New Jersey Center for Teaching and Learning

Progressive Science Initiative

Click to go to website:
www.njctl.org

PHYSICS
Solving Equations

April 2012

www.njctl.org

Solving for a Variable

Our goal is to be able to solve any equation for any variable that appears in it.

Let's look at a simple equation first.

$$s = \frac{d}{t}$$

The variables in this equation are s, d and t.

Solving for a variable means having it **alone** on the **left side**.

This equation is currently solved for "s".

The Rules

Like in any game there are a few rules.

1. To "undo" a mathematical operation, you must do the opposite.

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

3. If there is more than one operation going on, you must undo them in the opposite order in which you would do them, the opposite of the "order of operations."

4. You can always switch the left and right sides of an equation.

The Rules

Let's solve this equation for "d"

$$s = \frac{d}{t}$$

That means that when we're done
we'll have d **alone**
on the **left side** of the equation.

1 First, is d already ALONE? If not, what is with it?

- A s
- B d
- C t
- D it is already alone

$$s = \frac{d}{t}$$

2 What mathematical operation connects d and t?

- A d is added to t
- B d is multiplied by t
- C d is divided by t
- D t is subtracted from d

$$s = \frac{d}{t}$$

3 What is the opposite of dividing d by t?

- A dividing t by d
- B dividing by s into t
- C multiplying d by t
- D multiplying by t by d

$$s = \frac{d}{t}$$

Rule 1. To "undo" a mathematical operation, you must do the opposite.

4 What must we also do if we multiply the right side by t?

- A divide the left side by t
- B multiply the left side by t
- C divide the left side by d
- D divide the left side by d

$$s = \frac{d}{t}$$

Rule 2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

5 Is there more than one mathematical operation acting on "d"?

- ◯ **Yes**
- ◯ **No**

$$s = \frac{d}{t}$$

Rule 3. If there is more than one operation going on, you must undo them in the opposite order in which you would do them, the opposite of the "order of operations."

Applying Rules 1 and 2

1. To "undo" a mathematical operation, you must do the opposite.

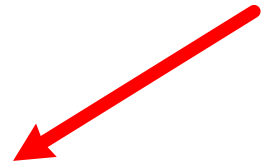
$$s = \frac{d}{t}$$

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other. So we undo d being divided by t, by multiplying both sides by t.

$$(t)\ s = \frac{d}{\cancel{t}}\ \cancel{(t)}$$

$$t\,s = d$$

$$st = d$$

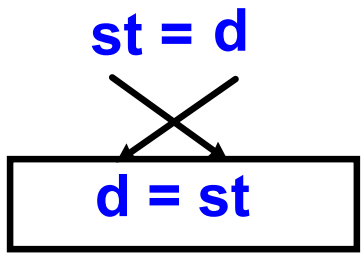


The order of the s and t doesn't matter, but we usually make them alphabetical so they look better.

Are we done?

Applying Rule 4

Rule 4. You can always switch the left and right sides of an equation.



We've now solved our equation for d.

A harder problem is to solve it for t.

Solving for t

Let's solve this equation for "t"

$$s = \frac{d}{t}$$

That means that when we're done
we'll have t **alone**
on the **left side** of the equation.

6 Is t already ALONE? If not, what is with it?

- A s
- B d
- C t
- D it is already alone

$$s = \frac{d}{t}$$

7 What mathematical operation connects d to t?

A t is being divided by d

B d is being divided by t

C d is being multiplied by t

D t is being subtracted from d

$$s = \frac{d}{t}$$

8 What is the opposite of dividing d by t?

- A dividing d by t
- B dividing s by t
- C multiplying d by t
- D multiplying t by d

$$s = \frac{d}{t}$$

Rule 1. To "undo" a mathematical operation, you must do the opposite.

9 What must we do if we multiply the right side by t?

- A divide the left side by t
- B multiply the left side by t
- C divide the left side by d
- D divide the left side by d

$$s = \frac{d}{t}$$

Rule 2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

10 Is there more than one mathematical operation acting on "d"?

- ◯ **Yes**
- ◯ **No**

$$s = \frac{d}{t}$$

Rule 3. If there is more than one operation going on, you must undo them in the opposite order in which you would do them, the opposite of the "order of operations."

Solving for T

1. To "undo" a mathematical operation, you must do the opposite.

$$s = \frac{d}{t}$$

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other. So we undo d being divided by t, by multiplying both sides by t.

$$(t)\ s = \frac{d}{\cancel{t}}\ \cancel{(t)}$$

$$st = d$$

Are we done?

