7.17 (a) $\frac{z+1}{2z} = \frac{1}{2} + \frac{1}{2z} = \frac{1}{2} + \frac{1}{2}z^{-1} \xleftrightarrow{Z} \boxed{\frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1]}$

(b) $\frac{z-1}{z-2} = \frac{z}{z-2} - \frac{1}{z-2} \xleftrightarrow{Z} \boxed{2^n u[n] - 2^{n-1} u[n-1]}$

(c) $\frac{2z+3}{z^2(z+1)} = \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z+1}$ $\quad Az(z+1) + B(z+1) + Cz^2 = 2z+3$

$= -\frac{1}{z} + \frac{3}{z^2} + \frac{1}{z+1}$ $\quad Az^2 + Az + Bz + B + Cz^2 = 2z+3 \quad B = 3$

$A + C = 0 \quad A + 3 = 2 \quad A = -1$

$C = 1$

$\boxed{-\delta[n-1] + 3\delta[n-2] + (-1)^{n-1} u[n-1]}$

(d) $\frac{z^2+3z}{z^2+3z+2} = \frac{2}{z+2} - \frac{2}{z+1} + 1 \xleftrightarrow{Z} \boxed{2(-2)^{n-1}u[n-1] - 2(-1)^{n-1}u[n-1] + \delta[n]}$

7.21 $\{1, 3, 2\}$ $\quad y[n] + 2y[n-1] = 4x[n] + 5x[n-1] \xleftrightarrow{Z} Y(z) + \frac{2}{z}Y(z) + y[-1] = 4(1 + 3z^{-1} + 2z^{-2})$

$x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2]$ $\quad X(z) = 1 + 3z^{-1} + 2z^{-2}$ $\quad + \frac{5}{z}(1 + 3z^{-1} + 2z^{-2})$

$Y(z) + \frac{2}{z}Y(z)$ $\quad = 4 + \frac{12}{z} + \frac{8}{z^2} + \frac{5}{z} + \frac{15}{z^2} + \frac{10}{z^3}$ $\quad Y(z)\left(\frac{z+2}{z}\right) = \frac{4z^3 + 17z^2 + 23z + 10}{z^3}$

$= \frac{4z^3 + 17z^2 + 23z + 10}{z^3}$ $\quad Y(z) = \frac{4z^4 + 17z^3 + 23z^2 + 10z}{z^3(z+2)}$

$Y(z) = 4\left(\frac{z^4 + 17/4\, z^3 + 23/4\, z^2 + 5/2\, z}{z^4 + 2z^3}\right) = 4\left[1 + \frac{z^4 + 17/4\, z^3 + 23/4\, z^2 + 5/2\, z - z^4 - 2z^3}{z^4 + 2z^3}\right] = 4\left[1 + \frac{9/4\, z^3 + 23/4\, z^2 + 5/2\, z}{z^4 + 2z^3}\right]$

$= 4 + \frac{9z^3 + 23z^2 + 10z}{z^3(z+2)}$ $\quad \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z^3} + \frac{D}{z+2}$ $\quad Az^2(z+2) + Bz(z+2) + C(z+2) + D(z^3)$

$Az^3 + 2Az^2 + Bz^2 + 2Bz + Cz + 2C + Dz^3 = 9z^3 + 23z^2 + 10z$ $\quad A + D = 9 \quad 2B + C = 10$

$2A + B = 23 \quad C = 0 \quad B = 5 \quad A = 9 \quad D = 0$

$= \frac{9}{z} + \frac{5}{z^2}$ $\quad Y(z) = 4 + \frac{9}{z} + \frac{5}{z^2}$ $\quad \boxed{y[n] = 4\delta[n] + 9\delta[n-1] + 5\delta[n-2]}$

7.23 (a) $H(z) = \frac{(z-3)(z-4)}{(z-1)(z-2)} = \boxed{\frac{z^2 + 7z + 12}{z^2 - 3z + 2}}$

(b) $x[n] = \delta[n] - 3\delta[n-1] + 2\delta[n-2]$ $\quad X(z) = 1 - 3z^{-1} + 2z^{-2} = 1 - \frac{3}{z} + \frac{2}{z^2} = \frac{z^2 - 3z + 2}{z^2}$

$Y(z) = X(z)H(z) = \frac{z^2 + 7z + 12}{z^2} = 1 + \frac{z^2 + 7z + 12 - z^2}{z^2} = 1 + \frac{7z + 12}{z^2} = 1 + \frac{7}{z} + \frac{12}{z^2}$

$y[n] = \delta[n] + 7\delta[n-1] + 12\delta[n-2] = \boxed{\{1, 7, 12\}}$

(c) $\frac{z^2 + 7z + 12}{z^2 - 3z + 2} = 1 + \frac{z^2 + 7z + 12 - z^2 + 3z - 2}{z^2 - 3z + 2} = 1 + \frac{10z + 10}{z^2 - 3z + 2} = 1 + \frac{30}{z-2} - \frac{20}{z-1}$

$\boxed{h[n] = \delta[n] + 30(2)^{n-1}u[n-2] - 20u[n-1]}$

Homework 10

7.23 (d) $H(z) = \frac{z^2+7z+12}{z^2-3z+2} = \frac{Y(z)}{X(z)}$ $\quad Y(z)(z^2-3z+2) = X(z)(z^2+7z+12)$

$Y(z)\left(1-\frac{3}{z}+\frac{2}{z^2}\right) \quad X(z)\left(1+\frac{7}{z}+\frac{12}{z^2}\right)$

$y[n]-3y[n-1]+2y[n-2] = x[n]+7x[n-1]+12x[n-2]$

7.24 (a) zeros: $z = 1, 6$ poles: $z = 2, 3$ not stable

(b) $H(z) = \frac{z^2-7z+6}{z^2-5z+6} = \frac{Y(z)}{X(z)}$ $\quad Y(z)(z^2-5z+6) = X(z)(z^2-7z+6)$

$Y(z)\left(1-\frac{5}{z}+\frac{6}{z^2}\right) = X(z)\left(1-\frac{7}{z}+\frac{6}{z^2}\right)$

$y[n]-5y[n-1]+6y[n-2] = x[n]-7x[n-1]+6x[n-2]$

(c) $x[n] = \delta[n]-5\delta[n-1]+6\delta[n-2]$ $\quad X(z) = 1-5z^{-1}+6z^{-2} = \frac{z^2-5z+6}{z^2}$

$H(z) = \frac{z^2-7z+6}{z^2-5z+6}$ $\quad Y(z) = X(z)H(z) = \frac{z^2-7z+6}{z^2} = 1-\frac{7}{z}+\frac{6}{z^2}$ $\quad y[n] = \delta[n]-7\delta[n-1]+6\delta[n-2]$

$y[n] = \{1, -7, 6\}$

(d) $H(z) = \frac{z^2-7z+6}{z^2-5z+6} = 1+\frac{z^2-7z+6-z^2+5z-6}{z^2-5z+6} = 1+\frac{-2z}{z^2-5z+6} = 1+\frac{4}{z-2}-\frac{6}{z-3}$

$h[n] = \delta[n]+4(2)^{n-1}u[n-2]-6(3)^{n-1}u[n-3]$