1. initial: $V_x = 100$ m/s →

$a(t) = (-20 - 10t)\,\vec{e}_y$ So moves constantly in x-direction.

1a) $v(t) = \int(-20-10t)\,dt = -20t - 5t^2 + c$, in this case, the c is the $V_x = 100$ m/s

Thus $\boxed{\vec{v}(t) = (-20t - 5t^2)\,\vec{e}_y + 100\,\vec{e}_x \quad \text{m/s}}$

1b) $x(t) = \int v(t) = \vec{e}_y \int(-20t - 5t^2)\,dt + \vec{e}_x \int 100\,dt$ starting at origin so $c_1 = c_2 = 0$.

$\boxed{\vec{r}(t) = \left(-10t^2 - \frac{5}{3}t^3\right)\vec{e}_y + 100t\,\vec{e}_x \quad \text{m}}$

1c) Only acts inside $0 \le x \le 2$.

Constant motion in x.

$2 = 100 \cdot t$ $\boxed{t = \frac{1}{50} \text{ second}}$

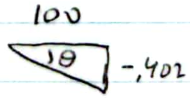
1d) deflection of electron is $r_y\left(\frac{1}{50}\right) = \boxed{-10 \cdot \left(\frac{1}{50}\right)^2 - \frac{5}{3} \cdot \left(\frac{1}{50}\right)^3 = -.004013 \text{ m}}$

1e) Find velocity components at $t = \frac{1}{50}$

$V_x\left(\frac{1}{50}\right) = 100$ m/s

$V_y\left(\frac{1}{50}\right) = -20 \cdot \frac{1}{50} - 5 \cdot \left(\frac{1}{50}\right)^2 = -.402$

Thus our velocity components are



$\tan(\theta) = \frac{-.402}{100}$

$\boxed{\theta = -.23^\circ}$

2 2.3 a) $f_{quad} = \kappa \varrho A v^2$

$f_{lin} = 3\pi\eta D v$

$$\frac{f_{quad}}{f_{lin}} = \frac{\kappa \varrho A v^2}{3\pi \eta D v}$$

for a sphere, $\kappa = \frac{1}{4}$. $A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$

$$\frac{\kappa \varrho A v^{\cancel{2}}}{3\pi \eta D \cancel{v}} = \frac{\frac{1}{4}\varrho \frac{\pi D^{\cancel{2}}}{4} v}{3\cancel{\pi}\eta \cancel{D}} = \frac{\varrho D v}{48\eta} = \underbrace{\frac{D v \varrho}{\eta}}_{R} \cdot \frac{1}{48} = \boxed{\frac{R}{48}}$$

b) $D = 2\,mm$, $v = 5\,cm/s$, $\varrho = 1.3\,g/cm^3$, $\eta = 12\,\frac{N\cdot s}{m^2}$

$D = .002\,m$, $v = .05\,m/s$, $\varrho = 1{,}300{,}000\,g/m^3 = 1300\,kg/m^3$, $\eta = 12\,\frac{N\cdot s}{m^2}$

$$R = \frac{Dv\varrho}{\eta} = \frac{.002 \cdot .05 \cdot 1300}{12} = \boxed{.0108\overline{3}}$$

unitless as $\dfrac{m \cdot m/s \cdot kg/m^3}{\frac{kg\cdot m}{s^2}\cdot\frac{s}{m^2}} = \dfrac{\frac{kg}{sm}}{\frac{kg}{ms}} = $ none

3 2.6 a) eqn 2.33: $v_y(t) = v_{ter}\left(1 - e^{-t/\tau}\right)$ $\qquad v_{ter} = \frac{mg}{b}$ $\qquad k = \frac{b}{m}$

$v_y(t) = v_{ter} - v_{ter}e^{-t/\tau}$ $\qquad \tau = \frac{1}{k} = \frac{m}{b}$

Taylor expansion around $t=0$: $v_y(0) = v_{ter} - v_{ter}e^0 = v_{ter} - v_{ter} = 0$

$\dot{v}_y(0) = -v_{ter}\,e^{-t/\tau} \cdot -\frac{1}{\tau} = \frac{v_{ter}}{\tau}e^0 = \frac{v_{ter}}{\tau}$

Only need the first 2 terms so I'll stop differentiating here.

