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quantum many bodies
classical paralellism
the area law for the entanglement entropy
entanglement and quantum criticality
three words about Conformal Field Theory
the computation of Holzhey, Larsen and Wilczek
entanglement of excited states
conclusions and outlook

Entanglement, Complexity, Quantum Criticality

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There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.

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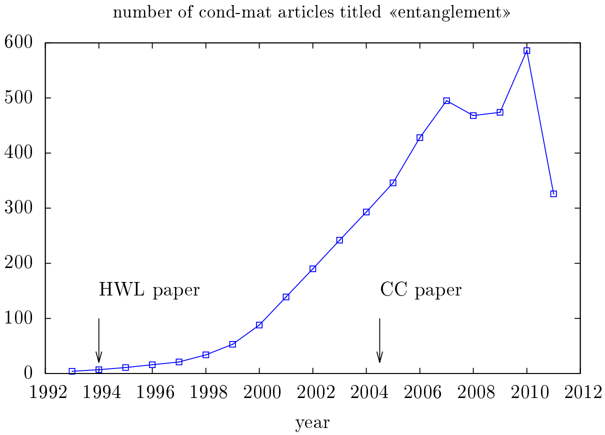

Outline

1. entanglement
2. quantum many bodies
   - classical paralellism
   - the area law for the entanglement entropy
3. entanglement and quantum criticality
   - three words about Conformal Field Theory
   - the computation of Holzhey, Larsen and Wilczek
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4. conclusions and outlook

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section 1: entanglement

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the EPR paradox [Einstein et. al. 1935]

à la Bohm: source of couples of spin-1/2 particles in singlets
$|s\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$
they are measured in A and B, far from each other

- if one measures σ_A^x (say) and obtains $+1$ then in B one will obtain -1 with prob. 1.
- if $\mathbf{a}\boldsymbol{\sigma}_A$ is measured then then the B particle will be eigenstate of $\mathbf{a}\boldsymbol{\sigma}_B$ (with unmeasurable consequences)

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how does the B particle "knows" what in A, arbitrarily far, was measured?
this is possible only if:

- measurments in A can condition without retard the state of B. "spukhafte Fernwirkung", in Einstein words. (entanglement)
- no spooky action at a distance: the spin in B is -1 also precedently wrt the measure of A. $|s\rangle$ is hence a phenomenological, *incomplete*, description (need for *hidden variables*)

Bell theorem, ASPECT experiments...
causality is respected (no information transmitted instantaneously)
but not *locality*

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entanglement

but this does not happen with all states (think about the triplet: $|\uparrow\uparrow\rangle$) an *entangled* state is needed, for which (Schroedinger, 1935):

> *...the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts, even though they may be entirely separate and therefore being possible of being best possibly know.*

in other words, the partial trace in one part

$$\rho_A \equiv \mathrm{tr}_B \rho$$

results in a *mixed* state.

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entanglement entropy

any state $|\psi\rangle$ can be expressed (Schmidt decomposition)

$$|\psi\rangle = \sum_n^{\min d_A, d_B} \lambda_n |\psi_n\rangle_A |\varphi_n\rangle_B$$

entanglement happens when more than one $\lambda_n \neq 0$ (not a product state)

and it is quantified with the entanglement (von Neuman) entropy,

$$S_1(A) = -\mathrm{tr}\, \rho_A \ln \rho_A = -\sum_n \lambda_n \ln \lambda_n$$

and with Rényi entropies,

$$S_n(A) = \frac{1}{1-n} \ln \mathrm{tr}\, \rho_A^n = \frac{1}{1-n} \ln \sum_n \lambda^n$$

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section 2: quantum many bodies

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quantum many bodies in 1D

consider n interacting spin-1/2 particles in 1D
the Hilbert space is $V = \{0,1\}^{\otimes n}$, 2^n-dimensional
a generic state: $\psi_{s_1 \dots s_n} |s_1 \dots s_n\rangle$

paradigm of a quantum chain: the quantum Ising model in a transverse field:

$$H_{ITF} = \sum_{j=1}^{n} \lambda\, \sigma_j^z \sigma_{j+1}^z + \sigma_j^x$$

- for $\lambda = \infty$ the g.s. is $|\uparrow\uparrow \dots \uparrow\rangle$ (and $|\downarrow\downarrow \dots \downarrow\rangle$)
- for $\lambda = 0$ the g.s. is $|\leftarrow\leftarrow \dots \leftarrow\rangle$
- in between, the g.s. is not a product state of two subparts

correlators $\langle \sigma_j^z \sigma_{j+r}^z \rangle \sim \exp(-r/\Delta)$ large r, $\Delta = E_1 - E_0$, gap

