SOLUTIONS

ECEn 380: Signals & Systems

Fall 2014

Professors Neal Bangerter and Brian Jeffs

Midterm #1

October 7 – 10, 2014

- **3 hour time limit. Please do not go over time.** You will be docked 1 point for every minute over time.
- Open book, open note (electronic books and/or notes or book on a tablet or smartphone allowed)
- Calculators allowed (okay to use tablet, smartphone, or e-book as calculator)
- **IMPORTANT: The exam is double sided, per testing center requirements**

- The exam consists entirely of multiple choice questions. **Please provide all answers on the scantron bubble sheet.**

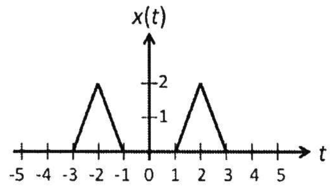
- There are 28 questions and 100 points possible in the exam, scored as follows:

    - Problems 1 – 12: 3 points each
    - Problems 13 – 28: 4 points each

- Manage your time carefully! Skip more difficult problems on your first pass through the exam, and return to them later (time permitting).

If you feel that something in the exam is not clear, please state your assumptions and work the problem based on those assumptions.

1. Find $x(2t) + x(-1+2t)$ given $x(t)$ shown.



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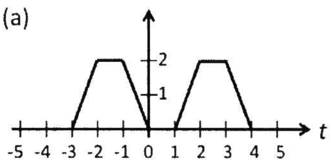

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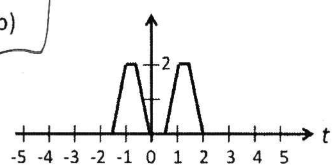

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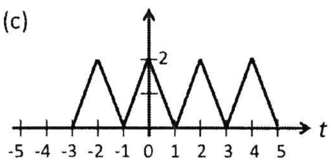

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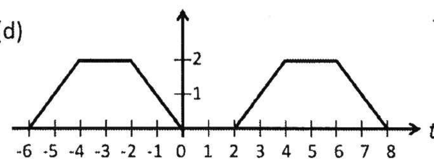
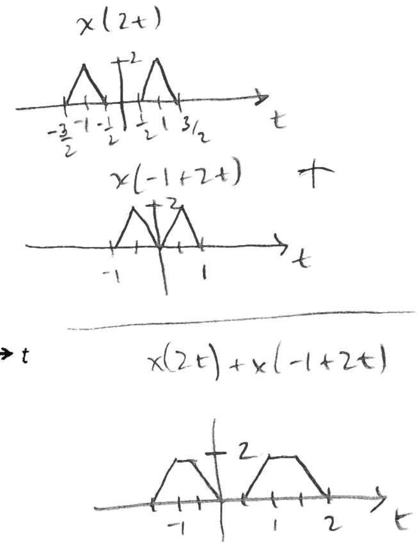
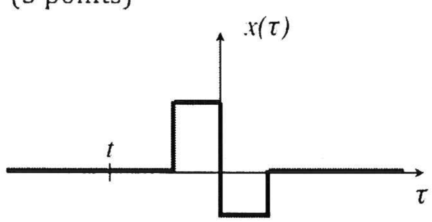


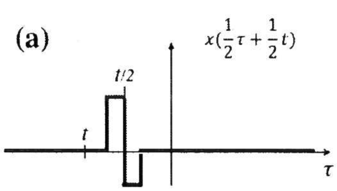
(e) None of the above



2. Which of the following drawings of $x(\frac{1}{2}\tau + \frac{1}{2}t)$ is correct for $t < 0$ given the function $x(\tau)$ shown below? (3 points)



(a)



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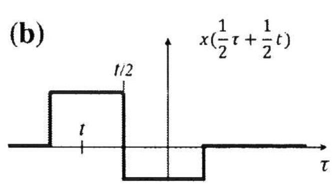

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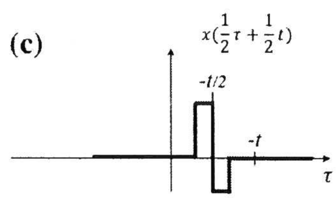

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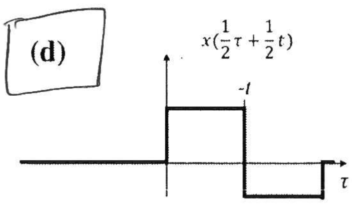

