Electric Current and DC Circuits

Batteries

Batteries maintain an electric potential difference, a voltage, between their terminals. The positive terminal is maintained at a higher voltage than the negative terminal; for instance, the positive terminal of a 6 volt battery is at an electric potential that is 6 volts higher than its negative terminal. This voltage difference is maintained by a chemical reaction that continues as long as the necessary chemicals are present in the battery.

You can envision a battery as being like an escalator for charges. The chemical reaction is like the motor of the escalator. It carries charge up to the higher voltage, but no higher; once the higher terminal has enough charge, the escalator stops; ideally a battery will maintain exactly the same potential difference between the terminals no matter what else is going on...no higher and no lower.

We will generally depict the voltage difference in a circuit as being due to a battery. Not all electric devices rely on batteries; for instance, some use solar cells or AC/DC converters. However, the principal stays the same. The ideal battery, or whatever serves in that role, maintains a constant voltage difference between the terminals.

Current and Circuits

An **electric circuit** is an external path that charges can follow between the terminals of a battery; it connects the two terminals together with a conducting material such as metal. The word circuit is directly related to the word circle; it represents a complete unbroken circle between the two terminals. The circle, or circuit, is completed inside the battery, by the chemical reaction that takes place there. **To be considered a circuit, the external path between the terminals must be complete and unbroken.**

When we talk about the flow of charges around a circuit we are forced to differentiate between what's actually flowing and the words we use to describe that flow. The problem goes back to Benjamin Franklin. He helped develop the theory that electric charges came in two forms, positive and negative. Unfortunately, experiments at the time couldn't determine if positive charges flowed one way or negative charges flowed the opposite way; for most purposes there is no way to determine which is actually happening. His model assumed that positive charges flowed from the positive to the negative terminal of a battery; we now know that is not the case. The flow of charge through a conductor is due to the movement of electrons, the carrier of negative charge. So what is actually happening is that electrons are flowing from the negative terminal of the battery (they are repelled by the negative terminal) to the positive terminal (to which they are attracted).

While the **actual current** is of electrons flowing from the negative to the positive terminal, much of the theory of electric circuits was complete before that was known. Hence, we speak of a **conventional current** consisting of hypothetical positive charges flowing from the positive to the negative terminal. Throughout this book, when we refer to the flow of current through a circuit we will be speaking of this conventional current. While this is not physically what is occurring (since the carrier of positive charge, the proton, does not move at all) this is a simple way of viewing what is going on in a circuit. It should be kept in mind that this is just a helpful image, not the underlying reality. But for the same reason that it took so long to discover the underlying reality, it turns out that all the predictions made by picturing a conventional current of positive charges going from the positive to the negative are identical to those that would be made of negative charges moving in the reverse direction.

The symbol for conventional current is the letter "I". It is defined as the amount of charge that flows past a location in a conductor per unit time.

$$\boldsymbol{I \equiv \frac{\Delta Q}{\Delta t}}$$

The "Δt" in the denominator denotes the time interval being measured while the "ΔQ" in the numerator denotes the amount of charge that flowed flow past that location in the circuit during that time interval.

Current is measured in amperes (often abbreviated to "amps"); an ampere of current is defined as one Coulomb of charge passing by a location in the circuit each second.

$$\mathbf{1\ A \equiv 1\ \boldsymbol{\frac{C}{s}}}$$

The Definition and Units of Current

Current is defined as the amount of charge per unit time that passes by a location.

$$\boldsymbol{I \equiv \frac{\Delta Q}{\Delta t}}$$

The ampere (A) is defined as 1 Coulomb of charge (Q) passing a location in a conductor each second (s).

1 Ampere ≡ 1 Coulomb per second

$$\mathbf{1\ A \equiv 1\ \boldsymbol{\frac{C}{s}}}$$

Ampere is often abbreviated as "amp"

Example 1: At a location in a circuit 4 C of charge passes by in 2s; what is the current flowing past that location in the circuit?

$$\boldsymbol{I \equiv \frac{\Delta Q}{\Delta t}}$$

ΔQ represents the amount of charge that flowed by and Δt represents the amount of time that it took for that to occur, so

$$I = \frac{4c}{2s}$$

$$I = 2\ \frac{c}{s}$$

$$I = 2\ A$$

Example 2: A current of 2A is flowing through a circuit. How long does it take 40 C of charge to travel through the circuit?

$$\boldsymbol{I} \equiv \frac{\Delta \boldsymbol{Q}}{\Delta \boldsymbol{t}}$$

First solve for the unknown, "Δt"

$$\Delta \boldsymbol{t} \equiv \frac{\Delta \boldsymbol{Q}}{\Delta \boldsymbol{I}}$$

Then substitute in the given values

$$\Delta t = \frac{40c}{2A}$$

$$\Delta t = 20\frac{C}{A}$$

$$\Delta t = \frac{20C}{\frac{C}{s}}$$

$$\Delta t = 20\ s$$

Resistance and Ohm's Law

Conductors resist the flow of current though them to a greater or lesser extent (superconductors being an exception to this rule that will be discussed later). The electrical resistance that a conductor presents to the flow of charge is called its **resistance (R)**. Resistance is measured in units called ohms, whose symbol is the Greek symbol omega "Ω".

