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3.7

(a) $x_1(t) = 25\cos(4\pi t + 30°)\,\delta(t) = 25\cos(30°)\,\delta(t)$ $\quad X(s) = \boxed{\frac{25\sqrt{3}}{2}}$

(b) $x_2(t) = 25\cos(4\pi t + 30°)\,\delta(t-0.2) = 25\cos(\frac{4\pi}{5} + 30°)\,\delta(t-0.2)$ $\quad \boxed{X(s) = 25\cos\left(\frac{29\pi}{30}\right) e^{-0.2s}}$

(c) $x_3(t) = 10\,\frac{\sin(3t)}{t}\,u(t)$ $\quad \frac{x(t)}{t}$ $\quad x(t) = 10\sin(3t)u(t)$ $\quad X(s) = 10\,\frac{3}{s^2+9} = \frac{30}{s^2+9}$

$X_3(s) = \int_s^\infty \frac{30}{s^2+9}\,ds = \boxed{-5\left(2\tan^{-1}\left(\frac{s}{3}\right) - \pi\right)}$

(d) $x_4(t) = \frac{d^2}{dt^2}\left[e^{-4t}u(t)\right]$ $\quad s^2 X(s) - s\,x(0^-) - x'(0^-)$, $\quad x(0^-) = 0$, $\quad x'(0^-) = 0$

$x(t) = e^{-4t}u(t)$ $\quad X(s) = \frac{1}{s+4}$ $\quad \boxed{X_4(s) = \frac{s^2}{s+4}}$

3.12 $X(s) = \frac{19 - e^{-s}}{s(s^2+5s+6)} = \frac{19-e^{-s}}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$ $\quad A: \frac{19 - e^0}{(0+2)(0+3)} = \frac{18}{6} = 3$

$B: \frac{19 - e^2}{(-2)(-2+3)} = \frac{19-e^2}{-2}$ $\qquad C: \frac{19 - e^3}{(-3)(-3+2)} = \frac{19-e^3}{3}$

$X(s) = \frac{3}{s} - \frac{(19-e^2)/2}{s+2} + \frac{(19-e^3)/3}{s+3}$ $\quad x(t) = 3u(t) - \frac{(19-e^2)}{2}e^{-2t}u(t) + \frac{(19-e^3)}{3}e^{-3t}u(t)$

$x(0^+) = 3 - \frac{19-e^2}{2} + \frac{19-e^3}{3} \approx -3.17$ $\qquad x(\infty) = 3 - 0 + 0 = 3$

3.15 a, b

(a) $X_1(s) = \frac{(s+2)^2}{s(s+1)^3} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}$ $\quad A(s+1)^3 + Bs(s+1)^2 + Cs(s+1) + D(s) = (s+2)^2$

$A(s^3 + 3s^2 + 3s + 1) + B(s^3 + 2s^2 + s) + C(s^2+s) + D(s) = s^2 + 4s + 4$ $\quad A=4 \quad B=-7 \quad C=3 \quad D=-4$

$A + B + C = 0 \quad 3A + 2B + C = 1 \quad 3A + B + C + D = 4 \quad A = 4$

$X_1(s) = \frac{4}{s} + \frac{-7}{s+1} + \frac{3}{(s+1)^2} + \frac{-4}{(s+1)^3}$ $\quad \boxed{x_1(t) = 4u(t) - 7e^{-t}u(t) + 3te^{-t} - 2t^2e^{-t}}$

(b) $X_2(s) = \frac{1}{(s^2+4s+5)^2}$ $\qquad \frac{-4 \pm \sqrt{16 - 4(5)}}{2} = -2+i,\ -2-i$

$= \frac{1}{(s-(-2+i))^2\,(s-(-2-i))^2} = \frac{A}{(s-(-2+i))} + \frac{B}{(s-(-2+i))^2} + \frac{C}{(s-(-2-i))} + \frac{D}{(s-(-2-i))^2}$

$A(s-(-2+i))(s-(-2-i))^2 + B(s-(-2-i))^2 + C(s-(-2+i))^2(s-(-2-i)) + D(s-(-2+i))^2 = 1$

$\boxed{-\frac{1}{4}e^{(-2-i)t}\left(e^{2it}t + t + ie^{2it} - i\right)u(t)}$

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3.18

(a) $\mathbf{X}_1(s) = \dfrac{(1-e^{-4s})(24s+40)}{(s+2)(s+10)} = \dfrac{A}{s+2} + \dfrac{B}{s+10}$ $\quad A: \dfrac{(1-e^{8})(-8)}{8} = e^{8}-1$

