ECEn 380
HW #2

1: 38, 39
2: 1, 2adg, 3, 4, 10, 11

Benjamin Bergeson

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1.38

(a) $x_1(t) = e^{-at}u(t)$ for $a > 0$ $\quad E = \int_{-\infty}^{\infty} e^{-at}u(t)\,dt = \int_0^{\infty} e^{-at}\,dt$

$n = -at \quad dn = -a\,dt \quad dt = -\frac{dn}{a}$ $\quad = -\frac{1}{a}\int_0^{\infty} e^n\,dn = -\frac{1}{a}e^n\Big|_0^{\infty} = -\frac{1}{a}e^{-at}\Big|_0^{\infty}$

$= -\frac{1}{a}e^{-a(\infty)} - -\frac{1}{a}e^{-a(0)} = 0 + \frac{1}{a} = \boxed{\frac{1}{a}}$

(b) $x_2(t) = e^{-a|t|}$ $\quad E = \int_{-\infty}^{\infty} e^{-a|t|}\,dt = \int_0^{\infty} e^{-at}\,dt = \boxed{\frac{1}{a}}$

(c) $x_3(t) = (1-|t|)\,\mathrm{rect}(t/2)$ $\quad E = \int_{-\infty}^{\infty}(1-|t|)\,\mathrm{rect}(t/2)\,dt = \int_{-1}^{1}(1-|t|)\,dt = 2\int_0^{1} 1-t\,dt$

$= 2\left(t - \frac{t^2}{2}\right)\Big|_0^1 = 2\left[\left(1-\frac{1}{2}\right) - 0\right] = 2\left(\frac{1}{2}\right) = \boxed{1}$

1.39

(a) $x_1(t) = e^{jat}$ $\quad P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|e^{jat}|^2\,dt = \lim_{T\to\infty}\frac{1}{T}\left[t\right]\Big|_{-T/2}^{T/2} = \lim_{T\to\infty}\frac{1}{T}\left(\frac{T}{2}+\frac{T}{2}\right)$

$= \lim_{T\to\infty}\frac{1}{T}(T) = \boxed{1}$

(b) $x_2(t) = (3-j4)e^{j7t}$ $\quad P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|(3-j4)e^{j7t}|^2\,dt = \lim_{T\to\infty}\frac{1}{T}\left[25t\right]\Big|_{-T/2}^{T/2}$

$= \lim_{T\to\infty}\frac{1}{T}\left(\frac{25T}{2}+\frac{25T}{2}\right) = \lim_{T\to\infty} 25 = \boxed{25}$

(c) $x_3(t) = e^{j3}e^{j5t}$ $\quad P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|e^{j3}e^{j5t}|^2\,dt = \lim_{T\to\infty}\frac{1}{T}\left[t\right]\Big|_{-T/2}^{T/2} = \boxed{1}$

2.1

(a) not linear; time invariant

(b) linear; not time invariant

(c) $\frac{dy}{dt} + t\cdot y(t) = x(t)$ $\quad \frac{d}{dt}cy + cyt = cx$ $\quad c\left[\frac{dy}{dt} + yt\right] = c(x)$

$\frac{dy_1}{dt} + ty_1 = x_1(t)$ $\quad \frac{dy_2}{dt} + ty_2 = x_2(t)$ $\quad \frac{dy_3}{dt} + ty_3 = x_3(t)$

linear; time invariant

(d) $\frac{dy}{dt} + 2y = 3\frac{dx}{dt}$ $\quad \frac{d}{dt}(cy) + 2(cy) = 3\frac{d}{dt}(cx) \Rightarrow c\left[\frac{dy}{dt}+2y\right] = c\left[3\frac{dx}{dt}\right]$ $\quad \frac{dy_1}{dt} + 2y_1 = 3\frac{dx_1}{dt}$

$\frac{dy_2}{dt} + 2y_2 = 3\frac{dx_2}{dt}$ $\quad \frac{dy_3}{dt} + 2y_3 = 3\frac{dx_3}{dt}$

Linear; time invariant

(e) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ $\quad y(t+T) = \int_{-\infty}^{t+T} x(\tau)\,d\tau$ $\quad \tau + T = t_2$

$\int_{-\infty}^{t} x(\tau+T)\,d\tau = \int_{-\infty}^{t+T} x(t_2)\,dt_2$

Nonlinear; time invariant

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2.1 (cont'd)

(f) $y(t) = \int_0^t x(\tau)\,d\tau$ — $\boxed{\text{Linear; time invariant}}$

(g) $y(t) = \int_{t-1}^{t+1} x(\tau)\,d\tau \qquad y(t+T) = \int_{t-1+T}^{t+1+T} x(\tau)\,d\tau$ — $\boxed{\text{Linear; time invariant}}$

$\int_{t-1}^{t+1} x(\tau+T)\,d\tau = \int_{t-1+T}^{t+1+T} x(t_1)\,dt_1 \qquad t_1 = \tau + T$

2.2

(a) $y(t) = 3x(t-1)$ If $3x_1(t-1) + 3x_2(t-1) = 3x_3(t-1)$ — $\boxed{\text{Linear; Time invariant}}$

$y_3(t) = y_1(t) + y_2(t) = 3x_1(t-1) + 3x_2(t-1) = 3x_3(t-1)$

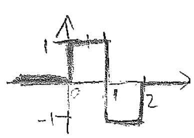
$y(t+T) = 3x(t+T-1) \qquad 3x(t+T-1)$

(d) $\frac{dy}{dt} + 2y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ — $\boxed{\text{Nonlinear; time invariant}}$