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reminder of path integral in field theory

the Wick rotation: $t \to t + i\epsilon$

$$\langle 0|\hat{O}(t_1)\dots\hat{O}(t_n)|0\rangle = \lim_{\epsilon\to 0}\frac{\int [dx]\, e^{iS[x]}\, O[x(t_1)]\dots O[x(t_n)]}{\int [dx]\, e^{iS[x]}} \tag{1}$$

generalizing for a d space-time dimensional bosonic field $\hat{\varphi}(t,\mathbf{x})$, $\mathbf{x}\in\mathbb{R}^{d-1}$

$$\langle\hat{\varphi}(t_1,\mathbf{x}_1)\dots\hat{\varphi}(t_n,\mathbf{x}_n)\rangle = \lim_{\epsilon\to 0}\frac{\int [d\varphi]\, e^{iS[\varphi]}\, \varphi(t_1,\mathbf{x}_1)\dots\varphi(t_n,\mathbf{x}_n)}{\int [d\varphi]\, e^{iS[\varphi]}} \tag{2}$$

where $[d\varphi] = \lim_{N\to\infty}\prod_{j=1}^{N-1}\prod_{j_1=1}^{N-1}\dots\prod_{j_{d-1}=1}^{N-1} d\varphi(t_j, x_{j_1},\dots,x_{j_{d-1}})$

and where $S[\varphi] = \int dt\,[\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)]$ is the field action

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imaginary time

$$t \to -i\tau \qquad \tau \in \mathbb{R}$$

$$S[\varphi] \to -iS_E[\varphi] \qquad S_E[\varphi] = \int_{-\infty}^{\infty} d\tau\, H[\varphi]$$

S_E, Euclidian action

$H[\varphi] = \int d^{\,d-1}\mathbf{x}\, \left[(\partial_\tau\varphi)^2 + (\nabla\varphi)^2 + V(\varphi)\right]$ is a Hamiltonian in $d-1$ spatial dimensions

hence:

$$\langle\hat{\varphi}(\mathbf{x}_1)\ldots\hat{\varphi}(\mathbf{x}_n)\rangle = \frac{\int [d\varphi]\, e^{-S_E[\varphi]}\, \varphi(\mathbf{x}_1)\ldots\varphi(\mathbf{x}_n)}{\int [d\varphi]\, e^{-S_E[\varphi]}} \tag{3}$$

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classical statmec

the partition function of a d-dim classical system in the continuum:

$$Z_{cl}(\beta) = \int [d\varphi]\, e^{-\beta E_{cl}[\varphi]} \tag{4}$$

E_{cl}, classical energy in the continuum. The correlators:

$$\langle \varphi(\mathbf{x}_1) \ldots \varphi(\mathbf{x}_n) \rangle_{cl} = \frac{1}{Z_{cl}(\beta)} \int [d\varphi]\, e^{-\beta E_{cl}[\varphi]}\, \varphi(\mathbf{x}_1) \ldots \varphi(\mathbf{x}_n) \tag{5}$$

example: for the 2D Ising model, $H_{Ising} = -J \sum_{i,j} s_i s_j$ we have (arround T_c),

$$E_{cl}[\varphi] = \int d^2\mathbf{x} \left[(\nabla\varphi)^2 + m\varphi^2 + O(\varphi^4) \right]$$

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the classical-quantum connection

comparing (3) with (5):
for each H_{cl}, statistical model in d-spatial dims.
there is a quantum H in $d-1$-spatial dims. such that they share correlators and phase diagram

2D classical Ising @ $\beta \leftrightarrow$ 1D ITF @ 0-T connection: $\lambda = \beta J \exp(\beta J)$

sums ovr. confs. in the "extra" dim $\leftrightarrow$ paths in imaginary time

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entanglement & many bodies: the area law...