$B: \dfrac{(1-e^{40})(-200)}{-8} = 25(1-e^{40})$ $\quad \mathbf{X}_1(s) = \dfrac{e^{8}-1}{s+2} + \dfrac{25(1-e^{40})}{s+10}$

$$\boxed{x_1(t) = (e^{8}-1)e^{-2t}u(t) + 25(1-e^{40})e^{-10t}u(t)}$$

(b) $\mathbf{X}_2(s) = \dfrac{s(s-8)e^{-6s}}{(s+2)(s^2+16)} = \dfrac{A}{s+2} + \dfrac{B}{s+4i} + \dfrac{C}{s-4i}$

$A: \dfrac{(-2)(-2-8)e^{12}}{20} = e^{12}$ $\quad B: \dfrac{(-4i)(-4i-8)e^{24i}}{(-4i+2)(-4i-4i)} \cdot \dfrac{(-16+32i)e^{24i}}{-32-16i} = -i e^{24i}$

$C: \dfrac{(4i)(4i-8)e^{-24i}}{(4i+2)(4i+4i)} = \dfrac{(-16-32i)e^{-24i}}{-32+16i} = i e^{-24i}$

$\mathbf{X}_2(s) = \dfrac{e^{12}}{s+2} + \dfrac{-i e^{24i}}{s+4i} + \dfrac{i e^{-24i}}{s-4i}$ $\quad \boxed{x_2(t) = e^{12}e^{-2t}u(t) + i\,2\cos(4t-24)\,u(t)}$

$= \dfrac{e^{12}}{s+2} + i\left(\dfrac{-e^{24i}}{s+4i} + \dfrac{e^{-24i}}{s-4i}\right)$ $\quad \theta = 24$, $b = 4$

(c) $\mathbf{X}_3(s) = \dfrac{4s(2-e^{-4s})}{s^2+9} = \dfrac{4s(2-e^{-4s})}{(s+3i)(s-3i)} = \dfrac{A}{s+3i} + \dfrac{B}{s-3i}$

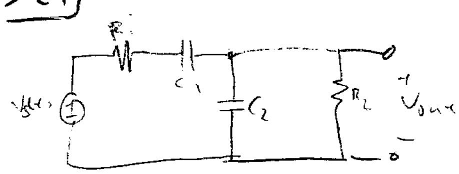
$A: \dfrac{4(-3i)(2-e^{12i})}{-3i-3i} = 2(2-e^{12i}) = 4-2e^{12i}$ $\quad B: \dfrac{4(3i)(2-e^{-12i})}{3i+3i} = 4-2e^{-12i}$

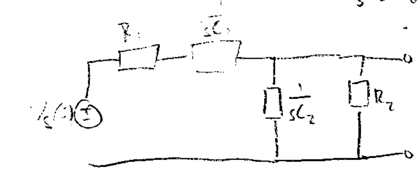
$\mathbf{X}_3(s) = \dfrac{4}{s+3i} - \dfrac{2e^{12i}}{s+3i} + \dfrac{4}{s-3i} - \dfrac{2e^{-12i}}{s-3i} = \dfrac{4}{s+3i} + \dfrac{4}{s-3i} - 2\left(\dfrac{e^{12i}}{s+3i} + \dfrac{e^{-12i}}{s-3i}\right)$ $\quad \theta = 12$, $b = 3$

$$\boxed{x_3(t) = 4e^{-3it}u(t) + 4e^{3it}u(t) - 2\left(2\cos(3t-12)\,u(t)\right)}$$

3.21

$V_s(s) = \mathcal{L}[V_s(t)] = 20$





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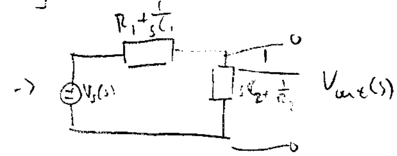

$V_s(s) \cdot \dfrac{\frac{1}{sC_2+\frac{1}{R_2}}}{R_1 + \frac{1}{sC_1} + \frac{1}{sC_2+\frac{1}{R_2}}} = V_s(s) \cdot \dfrac{\frac{1}{2s+2}}{1+\frac{1}{s}+\frac{1}{2s+2}} = V_s(s)\,\dfrac{1}{2s+2} \cdot \dfrac{s}{s+1+\frac{s}{2s+2}} = V_s(s)\,\dfrac{s}{2s^2+2s+2s+2+s}$

