ECEn 380
Homework 6

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5.19 P5.6: No P5.7: No P5.8: yes P5.9: no P5.10: no

P5.11: yes The ones that have Gibbs ringing have a jump discontinuity.

5.38

$$\int_{-\infty}^{\infty} x(t)\, e^{-jwt} = \int_0^3 \frac{5}{3} t e^{-jwt}\,dt = \frac{5}{3}\int_0^3 t e^{-jwt}\,dt = \frac{5}{3}\left[\left(\frac{t}{-jw}+\frac{1}{w^2}\right)e^{-jwt}\right]_0^3$$

$$= \frac{5}{3}\left[\left(\frac{3}{-jw}+\frac{1}{w^2}\right)e^{-3jw} - \frac{1}{w^2}\right] = \frac{5}{3}\left[\frac{3e^{-3jw}}{-jw} + \frac{e^{-3jw}}{w^2} - \frac{1}{w^2}\right]$$

$$= \boxed{\frac{5e^{-3jw}}{-jw} + \frac{5e^{-3jw}}{3w^2} - \frac{5}{3w^2}}$$

5.49

(a) $e^{-\alpha t}\sin(w_1 t)\cos(w_2 t)\,u(t)$

$x(t) = e^{-\alpha t}\sin(w_1 t)\,u(t)$ $x(t)\cos(w_2 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}\left[X(w-w_2) + X(w+w_2)\right]$

$$X(w) = \frac{w_1}{[(\alpha+jw)^2+w_1^2]} = \frac{4}{\left[\left(\frac{0.5}{5}+jw\right)^2+16\right]} = \frac{4}{\left(\frac{0.5}{jw}+jw\right)^2+16} = \frac{4}{-w^2-\frac{0.25}{w^2}+17} = \frac{-4}{w^2+\frac{0.25}{w^2}-17}$$

$$= \frac{-4w^2}{w^4-17w^2+0.25}$$

$$\boxed{\frac{1}{2}\left[\frac{-4(w-2)^2}{(w-2)^4-17(w-2)^2+0.25} - \frac{4(w+2)^2}{(w+2)^4-17(w+2)^2+0.25}\right]}$$

(b) $g(t) = te^{-\alpha t}$ $0 \le t \le 10\alpha$ $\alpha > \frac{0.5}{jw}$

$$\int_0^{10\alpha} te^{-\alpha t}e^{-jwt}\,dt = \int_0^{10\alpha} te^{-\alpha t - jwt} = \int_0^{10\alpha} te^{t(-\alpha-jw)}\,dt \qquad \text{Let } x = -\alpha - jw$$

$$\int_0^{10\alpha} te^{xt}\,dt = \left(\frac{t}{x}-\frac{1}{x^2}\right)e^{tx}\Big|_0^{10\alpha} = \left(\frac{10\alpha}{x}-\frac{1}{x^2}\right)e^{10\alpha x} - \left(\frac{0}{x}-\frac{1}{x^2}\right)$$

$$= \left(\frac{10\alpha}{-\alpha-jw} - \frac{1}{(-\alpha-jw)^2}\right)e^{10\alpha(-\alpha-jw)} + \frac{1}{(-\alpha-jw)^2} = \left(\frac{5}{\left(\frac{-0.5}{jw}-jw\right)} - \frac{1}{\left(\frac{-0.5}{jw}-jw\right)^2}\right)e^{\frac{5}{jw}\left(\frac{-0.5}{jw}-jw\right)} + \frac{1}{\left(\frac{-0.5}{jw}-jw\right)^2}$$

$$= \boxed{\left(\frac{5}{\frac{-0.5}{jw}-jw} + \frac{w^2}{(w^2-0.5)^2}\right)e^{\frac{-5(w^2-0.5)}{w^2}} - \frac{w^2}{(w^2-0.5)^2}}$$

5.58 $e^{-t}u(t) * \frac{\sin(t)}{\pi t}$ $\qquad E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(w)|^2\, dw$

$$y(t) = e^{-t}u(t) \qquad Y(w) = \frac{1}{1+jw} \qquad z(t) = \frac{\sin(t)}{\pi t} \qquad Z(w) = \text{rect}\left(\frac{w}{2}\right)$$

$$x(t) = y(t) * z(t) \qquad X(w) = Y(w)Z(w)$$

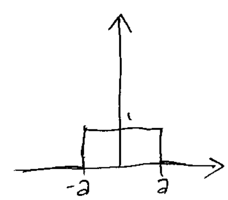
$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{\text{rect}(\frac{w}{2})}{1+jw}\right|^2 dw = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\text{rect}(\frac{w}{2})}{(1+jw)^2}\, dw = \frac{1}{2\pi}\int_{-1}^{1}\frac{1}{(1+jw)^2}\, dw$$

$$= \frac{1}{2\pi}\left[\frac{1}{w-j}\right]\Big|_{-1}^{1} = \frac{1}{2\pi}\left[\frac{1}{1-j} - \frac{1}{-1-j}\right] = \frac{1}{2\pi}[1] = \boxed{\frac{1}{2\pi}}$$

5.59 $\int_{-\infty}^{\infty}\frac{\sin^2(at)}{(\pi t)^2}\, dt = \int_{-\infty}^{\infty}\left(\frac{\sin(at)}{\pi t}\right)^2 dt = \frac{a}{\pi}$

$$x(t) = \frac{\sin(at)}{\pi t} \qquad X(w) = \text{rect}\left(\frac{w}{2a}\right)$$

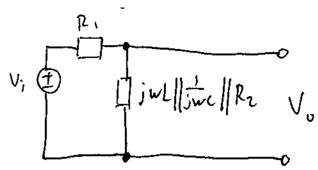
$\text{rect}\left(\frac{1}{2a}w\right)$



$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\text{rect}\left(\frac{w}{2a}\right)\right]^2 dw = \frac{a}{\pi}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\text{rect}\left(\frac{w}{2a}\right) dw = \frac{1}{2\pi}2a = \frac{a}{\pi}$$

6.4



$$V_o = V_i\frac{jwL\|\frac{1}{jwC}\|R_2}{\left(jwL\|\frac{1}{jwC}\|R_2\right)+R_1}$$

(a) $\boxed{H = \frac{jwL\|\frac{1}{jwC}\|R_2}{\left(jwL\|\frac{1}{jwC}\|R_2\right)+R_1}}$

(b) $$\frac{jwL\cdot\frac{1}{jwC}\cdot R_2}{jwL + \frac{1}{jwC} + R_2} = \frac{\frac{L}{C}R_2}{jwL + \frac{1}{jwC} + R_2} = \frac{jwLR_2}{-w^2LC + 1 + R_2jwC} = \frac{-jwLR_2}{w^2LC - R_2jwC - 1}$$

$$\frac{\frac{-jwLR_2}{w^2LC - R_2jwC - 1}}{\frac{-jwLR_2}{w^2LC - R_2jwC - 1} + \frac{R_1(w^2LC - R_2jwC - 1)}{w^2LC - R_2jwC - 1}} = \frac{-jwLR_2}{w^2LC - R_2jwC - 1}\cdot\frac{w^2LC - R_2jwC - 1}{R_1(w^2LC - R_2jwC - 1) - jwLR_2} = \frac{-jwLR_2}{R_1LCw^2 - R_1R_2Cjw - R_1 - LR_2jw}$$