Electric Charge and Force

Introduction

From ancient times it was known that when certain materials are rubbed together, they can form an attraction to one another. This is the same discovery that is made every time you take clothes out of the dryer and they cling together. (As you pull them apart you might even see sparks fly, like small lightning bolts.) One famous example of this was documented in ancient Greece. It was seen that when thread was spun, by passing it over a spindle made of amber, the thread - and the hair of the spinstress for that matter - was attracted to the spindle. The Greek word for amber was elektron, hence this force came to be called "electric".

Over the course of time, it was determined that *dissimilar* materials attract one another and *similar* materials repel each other. For example, if a piece of glass was rubbed by two different pieces of silk; either piece of silk would be attracted to the glass, but the two pieces of silk would repel each other; as would two pieces of glass that had each been rubbed by a piece of silk. The electric force could be either attractive or repulsive, with likes repelling and unlikes attracting.

Before being rubbed, the materials experience no force of attraction or repulsion towards one another. Also, after being rubbed, if the materials that attracted one another were brought into direct contact, the force between them would disappear. It was almost as if in the process of rubbing the glass with the silk, something had been taken from the glass and deposited upon the silk. Whatever that was, it seemed to "want" to get back to the glass, whence it came, thus creating an attraction between the silk and the glass. The only way this "substance" could get back to the glass was to bring the silk with it, creating an electric force of attraction between the silk and the glass.

This substance was later named "charge". Since it seemed to come in two types, that when brought together in equal amounts added to zero, it was determined that charge must come in two forms; forms that were to be named "positive" and "negative". A "neutral" object was one that contained an equal balance of positive and negative charge, so its total net charge would be zero. A positively charged object contains more positive charge than negative charge and the reverse is true for a negatively charged object. (It's important to note that neutral objects contain charge, just equal amounts of negative and positive charge.)

It was a great mystery as to which charge moved from one object to another when they were rubbed together. For example, when glass was rubbed by silk did a charge move from the glass to the silk or from the silk to the glass?

We now know that, in solid objects, only the negative charge can move. The positive charge is fixed into relatively permanent positions within the solid. To understand this better, we need to give you a brief overview of the atom.

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The Atom

The word atom also comes from ancient Greece where the verb “tomi” meant “to divide” and the prefix “a” meant “not”. So the word atom means “not divisible”. Atoms are supposed to represent the indivisible pieces that make up all matter. But as we’re about to see, what we call “atoms” are in fact divisible; atoms have parts.

Atoms are made up of mostly empty space with a very small solid center called the nucleus and even smaller particles called electrons buzzing about in the empty space. If an atom were somehow “blown up” to be the size of a gymnasium the nucleus would be about the size of a baseball and would be located in the center. The electrons would be smaller than gnats and would be flying around in random directions at high speeds. It’s hard to picture an atom as mostly empty space; when we picture an “empty” gymnasium it’s still filled with air; but there’s nothing to fill the space inside an atom. The space not occupied by the nucleus or the electrons in an atom is truly empty, or as empty as anything that we can picture can be. Since all matter is made of atoms, including you and me, that means that everything, including you and me, is mostly empty space.

One reason that protons and neutrons don’t fly around like electrons is that they are a lot more massive than electrons – about 2000 times more massive. So while the lightweight electrons are flying around through all that empty space, the protons and neutrons are stuck together in the center of the atom.

But getting back to electricity; electrons are the carriers of negative charge; the positive charge resides inside the nucleus. The nucleus is comprised of two types of particles, the neutron and the proton. While neutrons and protons have very similar masses, the proton has a positive charge and the neutron has no charge. So all the negative charge that we’ll be talking about is carried by electrons and all the positive charge is carried by the protons, inside the nucleus of the atom.

The magnitude of the charge of a single electron or a single proton is the same and is given the symbol “e”. So, an electron has a charge of “-e” and a proton has a charge of “+e”. In their “normal” state, atoms have neutral charge; that is they have an equal amount of negative and positive charge. That also means that they have an equal number of protons and electrons.

Atoms can sometimes become charged; by gaining an extra electron or losing one of their electrons; these charged atoms are called ions. A positive ion has fewer electrons than it normally would, while a negative ion has more electrons than it normally would. While the number of electrons can change in this manner – by adding or losing electrons – atoms don’t gain or lose protons.

