1) $V(r) = \frac{-\alpha}{r}$, use μ and L. energy E, $\vec{r}_{min}$.

$$r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos(\phi)}$$ using $r_{min} \Rightarrow E = \frac{-\alpha}{r_{min}} + \frac{L^2}{2\mu r_{min}^2}$ and $\dot{r} = 0$

From the hint, I'll start looking at the energy.

This potential $V(r) = -\frac{\alpha}{r}$ is simply the gravitational potential.

$$F_{eff} = \frac{-dV}{dr} + \frac{L^2}{\mu r^3}$$

$$V_{eff} = V(r) + \frac{L^2}{2\mu r^2}$$

just writing down equations where I figure out where to go with this.

kinetic energy $\Rightarrow$ $K = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2)$ we know $\dot{r} = 0$ at r_{min}.

$\Downarrow$ at r_{min}, $K = \frac{1}{2}\mu r_{min}^2 \dot{\phi}^2$

$$E = K + V = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{-\alpha}{r}$$

in HW 8 showed $\dot{\phi} = \frac{L}{\mu r^2} \Rightarrow L = \mu r^2 \dot{\phi}$

$$\Rightarrow E = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{\alpha}{r} = \frac{1}{2}\mu\dot{r}^2 + \frac{r^2\dot{\phi}^2\mu}{2} - \frac{\alpha}{r} = \frac{1}{2}\mu\dot{r}^2 + \frac{L^2}{2\mu r^2} - \frac{\alpha}{r}$$

$$E = \frac{1}{2}\mu\dot{r}^2 + \frac{L^2}{2\mu r^2} - \frac{\alpha}{r}$$

welp, I've basically just derived the hint, since $\dot{r} = 0$ at r_{min}.

The only thing I can think of is ~~[illegible]~~ solving for A in terms of E and r_{min}. Using hint we could find r_{min} and plug that in.

$$r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos\phi}$$

$$r(\phi)\left[\frac{\mu\alpha}{L^2} + A\cos\phi\right] = 1 \Rightarrow \frac{\mu\alpha}{L^2} + A\cos\phi = \frac{1}{r(\phi)}$$

$$A\cos\phi = \frac{1}{r(\phi)} - \frac{\mu\alpha}{L^2}$$

$$A = \frac{1/r(\phi) - \frac{\mu\alpha}{L^2}}{\cos\phi}$$

we showed in class $r(\phi) = \frac{c}{1 + \epsilon\cos\phi} = \frac{L^2/\mu\alpha}{1 + DL^2/\mu\alpha}$

shows $\phi = 0$ gives r_{min}.

with $\Phi = 0$ giving r_{min}, $\cos(\Phi) = \cos(0) = 1$, so we can simplify our equation to

$$A = \frac{1/r_{min} - \frac{\mu\alpha}{L^2}}{1} \Rightarrow \boxed{A = \frac{1}{r_{min}} - \frac{\mu\alpha}{L^2}}$$

So we just need r_{min}. Write it in terms of energy maybe?

$$E = \frac{-\alpha}{r_{min}} + \frac{L^2}{2\mu r_{min}^2}$$

$$E r_{min}^2 = -\alpha r_{min} + \frac{L^2}{2\mu}$$

$$E r_{min}^2 + \alpha r_{min} - \frac{L^2}{2\mu} = 0$$

I missed a negative

~~If we had minus here we'd have negative distance, which is impossible~~

$$r_{min} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4E\left(\frac{-L^2}{2\mu}\right)}}{2E}$$

~~$r_{min} = -\alpha + \sqrt{\alpha^2}$~~

$$r_{min} = \frac{-\alpha \pm \sqrt{\alpha^2 + 4E\frac{L^2}{2\mu}}}{2E}$$

~~$\sqrt{\alpha^2 + 4E\frac{L^2}{2\mu}}$ always larger than α, so I think we're free to take the negative to get smallest possible r for r_{min}.~~

$r_{min} =$ E should be negative!

with negative E, $\sqrt{\alpha^2 + 4E\frac{L^2}{2\mu}} = \sqrt{\alpha^2 - \text{something}} < \alpha$

Numerator will be negative regardless. I think we can choose the + to get smallest possible numerator. ← -larger + smaller

$$r_{min} = \frac{-\alpha + \sqrt{\alpha^2 + 4E\,L^2/2\mu}}{2E}$$

gives $$\boxed{A = \frac{2E}{-\alpha + \sqrt{\alpha^2 + 4E\,L^2/2\mu}} - \frac{\mu\alpha}{L^2}}$$