The resistance of an electrical circuit slows down the flow of charge through it. On the other hand, the voltage difference between the terminals to which the circuit is connected increases the flow of current. If there were no voltage difference, there would be no current. As the voltage difference increases, so does the "pressure" pushing the charge through the circuit.

Combining these two factors together yields a relationship between current, resistance and voltage that is named for the physicist, Ohm: Ohm's law.

$$\boldsymbol{I \equiv \frac{V}{R}}$$

The Definition and Units of Resistance (R)

Resistance can be defined by solving Ohm's Law for R

$$\boldsymbol{R = \frac{V}{I}}$$

Resistance represents the proportion between the voltage across a circuit and the current flowing through it. For instance, the higher the resistance, the less current will flow for a given applied voltage.

Resistance is measured in ohms (Ω).

Based on the above relationship it can be seen that :

$$\mathbf{1\Omega = 1\,\frac{V}{A}}$$

While this relationship can always be used to define the resistance of a material at a given current and voltage, it is not always linear: that is increasing the voltage does not always increase the current in direct proportion. However, it is very close to linear for the metal conductors that we will be using to construct circuits in this text. (Materials for which the relationship is linear are called ohmic; while materials for which this is not true (light bulbs, transistors, diodes, etc.) are called nonohmic.) You can assume that Ohm's law predicts a linear relationship between current and voltage, unless you are told otherwise.

Example 3: A 12-volt battery is connected to a 6 Ω external circuit; how much current will flow from the positive to the negative terminal of the battery.

$I = \frac{V}{R}$

$I = \frac{12V}{6\Omega}$

$I = \frac{2V}{\frac{V}{A}}$

$I = 2A$

Example 4: What is the voltage across a 4Ω circuit if there is a 5A current flowing through it?

$I = \frac{V}{R}$

Solve for the unknown, "V"

$V \;=\; IR$

$V \;=\; (5A)(4\Omega)$

$V \;=\; 20\,A\left(\frac{V}{A}\right)$

$V \;=\; 20\,V$

Resistance and Resistivity

The resistance of a conductor depends both on the material of which it is made and it's shape. Materials can vary widely in how well they conduct electricity. The measure of a material's resistance to conducing electricity is called its **resistivity (ρ)**. For example, silver conducts electricity about 6.7 times better than platinum. That means that if exactly the same wire were made out of both materials, the resistance of the platinum wire would be about 6.7 times greater than that of the silver wire.

But it is possible to make a platinum wire that has a lower resistance than a silver wire; this is where the shape of the conductor comes in. The longer the wire, the harder it is for charges to travel through it from one end to the other. The charges simply face a longer journey through the wire. Just like it's harder to push water through a very long hose than through a short hose; it's harder for a voltage to push charges through a longer conductor than a shorter one.

Similarly, the more cross-sectional area that a wire has, the easier it is to push charges through it. If you want to push a lot of water though a hose, use a bigger diameter hose. The hoses that are used by fire trucks are a lot wider than those used to water a garden. If you want to more easily push charges through a wire, use a thicker wire. Combining these two understandings together yields these reasonable assertions about the effect of a conductor's shape on its resistance.

- **The longer the conductor; the more resistance**
- **The wider the conductor (greater cross-sectional area); the less resistance**

Combining this together with the resistivity of the material yields the following:

$$\mathbf{R} = \frac{\rho L}{A}$$

This expression simply states that the resistance of a conductor depends on the material of which it is made, its length and its cross-sectional area: the longer, the more resistance; the wider, the less resistance.

The Definition and Units of Resistivity

We can solve the above equation for ρ to determine the units of resistivity.

$$\rho = \frac{RA}{L}$$

We can now substitute in the units of the variables on the right to get the units for "ρ".

$$\text{Units of } \rho = \frac{\Omega \cdot m2}{\text{m}}$$

Units of ρ = Ω·m

The resistivity of materials must be provided, either in a table or within a given problem, they cannot be determined theoretically. A few of the values for common materials are given below. Please note that in this table the resistivities are given in units of 10^{-8} Ω·m, not Ω·m. Thus the resistivity for silver is 1.59 x 10^{-8} Ω·m.

