Part 1: 1) mass m in $V(x) = \frac{V_0}{d^4}(x^4 - 2x^2d^2 + d^4)$

$$V'(x) = \frac{V_0}{d^4}(4x^3 - 4xd^2)$$

$$V'(x) = 0 = \frac{V_0}{d^4}(4x^3 - 4xd^2)$$

$$0 = 4x^3 - 4xd^2$$

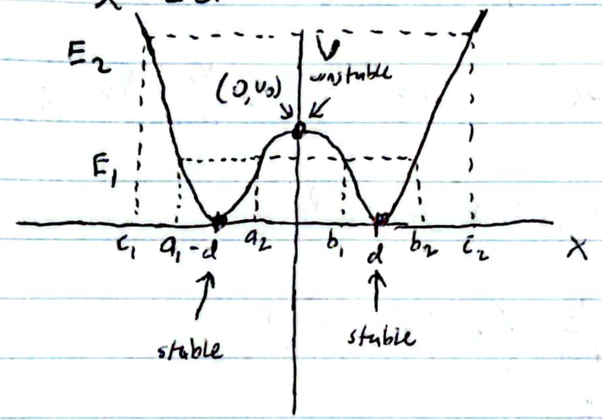
$$x^3 - xd^2 = 0$$

$$x^3 = xd^2 \leftarrow x=0 \text{ is a root}$$

$$x^2 = d^2$$

$$x = \pm d$$

our 3 roots are $x = -d, 0, d$



dots are equilibrium points

equilibrium points are $x = -d$, $x = 0$, $x = d$
stable, unstable, Stable

2) For a particle with energy E_1, it will be trapped in one of the 2 wells, and will oscillate around either $-d$ or $+d$. If the initial position is $a_1 \leq x_0 \leq a_2$ with $E = E_1$, then it will oscillate between a_1 and a_2. If the initial position is $b_1 \leq x_0 \leq b_2$ with $E = E_1$, then it will will oscillate between b_1 and b_2.

For a particle with energy E_2, it will oscillate between c_1 and c_2, and we know the position must start somewhere within this range, or else we'd have a potential energy greater than the total energy, which would imply a negative kinetic energy. So we'll get this oscillation, with max kinetic energy at $-d$ and $+d$.

3) This is pretty simple with conservation of energy. Note: I'll prove that it's a conservative force in part 5.

At $x=2d$, we have $V = \frac{V_0}{d^4}\left((2d)^4 - 2(2d)^2 d^2 + d^4\right) = \frac{V_0}{d^4}\left(16d^4 - 8d^4 + d^4\right) = 9V_0$.

At $x=d$, we trivially have $V = \frac{V_0}{d^4}\left(d^4 - 2d^4 + d^4\right) = 0$.

With a change in potential of a drop of $9V_0$, we know $KE = 9V_0$ at $x=d$.

since we started with $KE=0$

$$\frac{1}{2}mv^2 = 9V_0 \Rightarrow mv^2 = 18V_0 \Rightarrow \boxed{v = \sqrt{\frac{18V_0}{m}}}$$

4) $x=d$ with v_0. v_0 to reach $x=-d$.

The "hump" we have to get over is when $x=0$.

This potential is $V(x=0) = V_0$. ← This is the minimum kinetic energy we need, since $V(x=d)=0$, so we need to convert V_0 from ~~potential to~~ kinetic to potential on our way over the hump.

So $\frac{1}{2}mv_0^2 \geq V_0$

$$\Rightarrow \boxed{v_0 \geq \sqrt{\frac{2V_0}{m}}}$$

technically exclusively greater than, not greater than or equal to. If $v_0 = \sqrt{\frac{2V_0}{m}}$ then the particle would get stuck ~~freeze~~ at $x=0$ in unstable equilibrium.

$F = -\nabla V = -\nabla\left(\frac{V_0}{d^4}(x^4 - 2x^2d^2 + d^4)\right)$. part 1

we're in 1D and already calculated $\frac{d}{dx}V(x)$

5) $\boxed{F_{total} = 4x\frac{V_0}{d^2} - 4x^3\frac{V_0}{d^4}}$

It will be conservative. 1D and the potential depend only on the position.

$\boxed{a = \left(4x\frac{V_0}{d^2} - 4x^3\frac{V_0}{d^4}\right)/m}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F & 0 & 0 \end{vmatrix}$$

all terms are quite trivially 0.

$$\frac{\partial}{\partial z}F\hat{y} - \frac{\partial}{\partial y}F\hat{z} = 0 - 0 = 0.$$

$\boxed{\text{Yes, it is a conservative force!}}$ It depends only on the position and $\vec{\nabla}\times\vec{F} = 0$ which tells us that the line integral of the force around any closed loop is constant, and the work done by the force does not depend on the path.

