30318463

SOLUTIONS

RED

ECEn 380: Signals & Systems

Fall 2015

Professors Neal Bangerter and Brian Jeffs

Midterm #1

October 6 – 9, 2015

- 3 hour time limit
- Open book, open note (electronic books and/or notes or book on a tablet allowed)
- Calculators allowed (okay to use tablet or e-book as calculator)
- **IMPORTANT: The exam is double sided, per testing center requirements**

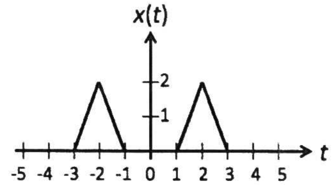
- The exam consists entirely of multiple choice questions. **Please provide all answers on the scantron bubble sheet.**

- There are 28 questions and 100 points possible in the exam, scored as follows:
  - Problems 1 – 12: 3 points each
  - Problems 13 – 28: 4 points each

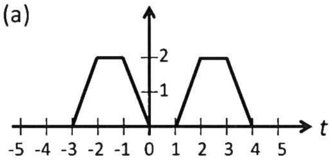
- Manage your time carefully! Skip more difficult problems on your first pass through the exam, and return to them later (time permitting).

If you feel that something in the exam is not clear, please state your assumptions and work the problem based on those assumptions.

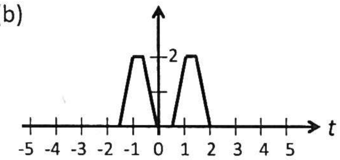
1. Find $x\left(\frac{1}{2}t\right) + x\left(1 - \frac{1}{2}t\right)$ given $x(t)$ shown.



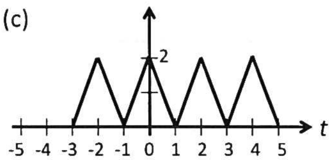
(a)



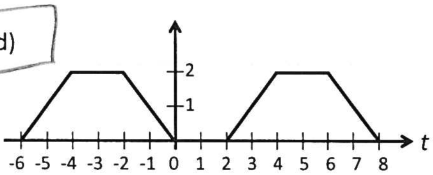
(b)



(c)

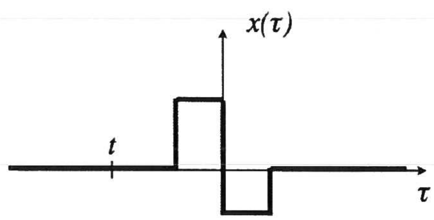


$\boxed{\text{(d)}}$



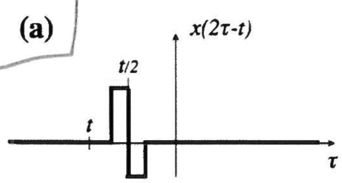
(e) None of the above

2. Which of the following drawings of $x(2\tau - t)$ is correct for $t < 0$ given the function $x(\tau)$ shown below? (3 points)

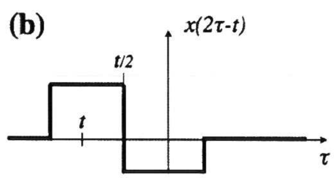


NOT TIME REVERSED, COMPRESSED IN TIME, NEW ZERO AT $\tau = t/2$!

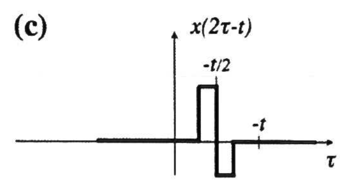
$\boxed{\text{(a)}}$



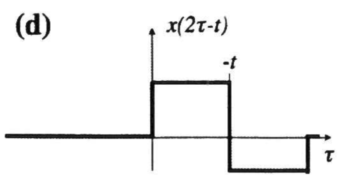
(b)



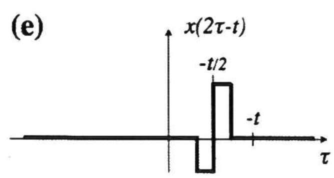
(c)



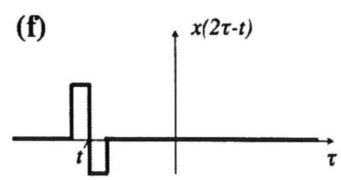
(d)



(e)



(f)



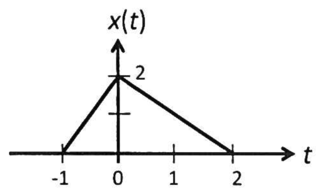
$T = \frac{2\pi}{2\omega_0} = \frac{\pi}{\omega_0}$ $T = \frac{2\pi}{\omega_0}$ ⇐ PERIODIC W/ PERIOD $\frac{2\pi}{\omega_0}$!

