LAB 2

Continuous-time LTI Impulse Response and Convolution

ECEn 380 Section 001

Task 1 signoff : [illegible] 9/21

Task 2 signoff : [illegible] 10/5/2015

Task 3 signoff : [illegible] 10/5/2015

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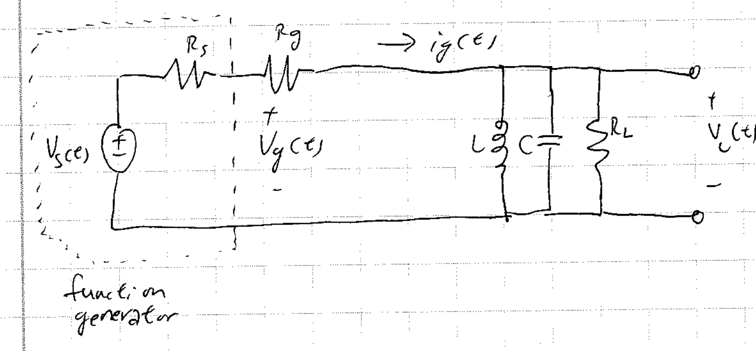
Objective: The purpose of this lab is to help us better understand LTI systems and the principles of convolution and impulse response. We will understand the mathematics & model LCCDEs and how they apply to analog circuits. We will build a crossover speaker

Task 1:



By using KCL, we get: $i_C(t) + i_{RL}(t) + i_L(t) = i_g(t)$

Using the equation for current for L, R_L, and C, and then differentiating the equation, we get: $C\frac{d^2}{dt^2}V_L(t) + \frac{1}{R_L}\frac{d}{dt}V_L(t) + \frac{1}{L}V_L(t) = \frac{d}{dt}i_g(t)$

Divide by C: $\frac{d^2}{dt^2}V_L(t) + \frac{1}{R_L C}\frac{d}{dt}V_L(t) + \frac{1}{CL}V_L(t) = \frac{1}{C}\frac{d}{dt}i_g(t)$

We find that $a_1 = \frac{1}{R_L C}$ $\quad a_2 = \frac{1}{CL}$

* We are using the values $R_L = 8\Omega$ $\quad L = 2.78\,mH$ $\quad C = 4.83\mu F$

$a_1 = \frac{1}{(8)(4.83E-9)} = 2.58799\times10^7$ $\qquad a_2 = \frac{1}{(4.83E-9)(2.78E3)} = 7.447\times10^7$

$P_1 = \frac{-a_1}{2} + \sqrt{\left(\frac{a_1}{2}\right)^2 - a_2}$ $\qquad P_2 = \frac{-a_1}{2} - \sqrt{\left(\frac{a_1}{2}\right)^2 - a_2}$

$= -2.87752$ $\qquad = -25879997$

$x(t) = \frac{1}{C}\frac{d}{dt}i_g(t)$ $\qquad i_g(t) = \frac{V_s(t)}{R_s + R_g}$

$x(t) = \frac{1}{C}\frac{d}{dt}\frac{V_s(t)}{R_s+R_g}$

$y(t) = V_L(t)$ $\quad a_1 = \frac{1}{R_L C}$ $\quad a_2 = \frac{1}{CL}$

$$\boxed{\frac{d^2}{dt^2}y(t) + a_1\frac{d}{dt}y(t) + a_2 y(t) = x(t)}$$

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Task 1

1 $h_c(t) = h_1(t) * h_2(t)$; $h_c(t)$ is the impulse response for a generic $x(t)$

$$\int_{-\infty}^{\infty} e^{p_1\tau}u(\tau)\cdot e^{p_2(t-\tau)}u(t-\tau)\,d\tau = \int_0^t e^{p_1\tau}\cdot e^{p_2(t-\tau)}\,d\tau$$

$$= \int_0^t e^{p_1\tau + p_2(t-\tau)}\,d\tau = \frac{1}{p_1-p_2} e^{p_1\tau+p_2(t-\tau)}\Big|_0^t = \frac{1}{p_1-p_2}\left[e^{p_1 t} - e^{p_2 t}\right]u(t)$$