$V_s(s)\,\dfrac{s}{2s^2+5s+2} = V_s(s)\,\dfrac{s}{(s+2)(2s+1)} = \dfrac{35}{s} - \dfrac{20}{s} = \dfrac{15}{s}$

$V_{out}(s) = \dfrac{15}{(s+2)(2s+1)} = \dfrac{10}{2s+1} - \dfrac{5}{s+2} = \dfrac{5}{s+0.5} - \dfrac{5}{s+2}$

$$\boxed{V_{out}(t) = \left[5e^{-\frac{1}{2}t} - 5e^{-2t}\right]u(t)}$$

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3.22 $H(s) = \dfrac{18s+10}{s^2+6s+5}$

(a) $x_1(t) = u(t) \quad X_1(s) = \dfrac{1}{s}$ $\qquad \dfrac{1}{s}\cdot\dfrac{18s+10}{s^2+6s+5} = \dfrac{18s+10}{s(s+5)(s+1)} = Y(s)$

$Y(s) = \dfrac{A}{s} + \dfrac{B}{s+5} + \dfrac{C}{s+1} \quad A: \dfrac{10}{(5)(1)} = 2 \quad B: \dfrac{18(-5)+10}{-5\,(-5+1)} = \dfrac{-80}{20} = -4$

$C: \dfrac{18(-1)+10}{-1(-1+5)} = \dfrac{-8}{-4} = 2 \qquad Y(s) = \dfrac{2}{s} + \dfrac{-4}{s+5} + \dfrac{2}{s+1}$

$$\boxed{y(t) = 2u(t) + -4e^{-5t}u(t) + 2e^{-t}u(t)}$$

(b) $x_2(t) = 2t\,u(t) \quad X(s) = \dfrac{2}{s^2} \quad X(s)H(s) = \dfrac{36s+20}{s^2(s+5)(s+1)} = Y(s) = \dfrac{A_1}{s} + \dfrac{B_1}{s^2} + \dfrac{C_1}{s+5} + \dfrac{D_1}{s+1}$

$\dfrac{36s+20}{s(s+5)(s+1)} = \dfrac{A_2}{s} + \dfrac{B_2}{s+5} + \dfrac{C_2}{s+1} \quad A: \dfrac{20}{5} = 4 \quad B: -8 \quad C: 4 \quad = \dfrac{4}{s} + \dfrac{-8}{s+5} + \dfrac{4}{s+1}$

$Y(s) = \left(\dfrac{4}{s} + \dfrac{-8}{s+5} + \dfrac{4}{s+1}\right)\dfrac{1}{s} = \dfrac{4}{s^2} + \dfrac{-8}{s(s+5)} + \dfrac{4}{s(s+1)} \qquad \dfrac{-8}{s(s+5)} = \dfrac{A}{s} + \dfrac{B}{s+5} \quad \dfrac{-8}{5} = A \quad B = \dfrac{-8}{-5} = \dfrac{8}{5}$

$\dfrac{4}{s(s+1)} = \dfrac{A}{s} + \dfrac{B}{s+1} \quad A = 4 \quad B = -4 \qquad Y(s) = \dfrac{4}{s^2} + \dfrac{-8/5}{s} + \dfrac{8/5}{s+5} + \dfrac{4}{s} + \dfrac{-4}{s+1} = \dfrac{4}{s^2} + \dfrac{12/5}{s} + \dfrac{8/5}{s+5} + \dfrac{-4}{s+1}$

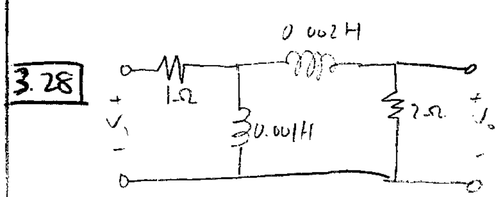
$$\boxed{y(t) = 4t\,u(t) + \tfrac{12}{5}u(t) + \tfrac{8}{5}e^{-5t}u(t) - 4e^{-t}u(t)}$$

(c) $x_3(t) = 2e^{-4t}u(t) \quad X(s) = \dfrac{2}{s+4} \qquad Y(s) = X(s)H(s) = \dfrac{36s+10}{(s+4)(s+5)(s+1)} = \dfrac{A}{s+4} + \dfrac{B}{s+5} + \dfrac{C}{s+1}$