3. Given $x(t) = 2 - e^{-j2\omega_0 t} + e^{j\omega_0 t}$, which of the following statements is correct?

(a) $x(t)$ is odd and periodic.
(b) $x(t)$ is even and periodic.
(c) $x(t)$ is odd, but not periodic.
(d) $x(t)$ is even, but not periodic.
$\boxed{\text{(e)}}$ $x(t)$ is neither even nor odd, but is periodic.
(f) $x(t)$ is neither even nor odd, and not periodic.
(g) None of the above is correct.

ODD OR EVEN?
CHECK $x(-t)$:
$x(-t) = 2 - e^{j2\omega_0 t} + e^{-j\omega_0 t}$
$\neq x(t)$ ⇐ NOT EVEN!
$\neq -x(t)$ ⇐ NOT ODD!

4. Express the following signal $x(t)$ in terms of ramp and/or step functions.



(a) $x(t) = 2r(t-1) - r(t)$
(b) $x(t) = 2r(t+1) - 3r(t)$
(c) $x(t) = r(t+1) - r(t) + r(t-2)$
$\boxed{\text{(d)}}$ $x(t) = 2r(t+1) - 3r(t) + r(t-2)$
(e) $x(t) = r(t+1) - 2r(t)$
(f) $x(t) = 2r(t+1) - 3r(t) + r(t-1)$
(g) $x(t) = [r(t-1) - 2r(t)]u(1-t)$

5. Express the following signal $x(t)$ in terms of ramp and/or step functions.

$$x(t) = \begin{cases} 0, & t < -2 \\ -1, & -2 < t < -1 \\ t, & -1 < t < 1 \\ 2-t, & 1 < t < 3 \\ 0, & t > 3 \end{cases}$$

(a) $x(t) = -u(t+2) + r(t+1) - 2r(t-1) + 2r(t-2)$
$\boxed{\text{(b)}}$ $x(t) = -u(t+2) + r(t+1) - 2r(t-1) + u(t-3) + r(t-3)$
(c) $x(t) = -u(t+2) - u(t+1) - 2r(t-1) + r(t-2)$
(d) $x(t) = -u(t+2) + r(t+1) - 2r(t-1) + r(t-3)$
(e) $x(t) = [-u(t+2) + r(t+1) - 2r(t-2)]u(2-t)$
(f) None of the above.

6. Evaluate the following integral:

$$\int_{-\infty}^{\infty} e^{j\pi t}\cos(2\pi t)u(t-2)\delta(2-2t)dt = \int_{-\infty}^{\infty} e^{j\pi t}\cos(2\pi t)u(t-2)\frac{1}{2}\delta(1-t)dt$$

$$= e^{j\pi t}\cos(2\pi t)u(t-2)\frac{1}{2}\Big|_{t=1}$$

$$= 0 \quad \text{, SINCE } u(-1)=0!$$

(a) -1

$\boxed{\text{(b)}}$ 0

(c) 1

(d) j

(e) -j

7. Evaluate the following integral:

$$\int_{-2}^{2}(2+t)^2\delta(3+2t)dt = \int_{-2}^{2}(2+t)^2\frac{1}{2}\delta(3/2+t)\,dt$$

$$= (2+t)^2\frac{1}{2}\Big|_{t=-3/2} = \left(\frac{1}{2}\right)^2\cdot\frac{1}{2} = \frac{1}{8}$$

(a) 1/16

$\boxed{\text{(b)}}$ 1/8

(c) 1/4

(d) 1/2

(e) 3/4

(f) 1

(g) 0

8. Find the impulse response $h(t)$ of the LTI system described by the following input/output relation:

$$y(t) = \int_{t-2}^{t} x(2\tau+1)d\tau$$

$\boxed{\text{(a)}}$ $h(t) = \frac{1}{2}u\left(t+\frac{1}{2}\right) - \frac{1}{2}u(t-\frac{3}{2})$

