LAB 3

Application of Laplace Transforms : Operational Amplifier Filters.

ECEn 380 Section 001

Task 1 signoff: [signature] 10/12

Task 2 signoff: [signature] 10/19

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Objective

The purpose of this lab is to solidify our understanding of Laplace Transform analysis in circuit design. We are also going to analyze, design, and experimentally verify the functionality of a basic filter design.

Task 1

Step 2

$$\frac{V_i - V_1}{R_1} + \frac{V_o - V_1}{1/sC_1} + \frac{V_o - V_1}{R_2} = 0 \quad (1) \qquad V_1 = V_o(sC_2R_2 + 1) \quad (2)$$

Substitute V_1 into equation (1)

$$\frac{V_i - V_o - V_o sC_2R_2}{R_1} + \frac{V_o - V_o - V_o sC_2R_2}{1/sC_1} + \frac{V_o - V_o - V_o sC_2R_2}{R_2} = 0$$

$$\frac{V_i}{R_1} - \frac{V_o}{R_1} - \frac{V_o sC_2R_2}{R_1} - V_o s^2 C_1C_2R_2 - V_o sC_2 = 0$$

$$\frac{V_i}{R_1} = \frac{V_o}{R_1} + \frac{V_o sC_2R_2}{R_1} + V_o s^2 C_1C_2R_2 + V_o sC_2 = 0$$

$$\frac{V_i}{R_1} = V_o\left(\frac{1}{R_1} + \frac{sC_2R_2}{R_1} + s^2C_1C_2R_2 + sC_2\right)$$

$$\frac{V_i}{R_1} = V_o\left(s^2C_1C_2R_2 + s\left(C_2 + \frac{C_2R_2}{R_1}\right) + \frac{1}{R_1}\right)$$

$$\frac{\left(\frac{1}{C_1C_2R_2}\right)}{\left(\frac{1}{C_1C_2R_2}\right)} \frac{\frac{1}{R_1}}{s^2C_1C_2R_2 + s\left(C_2 + \frac{C_2R_2}{R_1}\right) + \frac{1}{R_1}} = \frac{V_o}{V_i} = \boxed{\frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + \left(\frac{1}{C_1R_2} + \frac{1}{C_1R_1}\right)s + \frac{1}{R_1R_2C_1C_2}} = H(s)}$$

Step 3

$$\frac{w_c^4}{(s - w_c e^{j\frac{5}{8}\pi})(s - w_c e^{-j\frac{5}{8}\pi})(s - w_c e^{j\frac{7}{8}\pi})(s - w_c e^{-j\frac{7}{8}\pi})} : H_1(s)H_2(s)$$

$$= \frac{w_c^2}{(s - w_c e^{j5/8\pi})(s - w_c e^{-j5/8\pi})} \cdot \frac{w_c^2}{(s - w_c e^{j7/8\pi})(s - w_c e^{-j7/8\pi})}$$

$$H_1(s) = \frac{w_c^2}{s^2 + \sqrt{2-\sqrt{2}}\, s w_c + w_c^2} \qquad H_2(s) = \frac{w_c^2}{s^2 + \sqrt{\sqrt{2}+2}\, w_c s + w_c^2}$$

$$H_1(s) \simeq \frac{w_c^2}{s^2 + 0.7654 w_c s + w_c^2} \qquad H_2(s) \simeq \frac{w_c^2}{s^2 + 1.8478 w_c s + w_c^2}$$

$H_1(s)$ $\quad w_c = 2\pi(3 \times 10^3)$

$$H_1(s) = \frac{3.553 \times 10^8}{s^2 + 14427s + 3.553 \times 10^8} \qquad H_2(s) = \frac{3.553 \times 10^8}{s^2 + 34830s + 3.553 \times 10^8}$$

