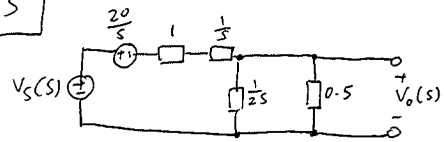
Homework 5 Benjamin Bergeson

4.3



$$V_o(s) = V_s(s) - \frac{20}{s}\left(\frac{\frac{1}{2s}\|0.5}{\left(\frac{1}{2s}\|0.5\right)+1+\frac{1}{s}}\right)$$

$$\frac{\frac{0.5}{2s}}{\frac{1}{2s}+0.5} = \frac{0.5}{1+s}$$

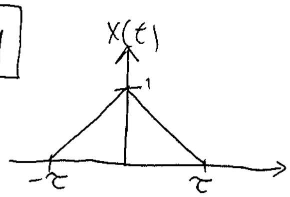
$$V_o(s) = \frac{15}{s}\left(\frac{\frac{0.5}{1+s}}{\frac{0.5}{1+s}+1+s}\right) = \frac{15}{s}\left(\frac{0.5}{0.5+(1+s)+\frac{1}{s}(1+s)}\right) = \frac{15}{s}\left(\frac{7.5}{0.5s+s+s^2+1+s}\right)$$

$$= \frac{7.5}{s^2+2.5s+1} = \frac{7.5}{(s+\frac{1}{2})(s+2)}$$

$$V_o(s) = \frac{A}{s+\frac{1}{2}} + \frac{B}{s+2} \qquad A: \frac{7.5}{(-0.5+2)} = \frac{7.5}{1.5} = \underline{5} \qquad B: \frac{7.5}{(-2+0.5)} = \frac{7.5}{-1.5} = \underline{-5}$$

$$V_o(s) = \frac{5}{s+\frac{1}{2}} - \frac{5}{s+2} \qquad \boxed{V_o(t) = 5e^{-\frac{1}{2}t} - 5e^{-2t}}$$

5.39



$$X(\omega) = \tau\,\text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$$

$$f(t) = 10\,x(t) \qquad \tau = 3$$

$$\boxed{F(\omega) = 30\,\text{sinc}^2\left(\frac{3\omega}{2\pi}\right)}$$

5.40

$$x(t) = \begin{cases} 0 & \text{for } t<0 \\ 12 & \text{for } 0<t<1 \\ 8 & \text{for } 1<t<2 \\ 4 & \text{for } 2<t<3 \\ 0 & \text{for } t>3 \end{cases}$$

$$X(\omega) = \int_0^1 12e^{-j\omega t}dt + \int_1^2 8e^{-j\omega t}dt + \int_2^3 4e^{-j\omega t}dt$$

$$= \frac{12e^{-j\omega t}}{-j\omega}\Big|_0^1 + \frac{8e^{-j\omega t}}{-j\omega}\Big|_1^2 + \frac{4e^{-j\omega t}}{-j\omega}\Big|_2^3$$

$$= \frac{12e^{-j\omega}}{-j\omega} - \frac{12}{-j\omega} + \frac{8e^{-2j\omega}}{-j\omega} - \frac{8e^{-j\omega}}{-j\omega} + \frac{4e^{-3j\omega}}{-j\omega} - \frac{4e^{-2j\omega}}{-j\omega}$$

$$= \frac{4e^{-3j\omega}+4e^{-2j\omega}+4e^{-j\omega}-12}{-j\omega}$$

$$= \boxed{\frac{4(e^{-3j\omega}+e^{-2j\omega}+e^{-j\omega}-3)}{-j\omega}}$$

5.47 $f(t) = 2\cos(5t-\frac{\pi}{5}) = 2\cos(5(t-\frac{\pi}{25}))$

$$x(t) = \cos(5t) \qquad X(\omega) = \pi[\delta(\omega-5)+\delta(\omega+5)]$$

$$y(t) = x(t-\tfrac{\pi}{25}) = \cos(5(t-\tfrac{\pi}{25})) \qquad Y(\omega) = e^{-\frac{\pi}{25}j\omega}\pi[\delta(\omega-5)+\delta(\omega+5)]$$

$$\boxed{F(\omega) = 2\pi e^{-\frac{\pi}{25}j\omega}[\delta(\omega-5)+\delta(\omega+5)]}$$