one is usually interested in the amount of entanglement between two spatial partitions A, B of a many-body system
gapped bipartite system $A \cup B$. for the ground state:

$$S_A \sim \partial(A), \textit{ area law } \text{[Sredniki 1993]}$$

physical "corner" in Hilbert space

1D, large but finite correlation lenght [Calabrese, Cardy 2004], $\ell = |A|$:

$$S_A \to \frac{c}{3} \ln \xi \qquad \ell \to \infty$$

c, conformal charge of the underlying CFT when $\xi \to \infty$

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...& complexity: MPS's

classical simulation of a 1d quantum system $\{0,1\}^{\otimes n}$, $|\Psi\rangle = \psi_{s_1 \dots s_n} |s_1 \dots s_n\rangle$

if only χ terms in the Schmidt decomposition $A : B$ $(1 \dots j : j+1 \dots n)$ are retained:

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi} (\lambda_\alpha)_j |\psi_\alpha\rangle_A |\varphi_\alpha\rangle_B$$

Ψ can be stored and simulated with:

- n $(2 \times \chi \times \chi)$-tensors $\Gamma^s_{\alpha\beta}$
- the n χ-vectors, λ_α

in s.a.w.t. time and memory resources are $R \sim n\,\mathrm{poly}(\chi)$

$R \sim \mathrm{poly}(n) \Rightarrow$ the maximum *Schmidt rank*, χ_n must be $\sim \mathrm{poly}(n)$

$\Rightarrow S_A < \ln \chi_n \sim \ln \mathrm{poly}(n)$ is required

MPS algorithms fulfill the 1D-area law by construction

a novel connection with complexity

Physical consequences of P$\neq$NP

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(Dated: Oct 13, 2010)

Computational complexity theory is applied to *simulations* of adiabatic quantum computation, providing predictions about the existence of quantum phase transitions in certain disordered systems. Moreover, bounds on their entanglement entropy at criticality are given. Concretely, physical consequences are drawn from the assumption that the complexity classes **P** and **NP** differ.

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J. Rodríguez-Laguna, submitted (2010)

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section 3: entanglement and quantum criticality

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critical entanglement

for 1D critical systems of length L [Holzley, Larsen, Wilczek 1994], [Cardy, Calabrese 2004]:

$$S_A(\ell) = \frac{c}{p} \ln \left[\frac{L}{\pi} \sin\left(\frac{\pi \ell}{L} \right) \right] + A$$

A non-universal. $p = 3, 6$ for PBC and OBC respectiveley. And (Rényi entropy $S_{A,n} = \frac{1}{1-n} \ln \mathrm{tr}\, \rho^n$):

$$\mathrm{tr}\, \rho_A^n(\ell) = \left[\frac{L}{\pi \epsilon} \sin\left(\frac{\pi \ell}{L} \right) \right]^{\frac{c}{6}\left(\frac{1}{n} - n\right)}$$

finite temperature $1/\beta$, PBC [Calabrese, Cardy 2004]:

$$S_A(\ell) = \frac{c}{p} \ln \left[\frac{\beta}{\pi} \sinh\left(\frac{\pi \ell}{\beta} \right) \right] + A$$

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generalizing scale invariance

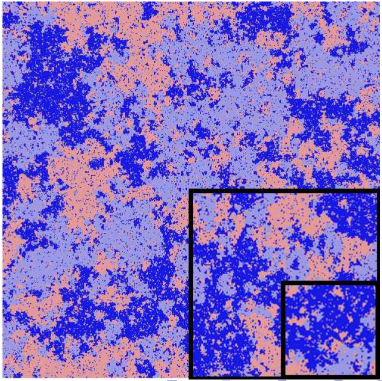
scaling hypothesis @ criticality:

$$\langle \Phi_1(\mathbf{r}_1)\ldots\ \Phi_n(\mathbf{r}_n)\rangle = a^{\sum_j x_j}\langle \Phi_1(a\mathbf{r}_1)\ldots\ \Phi_n(a\mathbf{r}_n)\rangle$$

may be generalized $\mathbf{r}'(\mathbf{r})\ \mathbb{R}^d \to \mathbb{R}^d$, and $b = \left|\frac{\partial \mathbf{r}'}{\partial \mathbf{r}}\right|$:

$$\langle \Phi_1(\mathbf{r}'_1)\ldots\ \Phi_n(\mathbf{r}'_n)\rangle = \prod_j b(\mathbf{r}_j)^{-x_j}\langle \Phi_1(\mathbf{r}_1)\ldots\ \Phi_n(\mathbf{r}_n)\rangle$$