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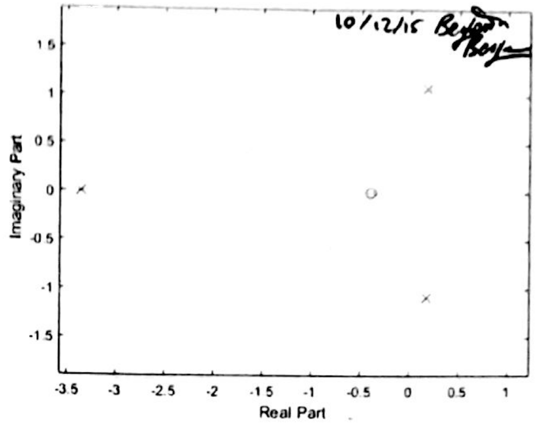
Task 1 step 4

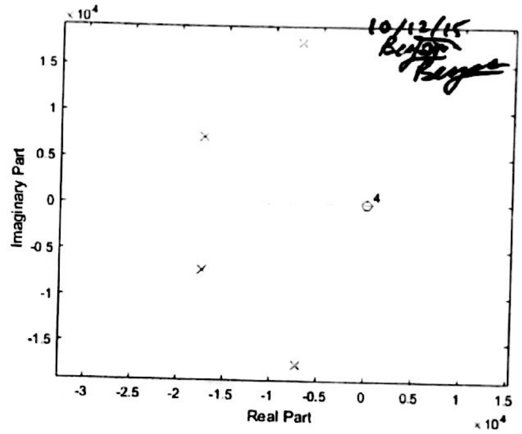
Sample code:
```
B = [0,0,5,2];
A = [1,3,0,4];
zplane(B,A);
```

SAMPLE GRAPH



Our Code:
```
A1 = [1, 14427, 3.553e8];
A2 = [1, 34830, 3.553e8];
A = conv(A1,A2);
B1 = [3.553e8];
B2 = [3.553e8];
B = conv(B1,B2);
zplane(B,A);
```

Our Graph



Task 1 Step 5

Our code for Bode plot.

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```
A1 = [1, 14427, 3.553e8];
A2 = [1, 34830, 3.553e8];

A = conv(A1, A2);

B1 = [3.553e8];
B2 = [3.553e8];

B = conv(B1,B2);