5.51

(a) $f(3t-2) = f(3(t-\frac{2}{3}))$ $\quad x(t) = f(3t)$ $\quad X(w) = \frac{1}{3}\left(\frac{5}{2+\frac{1}{3}jw}\right) = \frac{5}{6+jw}$

$y(t) = x(t-\frac{2}{3}) = f(3(t-\frac{2}{3}))$ $\quad Y(w) = \boxed{\frac{5e^{-\frac{2}{3}jw}}{6+jw}}$

(b) $x(t) = tf(t)$ $\quad Y(w) = j \cdot \frac{dF(w)}{dw} = j \cdot \left(\frac{-5}{(2+jw)^2}\right) = \boxed{\frac{-5j}{(2+jw)^2}}$

(c) $\boxed{\frac{5jw}{2+jw}}$

5.52 $\quad F(w) = \frac{e^{-jw}}{(2+jw)} + 1$

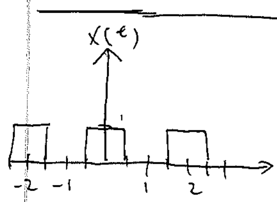
(a) $x(t) = f(\frac{5}{8}t)$ $\quad X(w) = \frac{8}{5}\left(\frac{1}{2+\frac{8}{5}jw}e^{-\frac{5}{8}jw} + 1\right) = \boxed{\frac{8}{10+8jw}e^{-\frac{5}{8}jw} + \frac{8}{5}}$

(b) $x(t) = f(t)\cos(2t)$ $\quad X(w) = \frac{1}{2}\left[X(w-2) + X(w+2)\right]$

$$X(w) = \frac{1}{2}\left[\left(\frac{e^{-j(w-2)}}{(2+j(w-2))} + 1\right) + \left(\frac{e^{-j(w+2)}}{(2+j(w+2))} + 1\right)\right]$$

$$= \frac{1}{2}\left[\frac{e^{-jw+2j}}{2-2j+jw} + 1 + \frac{e^{-jw-2j}}{(2+2j+jw)} + 1\right] = \boxed{\frac{e^{-jw+2j}}{4-4j+2jw} + \frac{e^{-jw-2j}}{4+4j+2jw} + 1}$$

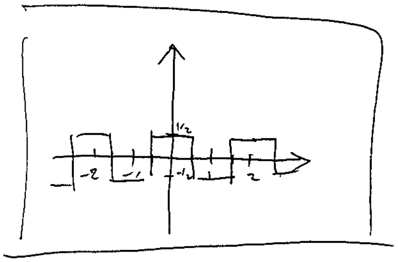
(c) $x(t) = \frac{d^3f(t)}{dt^3}$ $\quad \boxed{X(w) = -jw^3\left(\frac{e^{-jw}}{2+jw} + 1\right)}$



$$x_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jnw_0t}\,dt \qquad T_0 = 2 \quad W_0 = \pi$$

$$x_n = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-jn\pi t}\,dt = \frac{1}{2}\left[\frac{e^{-jn\pi t}}{-jn\pi}\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{-1}{2jn\pi}\left[e^{-jn\pi/2} - e^{jn\pi/2}\right]$$