$$\frac{1}{p_1-p_2}\left[e^{p_1 t} - e^{p_2 t}\right]u(t) = h_c(t)$$

$$\frac{dh_c(t)}{dt} = \frac{1}{p_1-p_2}\left[\frac{d}{dt}e^{p_1 t} - \frac{d}{dt}e^{p_2 t}\right] = \frac{1}{p_1-p_2}\left[p_1 e^{p_1 t} - p_2 e^{p_2 t}\right]u(t)$$

$$x(t) = \frac{1}{C}\frac{d}{dt}\frac{v_s(t)}{R_s+R_g}$$

$$\boxed{h(t) = \frac{1}{p_1-p_2}\left[p_1 e^{p_1 t} - p_2 e^{p_2 t}\right]\frac{1}{C}\cdot\frac{1}{R_s+R_g}u(t)}$$

$$p_1 = \frac{-1}{2R_L C} + \sqrt{\left(\frac{1}{2R_L C}\right)^2 - \frac{1}{CL}} = \frac{-1}{2R_L C} + \sqrt{\frac{1}{4R_L^2C^2} - \frac{1}{CL}}$$

$$p_2 = \frac{-1}{2R_L C} - \sqrt{\frac{1}{4R_L^2C^2} - \frac{1}{CL}}$$

For $8\Omega = R_L$ $\alpha = \frac{1}{2RC} = 12940 > \omega_0 = 1375$ so it's overdamped

For $R_L = 33\Omega$ $\alpha = \frac{1}{2RC} = 3136 > \omega_0$ so it's overdamped

For $R_L = 100\Omega$ $\alpha = \frac{1}{2RC} = 1035 < \omega_0$ so it's underdamped.

It looks like a RLC natural response because the capacitance and the inductance were designed with a 8Ω load in mind.

R = 8 ohms

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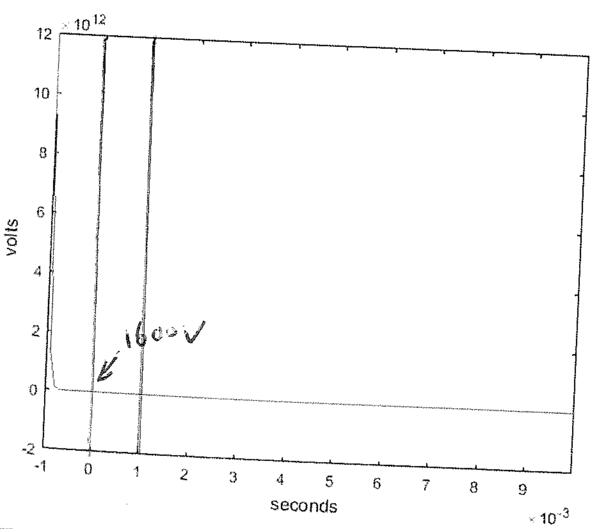

R = 33 ohms

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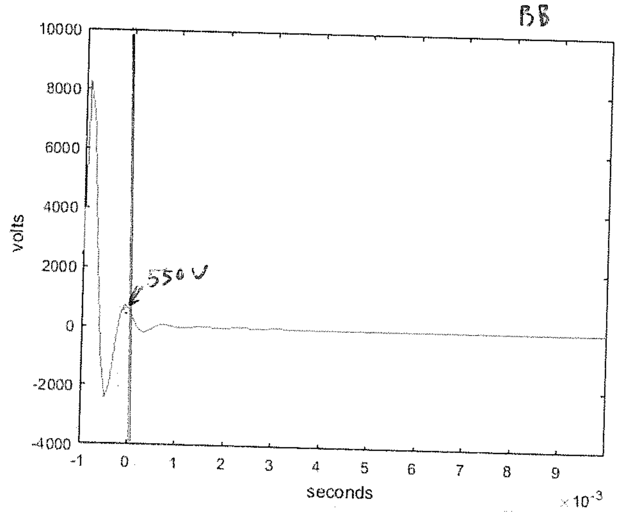