(b) $h(t) = u\left(t+\frac{1}{2}\right) - u\left(t-\frac{3}{2}\right)$

(c) $h(t) = u(t-2)$

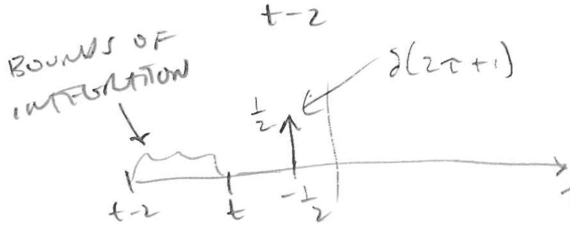
(d) $h(t) = \frac{1}{2}u(t-2) - \frac{1}{2}u(t)$

(e) $h(t) = u(t) - u(t-2)$

(f) $h(t) = u(t+2) - u(t)$

(g) $h(t) = 2u\left(t+\frac{1}{2}\right) - 2u(t-\frac{3}{2})$

$$h(t) = \int_{t-2}^{t}\delta(2\tau+1)d\tau$$



$$h(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ \frac{1}{2}, & -\frac{1}{2} < t < 3/2 \\ 0, & t > 3/2 \end{cases}$$

$T = \frac{2\pi}{2\pi} = 1$ $\quad$ $T = \frac{2\pi}{\frac{2\pi}{5}} = 5$

9. Determine the period of the following signal: $\quad x(t) = 2e^{-3j}(3+j)e^{-j2\pi t} + \cos\left(\frac{2\pi}{5}t\right)$

(a) 2

(b) 5

(c) 8

(d) 2π

(e) 5π

(f) The signal is not periodic

COMMON PERIOD = 5

$T = \frac{2\pi}{2/3} = 3\pi$ $\quad$ $T = \frac{2\pi}{\frac{2\pi}{3}} = 3$

10. Determine the period of the following signal: $\quad x(t) = \cos\left(\frac{2}{3}t\right) + \sin\left(\frac{2\pi}{3}t\right)e^{-3j} + 2$

RATIONAL

(a) 1

(b) 3

IRRATIONAL

(c) π

(d) 3π

NO COMMON MULTIPLE!

(e) 6π

(f) The signal is not periodic

11. Compute the total energy of the following signal: $x(t) = r(t) - 2r(t-1) + r(t-2)$

(a) 0

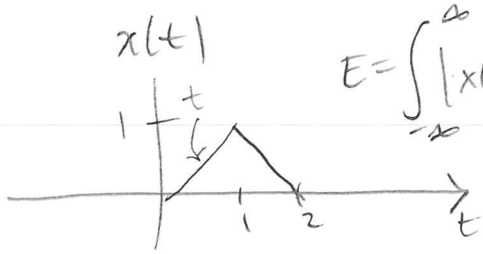
(b) 1/3

(c) 1/2

(d) 2/3

(e) 1

(f) None of the above



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_0^1 |t|^2 dt = 2\int_0^1 t^2 dt = \frac{2}{3}$$

12. Compute the average power of the following signal: $\quad x(t) = 2e^{-j\pi/7}(2-3j)e^{-j(4+3\pi t)}$

MAGNITUDE^2 = $4(4+9) = 52$

(a) 1

(b) $\sqrt{13}$

(c) $2\sqrt{13}$

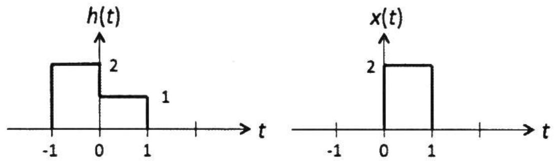
(d) $4\sqrt{13}$

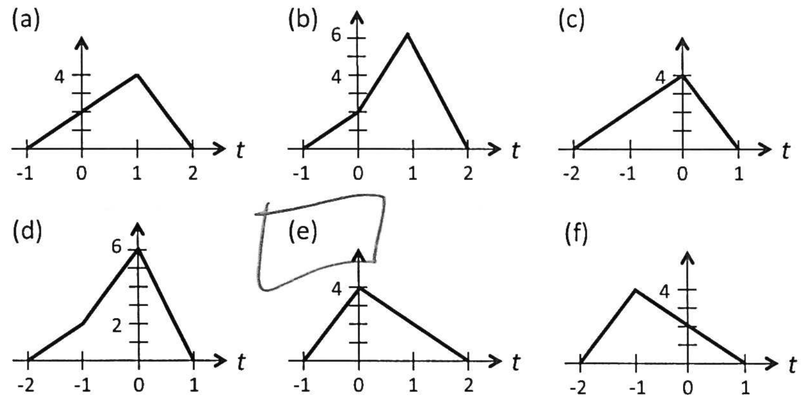
(e) 13

(f) 26

(g) 52

13. Find the output of the LTI system with impulse response $h(t)$ to the input $x(t)$ (both shown below):





14. Which of the following statements is true about the system described by the following input/output relation?

$$y(t) = (2 - t)x(t)$$

BREAKS TIME INVARIANCE!

