SOLUTIONS

ECEn 380: Signals & Systems

Fall 2014

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Midterm #2

November 18 – 21, 2014

- 3 hour time limit
- Open book, open notes
- Calculators allowed
- Tablets and/or computers are allowed for reading notes (but no Matlab)
- **IMPORTANT: The exam is double sided, per testing center requirements**

- The exam consists entirely of multiple choice questions. **Please provide all answers on the scantron bubble sheet.**

- You are welcome to use extra scratch paper for your work, but please include it when you hand in the exam.

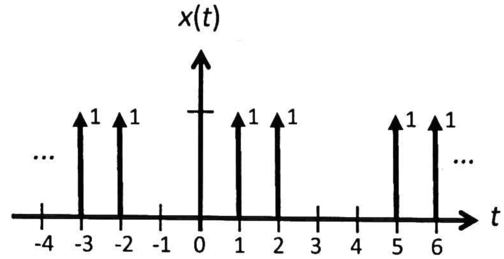
- There are 33 questions and 100 points possible on the exam. 32 of the problems are worth 3 points each, and one of the problems (the last one) is worth 4 points.

- **PLEASE NOTE:** The problems are not ordered in terms of difficulty. For example, if you really understand sampling and the Chapter 7 material, the second half of the exam may be significantly easier for you than the first half of the exam. You may want to skip difficult problems on your first pass through the test, and come back to them after you have answered all of the problems that are easy for you.

- Manage your time carefully! Skip more difficult problems on your first pass through the exam, and return to them later (time permitting).

If you feel that something in the exam is not clear, please state your assumptions and work the problem based on those assumptions.

1. Find the Fourier series coefficients x_n of the following **periodic** function $x(t)$: (3 pts)



$T_0 = 4$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$

(a) $x_n = \frac{e^{jn\frac{\pi}{2}} + e^{jn\pi}}{4}$ for all n

(b) $x_n = \frac{e^{jn\frac{\pi}{2}} + e^{jn\pi}}{5}$ for all n

(c) $x_n = \frac{-j-1}{4}$ for all n

$\boxed{\text{(d)}}$ $x_n = \frac{(-j)^n + (-1)^n}{4}$ for all n

$$x_n = \frac{1}{4}\int_0^4 \left(\delta(t-1) + \delta(t-2)\right) e^{-j\frac{\pi}{2}nt}\,dt$$

$$x_n = \frac{1}{4} e^{-j\frac{\pi}{2}n(1)} + \frac{1}{4} e^{-j\frac{\pi}{2}n(2)}$$

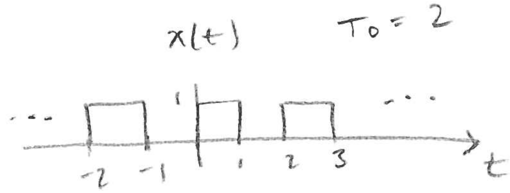
$$x_n = \frac{(-j)^n + (-1)^n}{4},$$ VALID FOR ALL n!

$\boxed{D}$

2. Find the Fourier series coefficients x_n of the following periodic function $x(t)$: (3 pts)

$$x(t) = \sum_{k=-\infty}^{\infty} [u(t-2k) - u(t-2k-1)]$$

$T_0 = 2$ $\quad \omega_0 = \frac{2\pi}{2} = \pi$



(a) $x_n = \begin{cases} \frac{1}{2}, & n = 0 \\ \left(-\frac{1}{2}\right)^n, & n \neq 0 \end{cases}$

(b) $x_n = \begin{cases} \frac{1}{2}, & n = 0 \\ \frac{1+e^{-j\pi n}}{j2\pi n}, & n \neq 0 \end{cases}$

$\boxed{\text{(c)}}$ $x_n = \begin{cases} \frac{1}{2}, & n = 0 \\ \frac{1-(-1)^n}{j2\pi n}, & n \neq 0 \end{cases}$

(d) $x_n = \frac{1}{j2\pi n}\left(1 - e^{-j\pi n}\right)$ for all n

$$x_n = \frac{1}{2}\int_0^1 1 \cdot e^{-j\pi nt}\,dt = \begin{cases} \frac{1}{2}, & n=0 \\ \frac{1-(-1)^n}{j2\pi n}, & n \neq 0 \end{cases}$$