2) $V(r) = \frac{-\alpha}{r}$. $F = -\frac{\alpha}{r^2}$, m, L.

$$\ddot{r} = -\frac{1}{m}\frac{dV(r)}{dr} + r\dot{\phi}^2 = \frac{-\alpha}{mr^2} + r\dot{\phi}^2$$

$$\dot{\phi} = \frac{L}{mr^2}$$

a) r_{min} of circular orbit. At r_{min}, $\ddot{r} = \dot{r} = 0$.

$$V_{eff} = V(r) + \frac{L^2}{2mr^2}$$

$$V_{eff} = \frac{-\alpha}{r} + \frac{L^2}{2mr^2}$$

at r_{min}, $\frac{dV_{eff}}{dr} = 0$.

$$\frac{\alpha}{r^2} - \frac{2L^2}{2mr^3} = 0$$

$$r\alpha - \frac{L^2}{m} = 0 \Rightarrow \boxed{r_{min} = \frac{L^2}{m\alpha}}$$

b) $\dot{r} = \ddot{r} = 0$. $\dot{\phi} = \frac{L}{mr_{min}^2}$ Not sure why we can't just plug in $r_{min} = \frac{L^2}{m\alpha}$

This would give $\dot{\phi} = \frac{L}{m\left(\frac{L^2}{m\alpha}\right)^2}$

$$= \frac{L}{m \cdot \frac{L^4}{m^2\alpha^2}} = \frac{L}{\frac{L^4}{m\alpha^2}} = \frac{m\alpha^2}{L^3}$$

We've only found magnitude, so $\boxed{\dot{\phi} = \pm\frac{m\alpha^2}{L^3}}$

Using method mentioned in problem, $\ddot{r} = 0 = \frac{F}{m} + \dot{\phi}^2 r$

$$\dot{\phi}^2 r = \frac{-F}{m} = \frac{\alpha}{mr^2}$$

$$\dot{\phi}^2 = \frac{\alpha}{mr^3}$$

$$\dot{\phi} = \pm\sqrt{\frac{\alpha}{mr^3}}$$

plugging in $r = r_{min} = \frac{L^2}{m\alpha}$, we get

$$\dot{\phi} = \pm\sqrt{\frac{\alpha}{m\frac{L^6}{m^3\alpha^3}}} =$$

which is exactly what I had found before

$$\dot{\phi} = \pm\sqrt{\frac{\alpha}{\frac{L^6}{m^2\alpha^3}}} = \pm\sqrt{\frac{m^2\alpha^4}{L^6}} = \boxed{\pm\frac{m\alpha^2}{L^3}}$$

c) Effective spring constant, k.

We may have talked about this in class, but I forgot so I looked through some old textbooks online. Found that k equals second derivative of the potential evaluated at the equilibrium point.

For our effective spring constant, we'll need the second derivative of $V_{eff} = V(r) + \frac{L^2}{2\mu r^2}$

$$= \frac{-\alpha}{r} + \frac{L^2}{2\mu r^2}$$

$$\frac{d}{dr}V_{eff} = \frac{\alpha}{r^2} - \frac{2L^2}{2\mu r^3} = \frac{\alpha}{r^2} - \frac{L^2}{\mu r^3}$$

$$\frac{d^2}{dr^2}V_{eff} = -\frac{2\alpha}{r^3} + \frac{3L^2}{\mu r^4}$$

I wrote μ, but it's actually m in this case.

Our equilibrium point is just r_{min} as found in part a, $r_{min} = \frac{L^2}{m\alpha}$

$$\Rightarrow \frac{-2\alpha}{\left(\frac{L^2}{m\alpha}\right)^3} + \frac{3L^2}{m\cdot\left(\frac{L^2}{m\alpha}\right)^4} = \frac{-2\alpha^4 m^3}{L^6} + \frac{3L^2}{m\cdot\frac{L^8}{m^4\alpha^4}}$$

$$= \frac{-2\alpha^4 m^3}{L^6} + \frac{3L^2\cdot m^3\alpha^4}{L^8} = \frac{-2\alpha^4 m^3}{L^6} + \frac{3m^3\alpha^4}{L^6}$$

$$= \boxed{\frac{m^3\alpha^4}{L^6}} \leftarrow \frac{m^3\alpha^4}{L^6}$$

I originally dropped a negative and ended with -5. This,

d) Should be able to just use $\omega = \sqrt{\frac{k}{m}}$

$$\omega = \sqrt{\frac{m^3\alpha^4/L^6}{m}} = \sqrt{\frac{m^2\alpha^4}{L^6}} = \boxed{\frac{m\alpha^2}{L^3}}$$