BUT STILL LINEAR.

(a) The system is both linear and time-invariant

(b) The system is linear, but not time-invariant

(c) The system is not linear, but it is time-invariant

(d) The system is neither linear nor time-invariant

(e) None of the above

15. Which of the following statements is true about the system described by the following input/output relation?

$$y(t) = \int_{t+2}^{\infty} 2x(\tau - 3)d\tau$$

LTI (CONVINCE YOURSELF)

TO CHECK CAUSALITY AND STABILITY, LOOK AT $h(t)$!

(a) The system is linear and time-invariant, but not causal and not stable

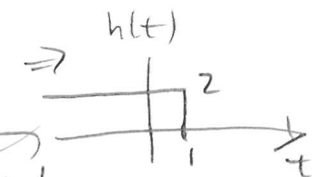
(b) The system is linear, but not time-invariant, not causal, and not stable

(c) The system is linear, time-invariant, causal, and stable

(d) The system is not linear, but it is time-invariant, causal, and stable

(e) None of the above is a true statement

$h(t) = \int_{t+2}^{\infty} 2\delta(\tau-3)d\tau \Rightarrow$



NOT CAUSAL!

NOT ABSOLUTELY INTEGRABLE!

16. Let $x(t)$ be the input of an LTI system with impulse response $h(t)$, where $x(t)$ and $h(t)$ are given by:

$$x(t) = \begin{cases} 0, & t < 0 \\ \sin(2\pi t), & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$$

$$h(t) = u(t)$$

Let $y(t)$ be the output. Find the output at time $t = 1.5$. That is, find $y(1.5)$.

$\boxed{\text{(a)}}$ $y(1.5) = 0$

(b) $y(1.5) = \frac{1}{\pi}$

(c) $y(1.5) = 1$

(d) $y(1.5) = \pi$

(e) None of the above

$$y(1.5) = \int_{-\infty}^{\infty} x(\tau)h(1.5-\tau)\,d\tau$$

$$= \int_0^1 \sin(2\pi\tau)\,d\tau = 0$$

17. Perform the following convolution: $t\cos(\omega_0 t) * [\delta(t) + \frac{1}{2}\delta(t-2)]$

(a) $\frac{1}{2}\cos(2\omega_0)$

(b) $t\cos(\omega_0 t) + \frac{1}{2}t\cos(\omega_0 t)$

(c) $t\cos(\omega_0 t) + \frac{1}{2}(t-2)\cos(\omega_0 t - 2)$

$\boxed{\text{(d)}}$ $t\cos(\omega_0 t) + \frac{1}{2}(t-2)\cos(\omega_0 t - 2\omega_0)$

(e) None of the above

CONVOLVE w/ EACH DELTA.

$t\cos(\omega_0 t) * \delta(t) = t\cos(\omega_0 t)$

$t\cos(\omega_0 t) * \frac{1}{2}\delta(t-2) = \frac{1}{2}(t-2)\cos(\omega_0 t - 2\omega_0)$

18. Which of the following is the Laplace Transform of the $x(t)$ shown below?

$$x(t) = 10(t+0.8)^2\cos(6\pi t - \tfrac{\pi}{5})\delta(t-0.2)$$

(a) $\dfrac{\frac{\sqrt{3}}{2}(s+3)+2}{(s+3)^2+36}$

(b) $\dfrac{s\cos(\frac{\pi}{5})+6\pi\sin(\frac{\pi}{5})}{s^2+(36\pi)^2}$

(c) $\dfrac{s\cos(\frac{\pi}{5})-6\pi\sin(\frac{\pi}{5})}{s^2+(36\pi)^2}$

$\boxed{\text{(d)}}$ $-10e^{-0.2s}$

(e) $10e^{0.2s}$

(f) None of the above

$x(t) = 10(0.2+0.8)^2\cos(6\pi(0.2) - \frac{\pi}{5})\delta(t-0.2)$

$x(t) = 10(1)^2\cos(\underbrace{\frac{6\pi}{5} - \frac{\pi}{5}}_{\pi})\delta(t-0.2)$