You’ll learn more about that when you learn about nuclear physics, but in terms of electricity; the only change that we’ll discuss with respect to atoms is when they gain or lose electrons: ionization. In fact, the names we give to atoms are based on the number of protons that they have; hydrogen atoms have one proton; helium has two protons; carbon has six; oxygen has eight; etc. Only nuclear reactions can change the number of protons; to learn more about that you'll have to wait for the chapter on nuclear physics.

An important principle of physics is **Conservation of Charge**. It is always the case that charge is conserved; charge is never created or destroyed. In any closed system, the total net charge never changes. So, whenever we speak of "charging a conductor" or giving something a charge, we mean that we are moving charge from one place to another, we are never creating it. To the extent that one object gains charge, another object must be losing an identical amount. Similarly, when we speak of "discharging" a conductor, we mean that we are moving charge to or from the conductor; we are not destroying charge in this process.

Solids

When solid matter is formed, the protons of the atoms that comprise the solid form a crystal with their positions very well defined with respect to each other. The only way to move the relative locations of the nuclei in a solid is through taking an outside action on it, like melting it; bending it; scraping it; etc. The reason that solids are "solid" is that they maintain their shape because the nuclei that comprise them are rigidly locked in position.

On the other hand, the electrons in a solid are very light and are not so tightly bound to one location. They are held in place by their attraction to the positive nucleus: Since they have an opposite charge to that of the nucleus, and opposites attract, most electrons stay with their nuclei. However, that is not always the case with all electrons. How tightly attached electrons are to their nuclei can vary; this leads to the two general types of materials we'll be discussing: **insulators** (sometimes called dielectrics) and **conductors** (the third type of solid, semiconductors, requires its own discussion that is beyond the scope of this chapter).

The electrons in insulators are very attached to their nuclei. Since the nuclei aren't moving, that means that the electrons in insulators also don't move around. On the other hand, some of the electrons in a conductor, called **conduction electrons**, easily move around inside of a solid. You can picture them like a fluid that can flow throughout the conductor, but can't leave it. Metals are great examples of conductors; they have conduction electrons that can move around the metal freely. (That's why wires that carry electricity are made of metal; the conduction electrons in the metal are free to move and can carry electricity from the wall to your television, radio, etc.)

Conductors

Let's now imagine a solid metal object. There are an equal number of immoveable positive nuclei scattered uniformly throughout it. Most of the electrons also can't move; but the conduction electrons can go wherever they want. If the sphere is neutral, there's not much more to talk about; the electrons will move about in it but it will have no net charge. The electrons will also be uniformly spread throughout the conductor as they are attracted to the nuclei as much as they are repelled by other electrons.

Now let's charge the conductor by adding some extra electrons to it; electrons that are free to move anywhere they want. Since the conductor was previously neutral, there is no overall attraction for the electrons towards the center. On the other hand, since all the added electrons have the same negative charge, they spread out so they can be as far from each other as possible (like charges repel). The best way to achieve this is for all the excess electrons to move to the surface of the sphere and spread out uniformly over that surface. More generally, all excess electrons on a conductor are uniformly distributed over its surface.

<u>Charging by rubbing</u>

As you'll recall the first known discovery of electric force goes back thousands of years ago. The very name electric comes from the Greek name for amber, *elektron*, and was due to the charge generated in amber when it was rubbed with cloth.

You experience the same thing when you rub your feet on carpeting. Electrons move between you and the carpeting, giving you a net charge; when you touch something that is grounded, like a doorknob, the electrons flow through your fingers, bringing you back to being electrically neutral. That's the shock you sometimes get when you touch a doorknob after walking on carpeting; it's due to the flow of electrons between you and the earth.

An easy way to charge some materials is to rub them against another one. Here are two famous examples: rubbing a plastic rod with animal fur gives the rod a negative charge and the fur a positive charge; rubbing a glass rod with silk gives the glass a positive charge and the silk a negative charge. Each is charged by the movement of electrons between the materials. So in the first case, the charge of the plastic rod is exactly equal and opposite to that of the fur and in the second case the charge of the glass rod is equal and opposite to that of the silk. Charge is not created or destroyed, it only moves from place to place.

