ECEn 370 | Day 8 | Benjamin Bergeson

Homework #3

2.1.5 Problem 6

$X_1 + X_2 + X_3 + X_4 = 100 \qquad X_1 \in \{1,2,3\cdots\}, X_2 \in \{2,3,4\cdots\}, X_3, X_4 \in \{0,1,2,3\cdots\}$

$Y_1 = X_1 - 1 \quad Y_2 = X_2 - 2 \qquad Y_1 + 1 + Y_2 + 2 + X_3 + X_4 = 100$

$Y_1 + Y_2 + X_3 + X_4 = 97 \qquad Y_1, Y_2, X_3, X_4 \in \{0,1,2,3\cdots\}$

$$\binom{n+k-1}{n-1} = \binom{100}{3}$$

Schaum's 1.39

$|S| = \binom{15}{5} = 3003$

(a) $\binom{5}{2}\binom{10}{3} = (10)(120) = 1200 \qquad \frac{1200}{3003} \approx 0.3996$

(b) $\binom{5}{0}\binom{10}{5} = 252 \qquad \frac{252}{3003} \approx 0.0839$

Schaum's 1.94

$|S| = \binom{12}{2} = 66$

$\binom{8}{2}\binom{4}{0} = 28 \qquad \frac{28}{66} \approx 0.\overline{42}$

DAY 9

3.1.6 Problem 1

(a) $R_X = \{0.2, 0.4, 0.5, 0.8, 1\}$

(b) $0.1 + 0.2 + 0.2 = 0.5$

(c) $0.2 + 0.2 = 0.4$

(d) $\frac{P(0.2)}{P(\{0.2, 0.4, 0.5\})} = \frac{0.1}{0.5} = \frac{1}{5}$

3.1.6 Problem 2

(a) $R_X, R_Y = \{1,2,3,4,5,6\} \qquad P_X(k) = \begin{cases} 1/6 & \text{for } X = 1,2,\cdots 6 \\ 0 & \text{otherwise} \end{cases} = P_Y(k)$

(b) $P(X=2) \cdot P(X=6) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

(c) $P(X>3) = P(\{4,5,6\}) = \frac{1}{2}$

ECEn 370
Homework #3

Benjamin Bergeson

(d) $R_Z = \{2, 3, \cdots 12\}$ $P_Z(k) = \begin{cases} 1/36 & \text{when } k = 2, 12 \\ 2/36 & \text{when } k = 3, 11 \\ 3/36 & \text{when } k = 4, 10 \\ 4/36 & \text{when } k = 5, 9 \\ 5/36 & \text{when } k = 6, 8 \\ 6/36 & \text{when } k = 7 \end{cases}$

(e) $P(X=4 \mid Z=8) = \frac{P(X=4, Z=8)}{P(Z=8)} = \frac{P(X=4)P(Y=4)}{P(Z=8)} = \frac{3/36}{5/36} = \frac{1}{5}$

Schaum's 2.1

(a) $\{1, 2, 3, 4, 5, 6\}$ (b) $\frac{1}{2}$

Schaum's 2.2

(a) $\{0, 1, 2, 3\}$ (b) $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$P(X=0) = (1-p)^3$ $P(X=1) = 3p(1-p)^2$ $P(X=2) = 3p^2(1-p)$ $P(X=3) = p^3$

Day 10

3.1.6 Problem 3

for $k=1$ $P(N=1) = \frac{1}{3}$ $P(N=2) = \frac{1}{3}\left(1 - \frac{1}{3}\right) = \frac{2}{9}$

$P(N=k) = \begin{cases} p(1-p)^{k-1} & \text{for } k = 1, 2, 3 \cdots \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{3}(2/3)^{k-1} & \text{for } k = 1, 2, 3 \cdots \\ 0 & \text{otherwise} \end{cases}$

3.1.6 Problem 4

Let $X = Y + 10$ $P_Y(k) = \begin{cases} \binom{10}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{10-k} & \text{for } k = 1, 2, \cdots 10 \\ 0 & \text{otherwise} \end{cases}$

$R_X = \{10, 11, \cdots 20\}$ $P_X(x) = P(X=x) = P(Y+10=x) = P(Y=x-10)$

$P_X(y) = \begin{cases} \binom{10}{10-k}\left(\frac{1}{4}\right)^{k-10}\left(\frac{3}{4}\right)^{20-k} & \text{for } x = 10, 11 \cdots 20 \\ 0 & \text{otherwise} \end{cases}$

3.1.6 Problem 6

$\lambda = 10\left(\frac{3}{2}\right) = 15$ $P(10 < X \le 15) = P(X=11) + P(X=12) + P(X=13) + P(X=14) + P(X=15)$

$= \left(\frac{e^{-15}(15)^{11}}{11!}\right) + \left(\frac{e^{-15}(15)^{12}}{12!}\right) + \left(\frac{e^{-15}(15)^{13}}{13!}\right) + \left(\frac{e^{-15}(15)^{14}}{14!}\right) + \left(\frac{e^{-15}(15)^{15}}{15!}\right) \approx$

$= e^{-15}\left(\frac{15^{11}}{11!} + \frac{15^{12}}{12!} + \frac{15^{13}}{13!} + \frac{15^{14}}{14!} + \frac{15^{15}}{15!}\right) = 0.4496$

ECEn 370
Homework #3

Benjamin Bergeson

3.1.6 Problem 8

(a) $|I| = \{0, 1, 4, 9\}$

(b) $P_I(y) = P(I=y) = P((X+1)^2 = y) = P(X = \sqrt{y} - 1)$

$P_I(0) = P(X=-1) = \frac{1}{8}$ $\quad P_I(1) = P(X=0) + P(X=-2) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

$P_I(4) = P(X = \pm 2 - 1) = P(X=1) + P(X=-3) = \frac{1}{4}$ $\quad P_I(9) = P(X = \pm 3 - 1) = P(X=2) + P(X=-4) = \frac{1}{4}$

$$P_I(y) = \begin{cases} \frac{1}{8} & \text{when } y=0 \\ \frac{3}{8} & \text{when } y=1 \\ \frac{1}{4} & \text{when } y=4 \\ \frac{1}{4} & \text{when } y=9 \\ 0 & \text{otherwise} \end{cases}$$

Schaum's 2.3

(a) $\{1, 2, 3\}$

(b) $P(X=1) = P(a) = \frac{1}{2}$ $\quad P(X=2) = P(b) = \frac{1}{4}$ $\quad P(X=3) = P(c) + P(d) = \frac{1}{4}$

$P(X>3) = P(\emptyset) = 0$

Schaum's 2.13

(a) $\sum_{i=0}^{\infty} \frac{3}{4}\left(\frac{1}{4}\right)^i = 1$

(b) (i) $P(X=2) = \frac{3}{4}\left(\frac{1}{4}\right)^2 = \frac{3}{64}$

(ii) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = \frac{3}{4} + \frac{3}{16} + \frac{3}{64} = \frac{63}{64}$

(iii) $P(X \ge 1) = 1 - P(X=0) = 1 - \frac{3}{4} = \frac{1}{4}$