5.1.6 Q1

(a) $\frac{1}{12} + \frac{1}{24} + \frac{1}{6} + \frac{1}{12} = \frac{2+1+4+2}{24} = \frac{9}{24} = \frac{3}{8}$

(b) $P_X(x) = \begin{cases} 1/6 & \text{for } x=1 \\ 3/8 & \text{for } x=2 \\ 11/24 & \text{for } x=3 \\ 0 & \text{otherwise} \end{cases}$ $P_Y(y) = \begin{cases} 1/2 & \text{for } y=2 \\ 1/4 & \text{for } y=4 \\ 1/4 & \text{for } y=5 \\ 0 & \text{otherwise} \end{cases}$

(c) $\frac{1/12}{1/12+1/24+1/4} = \frac{1}{2} = P(Y=2 \mid X=1)$

(d) $P(X=1)P(Y=2) = \frac{1}{12}$ $P(X=1)P(Y=4) = \frac{1}{24}$ $P(X=1)P(Y=5) = \frac{1}{24}$

$P(X=2)P(Y=2) = \frac{3}{16}$ not independent

Schaum's 3.14

(a) $\sum_{x_i}\sum_{y_j} P_{XY}(x_i,y_j) = \sum_{x_i=1}^{2}\sum_{y_j=1}^{2} k(2x_i+y_j) = k(2+1+2+2+4+1+4+2) = 18k = 1 \quad k = \frac{1}{18}$

(b) $P_X(x_i) = \sum_{y_j} P_{XY}(x_i,y_j) = \sum_{y_j=1}^{2} \frac{1}{18}(2x_i+y_j) = \frac{1}{18}(2x_i+1) + \frac{1}{18}(2x_i+2) = \frac{1}{18}(4x_i+3) \quad x_i = 1,2$

$P_X(x_i) = \begin{cases} 7/18 & \text{for } x_i = 1 \\ 11/18 & \text{for } x_i = 2 \\ 0 & \text{otherwise} \end{cases}$

$P_Y(y_j) = \sum_{x_i} P_{XY}(x_i,y_j) = \sum_{x_i=1}^{2} \frac{1}{18}(2x_i+y_j) = \frac{1}{18}(2+y_j) + \frac{1}{18}(4+y_j) = \frac{1}{18}(2y_j+6) \quad y_j = 1,2$

$P_Y(y) = \begin{cases} 4/9 & \text{for } y=1 \\ 5/9 & \text{for } y=2 \\ 0 & \text{otherwise} \end{cases}$

(c) $P(X=1)P(Y=1) = \frac{7}{18}\cdot\frac{4}{9} = \frac{14}{81}$ $P(X=1,Y=1) = \frac{1}{18}(2+1) = \frac{3}{18}$ not independent

Schaum's 3.15

(a) $\sum_x\sum_y P_{XY}(x,y) = \sum_{x=1}^{2}\sum_{y=1}^{3} kx^2y = k\left((1)(1)+(1)(2)+(1)(3)+(4)(1)+(4)(2)+(4)(3)\right) = 1$

$= k(1+2+3+4+8+12) = k(30) = 1 \quad k = \frac{1}{30}$

(b) $P_X(x) = \sum_y P_{XY}(x,y) = \sum_{y=1}^{3} \frac{1}{30}x^2y = \frac{1}{30}(x^2+2x^2+3x^2) = \frac{1}{30}(6x^2)$

$P_X(x) = \begin{cases} 1/5 & \text{for } x=1 \\ 4/5 & \text{for } x=2 \\ 0 & \text{otherwise} \end{cases}$

$$P_Y(y) = \sum_x P_{XY}(x,y) = \sum_{x=1}^{2} \frac{1}{30}x^2y = \frac{1}{30}(y+4y) = \frac{1}{30}(5y)$$

$$P_Y(y) = \begin{cases} 1/6 & \text{for } y=1 \\ 2/6 & \text{for } y=2 \\ 3/6 & \text{for } y=3 \\ 0 & \text{otherwise} \end{cases}$$

(c) $P_X(x)\,P_Y(y) = \frac{1}{30}(6x^2)\,\frac{1}{30}(5y) = \frac{x^2}{5}\cdot\frac{y}{6} = \frac{1}{30}(x^2y)$ independent

Schaum's 3.26

(a) $$P_{Y|X}(y|x) = \frac{P_{XY}(x,y)}{P_X(x)} = \frac{\frac{1}{18}(2x+y)}{\frac{1}{18}(4x+3)} = \frac{2x+y}{4x+3}$$

$$P_{X|Y}(x|y) = \frac{P_{XY}(x,y)}{P_Y(y)} = \frac{\frac{1}{18}(2x+y)}{\frac{1}{18}(2y+6)} = \frac{2x+y}{2y+6}$$

(b) $$P(Y=2|X=2) = \frac{2(2)+2}{4(2)+3} = \frac{4+2}{8+3} = \frac{6}{11}$$

$$P(X=2|Y=2) = \frac{2(2)+2}{2(2)+6} = \frac{6}{10} = \frac{3}{5}$$

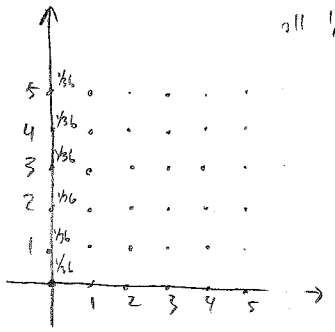
Schaum's 3.27

$$P_{Y|X}(y|x) = \frac{P_{XY}(x,y)}{P_X(x)} = \frac{1/30\,x^2y}{1/5\,x^2} = \frac{1}{6}y$$

$$P_{X|Y}(x|y) = \frac{P_{XY}(x,y)}{P_Y(y)} = \frac{1/30\,x^2y}{1/6\,y} = \frac{1}{5}x^2$$