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Task 1

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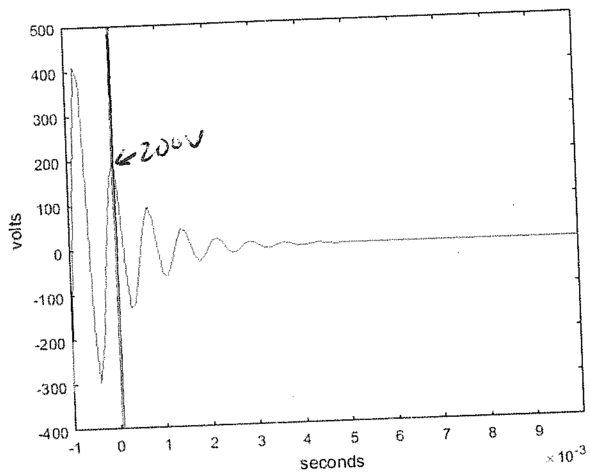
R = 100 ohms



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```
r = 8;
rs = 50;
rg = r*10;
l = 2.78e-3;
c = 4.83e-6;
t = -1:1e-4:10e-3;

p1 = (-1/(2*r*c))+ sqrt((1/(4*r^2*c^2)) -
1/(c*l));
p2 = (-1/(2*r*c))- sqrt((1/(4*r^2*c^2)) -
1/(c*l));
h_t = (1/(c*(p1-
p2)*(rs+rg)))*(p1*exp(p1*t)-
p2*exp(p2*t));

