ECEn 380	7: 27, 29, 31, 36, 38, 41, 42, 45, 47	
Homework 11		Benjamin Bergeson

1

7.27 $y[n] = x[n] + 0.5\,x[n-1] + x[n-2]$

(a) $H(e^{j\Omega}) = \sum_{n=0}^{\infty} h[n]\, e^{-j\Omega}$ $\quad \boxed{1 + 0.5e^{-j\Omega} + e^{-2j\Omega} = H(e^{j\Omega})}$

(b) $y[n] = \left|H(e^{j\Omega})\right| \cos(\tfrac{\pi}{2} n + \phi)$

$1 + 0.5e^{-\frac{\pi}{2}j} + e^{-2j\frac{\pi}{2}} = 1 + (-\tfrac{1}{2}j) - 1 = -\tfrac{1}{2}j = 90^\circ$

$\boxed{\tfrac{1}{2}\cos(\tfrac{\pi}{2}n - 90^\circ)}$

7.29 (a) $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$

$\boxed{1 + e^{-j\Omega} + e^{-2j\Omega} + e^{-3j\Omega} = H(e^{j\Omega})}$

(b) $x[n] = \cos(\tfrac{\pi}{2}n) + \cos(\pi n)$ $\quad n = 0 \dots 3 \quad 2, -1, 0, -1 \dots$

$y[n] = 0$

7.31 (a) $y[n] + y[n-1] = x[n] - x[n-1] \Rightarrow Y(z) + \tfrac{1}{z}Y(z) = X(z) - \tfrac{1}{z}X(z)$

$Y(z)\left(1 + \tfrac{1}{z}\right) = X(z)\left(1 - \tfrac{1}{z}\right)$ $\quad H(z) = \dfrac{1 + \frac{1}{z}}{1 - \frac{1}{z}} = \dfrac{z+1}{z-1}$

$\boxed{H(e^{j\Omega}) = \dfrac{e^{j\Omega} + 1}{e^{j\Omega} - 1}}$

(b) $x[n] = 3 + 4\cos(\tfrac{\pi}{2}n + \tfrac{\pi}{4})$ $\quad X(z) =$

ECEn 380 Homework 11	7: 27, 29, 31, 36, 38, 41, 42, 45, 47	Benjamin Bergeson

7.36 $x[n] = \{\underline{5}, 3, 1, 3\}$ $\quad X_k = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-jk\frac{2\pi}{N_0}n}$ $\quad X_0 = \frac{12}{4} = 3$

$X_1 = \frac{1}{4}\left(5 + 3e^{-j\frac{\pi}{2}} + e^{-j\pi} + 3e^{-j\frac{3\pi}{2}}\right) = \frac{1}{4}(5 - 3j - 1 + 3j) = \frac{1}{4}(4) = 1$

$X_2 = \frac{1}{4}\left(5 + 3e^{-j\pi} + e^{-j2\pi} + 3e^{-j3\pi}\right) = \frac{1}{4}(5 - 3 + 1 - 3) = 0$

$X_3 = \frac{1}{4}\left(5 + 3e^{-j\frac{3\pi}{2}} + e^{-j3\pi} + 3e^{-j\frac{9\pi}{2}}\right) = \frac{1}{4}(5 + 3j - 1 - 3j) = \frac{1}{4}(4) = 1$

$\boxed{x[n] = 3 + e^{j\frac{\pi}{2}n} + e^{j\frac{3\pi}{2}n}}$

7.38 $\{\underline{18}, 12, 6, 0, 6, 12\}$ $\quad X_k = \frac{1}{6}\sum_{n=0}^{3} x[n]\, e^{-jk\frac{\pi}{3}n}$

(a) $X_0 = \frac{1}{6}(18 + 12 + 6 + 6 + 12) = 9$

$X_1 = \frac{1}{6}\left(18 + 12e^{-j\frac{\pi}{3}} + 6e^{-j\frac{2\pi}{3}} + 6e^{-j\frac{4\pi}{3}} + 12e^{-j\frac{5\pi}{3}}\right) = \frac{1}{6}24 = 4$