Resistivities of some common conductors at 20°C

Material	**Resistivity (ρ) (10^{-8} Ω·m)**
Silver	1.59
Copper	1.68
Gold	2.44
Aluminum	2.65
Tungsten	5.60
Iron	9.71
Platinum	10.6
Mercury	98
Nichrome	100

Temperature and Resistivity

The table above shows the resistivity of materials at 20°C. Materials will have a greater resistivity at higher temperatures and a lower resistivity at lower temperatures. The extreme case of this is superconductivity. When materials reach a sufficiently low temperature, their resistance can completely vanish; electrical currents can run through them without any resistance.

The more common phenomenon is that the resistance of conductors rises when they get hot. That is one reason why Ohm's law can break down with even simple conductors, such as a tungsten wire in a light bulb. When the bulb is first turned on, it is cold, and its resistance is very low. It quickly gets hot enough to glow, and its resistance is much higher. However, the start-up rush of current though the bulb's initially low resistance makes it more likely to fail at the moment it is turned on.

The temperature dependence of resistivity can also be used to devise thermometers based on the flow of current through a known piece of wire. As the temperature of its environment changes, so too will its resistance; as will the current flowing though it. The measurement of the current than becomes a method of determining the temperature of the wire's environment.

Example 5: What is the resistance of a 100 m long piece of gold wire whose radius is 2.0mm?

R =$\frac{\rho L}{A}$

The area of a circle is πr^2 so this becomes

$R = \frac{\rho L}{\pi r^2}$

Converting r into meters and using the correct units for ρ, $10^{-8}\ \Omega{\cdot}m$

R = $\frac{(2.44\ x\ 10^{-8}\ \Omega{\cdot}m)(100m)}{(3.14)(2.0\ \mathrm{x}\ 10^{-3}\mathrm{m})^2}$

R = $\frac{(2.44\ x\ 10^{-6}\ \Omega{\cdot}m^2)}{(3.14)(4.0\ \mathrm{x}\ 10^{-6}\mathrm{m}^2)}$

R = $\frac{(2.44\ x\ 10^{-6}\ \Omega{\cdot}m^2)}{(12.6\ \mathrm{x}\ 10^{-6}\ \Omega{\cdot}m^2)}$

R = 0.19 Ω

Example 6: How long a piece of 2mm diameter nichrome wire has a resistance of 2Ω?

$R = \frac{\rho L}{A}$

Solving for L

$L = \frac{RA}{\rho}$

$L = \frac{R(\pi r^2)}{\rho}$

Remembering that 2mm diameter equals 1mm radius

L = $\frac{(2\Omega)(3.14)(1\ x\ 10^{-3}m)^2}{100\ x\ 10^{-8}\ \Omega{\cdot}m}$

L = $\frac{6.28\ \Omega(1\ x\ 10^{-6}m^2)}{1\ \mathrm{x}\ 10^{-6}\ \Omega{\cdot}\mathrm{m}}$

L = $\frac{(6.28\ \Omega)\left(1\ x\ 10^{-6}m^2\right)}{1\ \mathrm{x}\ 10^{-6}\ \Omega{\cdot}\mathrm{m}}$

L = 6.3 m

<u>Electric Power</u>

We can develop formulas for determining the power consumption of electrical circuits by combining two facts that we learned in previous chapters with what we just learned. First, from the chapter titled “Electric Field, Potential Energy and Voltage” we learned that the work necessary to move a charge Q through a voltage difference V is given by: $\boldsymbol{W = QV}$. Second, from the chapter titled “Energy” we learned that power is defined as the work done per unit time: $\boldsymbol{P = \frac{W}{t}}$. Combining these two equations together yields the following.

$\boldsymbol{P} = \frac{\boldsymbol{W}}{\boldsymbol{t}}$

Substituting $\mathbf{W = QV}$

$P = \frac{QV}{t}$

Rearranging

$P = \frac{QV}{t}$

$P = \left(\frac{Q}{T}\right) V$

Substituting $\boldsymbol{I} = \frac{\boldsymbol{Q}}{\boldsymbol{t}}$

P = IV

We see from this equation that the power consumed by a circuit, or portion of a circuit, is the product of the current flowing through it and the voltage difference across it. This makes sense in that energy is released when even a single charge is allowed to fall from a higher to a lower voltage. An electrical current represents a large number of charges falling through that potential difference per second. That released energy per second represents the power used by the circuit.

A useful image might be to think of water rushing along a stream bed and turning a water wheel. The water is flowing from a high point in the stream to a lower point; water only flows downhill. As it is flowing, it is releasing gravitational potential energy. The paddle wheel takes advantage of that released energy by being turned in a way that allows it to do work. Doing this requires slowing the water down a bit; the wheel represents a resistance to the water, but the results is that the power of the flowing water can be used to power saw mills, pumps, etc.