<u>Charging by Conduction</u>

Now let's see what happens if we have two identical metal spheres, one negatively charged (it has excess electrons) and one neutral (it has an equal number of electrons and protons) and we touch them together. Once we touch them together, the excess electrons are free to move anywhere on the two spheres. Once again, they'll spread out over the surfaces of the spheres. The best way to spread out is to cover both spheres with an equal amount of electrons, so the total charge will be divided in two, half on one sphere and half on the other. If we now separate the two spheres, so they are no longer touching, there's no way for the charge on the second sphere to get back to where it started; the second sphere remains charged. Giving something a charge by touching it with a second charged object and then separating them is called, charging it by conduction. The object that is being charged doesn't have to be identical to the second object; it's just that if they are identical the charge becomes equally divided. If they are not identical, more charge may end up on one conductor than the other.

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How about if one sphere is neutral and the other one has a positive charge (it's missing some electrons). In this case, there are more nuclei missing electrons on one sphere than on the other. When they are touched together electrons will flow from the neutral sphere to the positively charged sphere. If they are separated, the electrons can't flow back to the neutral sphere, so both spheres are missing the same number of electrons; they both have an equal positive charge.

Note that in the second case, the neutral sphere acquired a positive charge by touching a positively charged sphere. It looks like positive charges moved from the positive sphere to the neutral sphere, but that is not the case. The positive charges, the protons, didn't move. It's just that some of shortage of electrons is now spread between both spheres making them both positive.

Charging by Induction

It is possible to charge a neutral conductor without touching it with a charged conductor; but, in this case, you need to touch it with another conductor; you need a total of three conductors. Here's how it work.

Take the negatively charged sphere and move it close to the neutral sphere. The conduction electrons on the neutral sphere will be repelled by the excess electrons on the charged sphere. They can't leave the first neutral sphere since it's not touching anything, but they can move to the side opposite the charged sphere. They'll keep moving away from the charged sphere until the attraction of the abandoned nuclei near the charged sphere, which are now missing electrons because they moved to the other side, is equal to the repulsion from the excess electrons on the charged sphere. At that point, one side of the neutral sphere is positively charged and the other side is negatively charged; its total net charge is still zero. If, at this point, we moved the charged sphere away, the electrons would just flow back to where they started and that would be that.

But, what would happen if we touched the neutral sphere with another neutral sphere? The electrons, instead of just moving to the other side of the first neutral sphere will now move to the second neutral sphere; anything to get further from the charged sphere and spread out from one another. If we then take the second neutral sphere away, it will retain those extra electrons, giving it a negative charge. The first neutral sphere can't get those electrons back, and it was initially neutral, so now it has a positive charge. No change occurs to the initially charged sphere, since it never touched anything; it retains all of its extra electrons. We've charged two neutral spheres by using a third charged sphere which never touches the other two. Is it clear from this explanation that the induced positive charge on the second sphere is equal in magnitude to the induced negative charge on the third sphere?

This same process can work in reverse, with a nearby positively charged sphere attracting electrons which then are also drawn from a second neutral sphere. The effect is the same since one of the two neutral spheres ends up with a positive charge and the other ends up with an equal negative charge.

Grounding

The earth is an enormous conductor; that means that there's plenty of room on the earth for extra electrons to come or go without having any effect on the net charge of the earth. By attaching a wire between a conductor and the earth, a "ground wire", electrons will flow between the earth and the conductor, as needed, to keep the conductor neutral. This is called "grounding" the conductor. The third metal conductor on devices that you plug into the wall is a ground wire. It connects the device to the earth so that the device never accumulates a net charge that could give you a shock. For this to be effective, every building needs to have a metal stake of some sort driven into the ground and then ground wires, connected to that metal stake, need to be run throughout the building. As you plug a device, into the wall, it connects the device to the earth, making you safe from getting a shock.

When you're standing on the ground electrons are free to move from you to the earth or in the reverse direction; that keeps you grounded. So instead of needing a third conducting sphere to charge a conductor by induction, we can usually just touch it to have the same effect. Electrons flow through you to the earth or in the reverse direction, in order to make the conductor being touched neutral.

Electroscopes

Electroscopes work on the same principles of charging by induction and charging by conduction. An electroscope consists of a metal sphere attached to a rod from which are hanging moveable conductors, such as strips of aluminum foil. When the device is neutral the two hanging conductors experience no net force, they hang straight down due to their weight. However, if the electroscope is negatively charged by conduction, then the excess electrons will spread over all the surfaces of the conductors within the electroscope, including the aluminum leaves. Now that they each have a negative charge, they will repel each other, spreading them apart.

