SOLUTIONS

# Quiz #5

**ECEn 380: Signals & Systems**

Fall 2014

***Closed book, closed note, closed neighbor, no calculators allowed.*** *Time limit is 15 minutes. 20 points total possible.*

1. Consider a discrete-time system described by the input/output relationship:

$$y[n] = 3x[n] - 2x[n-1] + x[n-3]$$

   a. Is this system LTI? (2 pts) YES. SYSTEM IS A MOVING AVERAGE (MA) SYSTEM.

   b. Find the system's impulse response h[n]. (3 pts)

$$h[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-3] \qquad h[n] = \{\underline{3}, -2, 0, 1\}$$

   c. Find the output of the system if the input is $x[n] = \{\underline{1}, 3, -1\}$. (3 pts)

$$\underline{3} \quad -2 \quad 0 \quad 1$$

$$-1 \quad 3 \quad \underline{1}$$

$$y[n] = \{\underline{3}, 7, -9, 3, 3, -1\}$$

   d. Find the unilateral z-Transform of the signal $x[n]$ given in part (c). (3 pts)

$$X(z) = 1 + 3z^{-1} - z^{-2} = 1 + \frac{3}{z} - \frac{1}{z^2}$$

2. Find the unilateral z-Transform of the following signals, and **state the region of convergence (ROC).** In other words, state for what values of $z$ the z-Transform converges.

   a. $x[n] = 2\delta[n-3] - 3\delta[n-4]$ (3 pts)

$$X(z) = 2z^{-3} - 3z^{-4} = \frac{2}{z^3} - \frac{3}{z^4} \qquad \text{ROC: } z \neq 0$$

   b. $x[n] = \left(\frac{1}{2}\right)^{n+1} u[n]$ (3 pts)

$$= \frac{1}{2}\left(\frac{1}{2}\right)^n u[n]$$

$$X(z) = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{\frac{1}{2}}{z}\right)^n$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{2z}}\right] = \frac{z}{2z-1} \qquad \text{ROC: } |z| > \frac{1}{2}$$

   c. $x[n] = e^{j\pi n/2} u[n]$ (3 pts)

$$X(z) = \frac{z}{z - e^{j\pi n/2}} = \frac{z}{z - j^n} \qquad \text{ROC: } |z| > 1$$