2.14

$y(t) = x(t) * h(t) \qquad x(t) = \begin{cases} 0, & \text{for } t<0 \\ \sin \pi t, & \text{for } 0 \le t \le 1\,s \\ 0, & \text{for } t \ge 1\,s \end{cases} \qquad h(t) = u(t) \quad h(t-\tau) = u(t-\tau) \quad \begin{cases} 0, & \text{for } t<\tau \\ 1, & \text{for } t>\tau \end{cases}$

for $t<0 \quad y(t)=0$

for $0 \le t \le 1 \quad h(t-\tau) = 1 \quad x(\tau) = \sin \pi\tau \qquad \int_0^1 \sin \pi\tau \, d\tau = \frac{2}{\pi}$

for $t>1 \quad y(t)=0$

$$\boxed{y(t) = \begin{cases} 0, & \text{for } t<0 \\ 2/\pi, & \text{for } 0 \le t \le 1\,s \\ 0, & \text{for } t>1 \end{cases}}$$

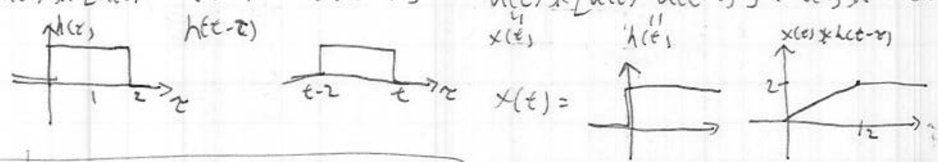
2.16

(a) $\delta(t-2) * [u(t) - 3u(t-1) + 2u(t-2)] = \boxed{u(t-2) - 3u(t-3) + 2u(t-4)}$

(b) $[\delta(t) + 2\delta(t-1) + 3\delta(t-2)] * [4\delta(t) + 5\delta(t-1)] = 4\delta(t) + 5\delta(t-1) + 8\delta(t-1) + 10\delta(t-2) + 12\delta(t-2) + 15\delta(t-3)$

$= \boxed{4\delta(t) + 13\delta(t-1) + 22\delta(t-2) + 15\delta(t-3)}$

(c) $u(t) * [u(t) - u(t-2) - 2\delta(t-2)] = \underset{x(t)}{u(t)} * \underset{h(t)}{[u(t) - u(t-2)]} + u(t) * -2\delta(t-2)$



$= r(t) - r(t-2)$

$$\boxed{A = r(t) - r(t-2) - 2u(t-2)}$$

2.17

(a) $e^{-t}u(t) * e^{-2t}u(t) \qquad \int_{-\infty}^{\infty} e^{-\tau} u(t) \cdot e^{-2(t-\tau)} u(t-\tau)\, d\tau = \int_0^t e^{-\tau} \cdot e^{-2t+2\tau} \, d\tau = \int_0^t e^{-2t} \cdot e^{\tau} \, d\tau$

$= e^{-2t} \cdot e^{\tau}\Big|_0^t = \boxed{e^{-2t}(e^{t}-1)}$

(b) $e^{-2t}u(t) * e^{-3t}u(t) = \int_{-\infty}^{\infty} e^{-2\tau} u(\tau) \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau = \int_0^t e^{-2\tau} \cdot e^{-3t+3\tau} \, d\tau = e^{-3t} \int_0^t e^{\tau} \, d\tau$

$= \boxed{e^{-3t}(e^{t}-1)}$

(c) $e^{-3t}u(t) * e^{-3t}u(t) = \int_{-\infty}^{\infty} e^{-3\tau} u(\tau) \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau = \int_0^t e^{-3\tau} \cdot e^{-3t+3\tau} \, d\tau = e^{-3t} \int_0^t 1 \, d\tau$

$= e^{-3t} \cdot \tau\Big|_0^t = \boxed{e^{-3t} \cdot t}$

2.22

(a) non causal, BIBO stable
(b) non causal, BIBO stable
(c) non causal, Not BIBO stable
(d) causal, Not BIBO stable
(e) causal, BIBO stable
(f) causal, BIBO stable

ECEn 380
Homework 3

2: 14, 16, 17, 22, 23, 30, 42
45

Benjamin Bergeson

2.23

(a) $y(t)=\frac{dx}{dt}$ $\boxed{\text{noncausal, Not BIBO stable} \quad x(t)=u(t) \quad y(t)=\delta(t)}$

(b) $y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau$ $\boxed{\text{causal,}}$ $\boxed{\text{Not BIBO}}$ $\boxed{x(t)=u(t) \quad y(t)=t}$

(c) $y(t)=\int_{-\infty}^{t} x(\tau)\cos(t-\tau)\,d\tau$ $\boxed{\text{causal}}$ $x(t) * \cos(t)$ $h(t)=\cos(t)u(t)$ $\boxed{\text{Not BIBO}}$ $\boxed{x(t)=\cos(t)}$

(d) $y(t)=x(t+1)$ $\boxed{\text{noncausal}}$ $\boxed{\text{BIBO stable}}$

(e) $y(t)=\int_{t-1}^{t+1} x(\tau)\,d\tau$ $\boxed{\text{noncausal}}$ $\boxed{\text{BIBO stable}}$

(f) $y(t)=\int_{t}^{\infty} x(\tau)e^{2(t-\tau)}\,d\tau$ $\boxed{\text{noncausal}}$ $x(t) * e^{2t}u(-t)$ $h(t)=e^{2t}u(-t)$ $\boxed{\text{BIBO stable}}$