The same effect occurs if the electroscope is charged by touching it with a positive conductor. The shortage of electrons spreads throughout the electroscope's conductors, including the hanging metal leaves, and they become positively charged. No matter what type of charge you put on an initially neutral electroscope, the leaves will move apart.

Electroscopes can be affected, as well as charged, by induction. For instance, if you move a negatively charged object near the sphere on the top of an electroscope, its conduction electrons will move away from the top, giving each of the hanging leaves a net negative charge; as a result they will repel each other and move apart. However, if without touching the electroscope you move that charged object away from it, the electrons will move back to where they started; the leaves will move back together. Do you see why the exact same thing will occur placing a positively charged conductor near the top of the electroscope; that the leaves will move apart?

If the leaves of an electroscope are spread apart, you know it has a charge; but that doesn't tell you if it's a positive or negative charge. However, if you have a charged object that you know to be positive, or negative, you can figure it out. If you place a positively charged object near the top of an electroscope that is positively charged, the leaves will spread further apart as the electrons on the leaves are attracted to the object; as a result, the leaves will be missing even more electrons, yielding greater repulsion between the leaves. However, if the leaves move closer together, the electroscope must have been negatively charged. The leaves had a net negative charge but some of those excess electrons will move to the top of the electroscope to get near the positively charged object.

Here's how you can charge an electroscope by induction. First, move a negatively charged object near a neutral electroscope; the leaves will move apart since electrons will flow to the leaves. While the electroscope is still neutral, the leaves will have a net charge and will move apart. If you then ground the electroscope while the leaves are spread apart, the electrons will move down the ground wire to the earth instead of to the leaves. At that point, the leaves will hang straight down, since they will be neutral, even though the net charge of the electroscope is positive; it's missing the electrons that flowed to the earth. If you disconnect the ground wire before moving the negatively charged object away, the electrons that left the electroscope will be stranded on the earth; they can't get back to the electroscope. As a result, the electroscope now has a net positive charge. When you move the negatively charged object away from the electroscope, the leaves will spread out, reflecting that they now share in the net positive charge of the electroscope; the leaves are both positively charged, so they repel each other.

Electric Force

If two plastic rods are rubbed with animal fur, they will both become negatively charged. If you arrange it so that one of the rods can move freely and the other can be brought close to it, you will see the first rod will move away; it will be repelled by the other rod. Objects that have the same charge repel each other. The same thing happens with two positively charged glass rods. If one charged glass rod is brought near another which is free to move, it will be repelled. Once again, objects that have the same charge repel each other.

As you might guess, the opposite occurs if rods with opposite charge are brought near each other; they are attracted to one another. So if a positively charged glass rod is free to move and a negatively charged plastic rod is brought near it; the glass rod will move towards the plastic rod. In the reverse case, where the plastic rod is free to move and the glass rod is brought near it, the plastic rod will move towards the glass rod. Objects with opposite net charge attract one another.

A more complicated case occurs when a neutral rod is free to move and a positively charged rod is brought near it. In this case, the electrons in the neutral rod will be attracted to the positively charged rod. If the neutral rod is a conductor the electrons will move to the side that is nearest the positive rod; if it is an insulator, the electrons can't leave their nuclei, but they can move to the side of their nuclei that is closest to the positive rod. Once they do this, the side of the neutral rod closest to the positive rod will have a net negative charge. Since, as we'll be discussing in the next section, the electric force is stronger if when charges are closer together, the attraction of those nearby electrons to the positive rod results in an attractive force; the neutral rod is attracted to a positively charged rod.

By the same reasoning, a neutral rod will be attracted to a negatively charged rod. The electrons will move away from the negatively charged rod, leaving the near side of the neutral rod with a net positive charge. A neutral rod will be attracted to a negative rod as well as a positive rod.

This effect can be seen if you place a charged rod near scraps of paper, your hair, or any other object that can move freely. Even though those objects have no charge, they will be attracted to the charged rod; regardless if it has a positive or negative charge. However, if they come in contact with the charged rod, they will quickly jump away. That's because once they touch the charged object, they pick up part of its charge by conduction. They are no longer neutral; they now have the same charge as the rod and are repelled by it.