This is the same magnitude as our solution for $\dot{\phi}$ in part b :)

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3] Same setup $V(r) = -\frac{\alpha}{r}$ & so that $A = \frac{m\alpha}{L^2}$ $r = \frac{1}{\frac{m\alpha}{L^2} + A\cos\theta}$ & $\overset{\text{same as before}}{}$

$$r = \frac{2r_0}{1+\cos\theta} \qquad r_0 = \frac{L^2}{2m\alpha}$$

a) Total energy $E = K+V$. Same equation as 1 where I used $E = K+V = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{\alpha}{r}$

Again, in HW 8 we showed $\dot{\phi} = \frac{L}{mr^2}$

we have $\dot{\phi}$ and r, so we can calculate $\dot{r}$ and plug everything in.

$r = \frac{2r_0}{1+\cos\theta}$ r_0 has no θ dependence, so

$$\dot{r} = 2r_0 \frac{d}{d\theta}\left[(1+\cos\theta)^{-1}\right]$$

$$\dot{r} = 2r_0\left(-1\cdot(1+\cos\theta)^{-2}\cdot -\sin\theta\right) = \boxed{\frac{2r_0\sin\theta}{(1+\cos\theta)^2}} \leftarrow \dot{r}$$

Plugging things in, $E = \frac{1}{2}m\left(\left(\frac{2r_0\sin\theta}{(1+\cos\theta)^2}\right)^2 + \left(\frac{2r_0}{1+\cos\theta}\right)^2\cdot\left(\frac{L}{mr^2}\right)^2\right) - \frac{\alpha(1+\cos\theta)}{2r_0}$

still have r here

$$E = \frac{1}{2}m\left[\left(\frac{2r_0\sin\theta}{(1+\cos\theta)^2}\right)^2 + \left(\frac{2r_0}{1+\cos\theta}\right)^2\cdot\left(\frac{L(1+\cos\theta)^2}{m4r_0^2}\right)^2\right] - \frac{\alpha(1+\cos\theta)}{2r_0}$$

$$= \frac{1}{2}m\left[\left(\frac{L^2\sin\theta}{m\alpha(1+\cos\theta)^2}\right)^2 + \left(\frac{L^2}{m\alpha(1+\cos\theta)}\right)^2\cdot\left(\frac{L(1+\cos\theta)^2}{4m\left(\frac{L^4}{4m^2\alpha^2}\right)}\right)^2\right] - \frac{\alpha(1+\cos\theta)}{2r_0}$$

$$= \frac{1}{2}m\left[\left(\frac{L^2\sin\theta}{m\alpha(1+\cos\theta)^2}\right)^2 + \left(\frac{L^2}{m\alpha(1+\cos\theta)}\right)^2\cdot\left(\frac{m(1+\cos\theta)^2\cdot\alpha^2}{L^3}\right)^2\right] - \frac{\alpha^2(1+\cos\theta)\cdot m}{L^2}$$

$$= \frac{1}{2}m\left[\left(\frac{L^2\sin\theta}{m\alpha(1+\cos\theta)^2}\right)^2 + \left(\frac{L^2}{m\alpha(1+\cos\theta)}\right)^2\cdot\left(\frac{m\alpha^2(1+\cos\theta)^2}{L^3}\right)^2 - \frac{2\alpha^2(1+\cos\theta)}{L^2}\right]$$

$$= \frac{1}{2}m\left[\left(\frac{L^2\sin\theta}{m\alpha(1+\cos\theta)^2}\right)^2 + \left(\frac{\alpha(1+\cos\theta)}{L}\right)^2 - \frac{2\alpha^2(1+\cos\theta)}{L^2}\right]$$

$$= \frac{1}{2}m\left[\frac{L^4\sin^2\theta}{m^2\alpha^2(1+\cos\theta)^4} + \frac{\alpha^2(1+\cos\theta)^2}{L^2} - \frac{2\alpha^2(1+\cos\theta)}{L^2}\right]$$

$$= \frac{m}{2}\cdot\frac{1}{L^2m^2\alpha^2(1+\cos\theta)^4}\left[L^6\sin^2\theta + m^2\alpha^4(1+\cos\theta)^6 - 2m^2\alpha^4(1+\cos\theta)^5\right]$$

After some nasty math, this should equal zero.

