ϕ? If yes, prove it. Otherwise, find a condition on ω such that the discrete-time cosine signal is periodic with the fundamental period N. Does the phase parameter ϕ have an effect on the periodicity.
- d) If a discrete-time cosine signal is periodic with fundamental period N, prove that it is also periodic with kN, for any positive integer valued k.
- e) Check whether the following discrete-time signals are periodic or not using (10). If they are periodic, please report the fundamental periods.
  - $x_8[n] = cos(3n)$
  - $x_9[n] = cos(1.01\pi n - 0.91)$
  - $x_{10}[n] = cos(\alpha n^2)$