figure(1);
plot(t, h_t);
xlim([-1e-3 1e-2]);
xlabel('seconds');
ylabel('volts');
```

Using the matlab code above, we plotted the impulse response of the RLC circuit with R= 8, 33, 100.

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TASK 2

We are building the circuit described from task 1.
We used $L = 2.78\,mH$, $C = 4.83\,\mu F$

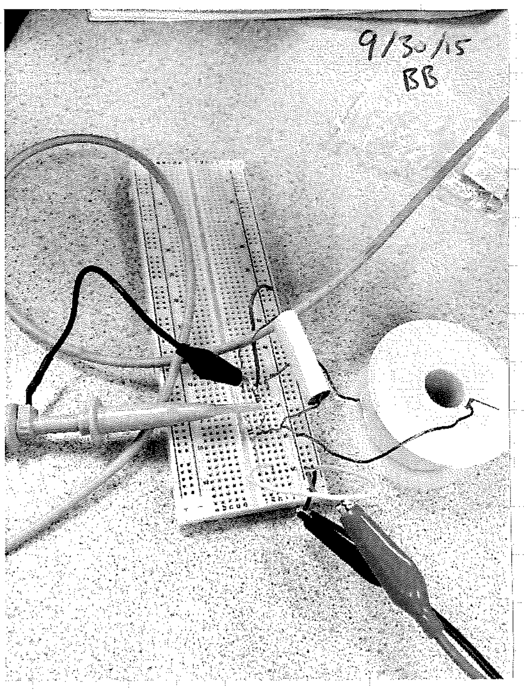

Photo of our circuit

Using the oscilloscope and a pulse from the function generator, we obtained the waveforms below. They represent the voltage drops across the load resistor. We took the calculated values and divided them by the ~~actu~~ measured value to obtain the scaling factor.

R = 8.2 ohms

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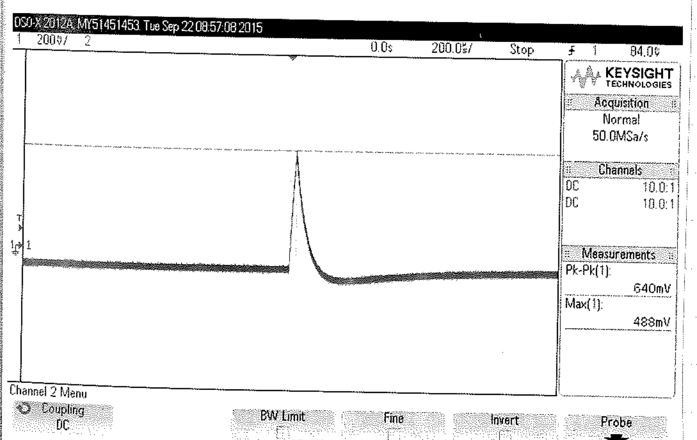

Scaling factor:

$$\frac{1600\,V}{0.5\,V} = 3200$$

R = 33 ohms

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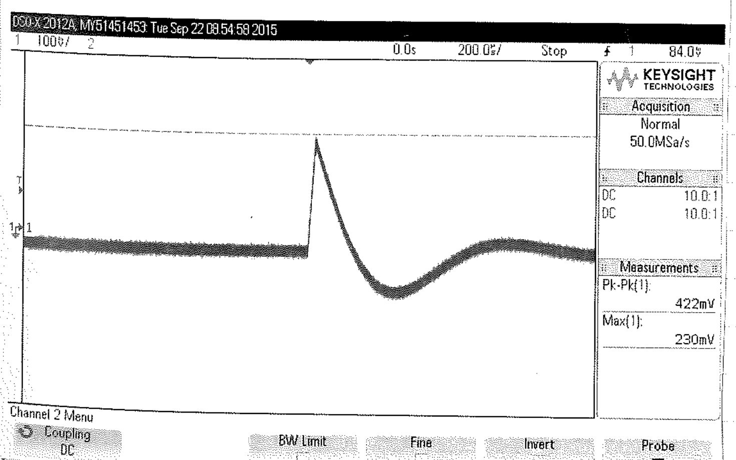

Scaling factor:

$$\frac{550\,V}{0.230\,V} = 2391$$

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Task 2

R = 100 ohms