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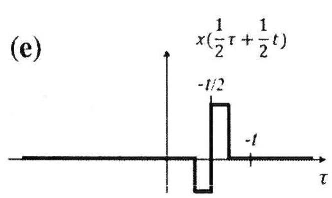
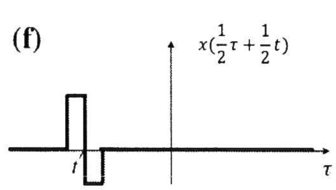


(f)



3. Given $x(t) = e^{-j\omega_0 t} - e^{j\omega_0 t}$, which of the following statements is correct?

   (a) $x(t)$ is odd and periodic.
   (b) $x(t)$ is even and periodic.
   (c) $x(t)$ is odd, but not periodic.
   (d) $x(t)$ is even, but not periodic.
   (e) $x(t)$ is neither even nor odd, but is periodic.
   (f) $x(t)$ is neither even nor odd, and not periodic.
   (g) None of the above is correct.

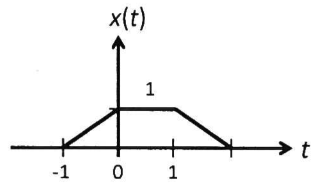
$e^{-j\omega_0 t} - e^{j\omega_0 t}$

$= -\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$

$= -2j \sin(\omega_0 t)$

PERIODIC AND ODD!

4. Express the following signal $x(t)$ in terms of ramp and/or step functions.



$r(t+1) - r(t) - r(t-1) + r(t-2)$

   (a) $x(t) = r(t-1) - r(t)$
   (b) $x(t) = r(t+1) - r(t)$
   (c) $x(t) = r(t+1) - r(t) + r(t-1)$
   (d) $x(t) = r(t-1) - 2r(t)$
   (e) $x(t) = r(t+1) - r(t) - r(t-1) + r(t-2)$
   (f) $x(t) = r(t+1) - 2r(t) + r(t-1)$
   (g) $x(t) = [r(t-1) - 2r(t)]u(1-t)$

5. Express the following signal $x(t)$ in terms of ramp and/or step functions.

$$x(t) = \begin{cases} 0, & t < -1 \\ 1, & -1 < t < 0 \\ 2, & 0 < t < 1 \\ -2t + 4, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

   (a) $x(t) = u(t+1) + r(t) - 3r(t-1) + 2r(t-2)$
   (b) $x(t) = u(t+1) + u(t) - 3r(t-1)$
   (c) $x(t) = u(t+1) - u(t) + r(t) - 2r(t-1) + r(t-2)$
   (d) $x(t) = u(t+1) + u(t) - 2r(t-1) + 2r(t-2)$
   (e) $x(t) = [u(t+1) + u(t) - 2r(t-2)]u(2-t)$
   (f) None of the above.

6. Evaluate the following integral: (Yes, the correct answer is there...☺)

$$\int_{0}^{\infty} e^{-j\frac{\pi}{4}t}u(t+1)\delta(2-t)dt \quad = \quad e^{-j\frac{\pi}{4}t}\,u(t+1)\Big|_{t=2}$$

$$= e^{-j\frac{\pi}{4}\cdot 2} = e^{-j\frac{\pi}{2}} = -j$$

(a) -1

(b) 0

(c) 1

(d) j

$\boxed{\text{(e) -j}}$

7. Evaluate the following integral:

$$\int_{-1}^{3} 3t^3\delta(3t-2)dt \quad = \int_{-1}^{3} 3t^3\,\tfrac{1}{3}\,\delta\!\left(t-\tfrac{2}{3}\right)dt$$

$$= \tfrac{1}{\cancel{3}}\cancel{3}\,t^3\Big|_{t=\frac{2}{3}} = \frac{8}{27}$$

(a) 4/9

(b) 4/27

(c) 1/9

$\boxed{\text{(d)}}$ 8/27

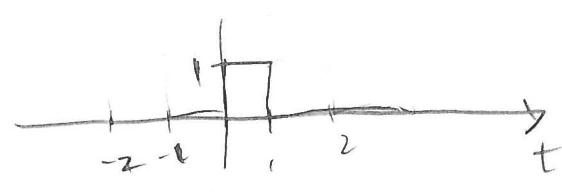
(e) 1

(f) None of the above

8. Find the impulse response $h(t)$ of the LTI system described by the following input/output relation:

$$y(t) = \int_{t}^{t+1} x(\tau-1)d\tau$$

$h(t)$



(a) $h(t) = u(t)$

$\boxed{\text{(b)}}$ $h(t) = u(t) - u(t-1)$

(c) $h(t) = u(t+1)$

(d) $h(t) = u(t-1) - u(t+1)$

(e) $h(t) = u(t) - u(t-2)$

(f) $h(t) = u(t+2) - u(t)$

(g) None of the above

LET $\tau' = \tau - 1$ $\quad \tau = \tau' + 1$

$d\tau' = d\tau$

$$y(t) = \int_{\tau'=t-1}^{t} x(\tau')\,d\tau'$$

so:

$$h(t) = \int_{t-1}^{t} \delta(\tau')\,d\tau' = u(t) - u(t-1)$$

9. Determine the period of the following signal: $x(t) = 2\cos(\frac{2}{5}t) + 4e^{-3j}e^{-j\pi t}$

PERIOD IS 5π PERIOD IS 2

NO L.C.M.!

(a) 2

(b) 5

(c) 10

(d) 2π

(e) 5π

(f) The signal is not periodic

10. Determine the period of the following signal: $x(t) = 2\cos(\frac{2}{3}\pi t) + (1-2j)\sin(2\pi t)$

PERIOD IS 3 PERIOD IS 1

L.C.M. IS 3!

(a) 1

(b) 3/2

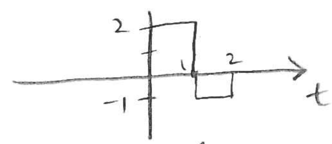
(c) 3

(d) 3π

(e) 6π

(f) The signal is not periodic

11. Compute the total energy of the following signal: $x(t) = 2u(t) - 3u(t-1) + u(t-2)$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 = \int_0^1 4\,dt + \int_1^2 1\,dt$$
$$= 4 + 1 = 5$$

(a) 0

(b) 1

(c) 2

(d) 5

(e) 8

(f) None of the above

12. Compute the average power of the following signal: $x(t) = e^{-j5\pi}e^{-j(3-7\pi t)}$

$$|x(t)|^2 = 1$$

SO:

$$P_{AV} = \frac{1}{T}\int_0^T |x(t)|^2\,dt = 1!$$

(a) 0

(b) 1

(c) $\sqrt{5}$

(d) 5

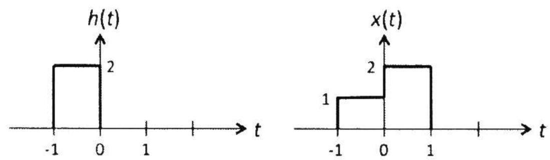
(e) 25

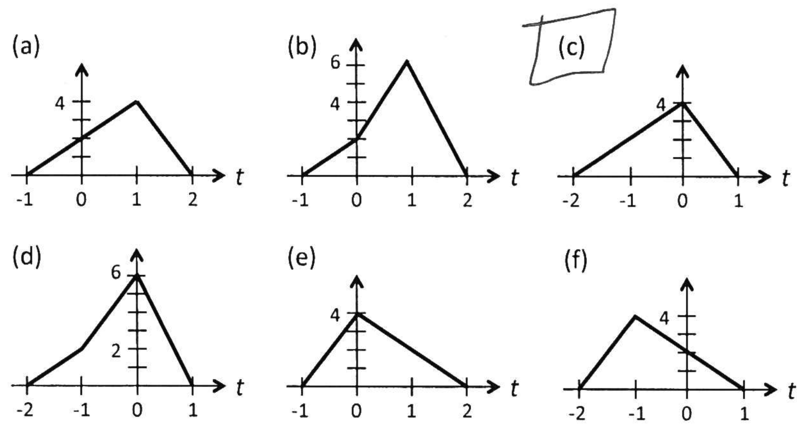
(f) $49\pi^2 + 9$

(g) $\sqrt{49\pi^2 + 9}$

(h) None of the above

13. Find the output of the LTI system with impulse response $h(t)$ to the input $x(t)$ (both shown below):





14. Which of the following statements is true about the system described by the following input/output relation?

$$y(t) = [2x(t)]^2$$

(a) The system is both linear and time-invariant

(b) The system is linear, but not time-invariant

(c) The system is not linear, but it is time-invariant

(d) The system is neither linear nor time-invariant

(e) None of the above

15. Which of the following statements is true about the system described by the following input/output relation?

$$y(t) = \int_{t-1}^{t+2} x(\tau + 3)d\tau$$

LOOK AT h(t) FOR CAUSALITY:

$$h(t) = \int_{t-1}^{t+2} \delta(\tau+3)\,d\tau$$

$$h(t) = \begin{cases} 0, & t+2 < -3 \\ 1, & \text{else} \\ 0, & t-1 > -3 \end{cases}$$

(a) The system is linear and time-invariant, but not causal

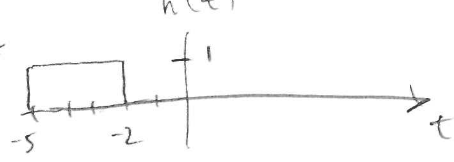
(b) The system is linear, but not time-invariant and not causal

(c) The system is linear, time-invariant, and causal

(d) The system is not linear, but is time-invariant and causal

(e) None of the above

NOT CAUSAL!!



