ON BOARD:

$e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} 2\pi\,\delta(\omega-\omega_0)$

SOLUTIONS

Quiz #3

ECEn 380: Signals & Systems

Fall 2014

Closed book, closed note, closed neighbor, no calculators allowed. *Time limit is 15 minutes. 20 points total possible.*

1. Find the Fourier transform $X(j\omega)$ of $x(t) = \cos(10t)\cos(t)$. (4 pts)

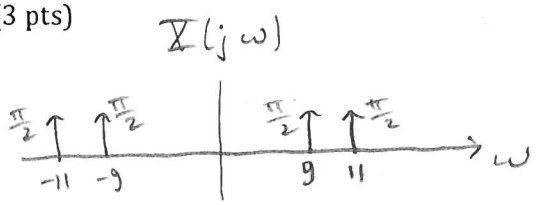
IF YOU KNOW THAT: $\cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$

ELSE USE EULER AND WHAT I WROTE ON BOARD: $\cos(\omega_0 t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$

THEN: $X(j\omega) = \frac{1}{2\pi}\mathcal{F}[\cos(10t)] * \mathcal{F}[\cos(t)]$

$$X(j\omega) = \frac{\pi}{2}\left[\delta(\omega-11)+\delta(\omega-9)+\delta(\omega+11)+\delta(\omega+9)\right]$$

2. Sketch a graph of $X(j\omega)$ that you found in the first problem. Please carefully label your axes. Note that $X(j\omega)$ is real-valued in this case, so you don't need to worry about plotting both magnitude and phase. (3 pts)



3. The impulse response of an LTI system is $h(t) = rect(t)$.

   a. What is the frequency response $H(j\omega)$ of the system? (3 pts)

$$H(j\omega) = \text{sinc}\left(\frac{\omega}{2}\right) = \frac{\sin(\omega/2)}{\omega/2} = 2\frac{\sin(\omega/2)}{\omega}$$

← ANY OF THESE ARE OKAY

IF YOU FORGOT, JUST DO INTEGRAL:

$$H(j\omega) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1\cdot e^{-j\omega t}\,dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{2}{\omega}\left(\frac{e^{j\frac{1}{2}\omega} - e^{-j\frac{1}{2}\omega}}{2j}\right) = \frac{2}{\omega}\sin(\omega/2)$$

Suppose the input to this system is $x(t) = e^{j3t}$.

   b. Find the Fourier transform of this input signal. (3 pts)

$$X(j\omega) = 2\pi\,\delta(\omega-3)$$

← IT WAS ON THE BOARD

   c. Find the output of the system in the frequency domain. That is, find $Y(j\omega)$. (3 pts)

$$Y(j\omega) = X(j\omega)H(j\omega) = 2\pi\,\delta(\omega-3)\cdot\frac{2}{\omega}\sin\left(\frac{\omega}{2}\right)$$

$$= \frac{4}{3}\pi\sin(3/2)\,\delta(\omega-3)$$

4. Describe how you would find the total energy of the signal $x(t) = \sin(t)/t$. Don't do it. Just tell me the steps. (4 pts)

USE PARSEVAL'S. FIND $X(j\omega)$, WHICH WILL BE A RECT IN THE FREQUENCY DOMAIN. THEN INTEGRATE THE RECT^2 IN THE FREQUENCY DOMAIN (EASY)!

↑ JUST LIKE THE HOMEWORK PROBLEM. :)