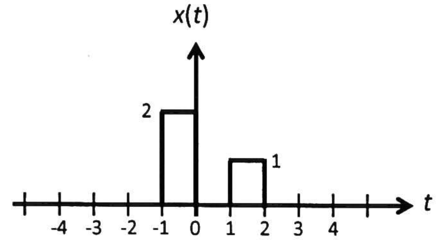
$$= \frac{1}{2} \left.\frac{e^{-j\pi nt}}{-j\pi n}\right|_0^1$$

$$= \frac{e^{-j\pi n} - 1}{-2j\pi n}$$

$$= \frac{1-(-1)^n}{j2\pi n}, \quad n \neq 0$$

$\boxed{C}$

3. Find the Fourier transform $X(j\omega)$ of the following function $x(t)$: (3 pts)



$\boxed{(a)}$ $\frac{2e^{j\omega}-2+e^{-j\omega}-e^{-j2\omega}}{j\omega}$

(b) $2e^{j\omega} - 2 + e^{-j\omega} - e^{-j2\omega}$

(c) $\frac{2\sin(2\omega)+\sin(\omega)}{j\omega}$

(d) $\frac{e^{j\omega}-1+e^{-j\omega}-e^{-j2\omega}}{j\omega}$

(e) $\frac{\sin(2\omega)+\sin(\omega)}{\omega}$

$$\mathbb{X}(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{0} 2e^{-j\omega t}\, dt + \int_{1}^{2} e^{-j\omega t}\, dt$$

$$= \frac{2e^{-j\omega t}}{-j\omega}\Big|_{-1}^{0} + \frac{e^{-j\omega t}}{-j\omega}\Big|_{1}^{2}$$

$$= \frac{2-2e^{j\omega}+e^{-j2\omega}-e^{-j\omega}}{-j\omega}$$

$\boxed{A}$

4. Which of the following is the Fourier transform of a periodic function $x(t)$? (3 pts)

(a) $X(j\omega) = \text{rect}\left(\frac{\omega}{2\omega_0}\right)$

(b) $X(j\omega) = \text{sinc}(\omega)$

$\boxed{(c)}$ $X(j\omega) = \sum_{n=-\infty}^{\infty} \pi(-1)^n\, \delta(\omega - 2n)$ ← HAS TO BE A DELTA TRAIN IN ω DOMAIN! (SINCE PERIODIC IN $x(t)$)

(d) $X(j\omega) = \sum_{n=-\infty}^{\infty} \pi(-1)^n\, rect(\omega - 2n)$

$\boxed{C}$

5. Find the Fourier transform of $x(t) = \text{rect}(t) * e^{j2\pi t}$ (where the * denotes convolution). (3 pts) **Yes, the answer is there...**

$\boxed{(a)}$ 0

(b) $2\pi\, \text{sinc}(2)\, \delta(\omega - 2)$

(c) $\frac{1}{2\pi}\, \text{sinc}\left(\frac{\omega}{2} - 2\pi\right)$

(d) $\frac{1}{2\pi}\, \text{sinc}\left(\frac{\omega-2\pi}{2}\right)$

$$\mathbb{X}(j\omega) = \text{sinc}\left(\frac{\omega}{2}\right)\cdot 2\pi\, \delta(\omega - 2\pi)$$

$$= \text{sinc}\left(\frac{2\pi}{2}\right)\cdot 2\pi\, \delta(\omega - 2\pi)$$

$$= \text{sinc}(\pi)\cdot 2\pi\cdot \delta(\omega - 2\pi)$$

$$= \frac{\sin(\pi)}{\pi}\, 2\pi\cdot \delta(\omega - 2\pi)$$

$= 0!$

$\boxed{A}$

6. Find the **inverse** Fourier transform of $X(j\omega) = \pi[\delta(\omega) + \delta(\omega - 2\omega_0)]$. (3 pts)

(a) $x(t) = \sin(\omega_0 t)$

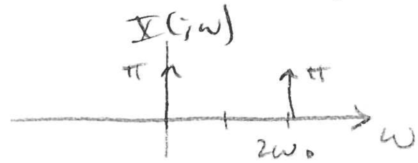
(b) $x(t) = 1 + e^{j2\omega_0 t}$

(c) $x(t) = e^{-j\omega_0 t}\cos(\omega_0 t)$

(d) $x(t) = e^{j2\omega_0 t}\cos(\omega_0 t)$

(e) $x(t) = e^{j\omega_0 t}\cos(2\omega_0 t)$

$\boxed{\text{(f)}}$ $x(t) = e^{j\omega_0 t}\cos(\omega_0 t)$



THIS IS A COSINE SPECTRUM SHIFTED RIGHT BY ω_0!