%creates a vector with 500 points between 0 and 5 that are logrithmically
%equally spaced
f = logspace(0,5,500);
%creates a Laplace Tranform frequency plot evaluated at 2*pi*f
[H,w] = freqs(B,A,2*pi*f);
%plots x axis logrithmically and y normally.
semilogx(f,20*log10(abs(H)))
xlabel('Frequency in Hz'); %labels the x axis
ylabel('Magnitude response in dB'); %labels the y axis
```

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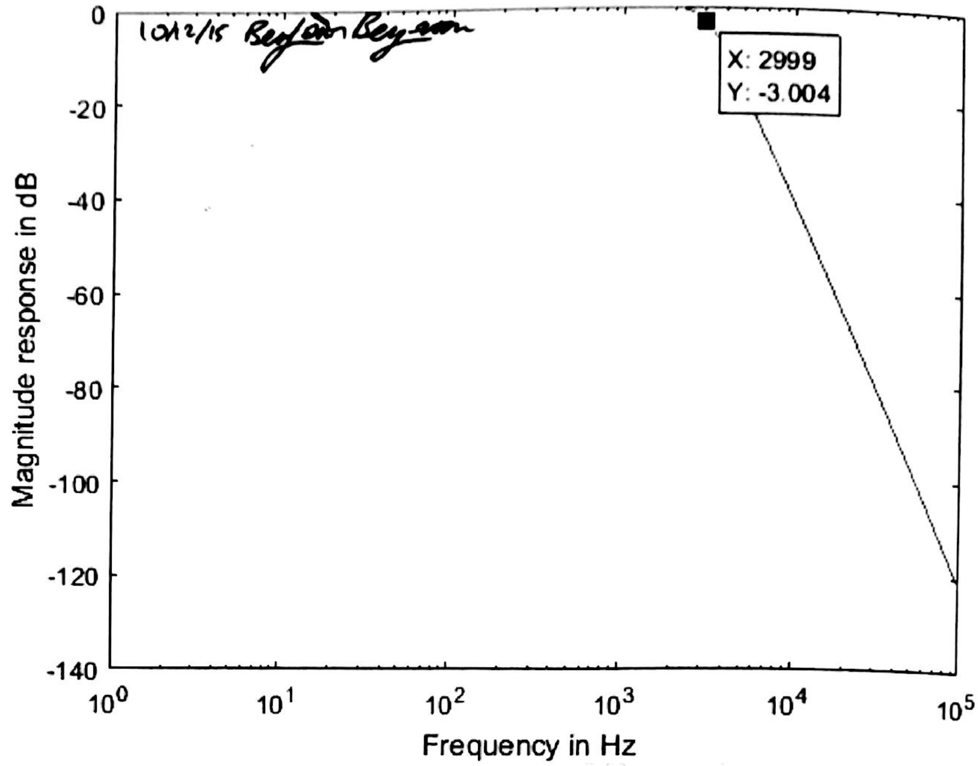
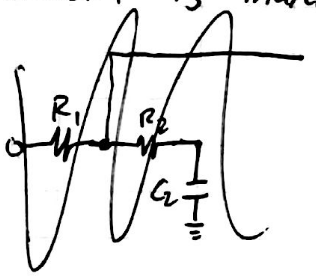
Task 1 Step 5



Task 1 step 6

$H(s) = H_1(s) \cdot H_2(s)$ because convolution in the time domain is multiplication in the s-domain.

Task 1 step 7



For $H_1(s)$: $\frac{1}{C_1C_2R_1R_2} = 3.553\times10^8 = \frac{1}{(1k)(1k)C_1C_2}$

$\frac{1}{C_1R_1} + \frac{1}{C_1R_2} = 14427 = \frac{1}{(1k)C_1} + \frac{1}{(1k)C_1}$

$$\boxed{\begin{array}{ll} C_1 = 1.386\times10^{-7} & = 0.1386\,\mu F \\ C_2 = 2.03\times10^{-8} & = 0.0203\,\mu F \end{array}}$$

For $H_2(s)$: $\frac{1}{(1k)(1k)C_1C_2} = 3.553\times10^8$

$\frac{1}{(1k)C_1} + \frac{1}{(1k)C_1} = 34830$

$$\boxed{\begin{array}{ll} C_1 = 5.74\times10^{-8} & = 0.0574\,\mu F \\ C_2 = 4.9\times10^{-8} & = 0.049\,\mu F \end{array}}$$

