Electric Field, Electric Potential Energy and Voltage

Electric Field

Electric field is a very important and useful idea that is easy to understand just by looking at a subject matter we've already discussed such as gravity, and seeing how the idea of gravitational field was developed.

When we were doing problems with the gravitational force, we found two different equations that could be used.

$F_G = G\frac{mM}{r^2}$ or $F_G = mg$; where $g = \frac{GM}{r^2}$

It's easy to see that these equations mean the same thing, but it's just that in the second pair of equations we have broken the first equation into two parts: the first part represents the force due to the gravitational field, mg, while the second part tells us how to calculate the gravitational field at that location, GM/r^2. Here's how you can get the first equation from the second pair of equations.

$F_G = mg$ where $g = \frac{GM}{r^2}$ so we can substitute that in for g.

$F_G = m\left(\frac{GM}{r^2}\right)$ By reorganizing this you get the first equation shown above.

$$F_G = G\frac{mM}{r^2}$$

There aren't any new concepts involved; it's just a different way of looking at the same thing. In the first case, $F_G = G\frac{mM}{r^2}$, the two masses, "m" and "M" are directly interacting. In the second case, mass "M" creates a gravitational field given by, $g = \frac{GM}{r^2}$, and the second mass, "m" interacts with that field.

It's easier to think of it that way. For instance, once you calculate g for the earth to be 9.8 m/s^2, you just have to multiply that times any mass to get the gravitational force, which is the object's weight near the earth. It's a lot simpler than going back to the equation, $F_G = G\frac{mM}{r^2}$ every time and substituting the radius and the mass of the Earth in to get the weight of an object.

The same exact approach works for the relationship between electric force and electric field. We can break down the equation for electric force into two equations; one tells us the force on a charge due to an electric field, while the second tells us how to calculate the field created by a point charge.

$F_E = k\frac{qQ}{r^2}$ or $F_E = qE$, where $E = \frac{kQ}{r^2}$

E is the symbol for electric field. Since force is a vector and q is a scalar, that means that the electric field must be a vector; it has both a magnitude and direction. Its magnitude is given by the equation above, but its direction has to be defined. In the case of gravity it was easy; all masses are attracted to one another so the gravitational field always points towards the mass that created it; it's the direction that another mass would fall if it were placed at that location.

Units of E, the Electric Field

$F_E = qE$; so that means that

$$E = \frac{F_E}{q}$$

The units of E must therefore be Newtons / Coulomb (N/C)

There is no separate unit for electric field.

There are positive and negative electric charges, so we need to define the direction of the electric field arbitrarily. The decision was made to define the direction of the electric field as being the direction of a positive test charge that would accelerate if it were placed at that location in space. If you were to place a negative test charge at that location, it would move in the opposite direction. That means that positive charges, by definition, fall in the direction of the electric field and negative charges move in the opposite direction.

Example 1
A 25 mC positive charge is located at the origin, x=0. What is the electric field generated by that charge at the following locations: a) x = -2m; b) x = +2m; and c) x = +4m?

a) First we need to calculate the magnitude of the field at x = -2m.

$$E = \frac{kQ}{r^2}$$

$$E = \frac{\left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right)(25 \times 10^{-3} C)}{(2m)^2}$$

$$E = 56.3 \times 10^6 \frac{N}{C}$$

The direction of the field is that which a positive charge would move if it were placed at that location. Since a positive test charge would move away from a 25 mC placed at the origin that means that it would move to the left since that is the direction away from the origin for a location on the negative x-axis. The answer then becomes:

E = 5.625 x 10^7 N/C to the left

b) Since the location, x = +2m is also a distance of 2 m from the charge; the magnitude of the electric field will be the same at this location. However, since this location is on the positive x-axis, away from the origin is to the right. So the answer becomes:

E = 5.6 x 10^7 N/C to the right

c) In this case the field will also be directed to the right since a positive test charge placed at x = +4m would move away from the origin, which is to the right. But the distance from the charge to this location is twice as great. That means that the electric field will be ¼ as large as it was at x = +2m.

E = ¼ (5.6 x 10^7 N/C to the right)

E = 1.4 x 10^7 N/C to the right

If you want to check that result you can put Q and r into the original equation and confirm that you get this result.