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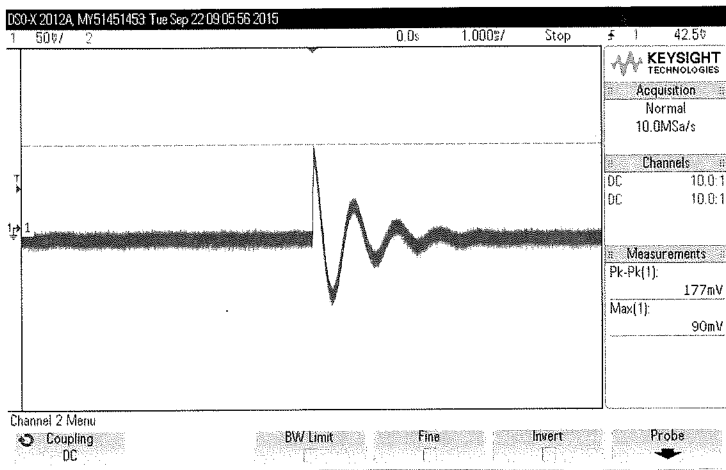

Scaling factor:

$\frac{200}{0.09} = 2222$

We changed the MATLAB code a little bit to account for the scaling factor. We used 2500. The plots can be seen below.

R=8

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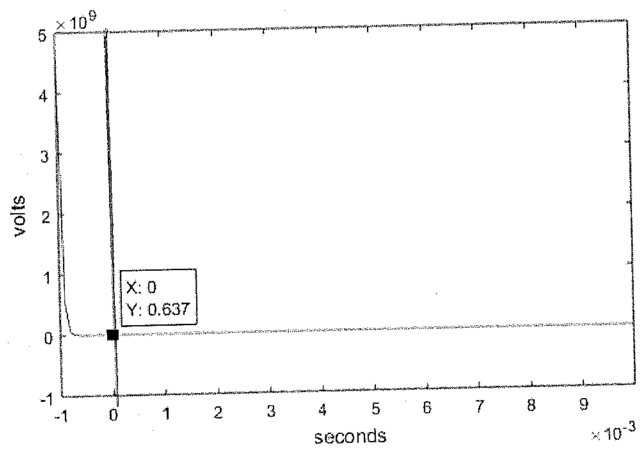

Calculated	Measured
0.637 V	0.488 V

R=33

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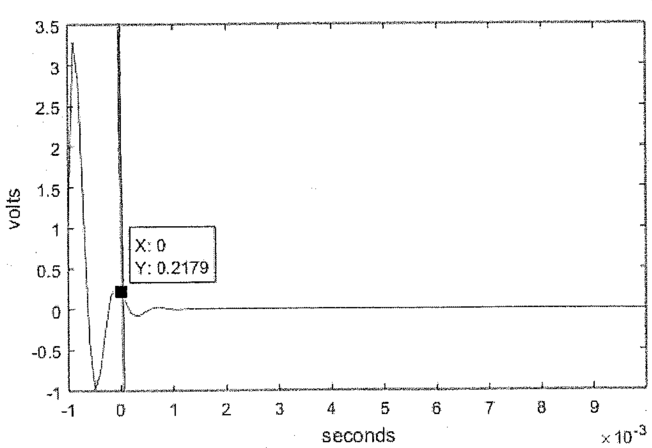

Calculated	Measured
0.2179 V	0.230 V

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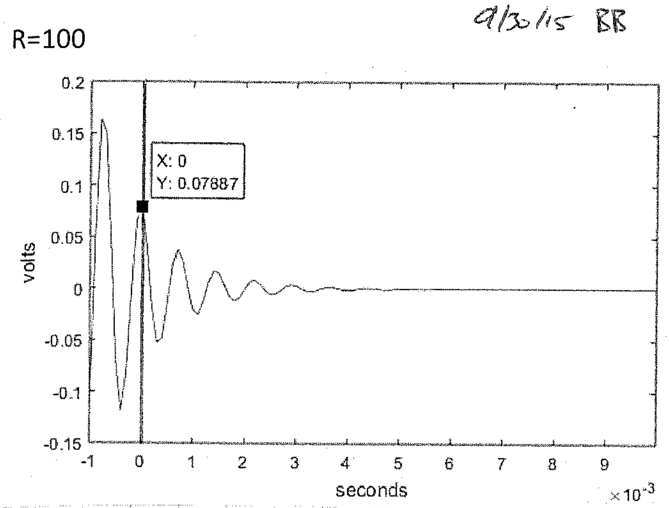

Task 2

Calculated	Measured
0.07887 V	0.09 V

The ~~calculated~~ MATLAB plots matched up fairly closely with the measured response. The errors could have been caused by several things, including: differences in actual values of the inductor and capacitor. We did not measure what the exact value of them. We just used the printed value and did not take into account the tolerance. It could also have been the noise from the O-scope.

TASK 3