$x(t) = e^{j\omega_0 t}\cos(\omega_0 t)$

$\boxed{F}$

7. Find the signal $x(t)$ if its Fourier transform $X(j\omega) = (j\omega)^2\delta(\omega - 3)$. (3 pts)

$\boxed{\text{(a)}}$ $x(t) = \frac{-9}{2\pi}e^{j3t}$

(b) $x(t) = -9\delta(t-3)$

(c) $x(t) = \frac{-9}{2\pi}\delta(t-3)$

(d) $x(t) = \frac{-9}{2\pi}\cos(3t)$

(e) 2π

$X(j\omega) = (j3)^2\delta(\omega-3)$

$X(j\omega) = -9\,\delta(\omega-3)$

OR USE DERIVATIVE PROPERTY!!

$x(t) = \frac{-9}{2\pi}e^{j3t}$

$\boxed{A}$

8. Suppose the Fourier transform of a signal $x(t)$ is $X(j\omega)$. Find the Fourier transform of $x(2t) * \cos(3t)$ (where the * denotes convolution). (3 pts)

(a) $\frac{\pi}{2}X\left(j\frac{\omega-3}{2}\right) + \frac{\pi}{2}X\left(j\frac{\omega+3}{2}\right)$

(b) $\frac{\pi}{2}X\left(j\frac{3}{2}\right) + \frac{\pi}{2}X\left(-j\frac{3}{2}\right)$

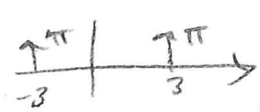
(c) $\frac{\pi}{2}X\left(j\frac{\omega}{2}\right) + \frac{\pi}{2}X\left(-j\frac{\omega}{2}\right)$

$\boxed{\text{(d)}}$ $\frac{\pi}{2}X\left(j\frac{3}{2}\right)\delta(\omega-3) + \frac{\pi}{2}X\left(-j\frac{3}{2}\right)\delta(\omega+3)$

(e) None of the above

$x(2t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}X\left(j\frac{\omega}{2}\right)$

$\cos(3t) \xleftrightarrow{\mathcal{F}}$



$\frac{1}{2}\cdot\pi\left[\delta(\omega-3) + \delta(\omega+3)\right]X\left(\frac{\omega}{2}\right)$

$= \frac{\pi}{2}X\left(j\frac{3}{2}\right)\delta(\omega-3) + \frac{\pi}{2}X\left(-j\frac{3}{2}\right)\delta(\omega+3)$

$\boxed{D}$

9. $\frac{\sin(10\pi t)}{\pi t}[2+2\cos(20\pi t)]$ is equal to which of the following? **HINT:** You don't need any trig identities to solve this! Just look at it in the frequency domain. (3 pts)

(a) $\text{rect}\left(\frac{t}{30\pi}\right)+\text{rect}\left(\frac{t}{10\pi}\right)$

(b) $\text{rect}(30\pi t)+rect(10\pi t)$

(c) $\text{rect}(30\pi t)$

(d) $\frac{\sin(30\pi t)+\sin(10\pi t)}{\pi t}$

(e) $\frac{2\sin(10\pi t)\cos^2(20\pi t)}{\pi t}$

(f) None of the above

$= \frac{\sin(10\pi t)}{\pi t} + \frac{\sin(10\pi t)}{\pi t}[1+2\cos(20\pi t)]$

$= \frac{\sin(10\pi t)}{\pi t} + \frac{\sin(30\pi t)}{\pi t}$

$\boxed{D}$

10. Suppose the Fourier transform of a signal $x(t)$ is $X(j\omega)=\frac{e^{-2j\omega}}{3+j\omega}$. What is the Fourier transform of $x(t+2)\cos(5t)$? (3 pts)

(a) $\frac{1}{3+j\omega}$

(b) $\frac{\pi\delta(\omega-5)+\pi\delta(\omega+5)}{3+j\omega}$

(c) $\frac{\pi\delta(\omega-5)}{3+j(\omega-5)}+\frac{\pi\delta(\omega+5)}{3+j(\omega+5)}$

(d) $\frac{\pi\delta(\omega-5)}{3+j5}+\frac{\pi\delta(\omega+5)}{3-j5}$

(e) $\frac{1}{2}\left[\frac{1}{3+j(\omega-5)}+\frac{1}{3+j(\omega+5)}\right]$

(f) None of the above

$x(t+2) \xleftrightarrow{\mathcal{F}} e^{j2\omega}\frac{e^{-j2\omega}}{3+j\omega}$