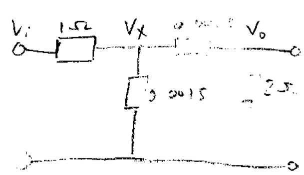
$A: \dfrac{36(-4)+10}{(-4+5)(-4+1)} = \dfrac{-134}{-3} = \dfrac{134}{3} \quad B: \dfrac{36(-5)+10}{(-5+4)(-5+1)} = \dfrac{-170}{4} = \dfrac{-85}{2} \quad C: \dfrac{36(-1)+10}{(-1+4)(-1+5)} = \dfrac{-26}{12} = -\dfrac{13}{6}$

$Y(s) = \dfrac{134/3}{s+4} + \dfrac{-85/2}{s+5} + \dfrac{-13/6}{s+1} \qquad \boxed{y(t) = \tfrac{134}{3}e^{-4t}u(t) - \tfrac{85}{2}e^{-5t}u(t) - \tfrac{13}{6}e^{-t}u(t)}$

3.28



$\Rightarrow$



$V_x = V_i \cdot \dfrac{0.001s}{1+0.001s}$

$V_o = V_x \cdot \dfrac{2}{2+0.002s}$

$V_o = V_i \cdot \dfrac{0.001s}{1+0.001s}\cdot\dfrac{1}{1+0.001s} \qquad \dfrac{V_o}{V_i} = \boxed{H(s) = \dfrac{0.001s}{(1+0.001s)^2}}$

(b) $H(s) = \dfrac{0.001s}{(1+0.001s)^2} = \dfrac{A}{(1+0.001s)} + \dfrac{B}{(1+0.001s)^2} \qquad A(1+0.001s) + B = 0.001s$

$A + B = 0$

$A = 1 \quad B = -1$

$= \dfrac{1000}{s+1000} - \dfrac{1\times10^6}{(s+1000)^2}$

$$\boxed{h(t) = \left[1000e^{-1000t} - 1\times10^6\,t\,e^{-1000t}\right]u(t)}$$

~~3.30~~

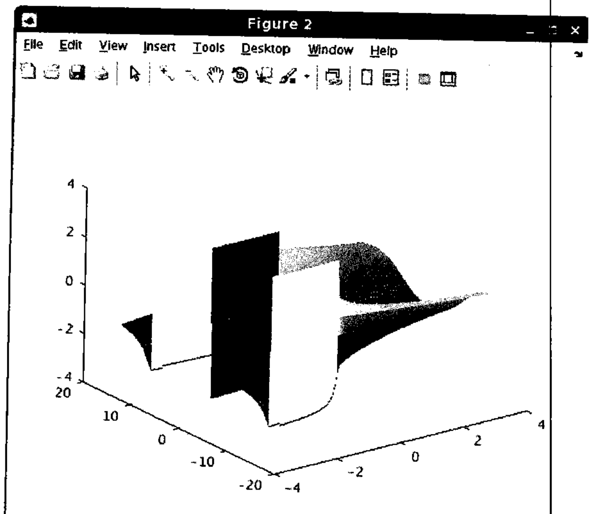
$\boxed{3.30}$

$$\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = \frac{dx}{dt} + 5x$$

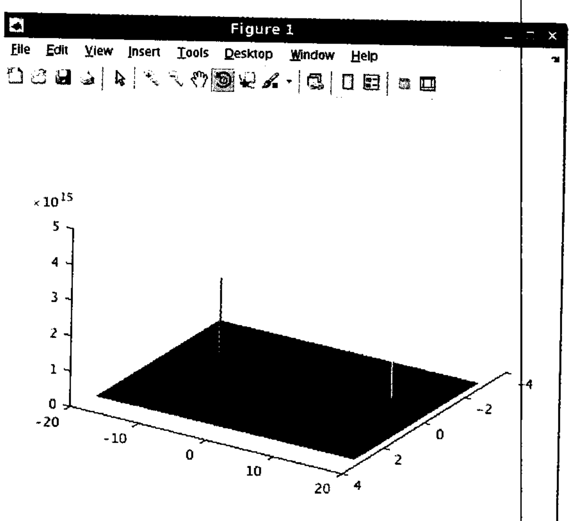
$$H(s) = \frac{s+5}{s^2-7s+12} = \frac{s+5}{(s-4)(s-3)}$$

poles @ $s = 4, 3$

$\boxed{\text{NOT BIBO stable}}$



1. zeroes: s = -9
   poles: s = -0.9 $\pm$ 4*pi*i
2. They are easy to identify.



3. Phase is odd. Magnitude is even.