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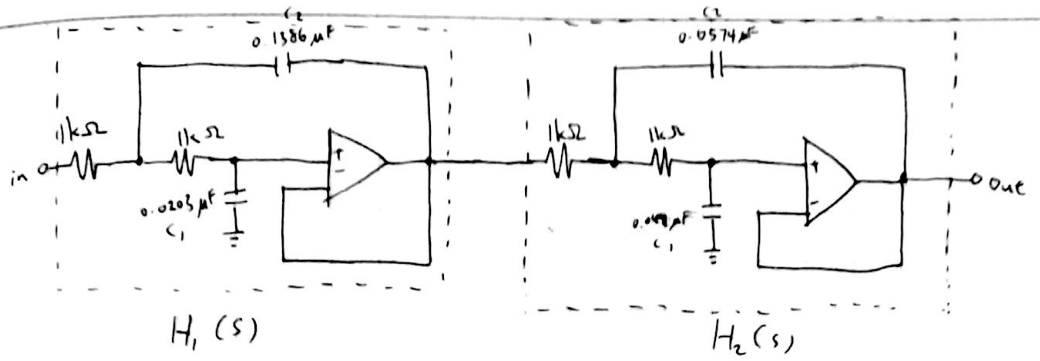
Task 2
Step 1



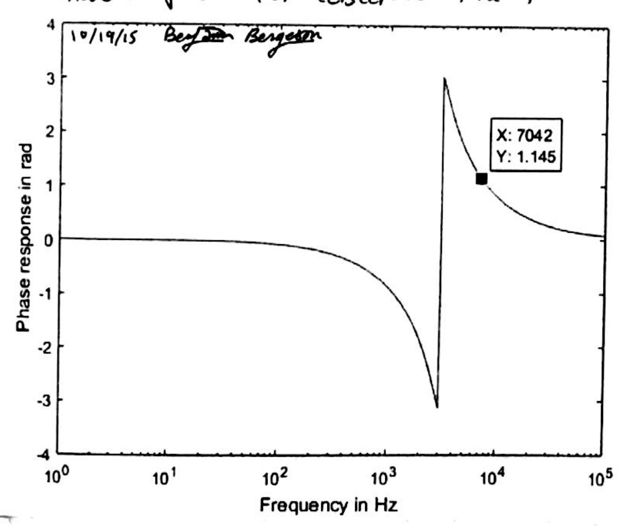
Actual capacitor values: $H_1(s)$: $C_1 = 138nF$ $C_2 = 24nF$ $H_2(s)$: $C_1 = 57nF$ $C_2 = 48nF$

Task 2 Step 2

It attenuates higher frequencies. When measuring each of the filters individually, they don't attenuate as much as when they are cascaded together. When using speech alone as an input, the words come through fine, but at a lower volume. We noticed than when playing music as the input, some of the higher notes got attenuated.

Task 2 Step 3 for cascaded filters

Phase Response for cascaded filters



The magnitude frequency response can be seen on page 26.

The ~~mea~~ MATLAB simulation yielded the following results:

@ 20 Hz: 0 rad ≈ 0°
@ 1 kHz: −0.8983 rad. ≈ −51.47°
@ 3 kHz: −π rad ≈ −180°
@ 5 kHz: 1.67 rad ≈ 95.68°
@ 7 kHz: 1.145 rad. ≈ 65.6°

Task 2 Step 4 for cascaded filters

@ 20 Hz

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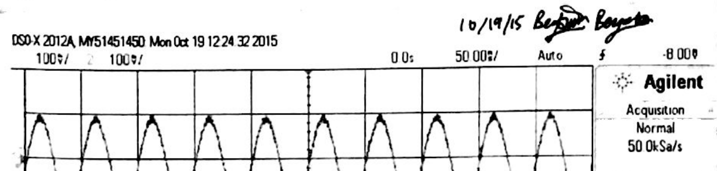

Measured Phase = 1.4° ≈ 0°
Measured Gain = 0

The measured phase and magnitude are very similar to the MATLAB simulation.

Calc Gain: 0

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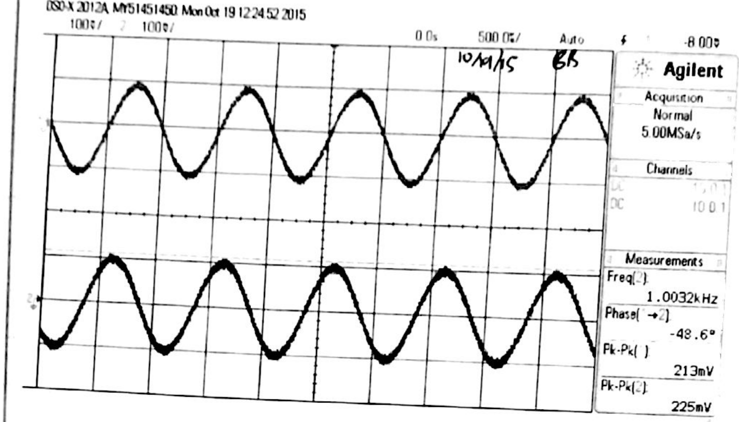