<u>Coulomb's Law</u>

It had been known for a number of years that objects that have a positive charge attract negatively charged objects and repel other positively charged objects; like charges repel and opposites attract. Benjamin Franklin and Joseph Priestly were the first to reason, in the 1760's, that the electric force between two charged objects must decrease as the inverse square of the distance between two charged objects. Franklin had made the observation that excess electrons move to the surface of a conductor. Priestly then proved, mathematically, that that could only occur if the repulsion between electrons decreases as the inverse square of their separation. They thus showed that the electric force is proportional to $1/r^2$: $F_E \propto 1/r^2$. That means that if the distance between the centers of two charged objects is doubled, the electric force is one-fourth as great; triple the distance, you get one-ninth the force; halving the distance yields four times the force.

In the 1780's Charles Coulomb conducted a series of experiments that confirmed that result and also showed that the force is proportional to the charge of each object. He did this by using identical metal spheres and halving the charge on each one by touching them together with neutral spheres. This allowed him to vary the charge on each sphere. He then used a torsion balance to measure the force.

The symbol for charge is "q" so this yields the formula for the electric force:

$$F_E = k\frac{q_1 q_2}{r^2}$$

The direction of this force is on a line between the point charges, or between the centers of spherical objects. If the result is positive, the force is repulsive; while if the result is negative the force is attractive.

In this formula:

k is a constant and equals 9.0×10^9 N·m^2/C^2

q_1 is the net charge on one of the objects

q_2 is the net charge on one of the other object

r is the distance between the objects, if they are point charges, or between the centers of the objects, if they are spherical

Units of Charge and the size of “e”

The units of the constant k depends on the units of charge. If charge is measured in Coulombs (C), named after Charles Coulomb, then the units of k are:

$$\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Given the choice of units for k, the **magnitude** of the charge of both electrons and protons is given by:

$e = 1.6 \times 10^{-19}$ C

Protons have a charge of $+1.6 \times 10^{-19}$C and electrons have a charge of -1.6×10^{-19}C.

It’s important to note that an object can only have a charge that is equal to some multiple of “e” since the charge on any object can only be due to the number of electrons and protons within it.

The formula for electric force is similar in form to that for gravitational force.

Electric Force

$$F_E = k\frac{q_1 q_2}{r^2}$$

Gravitational Force

$$F_G = G\frac{m_1 m_2}{r^2}$$

The key difference between these two formulas is that mass can only be positive, while charge can be positive or negative. If you include the sign of the charge in the formula for electric force, a negative result means that the force is attractive; a positive result indicates a repulsive force. In either case, the force is directed along the line between the centers of point charges, or the centers of spherical objects (this is the same as in the formula for gravitational force).

The electric force is much more powerful than the gravitational force. In fact, even small imbalances in net charge can create very large forces. The reason that the gravitational force has so much effect is due to the fact that it is always attractive; mass is always positive. On the other hand, only the imbalance of charge on an object, its net charge, can generate an electric force. However, even small net charges can generate significant amounts of force. As a result, net charges are rarely given in Coulombs (C), instead they are usually given in milli-Coulombs (mC) which equals 10^{-3} C; micro-Coulombs (μC) which equals 10^{-6} C; and nano-Coulombs (nC) which equals 10^{-9} C.

Example 1: Two point charges are located 10 cm apart. One has a charge of +20 μC *and the other has a charge of -30* μC*. What is the force on each due to the other?*

$$F_E = k\frac{q_1 q_2}{r^2}$$

$$F_E = (9.0 \times 10^9 \frac{N \cdot m^2}{C^2}) \frac{(20 \times 10^{-6} C)(-30 \times 10^{-6} C)}{(10 \times 10^{-2} m)^2}$$

$$F_E = -540N$$

The negative sign tells us that this is an attractive force so the answer should be given as
F_E = 540 N directly towards each another.

Example 2: Two electrons are located 2.0m apart. What is the force on each electron due to the other?

