A hungry graduate student
(Cartoon from Liana Sega's website)

Grad. Algebra II

MATH B504

Course info ———

📅 Tuesday & Thursday

🕐 9:55–11:15a

📍 Park Science 328

Instructor ———

👤 Professor John Bergdall

📍 Park Science 334

@ jbergdall@brynmawr.edu

☎ x5356

🕐 Office Hrs:
Mon TBD
Wed TBD

Problem session -

📅 Wednesday

🕐 11:30a - 1:00p

📍 Room TBD

Essential statement

We welcome your participation in whatever form it takes, with the hope that you enrich our space with your identities.

We pledge our attention and a sincere approach to your experiences. We ask that you also acknowledge the different identities and experiences of your peers, and you pledge to respect each of them fully.

As a class, we will develop a strong system of norms to observe. If at any point you feel less than empowered in your learning or less than comfortable in the environment, we encourage you to interface with us in order to provide myself, yourself, and your peers an opportunity to recommit and revisit these norms.

Essential questions

The essential questions in MATH B504 are the same as MATH B503:

- How are mathematical structures organized?
- How are definitions in mathematics given and used?
- How are mathematical objects built from one another?

Overview

This is a 2nd semester course in algebra at the graduate level. In the 1st semester course we studied groups, rings, and modules with a high emphasis on definitions and structural relationships through the language of category theory.

In this course we take two further steps. The first is that we will study the structure of fields and the correspondence theorem provided at the heart of *Galois theory*. Though first understood in the middle of the 19th century, it is a landmark result and a prototype for results in other branches of mathematics. The second is the introduction of a second layer of categorical thinking, represented by the notion of *functors*.[1]

Functors are to categories as morphisms are to objects in a category: mechanisms by which information transfer takes place. They appear nearly everywhere in modern mathematics, though on a practical level the language and use of functors is more likely implicit rather than explicit. A typical example you may encounter is the ring of smooth real-valued functions on a smooth manifold, which is a functor on the category of smooth manifolds. Or, you may have heard (in seminar talks, say) statements like

…we can associate to object X a natural (or canonical) group $G(X)$.

The evoked word "natural" is the implicit reference to a functor $G(-)$. Accurately using words like natural, or canonical, is an important rite of passage this semester.

We will further study, with the aid of the language of functors, the ideas behind *multilinear algebra*, which lies at the heart of differential forms in geometry and topology, and *homological algebra*. In fact, both multi-linear algebra and homological algebra are techniques of calculation and organizing (algebraic) information that lies at the heart of geometry and topology. The typical homological-algebraic calculation you need to make in these fields involves *derived functors* Ext and Tor, where the word derived is used in the tradition of calculus. (Seriously.) It is no wonder that, despite the initial appearance of pure abstraction, these topics remain part and parcel of early career training for graduate students.

Learning goals

In MATH B504, you will:

- Develop advanced skills in reading, writing, and discussing mathematics.
- Learn how mathematical objects are defined by their relationships with others.
- Practice categorical thinking and recognizing *content* versus *formality* in math.
- Study the correspondence theorem of Galois theory, multi-linear algebra, and homological algebra.

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[1] The reader who revisits this syllabus at the end of the semester may find a hilarious pun.

FAQs

? How should we address you?

! If you need to name me, I go by Professor Bergdall or John (not Dr.). If you write me an email, include an appropriate greeting (Dear/Hello...) and closing (Sincerely/Thank you...). If you need to reference me with pronouns, I use he/him/his.

? Why are policies different between 503 and 504?

! There are at least two reasons. First, as you learn things your perspective changes and you grow. This is true for professors as well as students. That explains, at least, the possibility of shifting policies. The second clear reason is that the first semester is the introduction the cast of characters in abstract algebra, while the second semester is more like a theatrical performance put on by these characters. So, the second semester necessarily is more subjective and thus demands a slightly more relaxed perspective on progress.

? Will you repeat that? Can you say it more directly?

! Yes, gladly! Part of mathematical discourse for me is EXCITEMENT! Please *selfishly* ask me to repeat or re-explain anything you have not grok'd.

Prerequisites

The main prerequisite is the material in MATH B503. If you are unsure about your preparation, please discuss with me.

Material

Text

Allufi, P., *Algebra: Chapter 0*. 1st edition. American Mathematical Society.

The author maintains a list of errors at

> https://www.math.fsu.edu/ aluffi/algebraerrata.2016/Errata.html

Please check this errata, especially if you've found an error in the exercises.

Our book has been chosen for its clarity and modern viewpoint.

Other recommended texts

The following texts should be on reserve in Collier Library, same as last semester.

- Dummit, D. and Foote, R. *Abstract Algebra*. John Wiley & Sons. ISBN: 0-471-43334-9.

- Lang, S. *Algebra*. Graduate Texts in mathematics, 211. Springer-Verlag. ISBN: 0-387-95385-X.

- Hungerford, T. *Algebra*. Graduate Texts in Mathematics, 73. Springer-Verlag. ISBN: 0-387-90518-9

- Isaacs, I. M. *Algebra*. Graduate Studies in Mathematics, 100. American Mathematical Society. ISBN: 0-8218-4799-2.

- Jacobson, N. *Basic Algebra I, II*. (These are printed these days by Dover).

You should not need these texts to do well in our course. They may be useful later in your life. Dummit and Foote is rife with examples, hence useful for first exposure to a topic. Lang is invaluable as a desk reference and tofu press. The others are common choices for learning and reference.