Trig substitutions and stuff. //

b) $x = x_0 - \frac{y^2}{R}$. Refer back to question 1 where I mentioned $r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$

we showed in class back on 3/31/23 that $E=0$ (from part a)

implies that $\varepsilon = 1$, so we have

$$r(\phi) = \frac{c}{1+\cos\phi}$$

We could also show $\varepsilon = 1$ since $\varepsilon = \frac{AL^2}{\alpha\mu}$, and with

$$A = \frac{m\alpha}{L^2} \Rightarrow \varepsilon = \frac{m\alpha}{L^2} \cdot \frac{L^2}{\alpha m} = 1 \checkmark$$

Either way, $r(\phi) = \frac{c}{1+\cos\phi}$ where $c = \frac{L^2}{\mu\alpha}$

all of this was done in class on 4/3/23

$$r(\phi) = \frac{c}{1+\cos\phi} \Rightarrow r(1+\cos\phi) = c$$

$$\Rightarrow r + r\cos\phi = c$$

$$\Rightarrow r + x = c \quad \text{since } x = r\cos\phi$$

$$r = c - x$$

$$r^2 = c^2 - 2cx + x^2$$

$$x^2+y^2 = c^2 - 2cx + x^2 \quad \text{since } r^2 = x^2+y^2$$

$$y^2 = c^2 - 2cx$$

$$y^2 = \left(\frac{L^2}{\mu\alpha}\right)^2 - 2\left(\frac{L^2}{\mu\alpha}\right)x$$

Want to rewrite in terms of x, so I'll actually go back to $y^2 = c^2 - 2xc$ and put c in after.

I keep using μ on accident, but it's actually m.

$$-2cx = y^2 - c^2$$

$$x = \frac{-y^2}{2c} + \frac{c^2}{2c} = \frac{-y^2}{2c} + \frac{c}{2}$$

Now I'll plug in $c = \frac{L^2}{\mu\alpha}$

$$x = \frac{-y^2}{\frac{2L^2}{\mu\alpha}} + \frac{L^2/\mu\alpha}{2} \quad \frac{L^2}{2m\alpha} - \frac{y^2}{2L^2/m\alpha}$$

$\leftarrow$ should be m, not μ.

$$\boxed{x = \frac{L^2}{2\mu\alpha} - \frac{y^2}{2L^2/\mu\alpha}}$$

so $x = x_0 - \frac{y^2}{R}$ where $x_0 = \frac{L^2}{2\mu\alpha}$

and $R = \frac{2L^2}{\mu\alpha}$

$4r_0 = 4 \cdot \frac{L^2}{2\mu\alpha} = \frac{2L^2}{\mu\alpha}$.

in terms of $r_0 = \frac{L^2}{2m\alpha}$, $\boxed{x = r_0 - \frac{y^2}{4r_0}}$ $\leftarrow$ Final

4) $r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$, same as I've been using. for $\varepsilon > 1$, $\frac{(x-\delta)^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$

find α, β, δ in terms of c and ε.

we have $\varepsilon > 1$, $E > 0$

$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$

Denominator vanishes when $\cos\phi = \frac{-1}{\varepsilon}$, so it must diverge.

goes to 0

~~$x = \varepsilon\cos\phi$~~

$r(1+\varepsilon\cos\phi) = c$

$r + r\varepsilon\cos\phi = c$

using $x = r\cos\phi$, $r + \varepsilon x = c$ or $r = c - \varepsilon x$.

Squaring gives $r^2 = c^2 + \varepsilon^2 x^2 - 2c\varepsilon x$

since $r^2 = x^2 + y^2$, $x^2 + y^2 = c^2 + \varepsilon^2 x^2 - 2c\varepsilon x$

$x^2(1-\varepsilon^2) + y^2 + 2c\varepsilon x = c^2$

$x^2(\varepsilon^2 - 1) - y^2 - 2c\varepsilon x = -c^2$

If we define $\delta = \frac{c\varepsilon}{\varepsilon^2-1}$ and $\alpha = \frac{c}{\varepsilon^2-1}$, so $\delta = \varepsilon\alpha$.