$$\boxed{= \frac{-1}{2jn\pi}\left[(-i)^n - (i)^n\right]} \qquad \boxed{X_0 = \frac{1}{2}}$$



```
T = 5;
w = 2*pi/T;

t_beg = -1;
t_end = t_beg + T;
dt = 0.001;
t = t_beg:dt:t_end-dt;

x = zeros(1,length(t));
for m = 1:length(t)
    if t(m) <0
        x(m) = 0;
    elseif t(m) < 1
        x(m) = 1;
    elseif t(m) < 2
        x(m) = t(m)-1;
    elseif t(m) < 2.5
        x(m) = 3-t(m);
    elseif t(m) < 3
        x(m) = -2+t(m);
    else
        x(m) = 4-t(m);
    end
end

highest_harmonic = 10;
n = -highest_harmonic:highest_harmonic;
x_n = zeros(1,length(n));

for k = 1:length(n)
   x_n(k) = sum(x.*(1/T).*exp(-j*k*w.*t));
end

x_FS = zeros(1,length(x));

for k = 1:length(n)
    x_FS = x_FS + x_n(k).*exp(j*k*w.*t);
end

close all;
figure(1);
plot(t,x);
xlabel('time (s)');
ylabel('x(t)');

figure(2);
plot(t,real(x_FS),'r','LineWidth',2);
hold on
plot(t,x,'k:','LineWidth',2);
title('Comparison of Fourier Series approximation of x(t) with x(t)');
legend('x_F_S(t)','x(t)');
xlabel('time (s)');

phase_x_n = angle(x_n)*180/pi;
mag_x_n = abs(x_n);
```

```
figure(3);
subplot(1,2,1);
stem(n, mag_x_n);
title('Fourier Series Coefficients - Magnitude');
ylabel('|x_n|');
xlabel('n');

subplot(1,2,2);
stem(n, phase_x_n);
title('Fourier Series Coefficients - Phase');
ylabel('angle(x_n) (degrees)');
xlabel('n');
```