(g) $y(t) = \int_t^{2t} x(\tau)\,d\tau \qquad y(t+T) = \int_{t+T}^{2(t+T)} x(\tau)\,d\tau \qquad \int_t^{2t} x(\tau+T)\,d\tau = \int_{t+T}^{2t+T} x(t_1)\,dt_1$

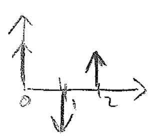
$\boxed{\text{Linear; time variant}}$

2.3



$h(t) = u(t) - 2u(t-1) + u(t-2)$

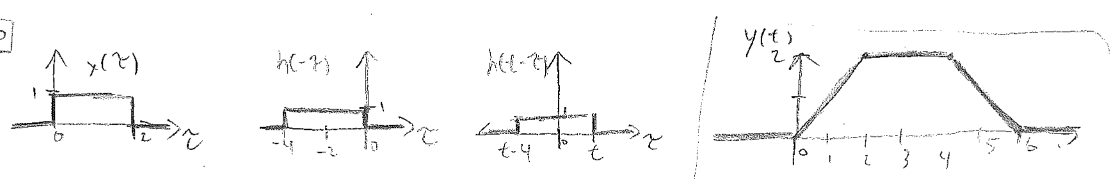
2.4



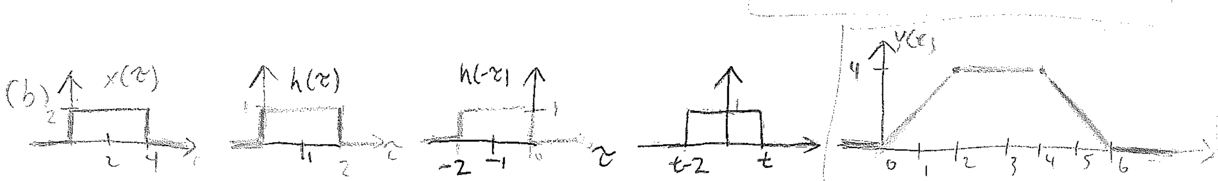
$h(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$

2.10

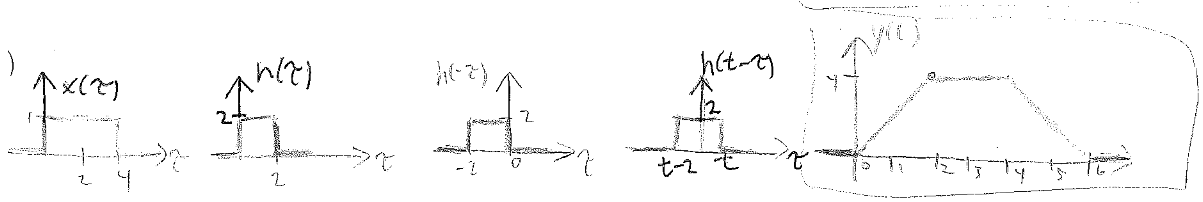
(a)



(b)



(c)



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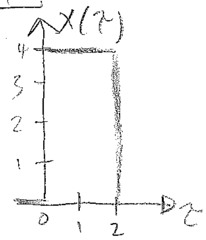
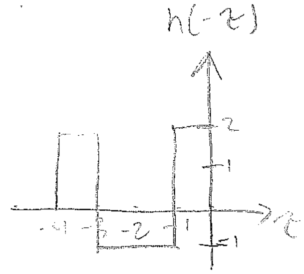
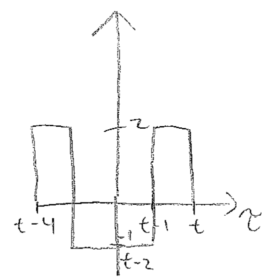
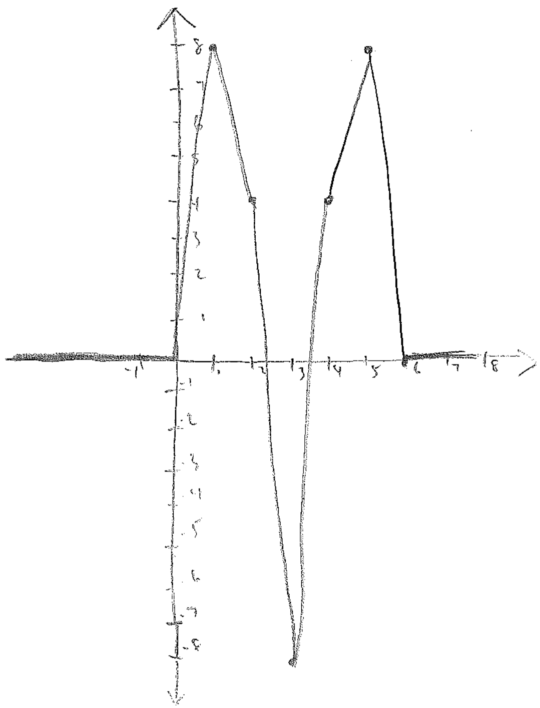
2.11









Editor - J:\ECEn 380\Homework\HW2\matlab.m

matlab.m

```
f = [1 10 50 1000];
t_start = 0;
t_end = 6;
delta_t = 0.0001;
t = t_start:delta_t:t_end;

y = cos(2*pi*f'*t);

E = 0;
for t = 1:t_end/delta_t
  E = E + delta_t * abs(y(:,t)).^2;
end

E

P = E .* f'

```

Command Window

```
>> matlab

E =     Energy

    3.0000   for f = 1
    3.0000   for f = 10
    3.0000   for f = 50
    3.0000   for f = 1000


P =   Average Power

   1.0e+03 *

    0.0030   for f = 1
    0.0300   for f = 10
    0.1500   for f = 50
    3.0000   for f = 1000
```

(4) (a) The frequency did not have any impact on the total energy of the signal. However higher frequencies resulted in higher average power.