$x(t) = -10\delta(t-0.2)$

$\boxed{X(s) = -10e^{-0.2s}}$

19. Which of the following is the Laplace Transform of the $x(t)$ shown below?

$$x(t) = 10(t-1)^3 e^{-2(t-1)} u(t-1)$$

(a) $\frac{60}{(s+2)^4}$

(b) $\frac{60e^{t}}{(t+2)^4}$

$\boxed{\text{(c)}}$ $\frac{60e^{-s}}{(s+2)^4}$

(d) $\frac{10}{s+2}$

(e) $\frac{10e^{-s}}{s+2}$

(f) None of the above

$$X(s) = 10e^{-s}\frac{3!}{(s+2)^4} = \boxed{\frac{60e^{-s}}{(s+2)^4}}$$

20. Which of the following is $x(t)$ given $X(s) = \frac{2s^2+4s-16}{(s+3)(s+2)(s-1)}$?

$\boxed{\text{(a)}}$ $\left[-\frac{5}{2}e^{-3t} + \frac{16}{3}e^{-2t} - \frac{5}{6}e^{t}\right]u(t)$

(b) $\left[\frac{3}{2}e^{-3t} + \frac{11}{3}e^{-2t} - \frac{5}{6}e^{t}\right]u(t)$

(c) $\left[-\frac{5}{2}e^{3t} + \frac{16}{3}e^{2t} - \frac{5}{6}e^{-t}\right]u(t)$

(d) $\left[3e^{-3t} - 2e^{-2t} + \frac{3}{2}e^{t}\right]u(t)$

(e) None of the above

$$\frac{2s^2+4s-16}{(s+3)(s+2)(s-1)} = \frac{A}{s+3} + \frac{B}{s+2} + \frac{C}{s-1}$$

$$A = \frac{2(-3)^2+4(-3)-16}{(-3+2)(-3-1)} = -\frac{5}{2}$$

$$B = \frac{2(-2)^2+4(-2)-16}{(-2+3)(-2-1)} = \frac{16}{3}$$

$$C = \frac{2(1)^2+4(1)-16}{(1+3)(1+2)} = -5/6$$

21. When the input to a system is $u(t)$, the output is:

$$y(t) = \left[2 + 4t - 2e^{-j2\pi t}\right]u(t).$$

Which of the following is the system Transfer Function?

(a) $\frac{2s^2 + 4\pi js + 8\pi j}{s^2(s+2\pi j)}$

(b) $\frac{(4+4\pi)s + 8\pi}{s(s+2\pi)}$

$\boxed{\text{(c)}}$ $\frac{(4+4\pi j)s + 8\pi j}{s(s+2\pi j)}$

(d) $\frac{(4+4\pi j)s + 8\pi j}{s^2(s+2\pi j)}$

(e) None of the above

$$Y(s) = \frac{2}{s} + \frac{4}{s^2} + \frac{-2}{s+j2\pi}$$

$$Y(s) = \frac{2s(s+j2\pi) + 4(s+j2\pi) - 2s^2}{s^2(s+j2\pi)}$$

$$Y(s) = \frac{2s^2 + 4\pi j s + 4s + 8\pi j - 2s^2}{s^2(s+j2\pi)}$$

$$Y(s) = \frac{(4+4\pi j)s + 8\pi j}{s^2(s+j2\pi)}$$

$$X(s) = \frac{1}{s}$$

$$H(s) = Y(s)/X(s) = \frac{(4+4\pi j)s + 8\pi j}{s(s+j2\pi)}$$

22. What is the impulse response of the system in the previous problem (Problem 21)?

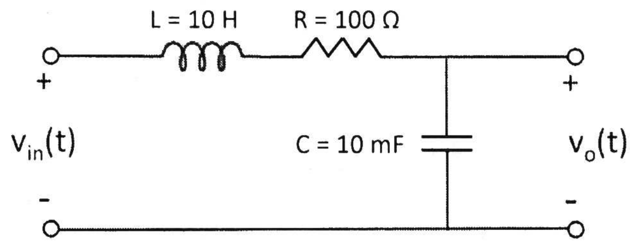
(a) $2\delta(t) + 4u(t) - 2e^{-j2\pi t}u(t)$

(b) $2\delta(t) + 4u(t) + 4\pi j e^{-j2\pi t}u(t)$

(c) $4u(t) + 4\pi j e^{-j2\pi t}u(t)$

(d) None of the above

23. Consider the circuit shown:



Which of the following is the s-domain Transfer Function of this circuit?

