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$\boxed{\text{Schaum's 7.16}}$

$\hat{Y}(x) = c \qquad E[(Y-c)^2] = \int_{-\infty}^{\infty} (y-c)^2 f(y)\, dy$

$c\int_{-\infty}^{\infty} f(y)\,dy = c = \int_{-\infty}^{\infty} y f(y)\, dy = E(Y) = c = \hat{Y}$

$\boxed{\text{Schaum's 7.17}}$

$\hat{Y}(x) = g(x) \qquad E[(Y-g(x))^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (y-g(x))^2 f(x,y)\, dx\,dy$

Since $f(x,y) = f(y|x)\, f(x)$

$\int_{-\infty}^{\infty} f(x) \int_{-\infty}^{\infty} (y-g(x))^2 f(y|x)\, dy\ dx \qquad$ minimum: $\int_{-\infty}^{\infty} (y-g(x))^2 f(y|x)\, dy$

$\hat{Y} = g(x) = \int_{-\infty}^{\infty} y\, f(y|x)\,dy = E[Y/X]$

$\boxed{\text{Schaum's 7.19}}$

$\hat{Y}(x) = X^2 \qquad E[Y|X] = E[X^2 \mid X = x] = x^2$

$E[(Y-X^2)^2] = E[(X^2-X^2)^2] = 0$

$\boxed{\text{6.2.6. Q3}}$

$E[X] = \frac{1}{\lambda} \qquad P(X \geq a) \leq \frac{1}{\lambda a}$

Actual Value of $P(X \geq a) = e^{-\lambda a} \qquad \frac{1}{\lambda a} \geq e^{-\lambda a}$

$\boxed{\text{6.2.6 Q4}}$

$E[X] = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2} \qquad P(|X - EX| \geq b) \leq \frac{1}{\lambda^2 b^2}$

$\boxed{\text{Schaum's 2.38}}$

$P(X \geq a) = \int_a^{\infty} f_X(x)\, dx \qquad E[X] = \int_0^{\infty} x f_X(x)\, dx \geq \int_a^{\infty} x f_X(x)\,dx \geq a\int_a^{\infty} f_X(x)\,dx$

$\int_a^{\infty} f_X(x)\,dx < \frac{E[X]}{a}$

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Schaum's 2.39

$$P(|X-\mu_X| \geq a) = \int_{-\infty}^{\mu_X - a} f_X(x)\,dx + \int_{\mu_X + a}^{\infty} f_X(x)\,dx = \int_{|x-\mu_X| \geq a} f_X(x)\,dx$$

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(x)\,dx \geq \int_{|x-\mu_X| \geq a} (x-\mu_X)^2 f_X(x)\,dx \geq a^2 \int_{|x-\mu_X| \geq a} f_X(x)\,dx$$

$$\int_{|x-\mu_X| \geq a} f_X(x)\,dx \leq \frac{\sigma_X^2}{a^2}$$

Schaum's 4.81

$E[\bar{X}_n] = \mu$ $\quad Var(\bar{X}_n) = \frac{\sigma^2}{n}$

$$P(|\bar{X}_n - \mu| > \varepsilon) \leq \frac{\sigma^2}{n\varepsilon^2} \qquad \lim_{n\to\infty} \frac{\sigma^2}{n\varepsilon^2} = 0 \qquad \lim_{n\to\infty} P(|\bar{X}_n - \mu| > \varepsilon) = 0$$

Schaum's 4.82

$$P\left(|\bar{X}_n - \mu| > \frac{\sigma}{10}\right) \leq \frac{\sigma^2}{n\frac{\sigma^2}{100}} = \frac{100}{n}$$

$$P\left(|\bar{X}_n - \mu| \leq \frac{\sigma}{10}\right) \geq 1 - \frac{100}{n} \qquad 1 - \frac{100}{n} = 0.95 \qquad \frac{100}{n} = 0.05 \qquad n = 2000$$

7.3.0 Q1

(a) $E[X_i] = \int_0^1 x f_X(x)\,dx = \int_0^1 x \, dx = \frac{1}{2}$ $\quad Var(X_i) = \int_0^1 x^2 dx - \left(\frac{1}{4}\right) = \frac{1}{12}$

$E[M_n] = \frac{1}{2}$ $\quad Var(M_n) = \frac{\sigma^2}{n} = \frac{1}{144n}$

(b) $$P\left(|M_n - \tfrac{1}{2}| \geq \tfrac{1}{100}\right) \leq \frac{1}{144n\frac{1}{10000}} = \frac{10000}{144n}$$

(c) $$\lim_{n\to\infty} \frac{10000}{144n} = 0$$