In the case of an electrical circuit, the circuit takes advantage of the flow of charge from a high to a low voltage; from the positive to the negative terminal of the battery. In the process of falling, the circuit gets the charge to do work (light a bulb, run a motor, etc.). This slows the charges down, as the circuit resists the flow of the charges, but it allows electrical power from the battery to become useful power for the circuit.

We can obtain two other convenient expressions for electrical power by using Ohm's Law, I = V/R, to eliminate either I or V from the above equation: P = IV.

P = IV	
Now substituting either $\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{R}}$ **or V = IR**	
$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{R}}$	**V = IR**
P = $\left(\frac{V}{R}\right) V$	$P = I(IR)$
P = $\frac{V^2}{R}$	$P = I^2 R$

These are three equivalent expressions that can be used to determine the power being consumed by a circuit, or any portion of a circuit. The choice as to which one will to use is purely based on convenience: which is quicker to use based on the given information.

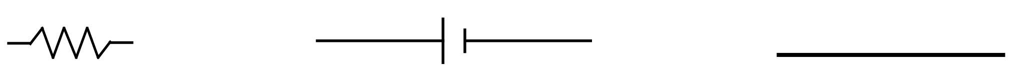
$$P = IV = \frac{V^2}{R} = I^2R$$

DC Circuits

A basic direct current (DC) circuit is made up of a battery connected to an external circuit made up of combinations of resistors; the resistors and the battery are connected together by conducting wires. Direct current means that the voltage source is not varying with time; a battery is a good example of a voltage source for a DC circuit. The alternative to DC circuits is alternating current (AC) circuits. Both are very commonly used; for instance, the outlets in your house are alternating current (AC). AC is a convenient way to transport electricity from place to place. However, many devices then convert the voltage back to DC before using it to power a circuit. How that is done, and how AC circuits operate, is beyond the scope of this text. We will focus on DC circuits that are built simply using resistors, wires and batteries.

A circuit diagram represents the pathways that charges follow in traveling from the positive to the negative terminal of the battery. Circuit diagrams do not attempt to look like the actual circuit once it has been built. It is a representation that is useful for analyzing the circuit electrically; it is not a diagram that shows where the parts will be physically located. The circuits we will be analyzing require the use of two symbols, shown below, connected together by lines that represent wires.

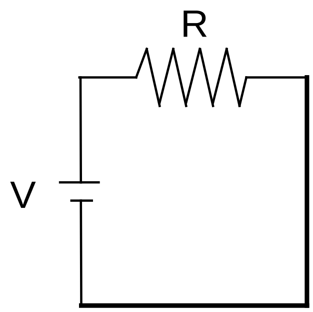
Symbols used in basic DC circuits diagrams



Resistor | Battery | Wire

The two vertical lines in the battery symbol depict the two terminals of the battery; the larger line represents the positive terminal and the smaller line the negative terminal

Example 7: The simplest circuit consists of a battery with a single resistor across its terminals. If, in the below diagram the resistor, R, has a value of 10 Ω and the battery has a voltage of 6 V. What is the direction and magnitude of the current?



The resistance of the circuit is given by that of the single resistor, 10 Ω, and the voltage is due to the battery, 6 V. We just need to use Ohm's law.

$$I = \frac{V}{R}$$
$$I = \frac{6V}{10\Omega}$$
$$I = 0.6\,A$$

The direction of conventional current is from the positive terminal to the negative, so the current goes clockwise through the circuit.

Example 8: If, using the same drawing, shown above, you are told that the voltage of the battery is 12 V and that a 2 A current is flowing out of the battery's positive terminal. What is the value of the resistor, R?

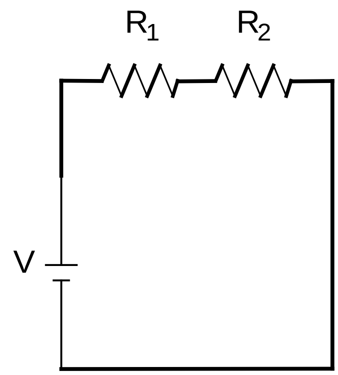
Once again, use Ohm's law, but this time solve for R.

$$I = \frac{V}{R}$$
$$R = \frac{V}{I}$$
$$R = \frac{12V}{2A}$$

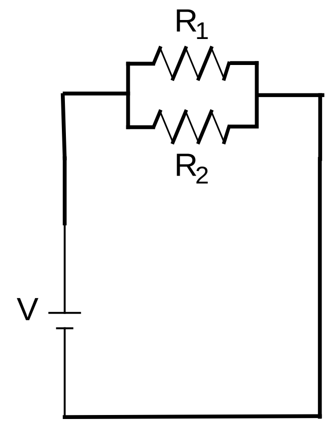
$$R = 6\,\Omega$$

Series and Parallel Circuits

There are two ways that a second resistor can be added to the above circuit: in series or in parallel.



