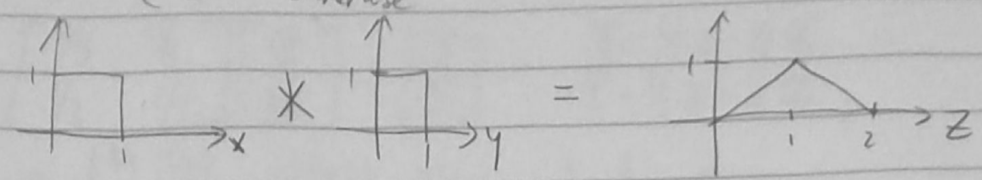
ECEn 370

Homework # 8   23, 24, 25   Benjamin Bergeson

$\boxed{\text{Schaum's 4.21}}$

$f_X(x) = \begin{cases} 1 & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$ $f_Y(y) = \begin{cases} 1 & 0<y<1 \\ 0 & \text{otherwise} \end{cases}$

$f_{XY}(x,y) = \begin{cases} 1 & 0<x<1,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$



$f_Z(z) = \begin{cases} z & 0<z<1 \\ 2-z & 1<z<2 \\ 0 & \text{otherwise} \end{cases}$

$\boxed{\text{Schaum's 4.18}}$

(a) $F_Z(z) = P(Z \le z) = P(X+Y \le z) = \int_{-\infty}^{\infty}\int_{-\infty}^{z-x} f_{XY}(x,y)\,dy\,dx$

$f_Z(z) = \frac{d}{dz}\int_{-\infty}^{\infty}\int_{-\infty}^{z-x} f_{XY}(x,y)\,dy\,dx = \int_{-\infty}^{\infty}\frac{d}{dz}\int_{-\infty}^{z-x} f_{XY}(x,y)\,dy\,dx$

$= \int_{-\infty}^{\infty} f_{XY}(x, z-x)\,dx$

(b) $\int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx$

$\boxed{\text{Schaum's 4.19}}$

$f_{ZW}(z,w) = f_{XY}(x,y)\,|J(x,y)|^{-1}$ $Z = X+Y$

$\begin{bmatrix} Z \\ W \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} Z \\ W \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$ $W = X$

$J = -1$

$f_{ZW}(z,w) = f_{XY}(x,y) = f_{XY}(x, z-w)$

$\int_{-\infty}^{\infty} f_{ZW}(z,w)\,dw = \int_{-\infty}^{\infty} f_{XY}(x, z-x)\,dx$

$\boxed{\text{Schaum's 4.25}}$

$Z = XY \qquad W = X$

$\begin{bmatrix} Z \\ W \end{bmatrix} = \begin{bmatrix} 0 & X \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ \frac{1}{X} & 0 \end{bmatrix} \begin{bmatrix} Z \\ W \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} \qquad X = W \qquad \frac{Z}{X} = Y$

$J = -\frac{1}{x} \qquad f_{ZW}(z,w) = \left|\frac{1}{x}\right| f_{XY}(x,y) = \left|\frac{1}{x}\right| f_{XY}(w, \frac{z}{w})$

$f_Z(z) = \int_{-\infty}^{\infty} f_{ZW}(z,w)\, dw = \int_{-\infty}^{\infty} \left|\frac{1}{x}\right| f_{XY}(w, \frac{z}{w})\, dw$

$\boxed{\text{5.2.5 Q5}}$ $h_1(r,\theta) = r\cos\theta \qquad h_2(r,\theta) = r\sin\theta$

$J = \begin{vmatrix} \frac{dh_1}{dr} & \frac{dh_1}{d\theta} \\ \frac{dh_2}{dr} & \frac{dh_2}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -\sin\theta\, r \\ \sin\theta & \cos\theta\, r \end{vmatrix} = r$

$f_{R\Theta}(r,\theta) = |r|\, f_{XY}(x,y) = r f_{XY}(r\cos\theta, r\sin\theta)$

$= \begin{cases} \frac{r}{\pi} & 0 < r < 1,\ -\pi < \theta < \pi \\ 0 & \text{otherwise} \end{cases}$

Scanned by CamScanner