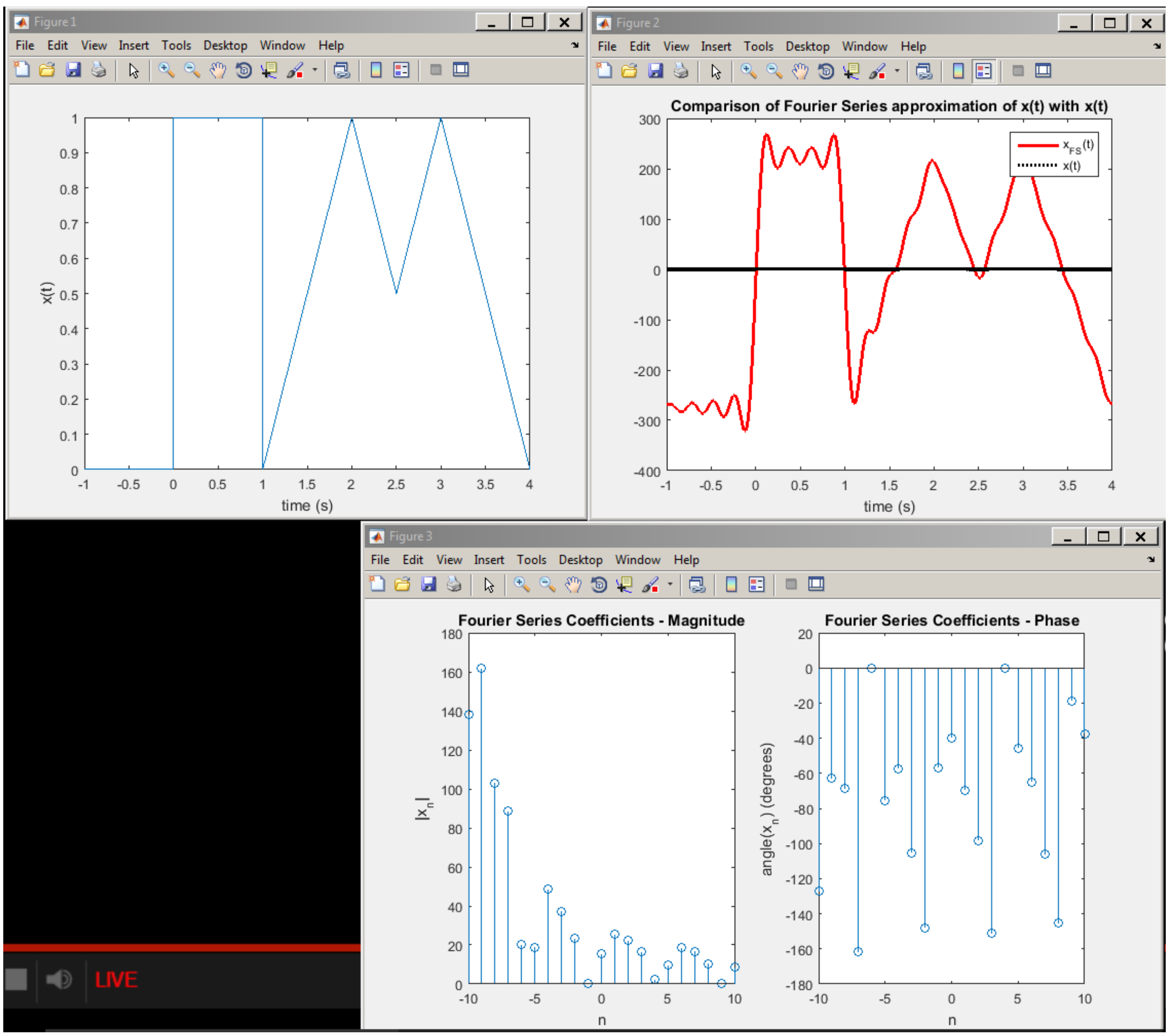
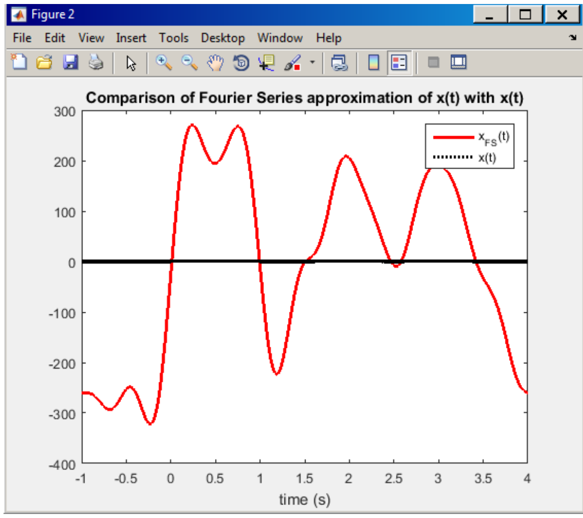


Figure 1 highest_harmonic = 10



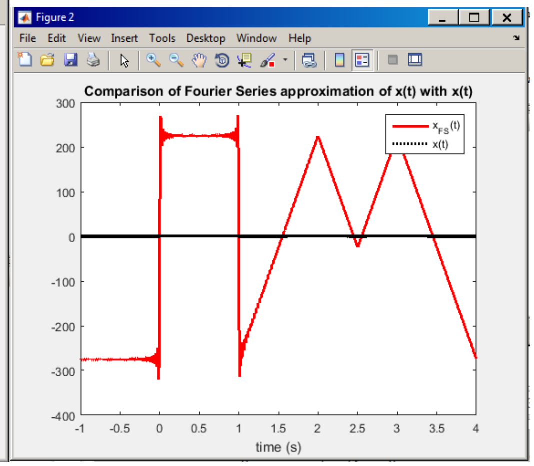


Figure 2 highest_harmonic = 5 **Figure 3 highest_harmonic = 100**

1) As highest_harmonic increases, the approximation is closer to the actual graph.
2) Points where there is discontinuity.