Using ~~the~~ $h(t) = \frac{1}{C\cdot(P_1-P_2)(r_s+r_g)} \cdot \left(P_1 e^{P_1 t} \cdot P_2 e^{P_2 t}\right)$, we did the convolution with a rect function.

$K = \frac{1}{C\cdot(P_1-P_2)(r_s+r_g)}$ for $t<t_0$ $\int_0^t K P_1 e^{P_1(t-\tau)} \cdot P_2 e^{P_2(t-\tau)}\, d\tau$

$$= K\left[e^{P_1(t-\tau)} \cdot e^{P_2(t-\tau)}\right]\Big|_0^t = K\left[e^{P_2 t} - e^{P_1 t}\right]$$

for $t>t_0$ $\int_0^{t_0} K P_1 e^{P_1(t-\tau)} P_2 e^{P_2(t-\tau)}\, d\tau = K\left[e^{P_1(t-\tau)} e^{P_2(t-\tau)}\right]\Big|_0^{t_0}$

$$= K\left[e^{P_1(t-t_0)} - e^{P_1 t} - e^{P_2(t-t_0)} + e^{P_2 t}\right]$$

We used the function generator to generate a rect function and recorded the output.

Output to a rect. function with R = 33 ohms

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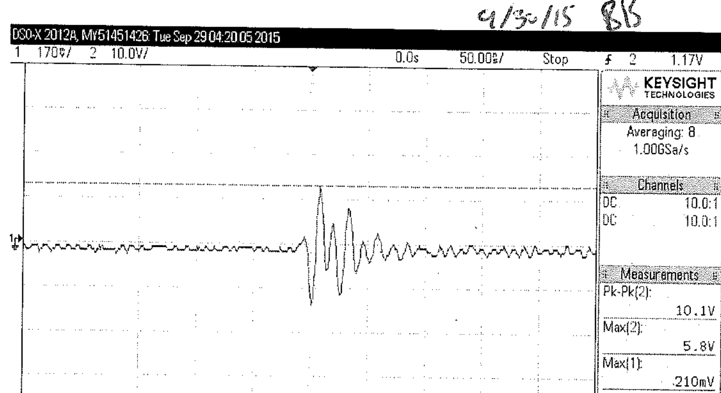

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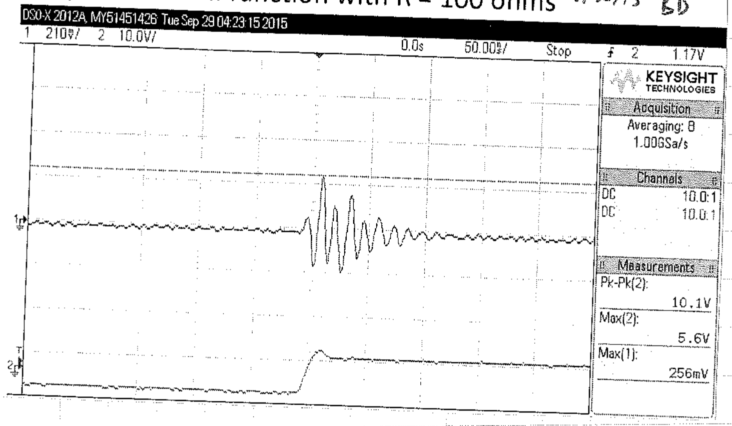
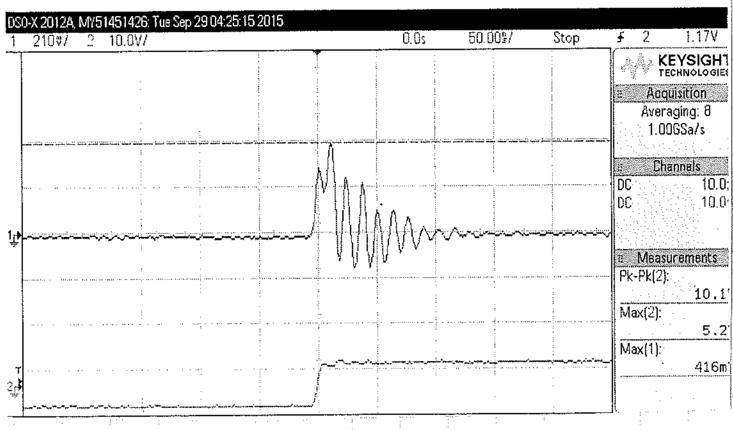
Task 3

Output to a rect. function with R = 100 ohms 9/30/15 BB



Output to a rect. function with R = 8 ohms 9/30/15 BB



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Convolution Code:

```
r = 8;
rs = 50;
rg = r*10;
l = 2.78e-3;
c = 4.83e-6;
t = -1:1e-4:10e-3;
scaleFactor = 2500;

p1 = (-1/(2*r*c))+ sqrt((1/(4*r^2*c^2)) - 1/(c*l));
p2 = (-1/(2*r*c))- sqrt((1/(4*r^2*c^2)) - 1/(c*l));
h_t = ((1/(c*(p1-p2)*(rs+rg)))*(p1*exp(p1*t)-
p2*exp(p2*t))) / scaleFactor;

t_0 = 1e-3;

t1 = t - t_0;
constant = 1/(c*(p1-p2)*(rs+rg));

funct1 = constant*(-exp(p1*(t)) +
exp(p2*(t))).*(t<t_0);
funct2 = constant*(-exp(p1*(t))+ exp(p2*(t)) +
exp(p1*(t1))-exp(p2*(t1))).*(t>=t_0);

Convolved = (funct1 + funct2).*(t>0).*10;

figure(1);
plot(t, Convolved);
xlim([-1e-3 1e-2]);
xlabel('seconds');
ylabel('volts');
```

We wrote MATLAB code to compute the convolution based on our integration. The plots can bee seen on the next page

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Task 3

Convolution of h_t and rect_t with R = 8 ohms
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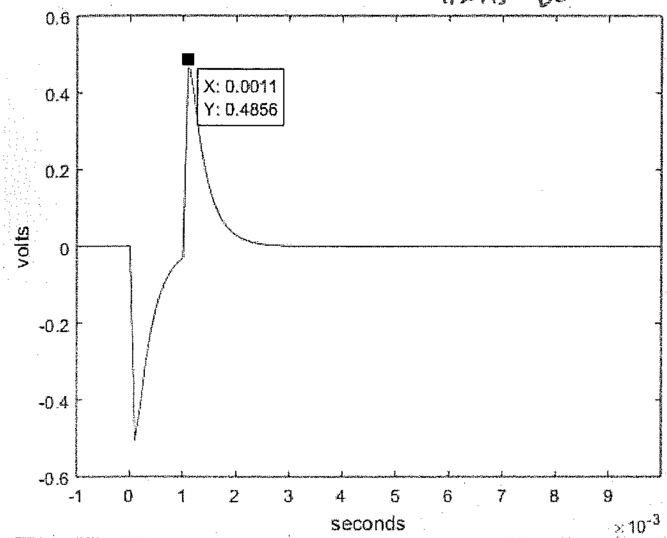
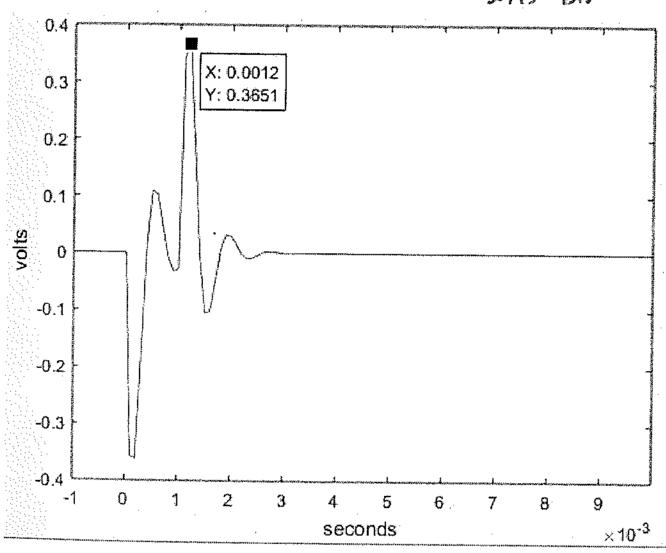


Convolution of h_t and rect_t with R = 33 ohms
9/30/15 BD



Convolution of h_t and rect_t with R = 100 ohms
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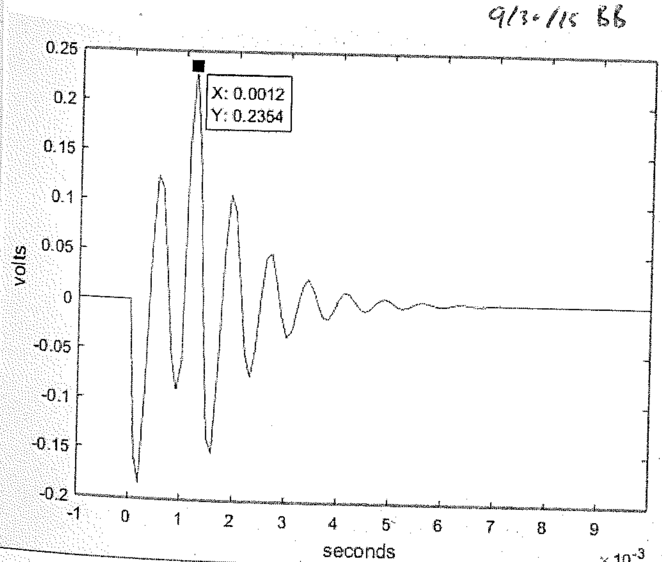

From the plots, we observed that when the load resistor is higher, the plot matches the measured waveform better. For R = 100Ω the plot matched the measurement pretty well, while the other two did not.

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Task 3

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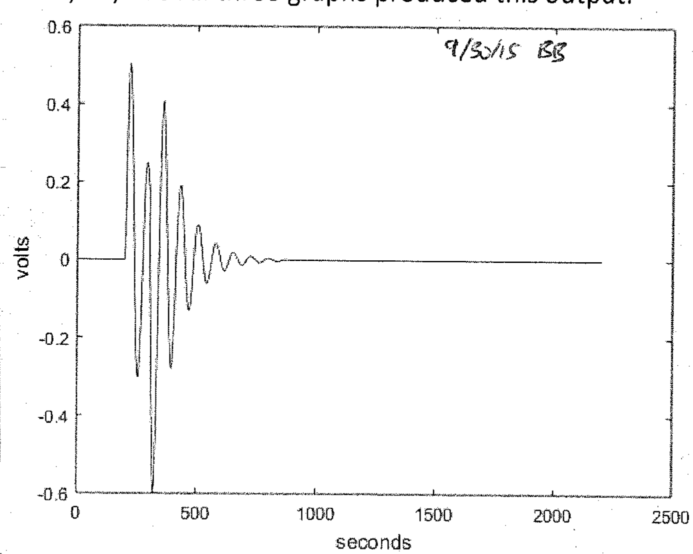
Matlab Code with conv funct.