$\cos(5t) \xleftrightarrow{\mathcal{F}}$

$\left(\frac{1}{2\pi}\cdot\frac{1}{3+j\omega}\right) * \left(\pi\delta(\omega+5)+\pi\delta(\omega-5)\right)$

$= \frac{1}{2}\left[\frac{1}{3+j(\omega-5)}+\frac{1}{3+j(\omega+5)}\right]$

$\boxed{E}$

11. Is the signal $x(t)=\frac{\sin(30\pi t)}{\pi t} * \frac{\sin(50\pi t)}{\pi t}$ band limited? If so, what is the value of B such that $X(f)=0$ for all $|f|>B$? (Note that * denotes convolution.) (3 pts)

(a) $x(t)$ is not band limited

(b) $x(t)$ is band limited, with B = 10 Hz

(c) $x(t)$ is band limited, with B = 15 Hz

(d) $x(t)$ is band limited, with B = 20 Hz

(e) $x(t)$ is band limited, with B = 30 Hz

(f) $x(t)$ is band limited, with B = 50 Hz

(g) $x(t)$ is band limited, with B = 80 Hz

MULTIPLICATION OF rects IN FREQ. DOMAIN!

SMALLEST RECT SETS BW.

$\boxed{C}$

12. Compute the total energy of the signal $\frac{\cos(4\pi t)\sin(2\pi t)}{\pi t}$. (3 pts)

NOTE: Amplitude scaling in the frequency domain will be important to solve this problem. Remember your factor of 1/2π!

(a) 0

(b) 1

(c) 2

(d) π

(e) 2π

(f) π^2

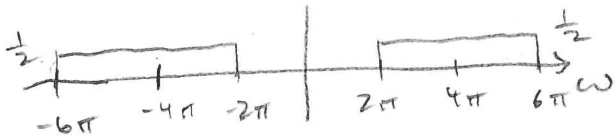
(g) $2\pi^2$

(h) $4\pi^2$

$$X(j\omega) = \frac{1}{2\pi}\left[\pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)\right] * \mathrm{rect}\left(\frac{\omega}{4\pi}\right)$$

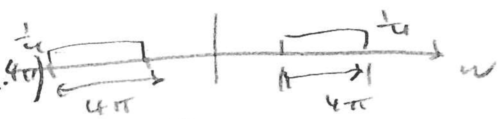
$$X(j\omega) = \frac{1}{2}\mathrm{rect}\left(\frac{\omega-4\pi}{4\pi}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{\omega+4\pi}{4\pi}\right)$$

$X(j\omega)$



$|X(j\omega)|^2$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$



$$= \frac{1}{2\pi}\left(\frac{1}{4}\cdot 4\pi + \frac{1}{4}\cdot 4\pi\right)$$

$$= \frac{1}{2\pi}\cdot 2\pi = 1$$

$\boxed{B}$

13. Evaluate the following integral: (3 pts)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{a^2+\omega^2} d\omega$$

BIG HINT: Notice that $\frac{1}{a^2+\omega^2} = \left|\frac{1}{a+j\omega}\right|^2$.

(a) 0

(b) 1/2a

(c) a

(d) 2a

(e) a/2

(f) π

(g) 2π

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{a+j\omega}\right|^2 d\omega$$

$$= \int_{-\infty}^{\infty} |e^{-at}u(t)|^2 dt$$

$$= \int_0^{\infty} e^{-2at} dt$$

$$= \left.\frac{e^{-2at}}{-2a}\right|_0^{\infty} = \frac{0-1}{-2a} = \frac{1}{2a}$$

$\boxed{B}$

14. Which of the following has an inverse Fourier transform that is real-valued and even? (3 pts)

HINT: What are the implications in the Fourier domain of a function being real-valued and even in the time domain?

(a) $X(j\omega) = e^{-j\omega} rect(\omega)$

(b) $X(j\omega) = \sin(\omega)\cos(\omega)$

(c) $X(j\omega) = e^{-j\frac{\pi}{2}} rect(\omega)$

(d) $X(j\omega) = e^{-j\pi} rect(\omega)$ ← REAL & EVEN IN ω!