$X_2 = \frac{1}{4}\left(18 + 12e^{-j2\frac{\pi}{3}} + 6e^{-j2\frac{\pi}{3}2} + 6e^{-j2\frac{\pi}{3}4} + 12e^{-j2\frac{5\pi}{3}}\right) = \frac{1}{6}0 = 0$

$X_3 = \frac{1}{4}\left(18 + 12e^{-j\frac{3\pi}{2}} + 6e^{-j\frac{3\pi}{2}2} \cdots\right) = \frac{1}{6}6 = 1$

$X_4 = 0 \qquad X_5 = 4$

$x[n] = 9 + 4e^{j\frac{\pi}{3}n} + 0e + e^{j\frac{3\pi}{3}n} + 0e + 4e^{j\frac{5\pi}{3}n}$

$\boxed{x[n] = 9 + 4e^{j\frac{\pi}{3}n} + e^{j\pi n} + 4e^{j\frac{5\pi}{3}n}}$

(b) $\frac{1}{6}\sum_{n=0}^{5}|x[n]|^2 = \frac{1}{6}(324 + 144 + 36 + 36 + 144) = \boxed{114}$

(c) $\sum_{k=0}^{5}|X_k|^2 = 81 + 16 + 1 + 16 = \boxed{114}$

7.41 $x[n] = \{\underline{4}, 2, 1, 6\}$ $\quad y[n] = \{\underline{10}, 4, 10, 4\}$

1	0	$\underline{4}$	2	1	0
c	b	$\underline{a}$			

$4a + c = 10$
$4b + 2a = 4$
$4c + 2b + a = 10$
$2c + b = 4$

$\boxed{h = \{\underline{2}, 0, 2\}}$

7.42 (a) $\{1, 1, \underline{1}, 1, 1\} = \delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2]$

$e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-2j\Omega} = \boxed{1 + 2\cos(2\Omega) + 2\cos(\Omega)}$

(b) $\{3, \underline{2}, 1\}$ $\quad 3\delta[n+1] + 2\delta[n] + \delta[n-1]$

$3e^{j\Omega} + 2 + e^{-j\Omega} = \boxed{2 + 4\cos(\Omega) + 2\sin(\Omega)j}$

ECEn 380
Homework11
~~12/4/15~~

Benjamin Bergeson
~~ECEn 340 Lecture Notes~~

7.45

(a) $\frac{1}{2\pi}\int_{\pi}^{\pi} \frac{e^{j\Omega} e^{j3\Omega}}{e^{j\Omega}-\frac{1}{2}}\, d\Omega$ $\quad X(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega}-\frac{1}{2}}$ $\quad n=3 \quad X[3] = \left(\frac{1}{2}\right)^n u[n] = \boxed{\frac{1}{8}}$

(b) $\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j3\Omega}\, 2\cos(3\Omega)\, d\Omega$ $\quad X(e^{j\Omega}) = 2\cos(3\Omega)|n=3$

$= e^{j3\Omega} + e^{-j3\Omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \delta[n+3] + \delta[n-3]$

$X[3] = \delta[n+3] + \delta[n-3] = \boxed{1}$

7.47 (a) $\{\underline{12}, 8, 4, 8\}$ $\quad X_k = \sum_{n=0}^{3} x[n]\, e^{-jk\frac{\pi}{2}n}$

$X_0 = 12 + 8 + 4 + 8 = 32$

$X_1 = 12 + 8e^{-j\frac{\pi}{2}} + 4e^{-j\frac{\pi}{2}2} + 8e^{-j\frac{\pi}{2}3} = 8$

$X_2 = 12 + 8e^{-j\pi} + 4e^{-j2\pi} + 8e^{-j3\pi} = 0$

$X_3 = 12 + 8e^{-j\frac{3\pi}{2}} + 4e^{-j3\pi} + 8e^{-j\frac{3\pi}{2}\cdot 3} = 8$

$\boxed{\{\underline{32}, 8, 0, 8\}}$

(b) $\{\underline{16}, 8, 12, 4\}$ $\quad X_k = \sum_{n=0}^{3} x[n]\, e^{-jk\frac{\pi}{2}n}$