Accessibility

Bryn Mawr College is committed to providing equal access to students with a documented disability. Students needing academic accommodations for a disability must first register with Access Services. Students can call 610-526-7516 to make an appointment with the Access Services Director, Deb Alder, or email her the address dalder@brynmawr.edu to begin this confidential process. Once registered, students should schedule an appointment with the professor as early in the semester as possible to share the verification form and make appropriate arrangements. Please note that accommodations are not retroactive and require advance notice to implement. More information can be obtained at the Access Services website whose URL is http://www.brynmawr.edu/access-services/

Any student who has a disability-related need to tape record this class first must speak with the Access Services Director and to me, the instructor. Class members need to be aware that this class may be recorded.

Academic Integrity

The Bryn Mawr College Honor Code is in effect for all students enrolled in this course. We also provide some specific guidelines for different assignments in *this* course below.

Course components

Lectures & problem session

This course is fast-paced and challenging. Remember, you are preparing for modern mathematical research through encounters with algebraic structures. There are 3 80-minute meetings each week. Lectures take place on Tuesday and Thursday. The problem session scheduling will occur during the first class meeting. We decided it will take place on:

- Problem session day and time: Wednesday mornings 9:30-11:00.

Lectures will (roughly) cover one section of the text. We will amplify crucial components and illustrate key steps, but some details will not be handled explicitly. We will dwell on definitions and move briskly through proofs. Part of a students' job is to develop skills reading and understanding mathematical arguments on your own. Having trouble is normal, so please communicate occluded arguments, in whatever way you feel comfortable, so we may share our experiences.

In problem session, we will discuss strategies to assigned exercises. Discussions will be student-led. We prefer that the leader not repeat, so come prepared to lead on at least one problem and ideally multiple ones. Note *lead*, not *solve*: leading can mean making partial progress, discussing the link between a hypothesis and a conclusion, or explaining a route you have tried.

Problem sets

Problem sets will be delivered on Thur. night or Fri. morning. The structure will be discussed on the first day of class:

- Approximate # of required problems per week: 7-8 problems.
- Problem sets are due: Thursday at 7pm through moodle.
- Should there be optional problems? Yes, a few extra suggested ones.

You will receive either written or oral feedback on your solutions 1 week later.[2] You will not receive numerical scores. (Change.)

You are expected to turn in problem sets complete and on time. You are invited to skip a problem, but consider writing down your progress so you can get feedback. Turning a set in late will delay feedback, which you are owed, and it will not delay the release of the next problem set. If you systematically skip problems or turn them in late, we will conference to discuss and construct more specific, personal, guidelines.

Mathematical conversations

Having a quality 30 minute conversation with a fellow researcher is worth days of reading their writings. Visiting a collaborator for a week to do intense research will pay dividends roughly on the order of spending a year collaborating over email. So, one of the learning goals in this course is to develop your skill in discussing mathematics. Twice during the semester you will be invited to our office to have a brief conversation about course topics. The dates and lengths for these conversations will be:

- Midterm conversation: March 4-5 (30 minutes).
- Final conversation: April 29-30 (60 minutes).

This exam is optional if (i) you are not taking a graduate qualifying exam at Bryn Mawr, and (ii) you have satisfactorily turned in each problem set. If you are planning to take a qualifying exam elsewhere, in the future, this activity is highly recommended.

Final exam

The class will decide during the first day whether or not a written, take-home, 72-hour, final exam is appropriate.

- There WILL/WILL NOT be an optional written final exam.
- If there will be, it will take place: during the first week of finals week.

Final grades

Grading in a 500-level course depends on the capacity in which you enrolled.

- Students taking the course for graduate credit receive a grade of S (satisfactory) or NS (not satisfactory). You will receive S if you complete all the work, including the mathematical conversations (if necessary).
- Undergraduate students taking the course for elective credit receive a grade on the x.x scale, as usual. You will receive a 3.0, at least, if you meet the requirements of an 'S' above. Your grade beyond that will depend on the quality of your work. You will receive an estimated score after spring break.

Signature

After filling in the blanks above, please sign in ink to codify the policies: _____

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[2]In an ideal world, we would meet and discuss your problem sets in person. This is possible, depending on the size of the class.

Classroom norms (updated)

Some of these classroom norms were proposed by students in the previous semester of MATH B503. I have added new ones for this semester.

- If possible, I'd appreciate any and all feedback. I find that it's very important to have an outside point of view to improve.
- Respect all questions.
- Try not to assume everyone knows/remembers everything you do. Good to bring up old information, just don't take it for granted. (Something I used to do, so if you catch me doing it, glare at me or something.)
- Don't use language that minimizes problems/proofs such as "easy," "pretty," "cute," etc.
- Balance comments with a kind word. Relate openly with someone else on a confusing point. Strength in numbers!
- A discussion should include more than one student's voice (or a particular subset of students' voices).
- I'd like to emphasize not demeaning others' ideas and attempts, even if they're wrong.
- I never want to hear the world "tr*vial" again.
- Celebrate making mistakes and confusion rather than pretending they didn't happen.
- Kindness.

First day check-in

Some personal details

Preferred name:

Preferred pronouns (if you would like to share — you needn't):

Logistics

Are you registered for this course, definitely planning to register, or shopping?

☐ Registered   ☐ Planning to register   ☐ Shopping

My BiCo username is: ☐ @brynmawr.edu   ☐ @haverford.edu

Briefly describe your prior experience with Galois theory:

Goals

Describe one goal you have for *this* course:

Describe one goal you have for this *semester* (regardless of MATH B504):

Class formation

Briefly describe one norm you would like the class to observe during discussion. (These will be compiled and appended to this syllabus *anonymously*. A prior semester's norms are already on the other side of this page.)

In the remaining space, if you are comfortable, please indicate anything else you think it is important for us as your professor to know. (If you are not comfortable, please email us or come see us in office hours.)