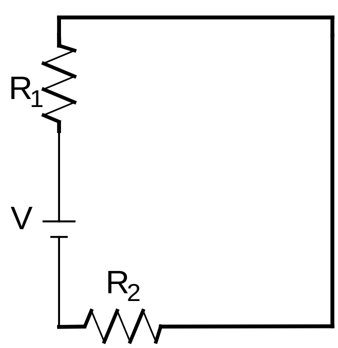
Series

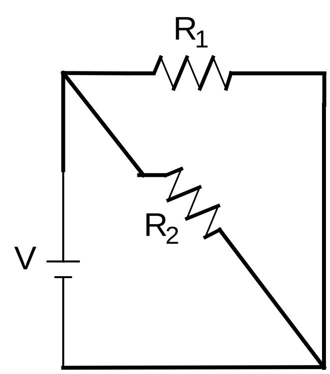


Parallel

The key difference between a series and parallel circuit is that in the case of the series circuit, all the charges must go through both resistors in order to travel from the positive to the negative terminals of the battery. If you trace a path from the positive plate of the battery in the series circuit to the negative plate, you must pass through both resistors; you have no choice. On the other hand, tracing the shortest path between the terminals in the parallel circuit only allows you to pass through R_1 or R_2, not both.

It is important to be able to determine if the components in a circuit are connected in series or in parallel. In the above cases, it's relatively clear, since the components in the parallel circuit are literally parallel to one another. But look at the following circuits.





Is it similarly clear that the resistors in the circuit on the left are in series? If not, just check to see if a charge moving from the positive to the negative terminal of the battery could make that trip without going through both resistors. You should see that that is not possible in the circuit on the left; that is a series circuit. On the other hand, a charge could make a complete trip around the circuit on the right by going through either R_1 or R_2. Since the charge would have a choice of paths, the circuit on the right is a parallel circuit. Your first step in interpreting circuit diagrams is to determine which components are in parallel and which are in series.

Equivalent Resistance of Circuits

From the viewpoint of the battery, all the circuits that we will be studying can be reduced to the simple form of a single resistor and battery. That single resistor, and the voltage of the battery, will determine the current flowing out of the battery. Thus the life of the battery, the power consumed by the circuit, etc. can all be studied in a very simple way. Every problem becomes as simple as Examples 7 and 8. The value of the single resistor that could replace the entire circuit, for this purpose, is called the equivalent resistance of the circuit.

The equivalent resistance (R_{eq}) of two or more resistors is the value that a single resistor would need to have to replace them in the circuit without changing the circuit from the viewpoint of the battery.

We will be studying three types of circuits: Series, Parallel and Combinations. In each case, we will develop a strategy to find the equivalent resistance of the circuit and then use that information to analyze it.

Series Circuits

There are two key principles that apply to all series circuits. These will provide the foundation for analyzing those circuits.

- **The current passing through all parts of a series circuit is the same.**
- **The sum of the voltage drops across each of the resistors in a series circuit equals the voltage of the battery.**

The first statement follows from the fact that each charge must pass through all the resistors to go from one terminal to the other; that's actually our definition of a series circuit. If each charge must do that, so must the current, which is simply the sum of the charges making that trip.

The second statement requires an understanding of the term "voltage drop". When a current flows through a resistance, there is a voltage difference between the two sides of the resistor. The side closest to the positive terminal of the battery is higher by an amount given by Ohm's law: $V= IR$, where I is the current through the resistor and R is the value of that resistance.

The charges in the circuit are traveling from the positive (high) terminal to the negative (low) terminal. Along their path the voltage drops each time they pass through a resistor. Eventually, when they reach the negative, low, terminal, the voltage is zero. You can see from this that the sum of all the drops must equal the total drop. So if there are three resistors in a series circuit then $V= V_1 + V_2 + V_3$; where V is the voltage of the battery.

One way to picture this is to think of a pretty flat stream of water going over a series of small waterfalls. The stream has to be pretty flat for this image to work as we are assuming that the wires in our circuits have no resistance, so there's no voltage drop between resistors, just when going through each resistor. You can then think about the water having to fall from a height down to ground level in steps...each step representing a resistor. The current of water is like the current of electricity. Since all the water has to make it to the bottom, the current is the same everywhere, but the individual drops for each waterfall can be different. But the sum of all those drops has to equal the total height that the water falls.