11 Is t ALONE? If not, what is with it?

- A s
- B d
- C t
- D it is already alone

st = d

12 What mathematical operation connects s to t?

- A t is being divided by d
- B t is being divided into s
- C t is being multiplied by s
- D t is being subtracted from s

$st = d$

13 What is the opposite of multiplying t by s?

A dividing t by s

B dividing t by t

C multiplying t by t

D multiplying by t by s

$st = d$

Solving for t

1. To "undo" a mathematical operation, you must do the opposite.

$$st = \frac{d}{s}$$

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

$$\frac{\cancel{s}t}{\cancel{s}} = \frac{d}{s}$$

$$t = \frac{d}{s}$$

14 Is t ALONE on the left?

- A s
- B d
- C t
- D it is alone

$$t = \frac{d}{s}$$

Solving for v_o

Let's solve this equation for "v_o"

$$\mathbf{v = v_o + at}$$

That means that when we're done

we'll have v_o **alone**

on the **left side** of the equation.

15 **Is v_o already ALONE? If not, what is with it?**

- A only a
- B only t
- C at
- D it is already alone

$v = v_o + at$

16 What mathematical operation connects at to v_o?

A "at" is being divided by v_o

B "at" is being added to v_o

C v_o is being multiplied by "at"

D v_o is being divided by "at"

$v = v_o + at$

17 What is the opposite of adding at to v_o?

A dividing by v_o by "at" into t

B subtracting v_o from "at"

C subtracting "at" from v_o

D dividing "at" by v_o

$v = v_o + at$

18 What must we do, if we subtract "at" from the right side?

- A add "at" to the left side
- B multiply the left side by "at"
- C subtract "at" from the left side
- D divide the left side by v_o

$v = v_o + at$

19 Is there more than one mathematical operation acting on "v_o"?

- ◯ **Yes**
- ◯ **No**

$v = v_o + at$

Solving for v_o

1. To "undo" a mathematical operation, you must do the opposite.

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

$$v = v_o + at$$
$$-at \qquad -at$$

$$v - at = v_o$$

$$\boxed{v_o = v - at}$$

Solving for a

Let's solve this equation for "a"

$$v = v_o + at$$

That means that when we're done
we'll have a **alone**

on the **left side** of the equation.

20 Is a already ALONE? If not, what is with it?

- A only v_o
- B only t
- C v_o and t
- D it is already alone

$v = v_o + at$

21 What mathematical operation connects v_o to at?

- A "at" is being divided by v_o
- B v_o is being added to "at"
- C v_o is being multiplied by "at"
- D v_o is being subtracted by "at"

$v = v_o + at$

22 What is the opposite of adding v_o to at?

- A dividing by v_o by at into t
- B subtracting v_o from at
- C subtracting at from v_o
- D dividing at by v_o

$v = v_o + at$

23 What mathematical operation connects t to a?

- A a is added to t
- B a is multiplied by t
- C a is divided by t
- D t is subtracted from a

$v = v_o + at$

24 What is the opposite of multiplying a by t?

- A dividing a by t.
- B dividing t by a.
- C multiplying a by t
- D multiplying t by a

$v = v_o + at$

25 What must we do, if we divide by t from the right side?

- A divide the left side by a
- B multiply the left side by a
- C divide the left side by t
- D multiply the left side by t

$v = v_o + at$

26 Is there more than one mathematical operation acting on "a"?

- ◯ Yes
- ◯ No

$v = v_o + at$

27 Which operation should we undo first?

- A divide a by t
- B subtract v_o from at

$v = v_o + at$

28 Which operation should we undo second?

- A divide a by t
- B subtract v_o from at

$v = v_o + at$

Solving for a

1. To "undo" a mathematical operation, you must do the opposite.

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

3. If there is more than one operation going on, you must undo them in the opposite order in which you would do them, the opposite of the "order of operations."

$$v = v_o + at$$
$$- v_o \quad - v_o$$

$$\frac{v - v_o}{t} = \frac{at}{t}$$

$$\frac{v - v_o}{t} = a$$

Are we done?

Solving for t

Let's solve this equation for "t"

$$v = v_o + at$$

That means that when we're done
we'll have t **alone**

on the **left side** of the equation.

Solving for t

1. To "undo" a mathematical operation, you must do the opposite.

2. You can do anything you want (except divide by zero) to one side of an equation, as long as you do the same thing to the other.

3. If there is more than one operation going on, you must undo them in the opposite order in which you would do them, the opposite of the "order of operations."

$$v = v_o + at$$