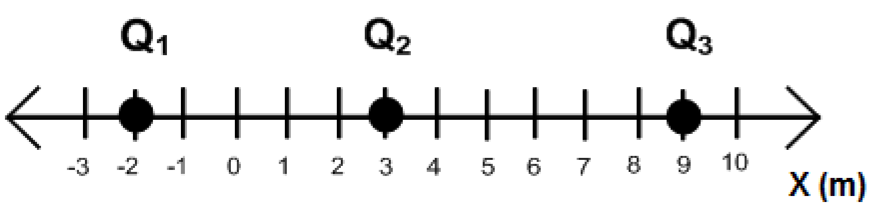
$$F_E = k\frac{q_1 q_2}{r^2}$$

$$F_E = (9.0 \times 10^9 \frac{N \cdot m^2}{C^2}) \frac{(-1.6 \times 10^{-19} C)(-1.6 \times 10^{-19} C)}{(2\ m)^2}$$

$$F_E = +5.76 \times 10^{-29} N$$

The positive sign tells us that this is a repulsive force so the answer should be given as
F_E = 5.8 x 10^{-29} N directly away from one another

Example 3: Three charges are arranged as shown below. A positive charge Q_1 = 10 μC is located at a point X_1 = -2 m, a negative charge Q_2 = -20 μC is located at a point X_2 = 3 m and a positive charge Q_3 = +30 μC is located at a point X_3 = 9 m. Find the net force acting on each charge.



It's important to remember that forces are vectors, they have magnitude and direction. If only two charges are present, the direction can be defined as towards or away from each other. But in this case, we need to determine all the forces acting on the charges and determine if they are positive (to the right) or negative (to the left). Only then can the forces be added together to get the net force.

Let's determine the net force acting on Q_1 first. The first step is to do a free body diagram. In the below diagram, you can see that there are two forces acting on Q_1, F_{12} to the right and F_{13} to the left. The force on

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Q_1 due to Q_2 is attractive so it will pull Q_1 to the right and the force on Q_1 due to Q_3 is repulsive so it will push Q_1 to the right.

Now we need to determine the magnitude of those two forces. We won't put in the sign of each charge since we'll use our free body diagram to determine the direction.

$$F_{12} = k\frac{q_1 q_2}{r^2}$$

$$F_{12} = (9.0 \times 10^9 \frac{N \cdot m^2}{C^2})\frac{(10 \times 10^{-6}C)(20 \times 10^{-6}C)}{(2\,m)^2}$$

$$F_{12} = 72 \times 10^{-3} N$$

Now we'll do the same thing for the force on Q_1 due to Q_3.

$$F_{13} = k\frac{q_1 q_2}{r^2}$$

$$F_{13} = (9.0 \times 10^9 \frac{N \cdot m^2}{C^2})\frac{(10 \times 10^{-6}C)(30 \times 10^{-6}C)}{(11\,m)^2}$$

$$F_{13} = 22.3 \times 10^{-3} N$$

From our free body diagram we see that the net force on Q_1 will be given by:

$F_1 = F_{12} - F_{13}$

$F_1 = 72 \times 10^{-3}N – 22.3 \times 10^{-3}N$

$F_1 = 49.7$ mN

We can now repeat this process for Q_2 and Q_3. However, we can use some of our previous results by using Newton's third law. We know that $F_{12} = - F_{21}$; the magnitude of those two forces is equal. Similarly, $F_{13} = - F_{31}$. So the only force whose magnitude still needs to be calculated is F_{23} which is also equal in magnitude to F_{23}.

$$F_{23} = k\frac{q_1 q_2}{r^2}$$

$$F_{23} = (9.0 \times 10^9 \frac{N \cdot m^2}{C^2})\frac{(20 \times 10^{-6}C)(30 \times 10^{-6}C)}{(6m)^2}$$

$$F_{23} = 150 \times 10^{-3} N$$

Now to determine the net force on Q_2, we do a free body diagram for Q_2 noting that F_{21} is attractive so it will pull Q_2 to the left and F_{23} will pull Q_2 to the right.

$F_2 = -F_{21} + F_{23}$

$F_2 = -72\text{ mN} + 150\text{ mN}$

$F_2 = +78\text{ mN}$

Determining the net force on Q_3 the same way yields

$F_3 = +F_{31} - F_{32}$

$F_3 = +22.3\text{ mN} - 150\text{ mN}$

$F_3 = -127.7\text{ mN}$

If we want to check our answer, we know that the total net force on all the charges should be equal to zero, by Newton's third law. So:

$F_1 + F_2 + F_3 = -127.7\text{ mN}+78\text{ mN} +49.7\text{ mN} =0$

This verifies our results.