(a) $\frac{10s}{(s^2+10s+10)}$

(b) $\frac{10s}{(s+j10)(s-j10)}$

(c) $\frac{10}{s^2+10s+10}$

(d) $\frac{100s}{10s^2+100s+100}$

(e) None of the above

$$I(s) = \frac{V_{in}(s)}{10s+100+\frac{1}{.01s}} = \frac{V_{in}(s)}{10s+100+\frac{100}{s}}$$

$$V_o(s) = I(s)\cdot\frac{100}{s} = \frac{100}{s}\cdot\frac{V_{in}(s)}{10s+100+\frac{100}{s}}$$

$$V_o(s) = V_{in}(s)\cdot\frac{100}{10s^2+100s+100}$$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{100}{10s^2+100s+100} = \boxed{\frac{10}{s^2+10s+10}}$$

24. Which of the following is the Transfer Function of the **inverse** system of the system with impulse response:

$$h(t) = -2\delta(t-2) + 4e^{-2(t-2)}u(t-2)?$$

(a) $\frac{-\frac{1}{2}e^{2s}(s+2)}{s}$

(b) $\frac{-\frac{1}{2}e^{-2s}(s+2)}{s}$

(c) $\frac{-2e^{-2s}(s+2)}{s}$

(d) $\frac{-2e^{-2s}(s+2)}{s+1}$

(e) None of the above

$$H(s) = -2e^{-2s} + \frac{4e^{-2s}}{s+2} = -2e^{-2s}\left(1 - \frac{2}{s+2}\right)$$

$$H(s) = -2e^{-2s}\left(\frac{s+2-2}{s+2}\right) = -2e^{-2s}\left(\frac{s}{s+2}\right)$$

$$H_i(s) = \frac{1}{H(s)} = -\frac{1}{2}e^{2s}\left(\frac{s+2}{s}\right)$$

25. Which of the following is true for a system with impulse response

NOT BIBO STABLE

$$h(t) = (4+j5)e^{(2+j3)(t+1)}u(t+1) + (4-j5)e^{-(2-j3)t}u(t) \ ?$$

NOT CAUSAL

(a) The system is both BIBO stable and causal

(b) The system is BIBO stable, but **not** causal

(c) The system is **not** BIBO stable, but it is causal

$\boxed{\text{(d)}}$ The system is neither BIBO stable nor causal

26. The response of an LTI system to the input $x(t) = \delta(t)$ is

$$y(t) = 3\delta(t-1) - 4e^{-3t}u(t) + \cos(3\pi t).$$

Which of the following statements is true?

NOT CAUSAL
NOT BIBO STABLE

(a) The system is both BIBO stable and causal

(b) The system is BIBO stable, but **not** causal

(c) The system is **not** BIBO stable, but it is causal

$\boxed{\text{(d)}}$ The system is neither BIBO stable nor causal

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4+8}}{2} = -1 \pm \frac{\sqrt{12}}{2} = -1 \pm \sqrt{3}$$

27. Does the system with impulse response $h(t) = \delta(t) - 3te^{-t}u(t)$ have a BIBO stable **inverse** system?

$$H(s) = 1 - \frac{3}{(s+1)^2} = \frac{(s+1)^2 - 3}{(s+1)^2}$$

$$H(s) = \frac{s^2+2s+1-3}{(s+1)^2} = \frac{s^2+2s-2}{(s+1)^2}$$

$$H_i(s) = \frac{(s+1)^2}{(s+1)^2 - (\sqrt{3})^2} = \frac{(s+1)^2}{(s+1-\sqrt{3})(s+1+\sqrt{3})}$$

(a) Yes

$\boxed{\text{(b)}}$ No

(c) Not enough information to tell

28. An LTI system has Transfer Function $H(s) = \frac{s^4+3s^3+2s^2+19s-5}{s^3(s-4+2j)(s-4-2j)}$. Does this system have a BIBO stable **inverse** system?

$$H_i(s) = \frac{s^3(s-4+2j)(s-4-2j)}{s^4+3s^3+2s^2+19s-5}$$

NUMERATOR DEGREE BIGGER THAN DENOMINATOR DEGREE

(a) Yes

$\boxed{\text{(b)}}$ No

(c) Not enough information to tell