(e) $X(j\omega) = \sin(\omega)$

15. Which of the following are the pole locations of a tenth order Butterworth low-pass filter with a corner frequency of 10 Hz (cycles/s)? (3 pts)

(a) $\pm 20\pi,\ 20\pi e^{j\frac{3}{10}\pi},\ 20\pi e^{j\frac{4}{10}\pi},\ 20\pi e^{j\frac{6}{10}\pi},\ 20\pi e^{j\frac{7}{10}\pi}$

(b) $\pm 10,\ 10e^{j\frac{3}{10}\pi},\ 10e^{j\frac{4}{10}\pi},\ 10e^{j\frac{6}{10}\pi},\ 10e^{j\frac{7}{10}\pi}$

(c) $\pm 20\pi,\ 20\pi e^{\pm j\frac{2\pi}{20}},\ 20\pi e^{\pm j\frac{4\pi}{20}},\ 20\pi e^{\pm j\frac{6\pi}{20}},\ 20\pi e^{\pm j\frac{8\pi}{20}}$

(d) $\pm 10,\ 10e^{\pm j\frac{2\pi}{20}},\ 10e^{\pm j\frac{4\pi}{20}},\ 10e^{\pm j\frac{6\pi}{20}},\ 10e^{\pm j\frac{8\pi}{20}}$

(e) $20\pi e^{\pm j\frac{19}{20}\pi},\ 20\pi e^{\pm j\frac{17}{20}\pi},\ 20\pi e^{\pm j\frac{15}{20}\pi},\ 20\pi e^{\pm j\frac{13}{20}\pi},\ 20\pi e^{\pm j\frac{11}{20}\pi}$

(f) $10e^{\pm j\frac{19}{20}\pi},\ 10e^{\pm j\frac{17}{20}\pi},\ 10e^{\pm j\frac{15}{20}\pi},\ 10e^{\pm j\frac{13}{20}\pi},\ 10e^{\pm j\frac{11}{20}\pi}$

(g) None of the above

E

16. Which of the following is the transfer function of a fourth order Buttterworth low-pass filter with DC (or passband) gain of 1 and corner frequency 10π radians/s? (3 pts)

(a) $H(s) = \dfrac{10\pi}{(s^2-10\pi e^{j\frac{7}{8}\pi})(s-10\pi e^{-j\frac{7}{8}\pi})(s-10\pi e^{j\frac{5}{8}\pi})(s-10\pi e^{-j\frac{5}{8}\pi})}$

(b) $H(s) = \dfrac{1}{(s-10\pi e^{j\frac{7}{8}\pi})(s-10\pi e^{-j\frac{7}{8}\pi})(s-10\pi e^{j\frac{5}{8}\pi})(s-10\pi e^{-j\frac{5}{8}\pi})}$

(c) $H(s) = \dfrac{(10\pi)^4}{(s-10\pi e^{j\pi/2})(s-10\pi e^{j\pi})(s-10\pi e^{j3\pi/2})(s-10\pi e^{-j2\pi})}$

(d) $H(s) = \dfrac{10{,}000\pi^4}{(s-10\pi e^{j\frac{5}{8}\pi})(s-10\pi e^{j\frac{7}{8}\pi})(s-10\pi e^{j\frac{9}{8}\pi})(s-10\pi e^{j\frac{11}{8}\pi})}$

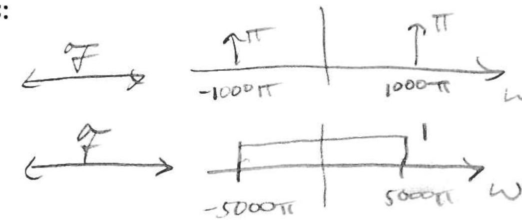
(e) None of the above

D

For **problems 17 – 20**, consider the following two signals:

$$x_1(t) = \cos(1{,}000\pi t)$$

$$x_2(t) = \frac{\sin(5{,}000\pi t)}{\pi t}$$



17. What is the Nyquist sampling rate for $x_2(t)$? (3 pts)

- (a) 5 kHz
- (b) 6 kHz
- (c) 8 kHz
- (d) 10 kHz
- (e) 15 kHz
- (f) 20 kHz
- (g) None of the above

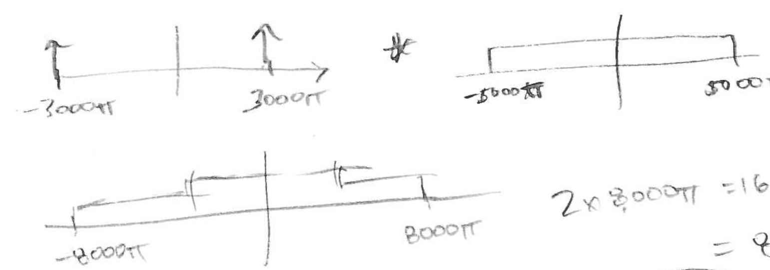
$2 \times 5000\pi = 10{,}000\pi$ rad./s.