So our Taylor Expansion gives us $v_y(t) = 0 + \frac{v_{ter}/\tau}{1!}(t-0) = \frac{v_{ter}}{\tau}\cdot t = \frac{mg/b}{m/b}\cdot t$

$= g \cdot t$

$\boxed{\text{So } v_y(t) = gt \text{ for small } t.}$

b) eqn 2.35: $y(t) = V_{ter}t + (V_{y_0} - V_{ter})\tau(1 - e^{-t/\tau})$

$V_{y_0} = 0 \Rightarrow y(t) = V_{ter}t - V_{ter}\tau(1 - e^{-t/\tau}) = V_{ter}t - V_{ter}\tau + V_{ter}\tau e^{-t/\tau}$

Taylor expand around $t=0$: $y(0) = 0 - V_{ter}\tau(1 - e^0) = -V_{ter}\tau(1-1) = 0$

$\dot{y}(0) = V_{ter} + V_{ter}\tau e^{-t/\tau} \cdot \frac{-1}{\tau} = V_{ter} - V_{ter}e^0 = V_{ter} - V_{ter} = 0$

note: $\dot{y}(t) = V_{ter} - V_{ter}e^{-t/\tau}$

$\ddot{y}(0) = -V_{ter}e^{-t/\tau} \cdot -\frac{1}{\tau} = \frac{V_{ter}}{\tau}e^0 = \frac{V_{ter}}{\tau}$

So a Taylor expansion gives us: $y(t) = 0 + \frac{0}{1}(t-0) + \frac{V_{ter}/\tau}{2}(t-0)^2$

$= \frac{V_{ter}}{2\tau}t^2 = \frac{mg/b}{2 \cdot m/b} \cdot t^2 = \frac{g}{2}t^2$

$\Rightarrow \boxed{y(t) = \frac{1}{2}gt^2 \text{ for small } t}$

4) 2.26) $c = .2 \ \frac{N\,m^2}{s^2}$, $m = 80$ kg

at $t=0$, $v_0 = 20$ m/s coasts under air resistance.

find $\tau = \frac{m}{cv_0}$ $\quad \tau = \frac{80}{.2 \cdot 20} = 20$ $\quad \boxed{\tau = 20s}$

We know $v(t) = \frac{v_0}{1 + t/\tau} = \frac{20}{1 + t/20} = v \Rightarrow v(1 + \frac{t}{20}) = 20 \Rightarrow \frac{20}{v} = 1 + \frac{t}{20}$

$\frac{20}{v} - 1 = \frac{t}{20}$

$t = \frac{400}{v} - 20$

Using this $t = \frac{400}{v} - 20$

to slow to $v = 15$ m/s: $\boxed{t = 6.\bar{6} \ s}$

to slow to $v = 10$ m/s: $\boxed{t = 20 \ s}$ known like this is τ

to slow to $v = 5$ m/s: $\boxed{t = 60 \ s}$

5) thrown from height h. air resistance proportional to v^2

v_0 at $t=t_0$ gravitational force.

$\vec{r}(t)$. origin at floor.

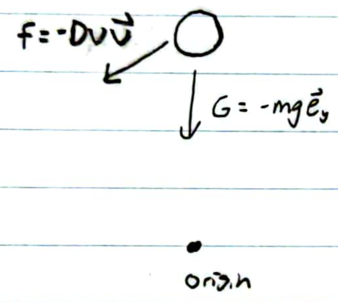
$$\vec{r}(t_0) = h\vec{e}_y$$

$$\vec{v}_0 = v_{x,0}\,\vec{e}_x + v_{y,0}\,\vec{e}_y$$

$$\vec{G} = -mg\,\vec{e}_y$$

$$\vec{f} = -Dv\vec{v}$$

a) forces acting on the ball are the gravitational force and the quadratic air resistance force.



arrow for $\vec{f}$ could be in a different direction depending on values of v_x and v_y. It always points opposite to $\vec{v}$. In this case, I'm assuming the ball was thrown up and to the right.

$$m\vec{a} = -mg\vec{e}_y - Dv\vec{v} \Rightarrow \boxed{\vec{a} = \frac{-mg\vec{e}_y - Dv\vec{v}}{m}} \quad \text{or} \quad \boxed{\vec{a}_x = \frac{-Dv_x|v|}{m} \qquad \vec{a}_y = \frac{-mg - Dv_y|v|}{m}}$$

b) falling in y-direction. integrate from t_0 to t and find v.

$$m\underset{\dot{v}}{a} = -mg - Dv^2$$

one dimensional with $+y$ being up. v is now only in the y-direction

$$v_{ter} = -\sqrt{\frac{-mg}{D}}$$

because $0 = -mg - Dv^2$

$$Dv^2 = -mg$$

$$v^2 = \frac{-mg}{D} \qquad v = \pm\sqrt{\frac{-mg}{D}}$$

in this case our terminal velocity should be negative

$$v_{ter} = -\sqrt{\frac{-mg}{D}}$$

$$-v_{ter} = \sqrt{\frac{-mg}{D}}$$

$$(-v_{ter})^2 = \frac{-mg}{D}$$

$$D = \frac{-mg}{v_{ter}^2}$$

$$m\dot{v} = -mg - \frac{-mg}{v_{ter}^2}v^2$$

$$\dot{v} = -g + \frac{g}{v_{ter}^2}v^2 = g\left(\frac{v^2}{v_{ter}^2} - 1\right)$$