2D $q = 3$ Potts model @ T_C
external & internal rectangles
are zoomed 1.25× and 1.5×



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first: conformal transformations in d dimensions

transformations that locally preserve the metric $g'_{\mu\nu}(\mathbf{r}') = \Omega(\mathbf{r}) g_{\mu\nu}(\mathbf{r})$

local traslations, rotations, scale transformations + "spetial conformal" $(x^\mu \to x^\mu + b^\mu x^2 - 2x^\mu b.x)$ transformations

a theory is said to be conformal invariant if, roughly speaking, there are some fields for which, $\mathbf{r}'(\mathbf{r})$ being conformal,

$$\langle \Phi_1(\mathbf{r}_1) \ldots \rangle_D = b(\mathbf{r}_1)^{\Delta_1} \langle \Phi_1(\mathbf{r}'_1) \ldots \rangle_{D'}$$

this constraints the 2 and 3-point correlators:

$$\langle \Phi_1(\mathbf{r}_1) \Phi_2(\mathbf{r}_2) \rangle = \begin{cases} \frac{c_{12}}{r_{12}^{2\Delta_1}} & \text{if } \Delta_1 = \Delta_2, \\ 0 & \text{else} \end{cases}$$

second: 2-D CFT's

- in 2D, all analytical transformations $f(x_0, x_1)$ ($\partial_0 f_0 = \partial_1 f_1$, $\partial_0 f_1 = -\partial_1 f_0$) are conformal
- in the complex plane $z = x_0 + ix_1$ this amounts for $w(z, \bar{z})$ to be $w(z)$, $\bar{w}(\bar{z})$
- theories whose action is scale, traslational & rotational invariant are also conformal invariant (the energy-momentum tensor is then: $T(z)$, $\bar{T}(\bar{z})$)
- in this case, there are some fields $\Phi_j(z, \bar{z})$ (*primary*) which transforms under conformal transformations:

$$\langle \Phi_j(z, \bar{z}) \ldots \rangle = \left(\frac{\partial w}{\partial z} \right)^{h_j} \left(\frac{\partial \bar{w}}{\partial \bar{z}} \right)^{\bar{h}_j} \langle \Phi_j(w, \bar{w}) \ldots \rangle$$

- the energy-momentum tensor is not primary:

$$T(z) = \left(\frac{\partial w}{\partial z} \right)^2 T(w) + \frac{c}{12} \mathrm{Sch}\{w, z\} \qquad \text{inside } \langle \ldots \rangle \tag{6}$$

c is the *central charge* of the theory and
$\mathrm{Sch}\{w, z\} = \frac{w'(z) w'''(z) - (3/2)(w''(z)^2)}{w'(z)^2}$ is the Schwartzian derivative

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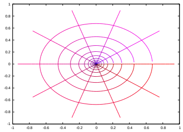
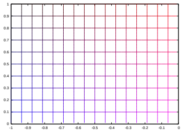
third: universality of finite-size scaling [Cardy 1984]

system with finite-size length L, the space-time is a cylinder of circumference L

$$w(z) = \frac{L}{2\pi} \ln z$$



z



w

takes the z plane to the $w = t + i\sigma$ cylinder of circumference L, $0 \leq \sigma \leq L$

using (6),

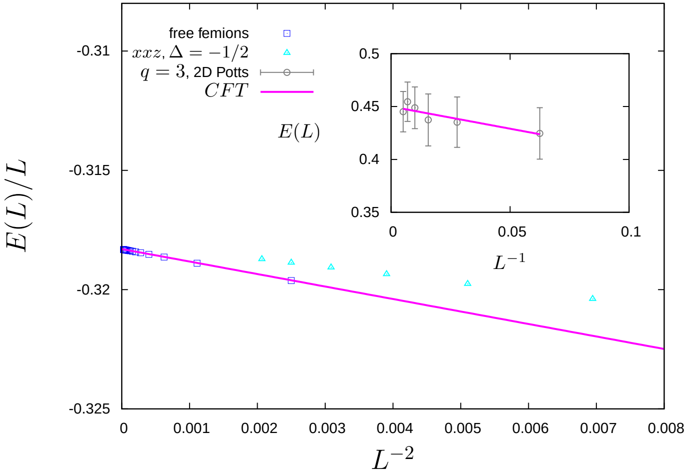
$$T(w)_{\mathrm{cyl}} = \left(\frac{2\pi}{L}\right)^2 \left(z^2 T(z)_{\mathrm{plane}} - \frac{c}{24}\right) \qquad \text{inside } \langle \ldots \rangle$$