Example 2
Determine the force that would be experienced by the following charges if they were placed at x = +2m in example 1: a) +20mC; b) -35μC: c) +15nC.

a) This is where the work we did in calculating the field makes life easy. We just need to use the equation: $F_E = qE$ *to find the magnitude of the force.*

$$F_E = qE$$

$$F_E = (20 \times 10^{-3} C)\left(5.6 \times 10^7 \frac{N}{C}\right)$$

$$F_E = 112 \times 10^4 N = 1.1 \times 10^6 N$$

But we also need the direction. Since the electric field points to the right at this location and this is a positive charge, which by definition falls in the direction of the field, the force is also to the right. So the answer is:

F_E = 1.1 x 10^6 N to the right

b) In this case we have to calculate the magnitude and be careful about the direction of the force, since this is a negative charge.

$$F_E = qE$$

$$F_E = (35 \times 10^{-6} C)\left(5.6 \times 10^7 \frac{N}{C}\right)$$

$$F_E = 196 \times 10^1 N = 2.0 \times 10^3 N$$

In this case the charge is negative so it moves in the direction opposite to the electric field. Since the field points to the right, the force on a negative charge is to the left.

F_E = 2.0 x 10^3 N to the left

c) In this case the charge is positive; so the force will be in the same direction as the field; to the right.

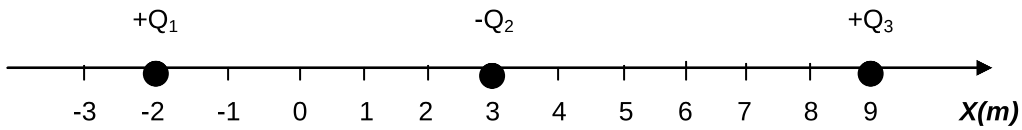
$$F_E = qE$$

$$F_E = (15 \times 10^{-9} C)\left(5.6 \times 10^7 \frac{N}{C}\right)$$

$$F_E = 84 \times 10^{-2} N = 0.84 N$$

Since electric field is a vector, the net electric field at a location, due to multiple charges is calculated by adding each of the vectors together. For instance if there are two electric fields at a location in space, 25 N/C to the left and 15 N/C to the right, then the net electric field at that location is 10 N/C to the left. Also, since direction counts when adding electric fields, it is possible to add two fields together and get zero electric field; just like two forces can add to zero.

Example 3



A positive charge, Q_1 = 20 μC, is located at a point x_1 = -2 m, a negative charge Q_2 = -10μC is located at a point X_2 = 3 m and a positive charge Q_3 = 30 μC is located at a point X_3 = 9 m.

a. *Find the magnitude and direction of the electric field at the origin due to Q_1.*
b. *Find the magnitude and direction of the electric field at the origin due to Q_2.*
c. *Find the magnitude and direction of the electric field at the origin due to Q_3.*
d. *Find the magnitude and direction of the net electric field at the origin.*

a) The distance to the origin is 2m, the magnitude of the charge is 20 μC and the charge is positive; so it will push a positive test charge at the origin to the right.

$$E = \frac{kQ}{r^2}$$

$$E = \frac{\left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right)(20 \times 10^{-6} C)}{(2m)^2}$$

$E = 45 \times 10^3 \frac{N}{C}$ *to the right*

b) The distance to the origin is 3m, the magnitude of the charge is 10 μC and the charge is negative; so it will pull a positive test charge at the origin to the right.

$$E = \frac{kQ}{r^2}$$

$$E = \frac{\left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right)(10 \times 10^{-6} C)}{(3m)^2}$$

$E = 10 \times 10^3 \frac{N}{C}$ *to the right*

c) The distance to the origin is 9m, the magnitude of the charge is 30 μC and the charge is positive; so it will push a positive test charge at the origin to the left.

$$E = \frac{kQ}{r^2}$$

$$E = \frac{\left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right)(30 \times 10^{-6} C)}{(9m)^2}$$