Task 2 Step 4
for cascaded filters

@ 1kHz

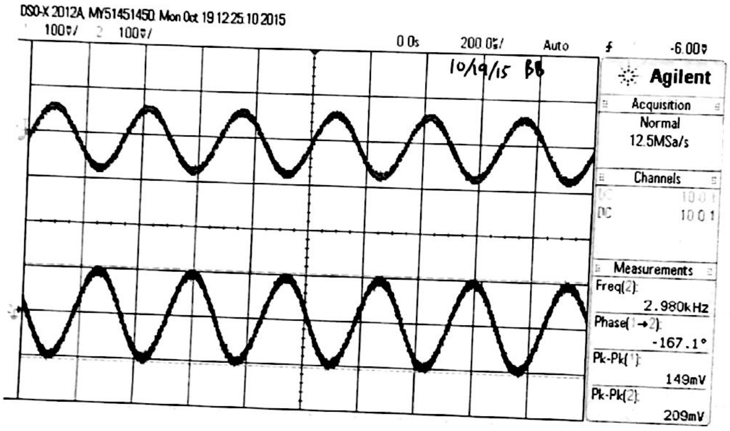


Measured phase: -48.6°
Measured Mag. ≈ 0.

Fairly close to phase of -51.4°

Calc Gain: -6.9×10^{-4}

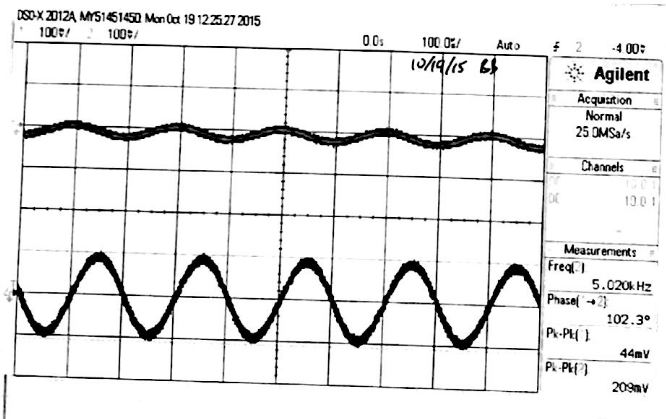
@ 3 kHz



Measured phase: -167.1°
Calc. phase: -180°

Measured Gain: -2.9
Calc Gain: -3

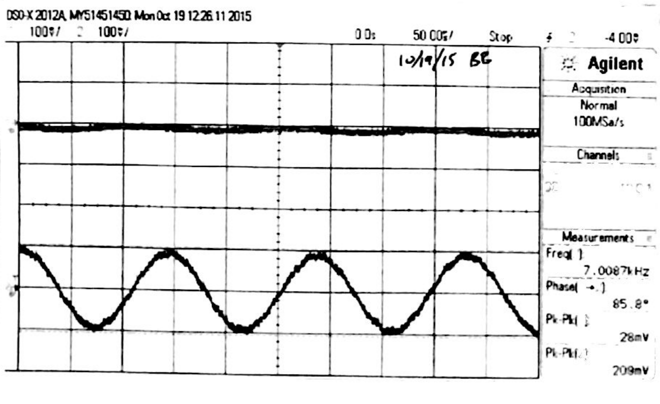
@ 5kHz



Meas. phase: 102.3°
Calc phase: 95.68°

Meas. Gain: -13.57
Calc Gain: -17.7

@ 7kHz



Meas. phase: 85.8°
Calc. phase: 65.6°

Meas. Gain: -17.5
Calc Gain: ≈ -24

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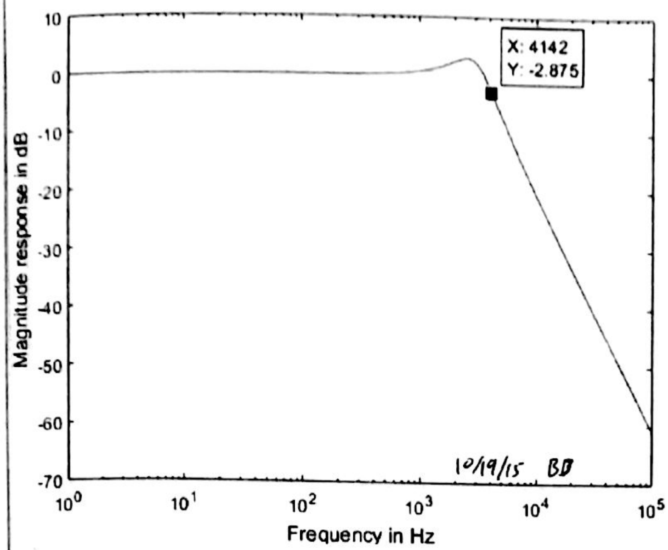
Task 2 step3 for single filter

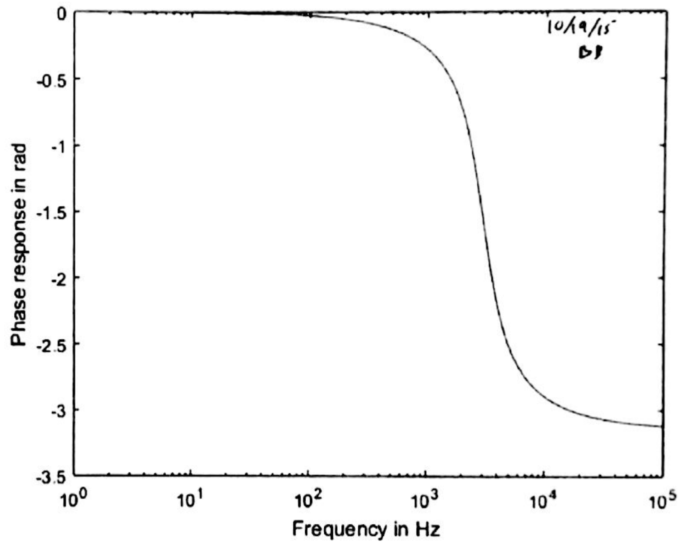


Corner Frequency

@ 20 Hz : 0
@ 1kHz : 0.67
@ 4.3 kHz : -3
@ 5 kHz : -7
@ 7 kHz : -13.5

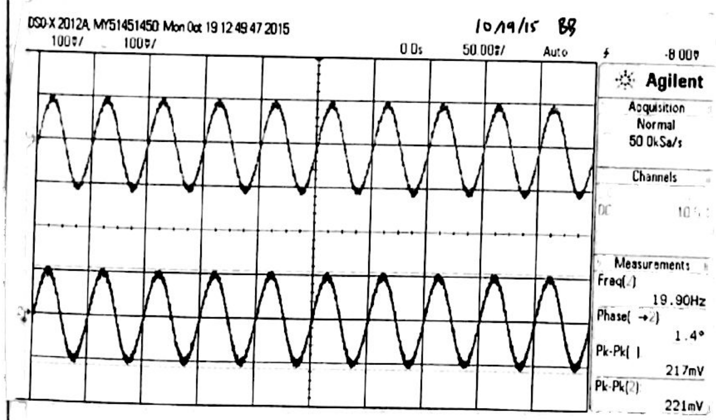
Phase Shift :

@ 20 Hz = ~~-.2765 rad~~ = 0 rad = 0°
@ 1kHz : -0.2765 rad = -15.8°
@ 4.3 kHz = -2.3 rad = -131.8°
@ 5 kHz : -2.516 rad = -144.1°
@ 7 kHz = -2.76 rad = -158.1°



@ 20 Hz

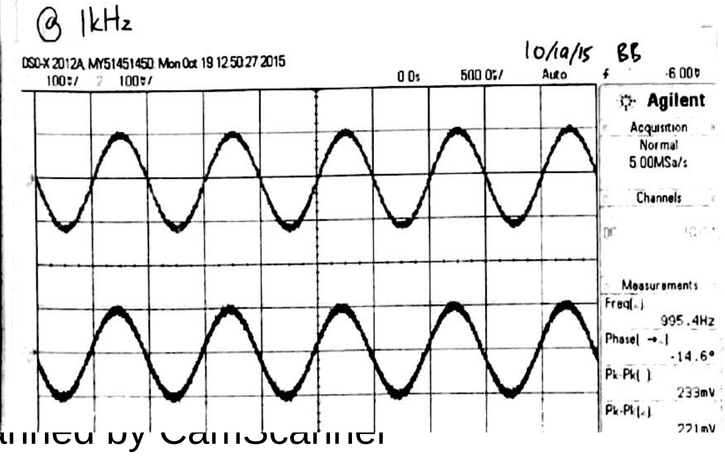
Task 2 Step4 for single filter



Measured Phase : 1.4° ≈ 0
Calc. Phase : 0

Measured Gain : -0.16
Calc Gain : 0

@ 1kHz



Measured Phase : -14.6°