$\beta = \frac{c}{\sqrt{\varepsilon^2-1}}$

Doing this allows us to rewrite as $(x-\delta)^2(\varepsilon^2-1) - y^2 = c^2 + \frac{\varepsilon^2 c^2}{\varepsilon^2-1}$

Which simplifies to $\frac{(x-\delta)^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$

So we have the desired form, with $\boxed{\alpha = \frac{c}{\varepsilon^2-1},\ \beta = \frac{c}{\sqrt{\varepsilon^2-1}} \text{ and } \delta = \frac{c\varepsilon}{\varepsilon^2-1} = \varepsilon\alpha}$

or $\dot{\vec{r}}$

Note when I write something like $\dot{r}_1$, it's actually a vector $\dot{\vec{r}}_1$, I just don't like writing dots and vector symbols at once.

5) $V(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6\right]$ $\quad r = |\vec{r}_i - \vec{r}_j|$

a) we start off with the usual $\boxed{\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}}$ and $\boxed{\vec{r} = \vec{r}_1 - \vec{r}_2}$

We quite trivially get $M\ddot{R} = 0$ where $M = m_1+m_2$, so $\dot{R}$ = constant

insert here

If we choose to set $\dot{R} = 0$, we then have $\mathcal{L}_{CM} = 0$ since we have $\mathcal{L}_{CM} = \frac{1}{2}M\dot{R}^2 = \frac{1}{2}M\cdot 0^2 = 0$.

Thus our total Lagrangian $\mathcal{L}$ is simply $\mathcal{L} = \mathcal{L}_{CM} + \mathcal{L}_{rel} = \mathcal{L}_{rel} = \frac{1}{2}\mu\dot{r}^2 - V(r)$

In general $\vec{L} = \vec{r}_1\times\vec{p}_1 + \vec{r}_2\times\vec{p}_2 = m_1\vec{r}_1\times\dot{\vec{r}}_1 + m_2\vec{r}_2 \times \dot{\vec{r}}_2$

We also showed very early on in class that $\vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r}$

$\vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}$

pretty straightforward from definitions of $\vec{R}$ and $\vec{r}$ above

all of this is justification for Lagrangian

This gives kinetic energy ~~[illegible]~~

$K = \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2)$ using standard kinetic energy $= \frac{1}{2}mv^2$ formula.

$= \frac{1}{2}\left(m_1\left[\dot{R} + \frac{m_2}{M}\dot{r}\right]^2 + m_2\left[\dot{R} - \frac{m_1}{M}\dot{r}\right]^2\right)$ by assuming $\frac{dm_i}{dt} = 0$

$= \frac{1}{2}\left(M\dot{R}^2 + \frac{m_1m_2}{M}\dot{r}^2\right)$

$= \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2$ using standard $\mu = \frac{m_1m_2}{M} = \frac{m_1m_2}{m_1+m_2}$

$\Rightarrow \mathcal{L} = K - V = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2 - V(r)$

call $\mathcal{L}_{CM} = \frac{1}{2}M\dot{R}^2$ and $\mathcal{L}_{rel} = \frac{1}{2}\mu\dot{r}^2 - V(r)$

If we plug $\vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r}$ and $\vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}$ ~~[illegible]~~

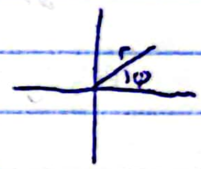
into $\vec{L} = m_1\vec{r}_1\times\dot{\vec{r}}_1 + m_2\vec{r}_2\times\dot{\vec{r}}_2$

in CM frame → we get $\vec{L} = \frac{m_1m_2}{M^2}\left(m_2\vec{r}\times\dot{\vec{r}} + m_1\vec{r}\times\dot{\vec{r}}\right) = \frac{\mu}{M}\left(m_2\vec{r}\times\dot{\vec{r}} + m_1\vec{r}\times\dot{\vec{r}}\right)$

$= \frac{\mu(m_1+m_2)}{M}(\vec{r}\times\dot{\vec{r}}) = \boxed{\mu(\vec{r}\times\dot{\vec{r}})}$