So we have some rules that we can use to analyze series circuits. If we use a subscript to denote the different resistors in the circuit and no subscript to denote the battery, then these can be written this way.

$$\boldsymbol{I = I_1 = I_2 = I_3} \ \ldots.$$

$$\boldsymbol{V} = \boldsymbol{V_1} + \boldsymbol{V_2} + \boldsymbol{V_3} + \cdots$$

We can use this to develop a formula to determine the equivalent resistance of a series circuit. Let's start with the second equation.

$$V = V_1 + V_2 + V_3 + \cdots$$

Now substitute, using Ohm's law, $V = IR$ for each term.

$$IR = I_1R_1 + I_2R_2 + I_3R_3 \ldots$$

But in a series circuit $I = I_1 = I_2 = I_3$ ....

$$IR = IR_1 + IR_2 + IR_3 + \cdots$$

Now divide by I and recognize that R is the total resistance of the circuit; it's equivalent resistance, R_{eq}

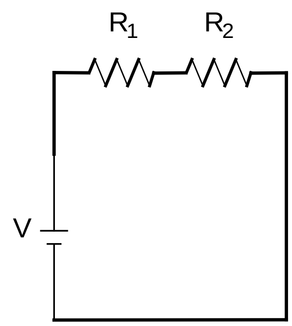
$$R_{eq} = R_1 = R_2 = R_3 \ldots.$$

This is an important result...and it seems reasonable. If a charge has to pass through additional resistors to complete its trip to the negative terminal, it will have to overcome more resistance.

Additional resistors always increase the resistance of a series circuit.

This also makes sense from the perspective of our equation for determining resistance: R = ρL/A. In the simplest case, you can think of each resistor as being a length of wire. Adding more resistors in series is just like making the wire longer. As can be seen from the above formula, a longer wire would result in a proportionally higher resistance.

One important way to check your results with series circuits is to recognize that the equivalent resistance of the circuit must always be **more than** the largest resistance in the circuit. That is because each additional resistor increases the resistance of the circuit from that initial starting point.



Example 9: In the circuit to the right, V= 12V; R_1 = 4Ω; and R_2 = 2 Ω.

First determine R_{eq}; I; V_1; and V_2.

Then determine the power consumption of the circuit, P, and that of each of the two resistors: P_1 and P_2.

$R_{eq} = R_1 = R_2 = R_3$ ….

$R_{eq} \ = 4\Omega + 2\ \Omega$

$Req = 6\Omega$

Note that this answer is more than the largest of the resistors, the 4Ω resistor.

$I \ = \frac{V}{R}$ For the whole circuit V is the voltage of the battery and R_{eq} is its resistance

$I \ = \frac{V}{R_{eq}}$

$I = \frac{12V}{6\Omega}$

$\boldsymbol{I \ = \ 2A}$

$V_1 \ = \ I_1 R_1$

In a series circuit, $I = I_1 = I_2 = I_3$ ….

$V_1 = IR_1$

$V_1 = (2A)(4\Omega)$

$\boldsymbol{V_1 = 8V}$

$V_2 = I_2 R_2$

In a series circuit, $I = I_1 = I_2 = I_3$ ...

$V_2 = IR_2$

$V_1 = (2A)(2\Omega)$

$\boldsymbol{V_1 = 4V}$

Note that these results are consistent with $\boldsymbol{V = V_1 + V_2 + V_3 + \cdots}$ since 12V = 8V + 4V. This is a good way to check your work.

We have three different equations that we can use to find the power: P = IV = V^2/R = I^2R
For the entire circuit, we use V to be the battery voltage and R to be $R_{eq.}$ Let's use all three equations just to confirm they give the same answers.

$P = IV$	$P = \frac{V^2}{R_{eq}}$	$P = I^2 R_{eq}$
$P = (2A)(12V)$	$P = \frac{(12V)^2}{6\Omega}$	$P = (2A)^2(6\Omega)$
$P = 24W$	$P = 24W$	$P = 24W$

To find the power used by R_1, we use V_1, R_1, and I; recognizing the I_1 = I.

$P = IV_1$	$P = \frac{V_1^2}{R_1}$	$P = I^2 R_1$
$P = (2A)(8V)$	$P = \frac{(8V)^2}{1.33\Omega}$	$P = (2A)^2(4\Omega)$
$P = 16W$	$P = 16W$	$P = 16W$

To find the power used by R_2, we use V_2, R_2, and I.

$P = I_2 V$	$P = \frac{V_2^2}{R_2}$	$P = I^2 R_2$
$P = (2A)(4V)$	$P = \frac{(4V)^2}{2\Omega}$	$P = (2A)^2(2\Omega)$
$P = 72W$	$P = 72W$	$P = 72W$

Note that not only are the results for power consumption equally valid with each of the three equations, but that **our results show that P = P_1 + P_2 +...** This makes sense since the power consumed by the total circuit must be consumed somewhere in the circuit. The only place that can occur is within the resistors.

In this example, we showed much more work than would be needed in solving a problem in that we used all three equations for power and solved for each resistor and the total circuit independently,. However, using a second method is a good way of checking your work.

Parallel Circuits

There are two key principles that apply to all parallel circuits. These will provide the foundation for analyzing those circuits.