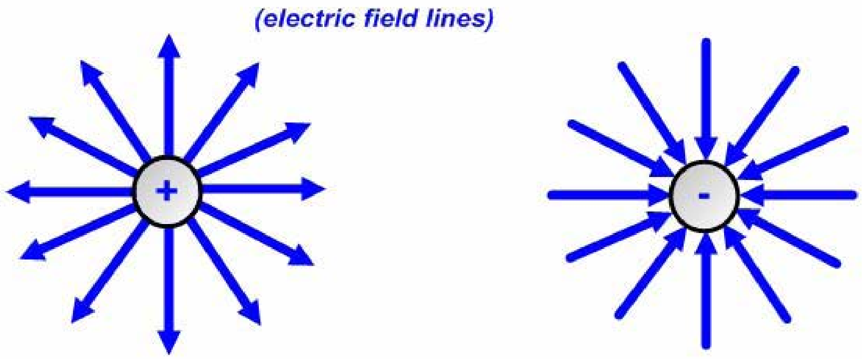
$E = 13 \times 10^3 \frac{N}{C}$ *to the right*

d) The three charges have created three electric fields at the origin. To get the net field at the origin, the three fields must be added as vectors. Let's designate the direction to the right as positive and to the left as negative so we can add them up.

$E_{net} = \Sigma E$
$E = E_1 + E_2 + E_3 + \ldots$
$E = E_1 + E_2 + E_3$
$E = +45 \times 10^3$ N/C + 10×10^3 N/C - 13×10^3 N/C
$E = (+45 + 10 - 13) \times 10^3$ N/C
$E = +42 \times 10^3$ N/C
$E = 4.2 \times 10^4$ N/C *to the right*

Lines of force

It is also useful to imagine the electric field as being present around a charge all the time, even when there are no other charges present and, therefore, no forces being generated. If you imagine a positive point charge, then no matter where you look around it, the electric field must be pointed away from it. That's because the definition of the direction of the electric field is the direction a small positive test charge would move if it were placed at that location. Any positive test charge would be repelled, and therefore move directly away, from a positive charge. So you can picture each positive charge as being surrounded by an electric field that would look like a porcupine, a series of arrows pointing away from it in all directions.



The electric field from an isolated positive charge. **The electric field from an isolated negative charge.**

You can also see that the lines spread out as they go further away from the charge. This reflects the fact that the field weakens as you travel away from it, it falls off as $\frac{1}{r^2}$. When drawing the lines of force, the density of the lines and how close together they are, tells you the strength of the electric field.
If you draw the lines of force surrounding a negative charge, the picture is very similar, but the arrows point towards the charge. That's because a positive test charge will always feel a force of attraction, pulling it towards the negative charge.

Uniform Electric Field – Charged Plates

While we derived the idea of electric field from looking at the force due to a point charge, it's also possible to create an electric field in other ways; for instance by charging a metal plate. If we are much closer to the plate than the plate is wide, we can think of the plate as being infinitely large. The result is that the net electric field due to the many millions of charges distributed over the plate points directly away from the plate. While charges on the top might also create some downward electric field, that is offset by the charges on the bottom that creates an upward electric field. The same cancellation occurs from charges on the left and right sides. The only direction that isn't canceled out is directly away from the plate. This also means that the lines of force point directly away from the plates; they don't spread out or get closer together. This also reflects the fact that the field does not weaken as you move away from the plate, it is uniform.
The force due to the electric field is still, $F_E = qE$. The only thing that changes is that the formula for E will not be the same as what was calculated for the point charge. We'll come up with one later. In the meantime, if you're given the magnitude of the electric field due to a metal plate, you can calculate the force on a charge near it by using that same formula, $F_E = qE$.

Very often, instead of having one charged plate, we put two next to each other with opposite charges, +Q and –Q. In fact, we accomplish this by moving electrons from one plate to the other. The plate with the extra electrons gets the charge –Q while the one that is missing electrons gets a charge +Q. In between these two plates the electric field will be uniform. The negative plate creates a uniform field that points towards it while the positive plate creates a uniform field that points away from it. Both of these fields point in the same direction inside the plates and cancel outside the plates. The result is a strong uniform electric field between the plates. This type of arrangement is called a capacitor and will be discussed later in this chapter.

Charged spheres

There is one other case where it is easy to determine the electric force and field: the case of a charged metal sphere. This is, once again, similar to the case of gravity. If you'll recall, for calculating the gravitational field or force due to a large sphere, like a planet, we have always measured from its center, not from its surface.