$$\frac{dv}{dt} = g\left(\frac{v^2}{v_{ter}^2} - 1\right)$$

$$\int_{v_0}^{v} \frac{dv}{\frac{v^2}{v_{ter}^2} - 1} = \int_{t_0}^{t} g\,dt$$

$$\int_{v_0}^{v} \frac{dv}{\frac{v^2}{v_{ter}^2} - 1} = \left[-V_{ter}\,\text{arctanh}\left(\frac{v}{v_{ter}}\right)\right]_{v_0}^{v} = -V_{ter}\,\text{arctanh}\left(\frac{v}{v_{ter}}\right) + V_{ter}\,\text{arctanh}\left(\frac{v_0}{v_{ter}}\right)$$

$$\int_{t_0}^{t} g\,dt = \left[gt\right]_{t_0}^{t} = gt - gt_0$$

$$\Rightarrow -V_{ter}\,\text{arctanh}\left(\frac{v}{v_{ter}}\right) + V_{ter}\,\text{arctanh}\left(\frac{v_0}{v_{ter}}\right) = gt - gt_0$$

$$V_{ter}\,\text{arctanh}\left(\frac{v}{v_{ter}}\right) = V_{ter}\,\text{arctanh}\left(\frac{v_0}{v_{ter}}\right) - gt + gt_0$$

$$\text{arctanh}\left(\frac{v}{v_{ter}}\right) = \frac{V_{ter}\,\text{arctanh}\left(\frac{v_0}{v_{ter}}\right) - gt + gt_0}{V_{ter}}$$

$$\frac{v}{V_{ter}} = \tanh\left(\frac{V_{ter}\,\text{arctanh}\left(\frac{v_0}{v_{ter}}\right) - gt + gt_0}{V_{ter}}\right)$$

$$\boxed{V(t) = V_{ter}\tanh\left(\frac{V_{ter}\,\text{arctanh}\left(\frac{v_0}{v_{ter}}\right) - gt + gt_0}{V_{ter}}\right)}$$

velocity at time t for an unknown v_0 at time t_0

if you had $v_0 = 0$, then $v = V_{ter}\tanh\left(\frac{gt_0 - gt}{V_{ter}}\right)$

if you had $v_0 = 0$ and $t_0 = 0$, then $v = V_{ter}\tanh\left(\frac{-gt}{V_{ter}}\right)$

often times, set $V_0 = 0$ and $t_0 = 0$

Our equation could also be simplified as

$$v(t) = \frac{V_{ter}^2\left(\frac{v_0}{v_{ter}}\right)}{V_{ter}} + V_{ter}\tanh\left(\frac{-gt + gt_0}{V_{ter}}\right) \Rightarrow \boxed{v(t) = V_0 + V_{ter}\tanh\left(\frac{-gt + gt_0}{V_{ter}}\right)}$$

$$\boxed{\text{often } v(t) = V_{ter}\tanh\left(\frac{-gt}{V_{ter}}\right)}$$

c) Find $y(t)$. starting at t_0. Find time to hit floor.

Setting $v_0 = 0$, our $v(t)$ simplifies to $v(t) = V_{ter} \tanh\left(\frac{gt_0 - gt}{V_{ter}}\right)$

to find $y(t)$, integrate $v(t)$

$$y(t) = \int V_{ter} \tanh\left(\frac{gt_0 - gt}{V_{ter}}\right) dt$$

$$\boxed{y(t) = \frac{-V_{ter}^2 \ln\left(\cosh\left(\frac{g(t-t_0)}{V_{ter}}\right)\right)}{g} + h}$$ ← where h is initial height

for time to hit floor, we want $y(t) = 0$

$$0 = \frac{\quad}{g} + h$$

$$-hg = -V_{ter}^2 \ln\left(\cosh\left(\frac{g(t-t_0)}{V_{ter}}\right)\right)$$

$$\frac{hg}{V_{ter}^2} = \ln\left(\cosh\left(\frac{g(t-t_0)}{V_{ter}}\right)\right)$$

$$e^{hg/V_{ter}^2} = \cosh\left(\frac{g(t-t_0)}{V_{ter}}\right)$$

$$\operatorname{arccosh}\left(e^{hg/V_{ter}^2}\right) = \frac{g(t-t_0)}{V_{ter}}$$

$$\boxed{t = \frac{V_{ter}}{g} \operatorname{arccosh}\left(e^{hg/V_{ter}^2}\right) + t_0}$$

t_0 is time when ball was thrown, so if you want the time elapsed between throw and the ball hitting the ground this term could be set to 0.