remember $\hat{H} = \frac{1}{2\pi} \int_0^L d\sigma (T + \bar{T})$. For the ground state we have:

$$\langle \hat{H} \rangle_{\mathrm{cyl}} = -\frac{\pi c}{6L}.$$

More generally, $E_{\mathrm{cyl}}(L)$, $P_{\mathrm{cyl}}(L)$ of excitations may serve to infer $h, \bar{h}$ of the excitation

...universality of finite-size scaling (2/2)



where $H_{xxz} = \frac{1}{2} \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z$

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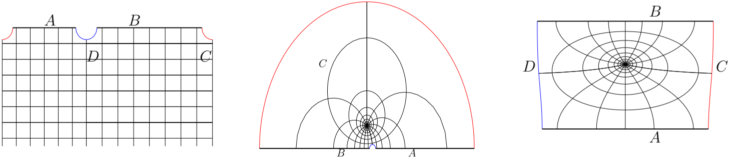
the HLW calculation [Holtzley et. al. 1994]

our space-time geometry: a semi-infinite cylinder
regularized with semi-disks if ϵ-radius separating A, B, $|A| = \ell$

it is mapped into a $\pi \times \left(d = 2\ln\left[\frac{L}{\pi}\sin\frac{\pi\ell}{L}\right]\right)$ strip

$$\zeta, \qquad w = -\frac{\sin\left(\frac{\pi(\zeta-\ell)}{N}\right)}{\sin\left(\frac{\pi\zeta}{N}\right)}, \qquad z = \log w$$



ζ $\qquad$ w $\qquad$ z

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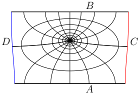
...the HLW calculation (2/4)

it is, for the g.s. w.f. & density matrix of A (when tracing in B):

$$\Psi_{XY} \propto \int_{\varphi(A)=X}^{\varphi(B)=Y} [d\varphi]\ e^{-S[\varphi]}$$

$\int$ in 

$$\rho_{XX'} = \int_{\varphi(A)=X}^{\varphi(A')=X'} [d\varphi] e^{-S[\varphi]}/Z(1)$$

$\int$ in a $2\pi \times d$ strip

and $Z(n)$ is the partition function on a $2\pi n \times d$ torus

$$\mathrm{tr}\ \rho^n = \frac{Z(n)}{Z(1)^n}$$

one now uses the CFT result: $Z_\tau = \mathrm{tr}\ q^{-c/12}$
for very small $q = e^{2\pi i\tau} = e^{-d/n}$

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...the HLW calculation (3/4)

one has:

$$\mathrm{tr}\ \rho^n = e^{\left(\frac{1}{n}-n\right)\frac{cd}{12}} = \left[\frac{L}{\pi\epsilon}\sin\frac{\pi\ell}{L}\right]^{\frac{c}{6}\left(\frac{1}{n}-n\right)}$$

or:

$$S_n(\ell) = \frac{c\,(n+1)}{6n}\ln\left[\frac{L}{\pi}\sin\left(\frac{\pi\ell}{N}\right)\right] + \gamma_n \qquad (7)$$

in particular: $(S_1 = \partial_n|_{n=1}\mathrm{tr}\ \rho^n)$

$$S_1(\ell) = \frac{c}{3}\ln\left[\frac{L}{\pi}\sin\left(\frac{\pi\ell}{L}\right)\right] + \gamma_1$$