16. Let $x(t)$ be the input of an LTI system with impulse response $h(t)$, where $x(t)$ and $h(t)$ are given by:

$$x(t) = \begin{cases} 0, & t < 0 \\ 3e^{-j\pi t}, & 0 \le t \le 2 \\ 0, & t > 1 \end{cases}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$h(t) = u(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)\,d\tau$$

Let $y(t)$ be the output. Find the output at time $t = 4$. That is, find $y(4)$.

$$y(4) = \int_{-\infty}^{\infty} x(\tau)u(4-\tau)\,d\tau = \int_0^4 x(\tau)\,d\tau = \int_0^2 x(\tau)\,d\tau = 0$$

(a) $y(4) = 0$

(b) $y(4) = \frac{1}{\pi}$

(c) $y(4) = 1$

(d) $y(4) = \pi$

ALSO OKAY!

(e) None of the above

17. Perform the following convolution: $te^{-j\omega_0 t} * [\delta(t-3) - \frac{1}{2}\delta(t-2)]$

(a) $te^{-j\omega_0(t-3)} - \frac{1}{2}te^{-j\omega_0(t-2)}$

$$(t-3)e^{-j\omega_0(t-3)} - \frac{1}{2}(t-2)e^{-j\omega_0(t-2)}$$

(b) $3e^{-j\omega_0(t-3)} - \frac{1}{2}2e^{-j\omega_0(t-2)}$

(c) $(t-3)e^{-j\omega_0(t-3)} - \frac{1}{2}(t-2)e^{-j\omega_0(t-2)}$

(d) $3e^{-j\omega_0 3} - \frac{1}{2}2e^{-j\omega_0 2}$

(e) None of the above

18. Which of the following is the Laplace Transform of the $x(t)$ shown below, expressed as a rational function?

$$x(t) = e^{-3t}\cos(4t + 30°)\,u(t)$$

(a) $\frac{\frac{\sqrt{3}}{2}(s+3)-2}{(s+3)^2+16}$

THIS IS:

$$e^{-at}\cos(bt-\theta)\,u(t)$$

(b) $\frac{0.866s-0.598}{s^2+6s+25}$

(c) $\frac{4s+12}{s^2+6s+25}$

WITH: $a = 3$

$b = 4$

$\theta = -30°$

$30°$, 1, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$

(d) None of the above

SO, FROM TABLE:

$$X(s) = \frac{(s+3)\cos(-30°) + 4\sin(-30°)}{(s+3)^2+16}$$

$$X(s) = \frac{(s+3)\sqrt{3}/2 - 2}{(s+3)^2+16}$$

19. Which of the following is the Laplace Transform of the $x(t)$ shown below?

PLUG IN $t=5$!

$$x(t) = \delta(t-5)e^{-j\pi t/5}\cos(\pi t)$$

$$x(t) = \delta(t-5)\underbrace{e^{-j\frac{\pi}{5}(5)}}_{-1}\underbrace{\cos(5\pi)}_{-1}$$

SO:

$$x(t) = \delta(t-5)$$

$$\boxed{X(s) = e^{-5s}}$$

(a) $\frac{e^{j\frac{\pi}{6}}}{s+j6\pi} + \frac{e^{j\frac{\pi}{6}}}{s-j6\pi}$

(b) e^{-5s} ✓

(c) $\frac{s\cos(30°)-6\pi\sin(30°)}{s^2+(6\pi)^2}$

(d) $-e^{-5s}$

(e) None of the above

20. Which of the following is $x(t)$ given $X(s) = \frac{\sqrt{2}(s+1)}{s^2+6s+13}$?

(a) $2e^{-2t}\cos(3t + 33.7°)$

(b) $2e^{-3t}\cos(2t + 45°)\,u(t)$ ✓

(c) $2\sqrt{2}e^{-3t}\cos(3t + 30°)$

(d) None of the above

21. When excited by $u(t)$, a system generates the output response:

$$y(t) = [5 - 10t + 20\sin(2t)]u(t).$$

$y_{step}(t)$

Which of the following is the system Transfer Function?