- **The voltage across all the resistors in a parallel circuit is the same**

- **The sum of the currents through each of the resistors in a parallel circuit equals the current of the battery**

The first statement follows from the fact that the battery is connected to both ends of each resistor. The battery keeps the voltage drop across each resistor the same as the voltage of the battery.

The second statement follows from the fact that each parallel resistor represents an alternative path that charges can take around the circuit. However, all the charges leaving the positive terminal eventually make it to the negative terminal: charges are not created, destroyed or stored in these circuits.

One way to picture this is to think of a stream of water that meets obstacles and divides to go around that obstacle on both sides. While it might divide into multiple streams, the total water flowing must be the same. When the streams eventually merge back together, all the water will still be flowing. So at any point, the total water flowing through all the separate paths must add up to be the same as the total current at the beginning or end.

So we have some rules that we can use to analyze parallel circuits. If we use a subscript to denote the different resistors in the circuit and no subscript to denote the battery, then these can be written this way.

$$\mathbf{I = I_1 + I_2 + I_3 ....}$$

$$\mathbf{V = V_1 = V_2 = V_3 = ...}$$

These formulas are very similar to the ones we developed for series circuits; the difference being that the "+" signs and the "=" signs have been interchanged. This profoundly changes the results we will obtain for the equivalent resistance of a parallel circuit. Let's start with the first equation.

$I = I_1 + I_2 + I_3 ....$

Now substitute, using Ohm's law, $I = \frac{V}{R}$ for each term.

$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots\ldots$

But in a parallel circuit $V = V_1 = V_2 = V_3 = \ldots.$

$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots\ldots$

Now divide by V and recognize that R is the total resistance of the circuit; it's equivalent resistance, R_{eq}

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots$

This is a more difficult equation than the one obtained for series circuits. The fact that this involves summing fractions both makes it more mathematically challenging as well as creating a result that might seem counter intuitive at first: the more resistors are added in parallel the lower the resistance of the circuit.

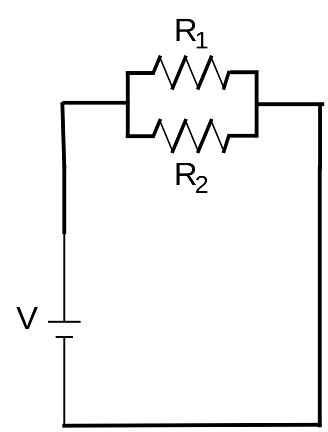
This result becomes more reasonable if one considers that each parallel resistor is an additional pathway that a charge can follow on its journey. The more pathways that a charge can follow, the less resistance it has to overcome. **Additional resistors always decrease the resistance of a parallel circuit.**

This also makes sense from the perspective of our equation for determining resistance: $R = \rho L/A$. In this case, you can think of the resistors as being lengths of wire that lie side by side. Adding more resistors in parallel is just like making the wire wider, increasing its diameter. As can be seen from the above formula, a wire with more cross-sectional area would have proportionally lower resistance.

One important way to check your results with parallel circuits is to recognize that the equivalent resistance of the circuit must always be **less than** the smallest resistance in the circuit. That is because each additional resistor reduces the resistance of the circuit from that initial starting point.

Example 10: In the circuit to the right, V= 12V; $R_1 = 4\Omega$; and $R_2 = 2\ \Omega$. First determine R_{eq}; I; I_1; and I_2.

Then determine the power consumption of the circuit, P, and of each of the two resistors.



$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2} + \ldots\ldots$

$\frac{1}{R_{eq}} = \frac{1}{4\Omega} + \frac{1}{2\Omega}$

Use the common denominator of 4Ω

$\frac{1}{R_{eq}} = \frac{1}{4\Omega} + \frac{2}{4\Omega}$

$\frac{1}{R_{eq}} = \frac{3}{4\Omega}$

Remember to solve for R_{eq} not $1/R_{eq}$: **a common mistake is to not invert the preliminary answer**

$R_{eq} = \frac{4\Omega}{3}$

$R_{eq} = 1.33\Omega$

Note that this answer is less than the smallest of the resistors, the 2Ω resistor.

$I = \frac{V}{R}$

For the whole circuit V is the voltage of the battery and R_{eq} is its resistance

$I = \frac{V}{R_{eq}}$

$I = \frac{12V}{1.33\Omega}$

$\boldsymbol{I = 9A}$

$I_1 = \frac{V_1}{R_1}$

In a parallel circuit, $\boldsymbol{V = V_1 + V_2 + V_3 + \cdots}$

$I_1 = \frac{V_1}{R_1}$

$I_1 = (12V)(4\Omega)$

$\boldsymbol{I_1 = 3A}$

$I_2 = \frac{V_2}{R_2}$

In a parallel circuit, $\boldsymbol{V = V_1 + V_2 + V_3 + \cdots}$

$I_2 = \frac{V}{R_2}$

$I_2 = (12V)(2\Omega)$

$\boldsymbol{I_2 = 6A}$

Note that these results are consistent with $\boldsymbol{I = I_1 = I_2 = I_3}$ **....** since$9A = 6A + 3A$. This is a good way to check your work.