2.30

(a) $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+7y=5\frac{dx}{dt}$; $x(t)=\cos(\omega t)$

If $\omega=2$ rad/sec. $y(t)=?$ $\quad (j\omega)^2H(\omega)e^{j\omega t}+2(j\omega)H(\omega)e^{j\omega t}+7H(\omega)e^{j\omega t}=5(j\omega)e^{j\omega t}$

$H(\omega)[(j\omega)^2+2j\omega+7]=5j\omega \quad H(\omega)=\frac{5j\omega}{-\omega^2+2j\omega+7}$

$H(2)=\frac{10j}{-4+4j+7}=\frac{10j}{3+4j}$ $\quad y(t)=\left|\frac{10j}{3+4j}\right|\cos(2t+\theta)$ $\quad \left|\frac{10j}{3+4j}\right|=\left|\frac{40+30j}{25}\right|=\frac{50}{25}=2$

$\theta=36.87°$ $\quad \boxed{y(t)=2\cos(2t+36.87°)}$

(b) $A\cos(\omega t)$: $\quad \frac{5j\omega}{-\omega^2+2j\omega+7}\cdot\frac{(-\omega^2-2j\omega+7)}{(-\omega^2-2j\omega+7)}=$ $\quad 5j\omega(-\omega^2-2j\omega+7) \Rightarrow \theta=0$

$=-5\omega^3j+10+35j\omega = 10+(35\omega-5\omega^3)j$

$\tan^{-1}\left(\frac{35\omega-5\omega^3}{10}\right)=0 \quad 35\omega-5\omega^3=0 \quad \boxed{\omega=\sqrt{7}}$

2.42 $\frac{d^2y}{dt^2}+B\frac{dy}{dt}+25y(t)=\frac{dx}{dt}+23x(t)$

(a) overdamped: $\xi>1$ $\quad \alpha=\frac{B}{2}$ $\quad \omega_0=\sqrt{25}=5$ $\quad \xi=\frac{\frac{B}{2}}{5}=\frac{B}{10}$ $\quad \frac{B}{10}>1$ $\boxed{B>10}$

(b) Underdamped: $\xi<1$ $\quad \frac{B}{10}<1$ $\boxed{B<10}$

(c) unstable: $h(t)=(C_1+C_2t)e^{-\alpha t}u(t)$ when $\alpha<0$ $h(t)$ is unstable

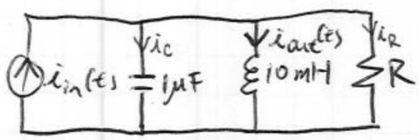
$B/2<0$ $\boxed{B<0}$

(d) $B=26$ $\quad \alpha=13$ $\quad \omega_0=5$ $\quad \xi=\frac{13}{5}>1$ overdamped $\quad p1=5\left[-\frac{13}{5}+\sqrt{(\frac{13}{5})^2-1}\right]=-1$

$p2=5\left[-\frac{13}{5}-\sqrt{(\frac{13}{5})^2-1}\right]=-25$ $\quad A_1=\frac{-1+23}{-1+25}=\frac{22}{24}=\frac{11}{12}$ $\quad A_2=\frac{-(-25+23)}{-1+25}=\frac{2}{24}=\frac{1}{12}$

$$\boxed{h(t)=\frac{11}{12}e^{-t}u(t)+\frac{1}{12}e^{-25t}u(t)}$$

2.45

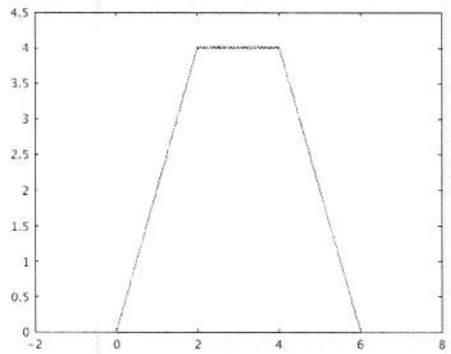


$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}} = 1 \qquad 2R = \sqrt{\frac{L}{C}} \qquad R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{10E\text{-}3}{1E\text{-}6}} = \boxed{50\Omega}$

$i_{in} = i_C + i_{out} + i_R \qquad i_C = C\frac{dv}{dt} \qquad i_{out} = \frac{1}{L}\int v\,dt \qquad i_R = \frac{V}{R}$

$i_{in} = C\frac{dv}{dt} + \frac{1}{L}\int v\,dt + \frac{V}{R} \qquad 0 = C\frac{d^2v}{dt} + \frac{V}{L} + \frac{d}{dt}\frac{V}{R} \qquad 0 = \frac{d^2v}{dt} + \frac{V}{RC}\frac{d}{dt} + \frac{V}{LC}$

$\alpha = \frac{1}{2RC} \qquad W_0 = \sqrt{\frac{1}{LC}}$



```
t_beg = -2;
d_t = 0.01;
t_end = 8;
t = t_beg:d_t:t_end;

y = zeros(1, length(t));

tau_beg = -10;
d_tau = 0.01;
tau_end = 10;


for m = 1:length(t)
    for tau = tau_beg:d_tau:tau_end
        y(m) = y(m) + x_t(tau) * h_t(t(m) - tau) * d_tau;
    end
end
y()
plot(t,y);
xlim([-2 8]);
```

3)

(a) 0 < tau < 4
(b)Yes, 0 < tau < t
(c)When d_tau is larger, the graph is less smooth and numerically less accurate. When d_tau is smaller, the graph is smoother and numerically more accurate.