SOLUTIONS

Quiz #3

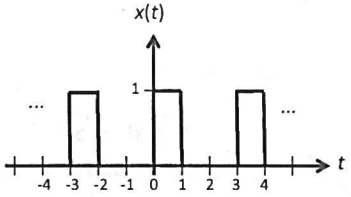
ECEn 380: Signals & Systems

Fall 2015

Closed book, closed note, closed neighbor, no calculators allowed. *Time limit is 15 minutes. 20 points + 2 extra credit points possible.*

HOMEWORK SECTION:

1. Will the periodic waveform $x(t)$ shown below exhibit Gibbs ringing when represented by a Fourier series expansion where only a finite number of terms in the series are preserved? (2 points)

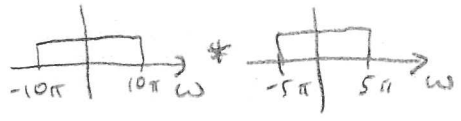


YES!

DISCONTINUITIES CAUSE GIBBS RINGING.

2. For what range of values of ω is the Fourier transform of the following signal zero? (5 points)

$$\frac{\sin(5\pi t)\sin(10\pi t)}{\pi^2 t^2}$$



$|\omega| > 15\pi$

=

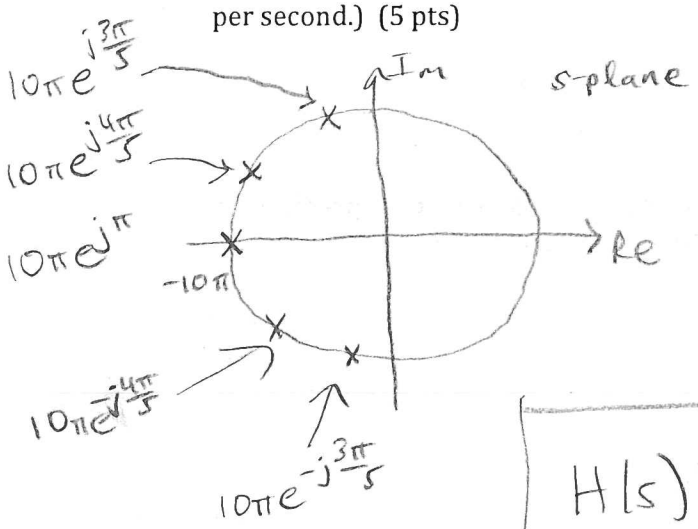
3. Evaluate the following integral. (5 points)

$$\int_{-\infty}^{\infty} \frac{\sin^2(5\pi t)}{(\pi t)^2}\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\text{rect}\left(\frac{\omega}{2\cdot 5\pi}\right)\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-5\pi}^{5\pi} d\omega = \frac{10\pi}{2\pi} = \boxed{5}$$

LECTURE SECTION:

4. Write the transfer function $H(s)$ for a 5th order Butterworth low pass filter with DC gain of 1 and a corner frequency of 5 Hz. (Note that the corner frequency is given in Hz, not radians per second.) (5 pts)



$\frac{\pi}{5}$ SPACING

$\frac{5\pi}{10} + \frac{\pi}{10} = \frac{6\pi}{10} = \frac{3\pi}{5}$

$$H(s) = \frac{(2\pi \cdot 5)^5}{(s-10\pi e^{j\pi})(s-10\pi e^{j3\pi/5})(s-10\pi e^{-j\frac{3\pi}{5}})(s-10\pi e^{j\frac{4\pi}{5}})(s-10\pi e^{-j\frac{4\pi}{5}})}$$

5. Is the signal $x(t) = \sin(5\pi t)/\pi t$ band limited? (2 pts)

↑ RECT IN ω DOMAIN

YES.

6. Above what frequency do you need to sample the signal in problem 5 in order to be able to perfectly reconstruct the continuous-time signal from its samples? (3 pts)

ABOVE 10π $\frac{\text{RADIANS}}{\text{SECOND}}$, OR ABOVE

5 Hz (SAMPLES/SECOND)