```
r = 33;
rs = 50;
rg = r*10;
l = 2.78e-3;
c = 4.83e-6;
t = -.001:1e-5:10e-3;
scaleFactor = 2500;

p2 = (-1/(2*r*c))+ sqrt((1/(4*r^2*c^2)) -
1/(c*l));
p1 = (-1/(2*r*c))- sqrt((1/(4*r^2*c^2)) -
1/(c*l));
h_t = ((1/(c*(p1-
p2)*(rs+rg)))*(p1*exp(p1.*t)-
p2*exp(p2.*t))) / scaleFactor.* (t >= 0);

t_0 = 1e-3;

rect_t = double(t>=0 & t <= t_0);

x = conv(rect_t, h_out);

figure(1);
plot(x);

xlabel('seconds');
ylabel('volts');
```

Above is the MATLAB code to plot the convolution of h(t) and rect(t). using the MATLAB function conv.

Below is the plot.

R = 8, 33, 100 All three graphs produced this output.



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Conclusion

Conclusion

In this lab we will able to derive the impulse response mathematically as well as measure it based on the circuit that we built. The graphs looked fairly similar except for the scaling factor. This is due to the ~~G~~ F.G. not being able to generate and infinite pulse. We were able to calculate the scaling factor and redraw the graphs in ~~MATA~~ MATLAB. This yielded plots that were much closer to the measured graph. We were then able to ~~do a convolution~~ convolve h(t) with rect(t) to predict the output. We then used the F.G. to create a rect function and measure the output ~~The~~ Using MATLAB to do the convolution based on our calculations yielded similar graphs for the output for 100 Ω, but less similar graphs for lesser resistance. Then using the conv function in MATLAB we plot the output again.

ct(t).

10/5/15