$= 5$ kHz

$\boxed{A}$

18. What is the Nyquist sampling rate for $x_1(3t)x_2(t)$? (3 pts)

- (a) 5 kHz
- (b) 6 kHz
- (c) 8 kHz
- (d) 10 kHz
- (e) 15 kHz
- (f) 20 kHz
- (g) None of the above



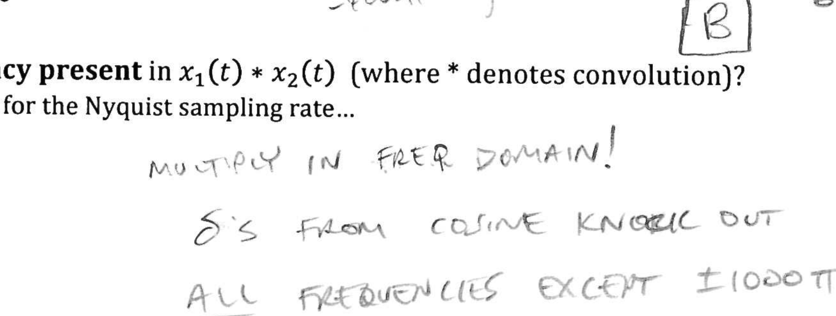
$2 \times 8000\pi = 16{,}000\pi$ rad/s.

$= 8$ kHz

$\boxed{C}$

19. What is the Nyquist sampling rate for $e^{j1{,}000\pi t}x_2(t)$? (3 pts)

- (a) 5 kHz
- (b) 6 kHz
- (c) 8 kHz
- (d) 10 kHz
- (e) 15 kHz
- (f) 20 kHz
- (g) None of the above



$2 \times 6{,}000\pi = 12{,}000\pi$

6 kHz

$\boxed{B}$

20. What is the **highest frequency present** in $x_1(t) * x_2(t)$ (where * denotes convolution)? (3 pts) **NOTE:** I'm **not** asking for the Nyquist sampling rate...

- (a) 500 Hz
- (b) 1 kHz
- (c) 2 kHz
- (d) 3 kHz
- (e) 5 kHz
- (f) 8 kHz
- (g) None of the above

MULTIPLY IN FREQ DOMAIN!

δ's FROM COSINE KNOCK OUT

ALL FREQUENCIES EXCEPT $\pm 1000\pi$ rad/s

$= \pm 500$ Hz

$\boxed{A}$

$\rightarrow 80\pi = \omega_s$

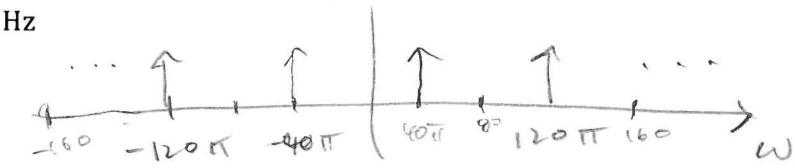
21. A 60 Hz cosine waveform, $\cos(120\pi t)$, is sampled at a sampling frequency of 40 Hz, and the samples are stored on a computer. The samples are then passed through a D/A converter which employs an ideal lowpass filter with cutoff frequencies of ±50 Hz. The reconstructed signal will be composed of a cosine signal or cosine signals at what frequencies? (3 pts)

(a) 20 Hz, 40 Hz

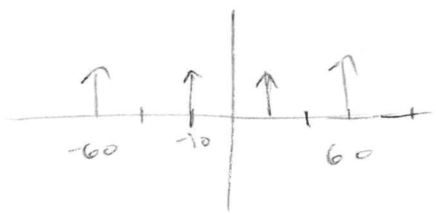
(b) 20 Hz

(c) 40 Hz

(d) 0 Hz, 20 Hz



$\boxed{B}$



22. We have an A/D converter capable of sampling at 20 kHz, which we wish to use to sample a signal $x(t)$. Which of the following would ensure that we prevent aliasing due to sampling of the signal? (3 pts)

(a) Pass $x(t)$ through an ideal low-pass filter with cutoff frequency less than 40 kHz prior to sampling

(b) Pass the discrete-time sampled version of $x(t)$ through an ideal low-pass filter with cutoff frequency less than 40 kHz immediately after sampling

(c) Pass $x(t)$ through an ideal low-pass filter with cutoff frequency less than 10 kHz prior to sampling

(d) Aliasing is possible for all of the above choices

(e) Wasn't "Aliasing" a show with Jennifer Garner?