Day 14

(a)



(b) $Z = X+Y$

$$P_Z(z) = \begin{cases} 1/36 & \text{for } z=0,10 \\ 2/36 & \text{for } z=1,9 \\ 3/36 & \text{for } z=2,8 \\ 4/36 & \text{for } z=3,7 \\ 5/36 & \text{for } z=4,6 \\ 6/36 & \text{for } z=5 \end{cases}$$

(c) $D = X-Y$

$$P_D(d) = \begin{cases} 1/36 & \text{for } d=-5,5 \\ 2/36 & \text{for } d=-4,4 \\ 3/36 & \text{for } d=-3,3 \\ 4/36 & \text{for } d=-2,2 \\ 5/36 & \text{for } d=-1,1 \\ 6/36 & \text{for } d=0 \end{cases}$$

(d) $$P_M(m) = \begin{cases} 1/36 & \text{for } m=0 \\ 3/36 & \text{for } m=1 \\ 5/36 & \text{for } m=2 \\ 7/36 & \text{for } m=3 \\ 9/36 & \text{for } m=4 \\ 11/36 & \text{for } m=5 \end{cases}$$

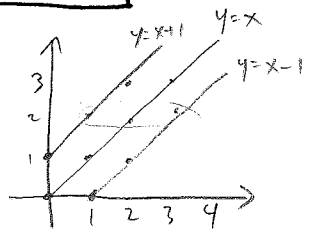
5.1.6 Q3

$$F_Z(z) = P(Z \le z) = P(\max(X,Y) \le z) = P(X \le z \text{ and } Y \le z) = P(X \le z)P(Y \le z) = F_X(z)F_Y(z)$$

$$F_W(w) = P(W \le w) = P(\min(X,Y) \le w) = 1 - P(\min(X,Y) > w) = 1 - P(X > w \text{ and } Y > w)$$
$$= 1 - P(X > w)P(Y > w) = 1 - (1 - F_X(w))(1 - F_Y(w)) = F_X(w) + F_Y(w) - F_X(w)F_Y(w)$$

5.1.6 Q4

(a)



(b) $P_X(0) = P_{XY}(0,0) + P_{XY}(0,1) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$P_X(1) = P_{XY}(1,0) + P_{XY}(1,1) + P_{XY}(1,2) = \frac{1}{6} + \frac{1}{12} + \frac{1}{12} = \frac{1}{3}$

$P_X(2) = P_{XY}(2,1) + P_{XY}(2,2) + P_{XY}(2,3) = \frac{1}{12} + \frac{1}{24} + \frac{1}{24} = \frac{1}{6}$

$$P_X(k) = P_Y(k) = \begin{cases} 1/3 & \text{for } k = 0 \\ \frac{1}{3 \cdot 2^{k-1}} & \text{for } k = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

(c) $P(X = Y \mid X < 2) = \dfrac{P(X=Y, X<2)}{P(X<2)} = \dfrac{P(0,0) + P(1,1)}{P_X(0) + P_X(1)} = \dfrac{1/4}{2/3} = \dfrac{3}{8}$

(d) $P(1 \le X^2 + Y^2 \le 5) = P(0,1) + P(1,0) + P(1,1) + P(2,1) + P(1,2) = \frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{7}{12}$

(e) $P(X = Y) = \frac{1}{3}$

(f) $E[X \mid Y=2]$ $\quad P_{X|Y}(k|2) = \dfrac{P_{XY}(k,2)}{P_Y(2)} = 6P_{XY}(k,2)$

$$P_{X|Y}(k|2) = \begin{cases} 1/2 & \text{for } k=1 \\ 1/4 & \text{for } k=2 \\ 1/4 & \text{for } k=3 \\ 0 & \text{otherwise} \end{cases}$$

$E[X \mid Y=2] = \frac{1}{2} + \frac{2}{4} + \frac{3}{4} = \frac{2+2+3}{4} = \frac{7}{4}$

(g) $\text{Var}(X \mid Y=2) = E[X^2 \mid Y=2] - (E[X \mid Y=2])^2 = \frac{1}{2} + \frac{4}{4} + \frac{9}{4} - \left(\frac{49}{16}\right) = \frac{11}{16}$

5.1.6 Q6

$EX = E[X|H]P(H) + E[X|T]P(T) = E[X|H]p + (1+EX)(1-p)$

$pEX = pE[X|H] + (1-p)$

$E[X|H] = E[X|HH]p + E[X|HT](1-p) = 2p + (2+EX)(1-p) = 2 + (1-p)EX$

$EX = \frac{1+p}{p^2}$

Schaum's 4.17

$Z = X + Y$ $\quad P_X(x) = \frac{e^{-\lambda_1}\lambda_1^x}{x!}$ $\quad P_Y(y) = \frac{e^{-\lambda_2}\lambda_2^y}{y!}$

$P_Z(z) = P(Z=z) = P(X+Y=z) =$ $\quad P_Z(z) = P(x) \cdot P(y) =$ $\quad \frac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{x+y}}{(x+y)!}$