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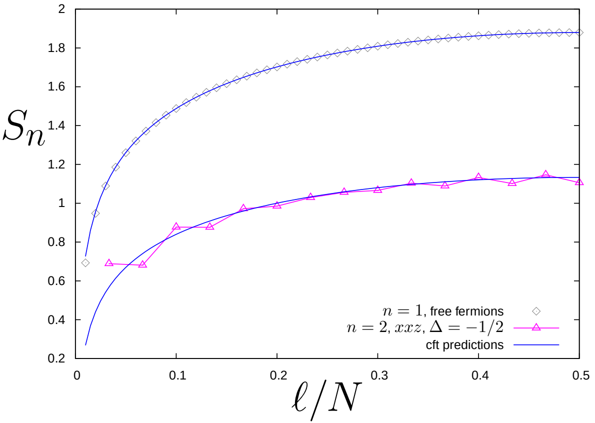
...the HLW calculation (4/4)



these results (7) were achieved in an alternative way in [Cardy, Calabrese 2004]

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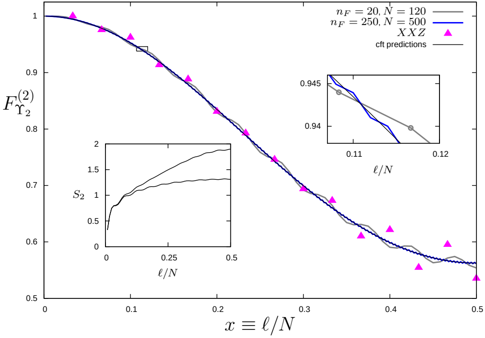
entanglement of excited states

generalization for excitations is possible [Alcaraz, Ibáñez, Sierra 2011] arXiv:1101.2881

by inserting in the infinite past the operator responsible for the excitation $|\Upsilon\rangle = \lim_{\zeta,\bar{\zeta}\to -i\infty} \Upsilon(\zeta,\bar{\zeta})\,|0\rangle$ (operator-state correspondence in *CFT*)

$$F_\Upsilon^{(n)}(x) \equiv \frac{\mathrm{tr}\,\rho_\Upsilon^n}{\mathrm{tr}\,\rho_{\Upsilon_0}^n} = \frac{n^{-2n(h+\bar{h})}\langle\prod_{j=0}^{n-1}\Upsilon(\frac{2\pi j}{n})\,\Upsilon^\dagger(\frac{2\pi(j+x)}{n})\rangle_{\mathrm{cy}}}{\langle\Upsilon(0)\,\Upsilon^\dagger(2\pi x)\rangle_{\mathrm{cy}}^n}$$



entanglement
quantum many bodies
classical paralellism
the area law for the entanglement entropy
entanglement and quantum criticality
three words about Conformal Field Theory
the computation of Holzhey, Larsen and Wilczek
entanglement of excited states
conclusions and outlook

outlook

- entanglement in cond-mat: the area law restricts the Hilbert space accessible to physical (ground) states
- the validity of algorithms for a given system its related to its amount of entanglement
- in 1D at criticality, the area law is violated and the entanglement depends on universal quantities
- the entanglement of excitations can give information on the CFT content describing them

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10^3 grazie

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reminder of the Feynman path integral

consider a system with one degree of freedom (q.m.), $\hat{x}$, $\hat{p}$
one has [Feynman, Hibbs 1965] ($U(t) = e^{itH}$)

$$\langle x_f | U(T) | x_i \rangle = \int_{x(0)=x_i}^{x(T)=x_f} [dx]\, e^{iS[x]} \tag{8}$$

propagator is a sum over trajectories $x(t)$
$[dx] = \lim_{N\to\infty} \prod_{j=1}^{N} \sqrt{\frac{mN}{2\pi jt}}\, dx_j$
$S[x]$, functional action, $\int dt\,[T - V] = \int dt\, \left[\dot{x}(t)^2/2m - V(x(t))\right]$

from (8), the *corresponding principle* emerges: divide all actions by $\hbar$, then $\hbar \to 0$:

$$\exp iS[x]/\hbar / \exp iS[x_c]/\hbar \to 0$$

where $x_c(t)$ maximizes $S[\,]$

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another result

$$S_1^{\Upsilon}(\ell) - S_1^{\mathrm{gs}}(\ell) \sim \frac{2\pi^2}{3}(h+\bar{h})\left(\frac{\ell}{N}\right)^2 + O\left(\frac{\ell}{N}\right)^{2\Delta_\Psi}$$