$\boxed{C}$

23. What is the fundamental period of $x[n] = \sin(.12n + 2)$? (3 pts)

(a) 25 samples

(b) 50 samples

(c) 12 samples

(d) $x[n]$ is not periodic, and thus does not have a fundamental period

(e) None of the above

$\boxed{D}$

24. What is the fundamental angular frequency of $x[n] = \cos(0.16\pi n) + (4 - j)e^{j3\pi n/25}$? (3 pts)

(a) 0.04π rad/sample

(b) 0.08π rad/sample

(c) 0.16π rad/sample

(d) 4π rad/sample

(e) $x[n]$ is not periodic, and thus does not have a fundamental angular frequency

(f) None of the above

$\frac{16}{100}\pi n = \frac{8}{50}\pi n = \frac{4}{25}\pi n$ $N_0 = 25$

$\frac{3\pi}{25} n$ $N_0 = 50$

$N_0 = 50$ $\Omega = \frac{2\pi}{N_0} = \frac{2\pi}{50} = \frac{4\pi}{100} = .04\pi$

$\boxed{A}$

25. What is the fundamental period of the following discrete-time signal? (3 pts)

$$x[n] = \sum_{k=-\infty}^{\infty} e^{j\frac{\pi}{2}k}\delta[n-k] = e^{j\frac{\pi}{2}n}$$

$N_0 = 4$

(a) 20 samples

(b) 10 samples

(c) 5 samples

(d) 4 samples

(e) 25π samples

(f) $x[n]$ is not periodic, and thus does not have a fundamental period

(g) None of the above

$\boxed{D}$

For problems 26 and 27, consider a discrete-time LTI system. The output to the system is { 6, $\underline{13}$, 8, -9, -8, -4 } when the input is:

↑
NON-CAUSAL OUTPUT!

$$x[n] = 2\delta[n] + 3\delta[n-1] - 4\delta[n-3]$$

← CAUSAL INPUT

26. Which of the following statements is true? (3 pts)

(a) The system is neither causal nor stable.

(b) The system is causal, but it is not stable.

$\boxed{\text{(c)}}$ The system is stable, but it is not causal.

(d) The system is both causal and stable.

(e) There is not enough information given to determine which of the statements is true.

$\boxed{C}$

27. What is the system's unit impulse response? (3 pts)

(a) { $\underline{1}$, 2, 3 }

(b) { $\underline{3}$, 2, 1 }

(c) { 1, $\underline{2}$, 3 }

$\boxed{\text{(d)}}$ { 3, $\underline{2}$, 1 }

(e) { 1, $\underline{2}$, 7 }

(f) { 7, $\underline{2}$, 1 }

(g) None of the above

$\underline{2}$ 3 0 -4

c $\underline{b}$ a

6 $\underline{13}$ 8 -9 -8 -4

$\boxed{D}$

$2a = 6$, $a = 3$

$2b + 3a = 13$, $2b + 9 = 13$, $2b = 4$, $b = 2$

$2c + 3b + 0a = 8$, $2c + 6 = 8$, $2c = 2$, $c = 1$

{ 3, $\underline{2}$, 1 }

28. Compute the following convolution: { 1, $\underline{3}$, -2 } * { 4, -1, $\underline{0}$, 1, -4 } (3 pts)

$\boxed{\text{(a)}}$ { 4, 11, -11, $\underline{3}$, -1, -14, 8 }

(b) { -8, 14, 1, $\underline{-3}$, 11, -11, -4 }

(c) { 4, 11, $\underline{-11}$, 3, -1, -14, 8 }

(d) { -8, 14, $\underline{1}$, -3, 11, -11, -4 }

(e) { $\underline{4}$, 11, -11, 3, -1, -14, 8 }

(f) { $\underline{-8}$, 14, 1, -3, 11, -11, -4 }

(g) None of the above

4 -1 $\underline{0}$ 1 -4

-2 $\underline{3}$ 1

{ 4, 11, -11, $\underline{3}$, -1, -14, 8 }

$\boxed{A}$

29. Consider the discrete-time system described by the following input/output relationship: (3 pts)

$$y[n] = \sum_{k=0}^{n} x[k]$$

THIS FIXED LOWER BOUND (NOT $-\infty$) KILLS TIME INVARIANCE!