- This result shows that the angular momentum can be interpreted as that of a single particle mass μ position $\vec{r} = \vec{r}_1 - \vec{r}_2$.
- By conservation of angular momentum, $\frac{d\vec{L}}{dt} = 0$ implies that $\mu(\vec{r} \times \dot{\vec{r}})$ is also constant. Since μ is constant (defined by masses) $\vec{r} \times \dot{\vec{r}}$ is also constant. This means (since the direction of $\vec{r} \times \dot{\vec{r}}$ is constant) that $\vec{r}$ and $\dot{\vec{r}}$ always lie in the same plane, we can just define ~~plane along~~ two dimensions (e.g. x and y) to span this plane, so we've reduced it to a problem in only two dimensions.

b) $r \in [0, \infty)$ $\phi \in [0, 2\pi]$. Show $K = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2)$



We know kinetic energy $K = \frac{1}{2}mv^2$.

~~We need v_x and v_y~~

We need v_r and v_ϕ instead of v_x and v_y

v_r is trivially $\dot{r}$

$v_\phi = r \cdot \dot{\phi}$, since it's tangential velocity.

In cartesian $K = \frac{1}{2}m(v_x^2 + v_y^2)$

In polar, $K = \frac{1}{2}m(v_r^2 + v_\phi^2) = \frac{1}{2}m(\dot{r}^2 + (r\dot{\phi})^2) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$

In part a we proved the 2 particles can be interpreted as a singular particle of mass μ at position $\vec{r}$,

thus $\boxed{K = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2)}$

c) I actually did this using generic potential energy in part a.

There I did $\mathcal{L} = K - V = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2 - U(r)$

Using part b, I can now write

$$\mathcal{L} = K - V = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\phi}^2\right) - V$$

$$\Rightarrow \boxed{\mathcal{L} = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\phi}^2\right) - 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]}$$

No, the potential energy is just $V = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$ which doesn't depend on ϕ.

d) $\frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} = 0$

$$\frac{\partial \mathcal{L}}{\partial r} = \mu r\dot{\phi}^2 - 4\varepsilon\left[\frac{6\sigma^6}{r^7} - \frac{12\sigma^{12}}{r^{13}}\right]$$ ~~[illegible]~~

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = \mu\dot{r}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} = \mu\ddot{r}$$

Plugging into $\frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} = 0$ gives

$$\mu r\dot{\phi}^2 - 4\varepsilon\left[\frac{6\sigma^6}{r^7} - \frac{12\sigma^{12}}{r^{13}}\right] - \mu\ddot{r} = 0$$

$$\Rightarrow \boxed{\mu\ddot{r} = \mu r\dot{\phi}^2 - 4\varepsilon\left[\frac{6\sigma^6}{r^7} - \frac{12\sigma^{12}}{r^{13}}\right]}$$

$$\frac{L^2}{\mu r^3} = \frac{\left(\mu r^2\dot{\phi}^2\right)^2}{\mu r^3} = \mu r\dot{\phi}^2$$

Yes, this is exactly what we'd expect. $\mu\ddot{r}$ is mass times acceleration, so force. The $\mu r\dot{\phi}^2$ is equal to the $\frac{L^2}{\mu r^3}$ that we expect to be present when looking at the center of mass frame ~~[illegible] energy term~~ (centripetal force) and the $-4\varepsilon\left[\frac{6\sigma^6}{r^7} - \frac{12\sigma^{12}}{r^{13}}\right]$ is the negative derivative of the potential, which is the force from that potential. So we essentially have mass times acceleration equals the forces present.

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$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \mu r^2 \dot{\phi} \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{d}{dt}(\mu r^2 \dot{\phi})$$

plugging this in we have $0 - \frac{d}{dt}(\mu r^2 \dot{\phi}) = 0$

can drop negative $\rightarrow$ $\boxed{-\frac{d}{dt}(\mu r^2 \dot{\phi}) = 0}$ $\leftarrow$ We could take this derivative, but this is a nice final form for part e.

$$\boxed{\frac{d}{dt}(\mu r^2 \dot{\phi}) = 0}$$

e) Using $\frac{d}{dt}(\mu r^2 \dot{\phi}) = 0$

Recall $\dot{\phi} = \frac{L}{r^2 \mu} \Rightarrow \frac{d}{dt}\left(\mu r^2 \cdot \frac{L}{r^2 \mu}\right) = 0 \Rightarrow \boxed{\frac{d}{dt}(L) = 0}$

Thus angular momentum is indeed conserved :)