6) linear momentum:

Linear momentum (p) conserved in $\frac{dp}{dt} = 0$. Since $p = mv$ and mass is constant, $\frac{dp}{dt} = 0$ if velocity is constant, meaning zero acceleration, and therefore 0 force. We calculated $F = 4x\frac{V_0}{d^2} - 4x^3\frac{V_0}{d^4}$, which has zeros when $x = -d, 0, +d$. We could have also read this from the equilibrium points in part 1.

$\Rightarrow$ In general, linear momentum is not conserved, it only is at the instantaneous points $x = -d, 0, d$.

angular momentum:

We know $\vec{L} = \vec{r}\times\vec{p}$ and $\frac{d\vec{L}}{dt} = \vec{r}\times\vec{F}$, so in this case $\frac{d\vec{L}}{dt} = \vec{r}\times x$ "x" cross product

$$\tau = \frac{d\vec{L}}{dt}$$

Now let's calculate the torque $\tau = \frac{d\vec{L}}{dt}$, and if it's zero then angular momentum is conserved.

6 cont.)

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 0 & 0 \\ F & 0 & 0 \end{vmatrix}$$

Force is only in x-direction.

$$\tau = 0+0+0-0-0-0=0.$$

All terms of the cross product are clearly zero since they all have a zero in what you're multiplying

Since $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$, angular momentum $\underline{\underline{\text{is}}}$ conserved.

Part 2

i) Minimum where $V(x)$ has 0 slope i.e. $\frac{dV}{dx} = 0$.

$$V(x) = \frac{-10}{x} + \frac{3}{x^2} + x = -10x^{-1} + 3x^{-2} + x$$

$$\frac{dV}{dx} = 10x^{-2} - 6x^{-3} + 1 = 0 = \frac{10}{x^2} - \frac{6}{x^3} + 1 = 0$$

$$\Rightarrow 10x - 6 + x^3 = 0$$

$$x^3 + 10x - 6 = 0 \Rightarrow x \approx .5804$$

cubic formula or realistically just looking it up online

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2) $V(x) = \frac{-10}{x} + \frac{3}{x^2} + x$

$F = -\nabla V = -\frac{d}{dx}V$ since we're in one dimension.

$$\Rightarrow F = -\left(\frac{10}{x^2} - \frac{6}{x^3} + 1\right) = \frac{-10}{x^2} + \frac{6}{x^3} - 1.$$

$$\vec{\nabla} \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F & 0 & 0 \end{vmatrix} = 0 + \frac{\partial}{\partial z}F\hat{y} + 0 - \frac{\partial}{\partial y}F\hat{z} - 0 - 0$$

Force only has x-component

$$= \frac{\partial}{\partial z}F \cdot \hat{y} - \frac{\partial}{\partial y}F \cdot \hat{z} = 0$$

Since F has no y or z, both are 0.

Since $\vec{\nabla} \times F = 0$, the force is energy conserving.

3) $v = 0$ at ~~x~~ $x = 2$.

We have no kinetic energy, so just potential.

$$E = V(2) = \frac{-10}{2} + \frac{3}{4} + 2 = \boxed{-2.25}$$ ← total energy

At this point, the particle will be accelerated in the $-x$ direction, and without any energy loss will oscillate around the ~~...~~ minimum $(x = 0.5804)$. The other point it will get up to will have the same potential energy, but will be closer ~~to~~ to the minimum since $V(x)$ is steeper on that side of the minimum.

Solving for this point: $V(x) = -2.25 = \frac{-10}{x} + \frac{3}{x^2} + x \Rightarrow x - \frac{10}{x} + \frac{3}{x^2} + 2.25 = 0$

$$\Rightarrow x^3 + 2.25x^2 - 10x + 3 = 0$$

$$\Rightarrow \boxed{x = 0.3277}$$

the other zero besides 2 is negative, so not what we're looking for.

Increasing x will <u>not</u> allow a particle with $KE_0 = 0$ to escape the ~~...~~ well. If we look at $F(x) = \frac{-10}{x^2} + \frac{6}{x^3} - 1$, the force will always be negative as you increase x, so the particle will always stay trapped.

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$F = \frac{-10}{x^2} + \frac{6}{x^3} - 1$

$F = ma$

$a = \frac{F}{m}$