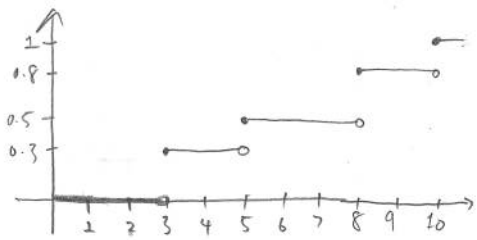
3.2.5 Q1

$$F_X(x) = \begin{cases} 0 & \text{for } x < 3 \\ 0.3 & \text{for } 3 \le x < 5 \\ 0.5 & \text{for } 5 \le x < 8 \\ 0.8 & \text{for } 8 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$



3.2.5 Q2

(a) $EX = 0.4 + 2(0.3) + 3(0.2) = 1.6$

(b) $\text{Var}(X) = E[X^2] - [EX]^2 = 0.4 + 4(0.3) + 9(0.2) - (1.6)^2 = 0.84$

(c) $EY = E(X-2)^2 = (-2)^2(0.1) + (-1)^2(0.4) + (0)^2(0.3) + (1)^2(0.2) = 1$

3.2.5 Q3

$P_Y(y) = P(Y=y) = P(X(X-1)(X-2)=y)$: $P_Y(0) = P(0) + P(1) + P(2) = 0.7$

$P_Y(6) = P_X(3) = 0.3$

$$P_Y(y) = \begin{cases} 0.7 & \text{for } y = 0 \\ 0.3 & \text{for } y = 6 \\ 0 & \text{otherwise} \end{cases}$$

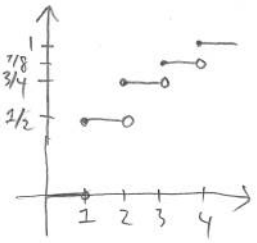
3.2.5 Q4

$E\left[\frac{1}{2^X}\right]$ $\qquad P_X(k) = \begin{cases} pq^{k-1} & \text{for } k = 1, 2, 3 \cdots \\ 0 & \text{otherwise} \end{cases}$ $\qquad q = 1-p$

$$E\left[\frac{1}{2^X}\right] = \sum_{k=1}^{\infty} \frac{1}{2^k} pq^{k-1} = \frac{p}{2} \sum_{k=1}^{\infty} \left(\frac{q}{2}\right)^{k-1} = \frac{p}{2} \frac{1}{1-\frac{q}{2}} = \frac{p}{2-q} = \frac{p}{2-1+p} = \frac{p}{p+1}$$

Schaum's 2.12

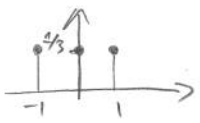
(a) $$F_X(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1/2 & \text{for } 1 \le x < 2 \\ 3/4 & \text{for } 2 \le x < 3 \\ 7/8 & \text{for } 3 \le x < 4 \\ 1 & \text{for } x \ge 4 \end{cases}$$



(b) (i) 0 (ii) $P(1 < X \le 3) = \frac{3}{8}$ (iii) $P(1 \le X \le 3) = \frac{7}{8}$

Schaum's 2.27

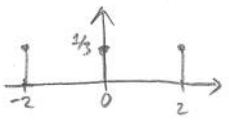
(a)



$EX = -1(1/3) + 1(1/3) = 0$ $\quad Var(X) = E[X^2] - [EX]^2 = 1(\frac{1}{3}) + 1(\frac{1}{3}) - 0$

$= \frac{2}{3}$

(b)



$EX = -2(\frac{1}{3}) + 2(\frac{1}{3}) = 0$ $\quad Var(X) = \frac{4}{3} + \frac{4}{3} - 0 = \frac{8}{3}$

Schaum's 2.28

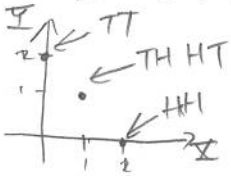
$EX = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = \frac{7}{2}$ $\quad Var(X) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} - \frac{49}{9} = \frac{35}{12}$

5.1.6 Q 2

$R_{XY} = \{(0,10)\ (1,9) \cdots (10,0)\}$ $\quad P_{XY} = \begin{cases} \dfrac{\binom{40}{i}\binom{60}{j}}{\binom{100}{20}} & \text{for } i+j=10,\ i,j \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Schaum's 3.1

(a) $R_X = \{0,1,2\}$ $\quad$ (b) $R_Y = \{0,1,2\}$ $\quad$ (c) $R_{XY} = \{(0,2)(1,1)(2,0)\}$



(d) $P(X=2, Y=0) = \frac{1}{4}$ $\quad P(X=0, Y=2) = \frac{1}{4}$

$P(X=1, Y=1) = \frac{1}{2}$

Schaum's 3.11

(a) $R_X = \{0,1\}$ $\quad R_Y = \{0,1\}$ $\quad R_{XY} = \{(0,0)(0,1)(1,0)(1,1)\}$.

(b) $P_{XY}(x,y) = \begin{cases} \frac{1}{4} & \text{for } x,y = 0,1 \\ 0 & \text{otherwise} \end{cases}$