$$h(t) = \frac{d}{dt}y_{step}(t) = 5\,\delta(t) - 10\cancel{t\,\delta(t)} - 10u(t) + 20\cancel{\sin(2t)\,\delta(t)} + 40\cos(2t)u(t)$$

$$h(t) = 5\delta(t) - 10u(t) + 40\cos(2t)u(t)$$

$$H(s) = 5 - \frac{10}{s} + \frac{40s}{s^2+4}$$

$$H(s) = \frac{5s^3+30s^2+20s-40}{s(s^2+4)}$$

(a) $\frac{(s+2)(s+10)^2}{s^3+5s^2+10s+400}$

(b) $\frac{5s^3+20s^2+10s+40}{s^2(s^2+4)}$

(c) $\frac{5s^3-10s^2+60s-40}{s(s^2+4)}$

YOU'D GET THIS IF YOU DIDN'T TAKE DERIVATIVE...

(d) $\frac{5s^3+30s^2+20s-40}{s^2(s^2+4)}$

(e) None of the above ✓

22. What is the impulse response of the system in the previous problem (Problem 21)?

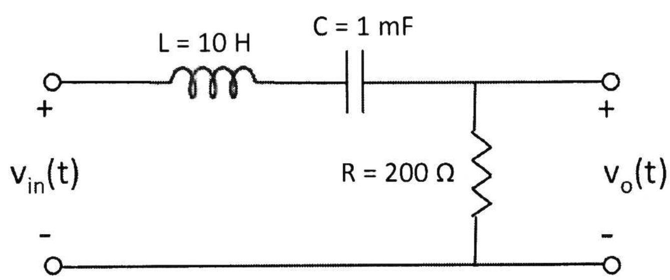
T (a) $5\delta(t) - 10[1 - 4\cos(2t)]u(t)$ ← SEE 21

(b) $[40\cos(2t) - 10]u(t)$

(c) $[5 + 40\sin(2t) - 10]u(t)$

(d) None of the above

23. Consider the circuit shown:



Which of the following is the s-domain Transfer Function of this circuit?

(a) $\frac{s}{(s+10)^2}$

(b) $\frac{20s}{(s+j10)(s-j10)}$

(c) $\frac{200}{(s+10)^2}$

T (d) $\frac{20s}{s^2+20s+100}$

(e) None of the above

$$I(s) = \frac{V_{in}(s)}{Ls + \frac{1}{Cs} + 200}$$

$$V_o(s) = I(s)\,200$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{20\cancel{0}}{1\cancel{0}s + \frac{100\cancel{0}}{s} + 20\cancel{0}}$$

$$H(s) = \frac{20s}{s^2+20s+100}$$

24. Which of the following is the **inverse** system impulse response, $h_i(t)$, for

$$h(t) = \delta(t) - 3e^{-3t}u(t)?$$

(a) $\delta(t) - 2e^{-t}u(t)$

(b) $\delta(t) - 2e^{-3t}u(t)$

T (c) $\delta(t) + 3u(t)$

(d) None of the above

$$H(s) = 1 - \frac{3}{s+3} = \frac{s+3-3}{s+3} = \frac{s}{s+3}$$

$$H^{-1}(s) = \frac{1}{H(s)} = \frac{s+3}{s} = 1 + 3/s$$

$$h_i(t) = \delta(t) + 3u(t)$$

25. Which of the following is true for a system with impulse response

$$h(t) = (4 + j5)e^{-(2+j3)t}u(t) + (4 - j5)e^{-(2-j3)t}u(t) \ ?$$

(a) The system is both BIBO stable and causal

(b) The system is BIBO stable, but **not** causal

(c) The system is **not** BIBO stable, but it is causal

(d) The system is neither BIBO stable nor causal

26. The response of an LTI system to the input $x(t) = \delta(t)$ is

$$y(t) = \delta(t) - 4e^{3t}u(t) \, .$$

Which of the following statements is true?

(a) The system is both BIBO stable and causal

(b) The system is BIBO stable, but **not** causal

(c) The system is **not** BIBO stable, but it is causal

(d) The system is neither BIBO stable nor causal

27. Does the system with impulse response $h(t) = \delta(t) - 3e^{-2t}u(t)$ have a BIBO stable **inverse** system?

(a) Yes

(b) No

(c) Not enough information to tell

28. An LTI system has a Transfer Function $H(s) = \frac{s^3+3s^2+2s}{(s^2+4)(s+4)}$. Is this system BIBO stable?

(a) Yes

(b) No

(c) Not enough information to tell