The same is true in the case of electric field or force. You use all the equations that were derived from a point charge, but simply measure from the center of the sphere. This will allow you to determine the electric force or field due to a metal sphere everywhere outside of that sphere.

Within the sphere, it will turn out that both the electric force and the field are always zero. But we'll talk more about that a bit later.

Electric Potential Energy

If two charges are infinitely far apart, they won't feel the attraction/repulsion from each other. If they are released, they won't move. That's because the force between them is given by Coulomb's Law:

$$F_E = k\frac{q_1 q_2}{{r_{12}}^2}$$

The force gets weaker as the inverse square of the distance increases between the charges. When they're twice as far apart you get ¼ the force, and when they're three times as far apart you get 1/9 the force. So if they are very far apart, there's no force at all.

However, if you hold one charge still and carry the second charge closer and closer to it, eventually you'll start getting some force. Let's first imagine that these are two positive charges, so they repel each other. As they get closer, you have to supply more force in order to keep them from flying apart. You have to do work in order to bring one closer to the other, since you are supplying a force over a distance parallel to that force:

Work = Force x $\text{distance}_{\text{parallel}}$

In this case, the force isn't uniform, so you need to use calculus to figure it out; but the result isn't complicated. The work it takes to move two charges from being infinitely far apart to a distance r apart is just: kQq/r. This is the same as the formula for the force between them, $F = kQq/r^2$, except instead of r^2, the denominator is r.

Since there was originally no energy in the system, the work that is done to bring these charges that close together is stored in the potential energy of the system. This can be seen if one releases them; they'll fly apart and that energy will turn into kinetic energy. So this gives us our formula for the potential energy due to two charges being near one another:

$$E_0 + W = E_f$$

$$0 + k\frac{qQ}{r} = E_f$$

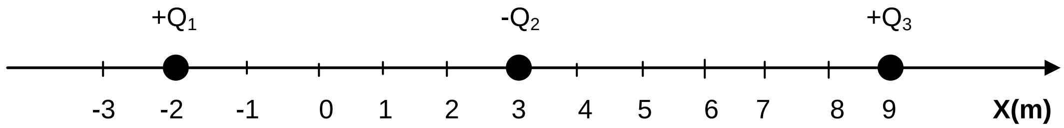
$$E_f = U_E = k\frac{qQ}{r}$$

$$U_E = k\frac{qQ}{r}$$

The formula for the potential energy due to two point charges being near one another is given by, $U_E = kQq/r$. There is no direction as energy is a scalar. However, there can be negative energy if one charge is positive and the other negative. That is because it takes an external force to keep them from getting closer together, not to keep them from flying apart. If you simply put the sign of the charge into the equation, that will take care of the sign of the energy.

If multiple charges are near one another, the energy must be computed due to each pair of charges in order to get the total energy of the system. These separate energies can just be added up arithmetically since there is no direction involved.

Example 4. Let's compute the energy of the system of charges described above, in Example 3.



A positive charge, Q_1 = 20 μC, is located at a point x_1 = -2 m, a negative charge Q_2 = -10μC is located at a point X_2 = 3 m and a positive charge Q_3 = 30 μC is located at a point X_3 = 9 m.

There are three pairs of energies that must be added together. In this case, the distances will be the distance between each pair of charges, not the distance to the origin.

$$U_{total} = U_{12} + U_{13} + U_{23}$$
$$U_{Total} = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_1q_3}{r_{23}}$$

We could factor out k here, but let's leave it in just to see what the individual energies are.

$$U_{Total} = \frac{\left(9x10^9\frac{Nm^2}{c^2}\right)(20x10^{-6}c)(-10x10^{-6}C)}{5m} + \frac{\left(9x10^9\frac{Nm^2}{c^2}\right)(20x10^{-6}c)(30x10^{-6}C)}{11m} + \frac{\left(9x10^9\frac{Nm^2}{c^2}\right)(-10x10^{-6}c)(30x10^{-6}C)}{6m}$$

U_{total} = -360 x 10^{-3} J + 491 x 10^{-3} J - 450 x 10^{-3} J
U_{total} = -319 x 10^{-3} J
U_{total} = -0.319 J

Electric Potential and Voltage

Just as it was useful to break the formula for electric force into two parts, with one equation to determine the electric field due to a charge and the other to determine the force on a second charge due to the field, it also proves useful to break the formula for Electric Potential Energy into two parts. One can be used for the Electric Potential of a location in space and the other to determine the energy of a charge at that location.

