SOLUTIONS

Quiz #3

ECEn 380: Signals & Systems

Fall 2013

Closed book, closed note, closed neighbor, no calculators allowed. *Time limit is 10 minutes. 20 points total possible.*

1. Write the Fourier integral (the integral equation defining the Fourier transform) and the inverse Fourier integral (the integral equation defining the inverse Fourier transform). (4 points)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

2. Find the Fourier transform of the following signals: (8 points)

$x_1(t) = \delta(t-3)$

$$X_1(j\omega) = \int_{-\infty}^{\infty} \delta(t-3)\, e^{-j\omega t}\, dt = \boxed{e^{-j3\omega}}$$

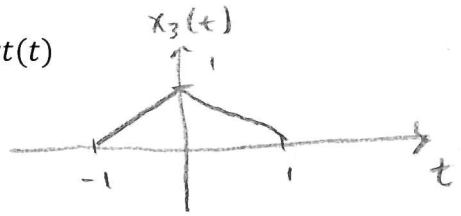
$x_2(t) = rect(t)$

$$X_2(j\omega) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j\omega t}\, dt = \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{e^{-j\omega\frac{1}{2}} - e^{j\omega\frac{1}{2}}}{-j\omega}$$

$$= \frac{2}{\omega}\left[\frac{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}{2j}\right] = \frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} = \boxed{\mathrm{sinc}\left(\frac{\omega}{2}\right)}$$

3. Sketch the signal: (4 points)

$x_3(t) = rect(t) * rect(t)$



4. Find the Fourier transform of $x_3(t) = rect(t) * rect(t)$. (4 points)

$$X_3(j\omega) = \left(\mathcal{F}[rect(t)]\right)^2 = \boxed{\mathrm{sinc}^2\left(\frac{\omega}{2}\right)}$$