(a) This system is linear, but not time invariant.

(b) This system is not linear, but it is time invariant.

(c) This system is neither linear nor time invariant.

(d) This system is both linear and time invariant.

(e) Not enough information is given to determine whether the system is linear and/or time invariant.

$\boxed{A}$

30. Consider the discrete-time system described by the following input/output relationship: (3 pts)

$$y[n] = \begin{cases} x[n], & x[n] < 0 \\ -x[n], & x[n] \geq 0 \end{cases}$$

NOT LINEAR, BUT TIME INVARIANT!

(a) This system is linear, but not time invariant.

(b) This system is not linear, but it is time invariant.

(c) This system is neither linear nor time invariant.

(d) This system is both linear and time invariant.

(e) Not enough information is given to determine whether the system is linear and/or time invariant.

$\boxed{B}$

CONSIDER $x_1[n] = 1 \rightarrow y_1[n] = -1$

$x_2[n] = -1 \rightarrow y_2[n] = -1$

$x_1[n] + x_2[n] = 0 \rightarrow 0 \neq y_1[n] + y_2[n]$!

31. Evaluate the following summation: (3 pts)

$$\sum_{n=-5}^{5} e^{j\frac{6\pi}{7}n}\delta[n-7]$$

$\delta[n-7] = 0$ FOR ALL $-5 \leq n \leq 5$!

(a) 0

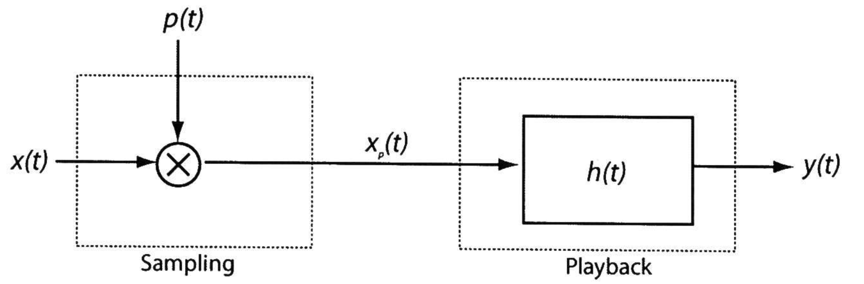
(b) 1

(c) -1

(d) None of the above

$\boxed{A}$

You have just finished recording and digitizing (sampling) a moving rendition of you singing "My Bonnie Lies over the Ocean" to send to your Scottish grandparents on their 50th wedding anniversary. The system you have devised for sampling and playback is shown below:



where $x(t)$ is the input audio signal (you singing), $x_p(t)$ is the sampled signal, $y(t)$ is the output upon playback, and $p(t)$ and $h(t)$ are given by:

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_S)$$

$$h(t) = \frac{T_S}{\pi t}\sin\left(\frac{\pi}{T_S}t\right)$$

$$T_S = 2.5 \times 10^{-5} \text{ seconds}$$

$$\omega_S = \frac{2\pi}{T_S} = 80{,}000\pi \text{ radians/sample}$$ $f_s = 40$ kHz

32. Assume that your singing (the signal $x(t)$) is band-limited to 18 kHz (angular frequency of 36,000π radians/second). Is $y(t)$ a perfect reconstruction of $x(t)$? (Answer this question theoretically. I'm not talking about degradation due to imperfections in the sound system components.) (3 pts)

(a) Yes
(b) No

$\boxed{A}$

33. Upon playing back your sampled recording, you notice that something is very wrong. Your recording exhibits an annoying high-pitched squeal at a frequency of 14 kHz. You deduce that your output signal is actually:

$$y(t) = x(t) + \cos(28{,}000\pi t)$$

You are puzzled until you remember your roommate showing you his new dog whistle, and you deduce that he must have been blowing it during your recording session. Assume that the whistle produces a single cosinusoidal signal $s(t) = \cos(2\pi f_0 t)$ at a frequency f_0 somewhere between 20 kHz and 30 kHz. What is the exact frequency f_0 of the dog whistle? (4 pts)

(a) 14 kHz
(b) 18 kHz
(c) 22 kHz
(d) 26 kHz
(e) 30 kHz

$\boxed{D}$