$U_E = \frac{KqQ}{r}$ OR $U_E = qV$ where $V = \frac{kQ}{r}$

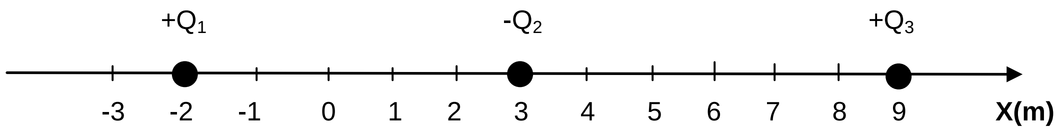
In this case, V is called the electric potential or the voltage. This is useful because in many applications the voltage at a location in space, such as the electrical outlet in your home, is determined independently of the device you connect to it. Establishing the voltage and then being able to put different amount of charge to work at that location is a better way to think about it.

It's easy to confuse the terms "electric potential energy" and "electric potential" so we're going to use the term "voltage" instead of "electric potential". Theoretically, there is a difference between the two since voltage is the difference between the electric potential at two locations. However, if we define both to be zero at infinity, they can be used interchangeably, so that's what we'll be doing.

Units of V, the Electric Potential or Voltage
U_E = qV; so that means that
V = $\frac{U_E}{q}$
The units of E must therefore be $\frac{Joules}{Coulomb}$ $(\frac{J}{C})$
Electric Potential, or Voltage, is used so frequently that it has its own unit; the volt.
Volt = $\frac{Joules}{Coulomb}$ $(\frac{J}{C})$

Voltage, like energy, is a scalar so there is no direction associated with it. However, it can be positive or negative since charges can be of either sign. Voltage is added arithmetically, like all scalars, so the net voltage at a location in space is simply the sum of the voltage due to each nearby charge.

Example 5. Let's compute the electric potential, the voltage, at the origin, for the charge distribution of Example 3.



A positive charge, Q_1 = 20 μC, is located at a point x_1 = -2 m, a negative charge Q_2 = -10μC is located at a point X_2 = 3 m and a positive charge Q_3 = 30 μC is located at a point X_3 = 9 m.

In this case, as in the case for electric field, we are computing a quantity for a location in space. We could choose any location. So the distance between the charges doesn't matter, only the distance to the location in space that we're studying. Unlike the case for electric field, direction doesn't matter.

$V = V_1 + V_{12} + V_3$

$$V = \frac{kq_1}{r} + \frac{kq_2}{r} + \frac{kq_3}{r}$$

We could factor out k here, but let's leave it in just to see what the individual voltages are

$$V = \frac{\left(9x10^9\frac{Nm^2}{c^2}\right)(20x10^{-6}c)}{2m} + \frac{\left(9x10^9\frac{Nm^2}{c^2}\right)(-10x10^{-6}c)}{3m} + \frac{\left(9x10^9\frac{Nm^2}{c^2}\right)(30x10^{-6}c)}{9m}$$

$V = +90 \times 10^3$ V - 30×10^3 V + 30×10^3 V
$V = +90 \times 10^3$ V
V = +90 kV

Please review all the examples and make sure that you're comfortable with the distances that are used in each case. The most confusing part of this chapter is the similarity in terms that have different meanings and the different values that are used for "r". In the case of force and potential energy "r" is the distance between charges. In the case of electric field and voltage "r" is the distance from each of the charges to a location in space. That's because force and energy are due to the interaction between charges while fields and voltage are created everywhere in space by all the nearby charges.

Example 6
How much work would be needed to carry an initially stationery 85 mC charge from infinity to the origin in Example 5?