$X_0 = 16 + 8 + 12 + 4 = 40$

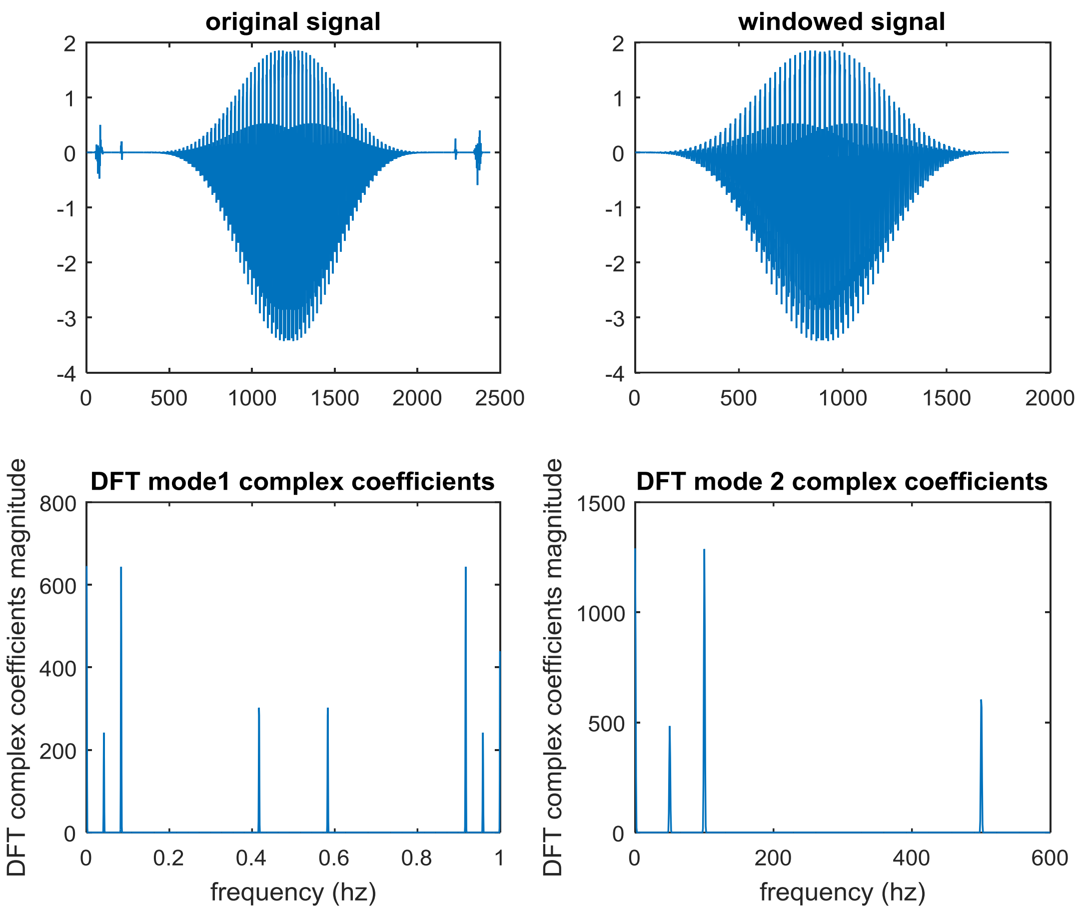
$X_1 = 16 + 8e^{-j\frac{\pi}{2}} + 12e^{-j\pi} + 4e^{-j3\pi/2} = 16 - 8j - 12 + 4j = 4 - 4j$

$X_2 = 16 + 8e^{-j\pi} + 12e^{-j2\pi} + 4e^{-j3\pi} = 16 - 8 + 12 - 4 = 16$

$X_3 = 16 + 8e^{-j\frac{3\pi}{2}} + 12e^{-j3\pi} + 4e^{-j9\pi/2} = 16 + 8j - 12 - 4j = 4 + 4j$

$\boxed{\{40, 4-4j, 16, 4+4j\}}$

Line of code that I changed: `Xk(k+1) = sum(signal(n).*exp(-1j.*(k).*2.*pi./No.*n));`



d) 0, 50, 100, 500 Hz