Calc. Phase : -15.8°

Measured Gain : 0.46
Calc Gain : 0.67

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Task 2 step 4
for single filter

@ 4.3 kHz

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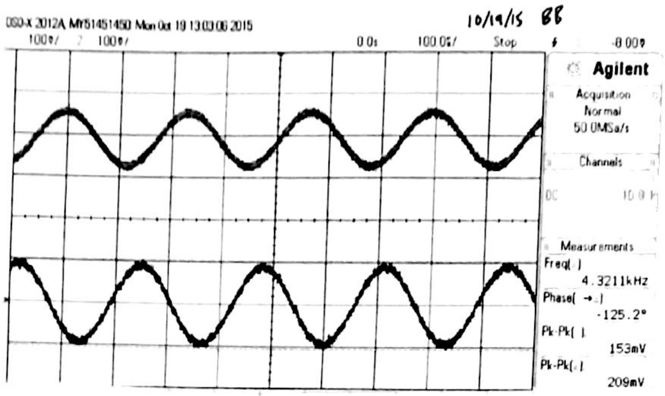

Measured Phase: -125.2°
Calc Phase: -131.8°

Measured Gain: -2.7
Calc Gain: -3

@ 5 kHz

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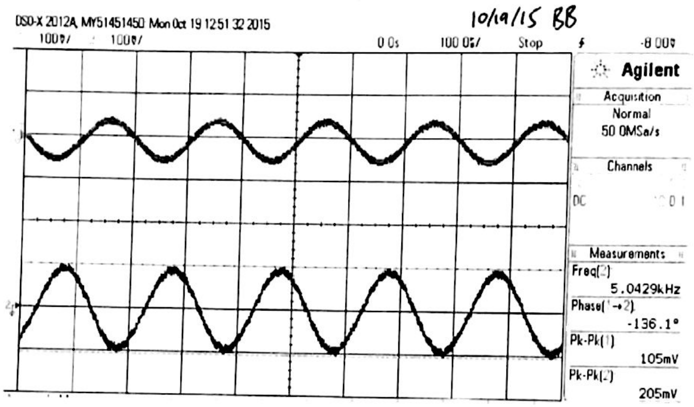

Measured Phase: -136.1°
Calc Phase: -144.1°

Measured Gain: -5.8
Calc Gain: -7

@ 7 kHz

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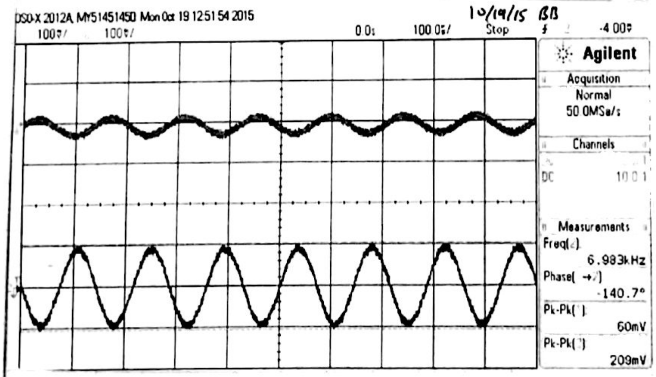

Measured Phase: -140.7°
Calc. Phase: -158.1°

Measured Gain: -10.8
Calc Gain: -13.5

In both the cascaded filters & the single filter, it seems that larger the frequency, the larger the discrepancies between measured values and calculated values.

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Conclusion

Conclusion

In this lab we were able to design a Butterworth filter with four poles and no zeros by cascading two Sallen-Key filters. Using KCL we were able to find the transfer function of the individual Sallen-Key filters. Knowing that this is a furth order Butterworth filter we're designing, we could figure out the transfer function. All that's left is to solve algebraically for the values of the resistors and capacitors of the two Sallen-Key filters. Using those values we were able to plot the poles and zeros in MATLAB using the zplane command.

Using these calculated values, we were able to simulate the frequency response for magnitude and the frequency response for phase for the ~~cascaded filters as~~ Butterworth filter and one of the Sallen-Key filters. We were then able to physically build the circuit and measure the outputs with the oscilloscope.

There were some small ~~errors~~ discrepencies between the simulated values and the actual measured values. The phase shifts were relatively close, with the margin of error being around 10% max. However we noticed that our physical circuit would not attenuate as much as the simulated circuit. However the corner frequencies were very close. We think that some of the error could be due in part to the fact that the capacitor value we choose weren't exactly the values that we calculated.

Overall, this was a very educational experience in learning how to build a low pass Butterworth filter from scratch.

10/26/15

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