The initial energy of the charge would be zero if it is infinitely far away from all other charges and at rest. Its final energy would be given by U_E= qV so:
$E_0 + W = E_f$
0 + W = qV
W = (85×10^{-3}C) ($+90 \times 10^3$V)
W = 7650 J

The Relationship between Electric Field and Voltage

There is an important relationship between electric field and voltage. The best way to picture it is once again by an analogy with gravity. In this case, let's consider the equation for the potential energy due to gravity near the surface of the earth, $U_G = mgh$. We can break that equation into two pieces, one piece due to the location, "gh" and the other due to the mass, "m". The mass m at that location gh, has the energy mgh.

Now let's picture a hill on the earth: it has a steep drop off on one side, a flat top, and a gentle slope on the other side going down to a flat plane. If you were to release a ball on the steep slope, it would roll down very fast. It would also roll down on the side with the gentler slope, just not as fast. However, if it were carefully placed on the flat top of the hill or the plane at the bottom it wouldn't roll at all. The steepness of the slope determines the net force pulling the ball down the hill, not how high in the air it is. It's all about how quickly the height goes down every time you travel a distance along the hill. If the height is constant, there is no net force. If the height is gradually dropping, a small net force. If the height is falling very quickly as you move along the path, the ball will feel a large net force pulling it down the hill

In the analogy with electrical potential, V serves the same role as "gh" does for the hill we were talking about. The electric field at a location is like the gravitational force making the ball roll down the hill. If the voltage everywhere around a charge is the same, there won't be any force acting on it, that means that the electric field at that location is zero. It's like being on the top of the flat hill. High voltage doesn't create a force on a charge, differences in voltage do. If the voltage is falling slowly as you travel a distance in space; there will be a small electric field at that location; if the voltage falls off quickly over a short distance, the electric field at that location will be very high. Electric field doesn't depend on the voltage. It depends on the difference in voltage over distance.

$$E = \frac{-\Delta V}{\Delta x} \quad \text{Or} \quad \Delta V = -E\,\Delta x$$

The negative sign just means that a positive charge will naturally fall from high voltage to low voltage. These two equations are always true at a location in space, but they are difficult to apply if the electric field is not uniform; you'd need to use calculus. However, with a **uniform electric field**, they are easy to use and are often simplified to be:

$V = -Ed$; for a uniform electric field

This also gives us a new unit for electric field

Units of E, the Electric Field

You'll recall from above that we derived the units of electric field to be:

$\frac{Newtons}{Coulomb}\left(\frac{N}{C}\right)$

But the relationship between the electric field and voltage gives us another unit for electric field: volts / meter $\left(\frac{V}{M}\right)$

We should be able to show them to be equivalent.

$1\frac{N}{C} = \frac{V}{m}$ but a volt = $\frac{Joules}{Coulomb}\left(\frac{J}{C}\right)$

$1\left(\frac{N}{C}\right) = \frac{\left(\frac{J}{C}\right)}{M}$ and a Joule = Newton · meter (N · m)

$1\left(\frac{N}{C}\right) = = \frac{\left(\frac{Nm}{C}\right)}{m}$

$1\left(\frac{N}{C}\right) = 1\left(\frac{N}{C}\right)$

So the two units are equivalent; either can be used as the unit for electric field.

Equipotentials and Lines of Force

On a contour map, curves are drawn connecting locations that are at the same height. They form closed curves in that as you move around a mountain, there will be some point in every direction that matches the height of the contour. If the contours get very close together, that means the height of the mountain is changing quickly at that location, and that it's a steep part of the mountain. If the contours spread apart, that means that it isn't very steep and you have to travel a long distance for your height to change very much.

The same sort of map is drawn for the voltage in a region of space. Instead of calling them contours, the curves are called equipotentials. They form closed curves where all the points on the curve are at the same voltage. When the equipotential curves are close together, the voltage is changing quickly and the electric field is very strong at that location. When the equipotentials spread out, that means the electric field is weaker there.

If you were to release a ball on a mountain, it would roll down the mountain, not sideways. It would not move along a contour, it would roll perpendicular to the contour, which is the fastest way down the mountain. Similarly, if you were to release a charge in a region of space, it would move perpendicular to the contours, not along a contour. But the direction of the electric field is the direction that a positive test charge would move if it were released at that location. So the electric field is always perpendicular to the equipotentials. We indicate the direction of the electric field by drawing the lines of force. As a result, the lines of force are always perpendicular to the equipotentials.