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1a) $\vec{F}_E = \dfrac{-GM_\odot M_E \,\vec{r}}{r^3}$ in 2D $r = \sqrt{x^2+y^2}$

get from assuming circular motion.

$M_E \dfrac{v^2}{r} = F = \dfrac{GM_\odot M_E}{r^2} \Rightarrow v^2 r = GM_\odot$

$\left(2\pi \frac{Au}{year}\right)^2 \cdot Au$

$GM_\odot = 4\pi^2 \, (Au)^3/yr^2$

$a_x = \dfrac{F_x}{M_E} = \dfrac{-GM_\odot}{r^3} x = \dfrac{dv_x}{dt}$

$a_y = \dfrac{F_y}{M_E} = \dfrac{-GM_\odot}{r^3} y = \dfrac{dv_y}{dt}$

$\frac{dx}{dt} = v_x \qquad \frac{dy}{dt} = v_y$

1b) units: $G = \dfrac{N \cdot m^2}{kg^2} = \dfrac{kg \cdot \frac{m}{s^2} \cdot m^2}{kg^2} = \dfrac{m^3}{kg \cdot s^2}$

Potential: $\dfrac{G \cdot M_{sun} \cdot M_{earth}}{r} = \dfrac{\frac{m^3}{kg \cdot s^2} \cdot kg \cdot kg}{m} = \dfrac{m^2 kg}{s^2} =$ Joules.

Kinetic: $\frac{1}{2}mv^2$. v measured in $\dfrac{Au}{year}$. multiplying by values for Au in m and year in s,

gives constant $\cdot M_{earth} \cdot \dfrac{m^2}{s^2} = kg \cdot \dfrac{m^2}{s^2} =$ Joules.

check: Earth has velocity 29,789.8 m/s, so $KE = \frac{1}{2}mv^2 =$

2a) Acceleration of Earth from sun's force was $\dfrac{-GM_\odot \vec{r}}{r^3}$

Came from $\vec{F}_E = \dfrac{-GM_\odot M_E \vec{r}}{r^3} = M_E \cdot a_E$

Mass of orbiting object cancels, so Jupiter will also have acceleration from sun of $a_J = \dfrac{-GM_\odot \vec{r}}{r^3}$ from $\vec{F}_J = \dfrac{-GM_\odot M_J \vec{r}}{r^3} = M_J \cdot a_J$

Now just add the acceleration from the force they exert on each other.

$\left\{ F_{EJ} = \dfrac{-GM_J M_E \,\vec{r}_{EJ}}{r_{EJ}^3} \quad \text{where } \vec{r}_{EJ} = \vec{r}_E - \vec{r}_J \right\}$ this is just my shorthand, actually for cartesian coordinates, ~~[illegible]~~

If we wanted to think in our cartesian coordinates:

Force between earth and Jupiter.

$$F_{EJx} = \frac{-GM_J M_E x_{EJ}}{r_{EJ}^3} \qquad x_{EJ} = x_E - x_J$$

$$F_{EJy} = \frac{-GM_J M_E y_{EJ}}{r_{EJ}^3} \qquad y_{EJ} = y_E - y_J$$

$$r_{EJ} = \sqrt{(x_E - x_J)^2 + (y_E - y_J)^2 + (z_E - z_J)^2}$$

$$F_{EJz} = \frac{-GM_J M_E \cancel{x_{EJ}} z_{EJ}}{r_{EJ}^3} \qquad z_{EJ} = z_E - z_J$$

Jupiter experiences force in opposite direction.

Putting it all together:

total for Earth

$$\frac{F_{Ex}}{M_E} = a_{Ex} = \frac{-GM_\odot x_E}{r^3} - \frac{GM_J(x_E - x_J)}{r_{EJ}^3}$$

$$= \frac{-4\pi^2 x_E}{r^3} - 4\pi^2\left(\frac{M_J}{M_\odot}\right) \cdot \frac{x_E - x_J}{r_{EJ}^3}$$

replace x with y or z for those directions

$$\cancel{\frac{F_{EJ}}{M_J}}$$

total for Jupiter

$$\frac{F_{Jx}}{M_J} = a_{Jx} = \frac{-GM_\odot x_J}{r^3} + \frac{GM_J(x_E - x_J)}{r_{EJ}^3}$$

$$= \frac{-4\pi^2 x_J}{r^3} + 4\pi^2\left(\frac{M_J}{M_\odot}\right) \cdot \frac{x_E - x_J}{r_{EJ}^3}$$

Again, replace x with y or z for those directions.

Note that in these equations r is for the planet being considered, since this is for that planet's interaction with the sun.

So in earth's acceleration $r = \sqrt{x_E^2 + y_E^2 + z_E^2}$

in Jupiter's acceleration $r = \sqrt{x_J^2 + y_J^2 + z_J^2}$

still have obviously $a_x = \frac{dv_x}{dt}$ $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$ for both planets.