We have three different equations that we can use to find the power: $P = IV = \frac{V^2}{R} = I^2R$

For the entire circuit, we use V to be the battery voltage and R to be $R_{eq.}$

$P = IV$	$P = \frac{V^2}{R_{eq}}$	$P = I^2R_{eq}$
$P = (9A)(12V)$	$P = \frac{12V^2}{1.33\Omega}$	$P = (9A)^2(1.33\Omega)$
$P = 108W$	$P = 108W$	$P = 108W$

To find the power used by R_1, we use V_1, R_1, and I.

$P = I_1V$	$P = \frac{V_1^2}{R_1}$	$P = I_1^2R^1$
$P = (3A)(12V)$	$P = \frac{(12V)^2}{4\Omega}$	$P = (3A)^2(4\Omega)$
$P = 36W$	$P = 36W$	$P = 36W$

To find the power used by R_2, we use V_2, R_2, and I.

$P = I_2V$	$P = \frac{V^2}{R_2}$	$P = I_2^2R_2$
$P = (6A)(12V)$	$P = \frac{(12V)^2}{2\Omega}$	$P = (6A)^2(2\Omega)$
$P = 72W$	$P = 72W$	$P = 72W$

Note that not only are the results for power consumption equally valid with each of the three equations, but that our results show that $P = P_1 + P_2 + \ldots$ This makes sense since the power consumed by the total circuit must be consumed somewhere in the circuit. The only place that can occur is within the resistors.

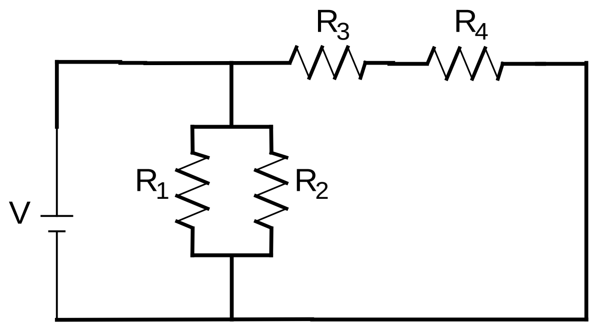
In this example, as in the previous one, we showed much more work than there would be needed in solving a problem in that we used all three equations for power and solved for each resistor and the total circuit independently. However, using a second method is a good way of checking your work.

Combination Circuits

Most circuits are made up of a combination of branches, each of those branches being either a set of components connected in series, parallel or a combination of the two. We don't have to develop any additional formulas to analyze these circuits; we just have to understand how to simplify each circuit step by step.

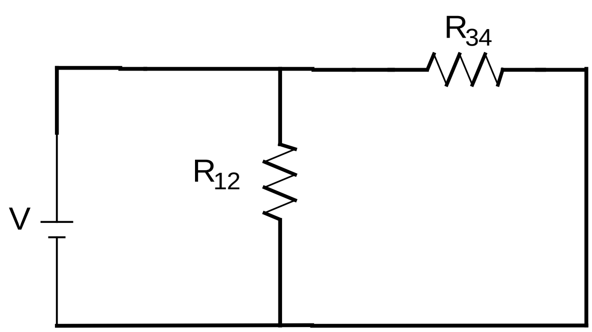
The below circuit consists of two branches. The branch on the left is a parallel circuit; charges have a choice as to how to proceed through that branch; they can pass through R_1 or R_2. The branch on the right is a series circuit, once a charge enters that branch and passes through R_3, it must also pass through R_4.

However, the branches themselves are parallel to one another. A charge may either go through the left branch, through either R_1 or R_2, or through the right branch, going through R_3 and R_4, but not through both branches. So these two branches are connected in parallel. The way to see that is to simplify the circuit step by step, which we will do in the following example.



Example 11: In the above diagram V = 24V; R_1 = 6Ω; R_2 = 12Ω; R_3 = 3Ω; and R_4 = 7Ω.
Determine R_{eq} and I.

Our first step will be to combine R_1 and R_2 in order to determine R_{12}, the equivalent resistance of that branch. We will then combine R_3 and R_4, to determine R_{34}, the equivalent resistance of the second branch. These subscripts will allow us to keep track of our work. They should be read as "R one two" and "R three four" not "R twelve" and R "thirty four". Our redrawn circuit will look like this:



Since R_1 and R_2 are in parallel we use the formula $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ to determine the value of R_{12}.

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{12}} = \frac{1}{6\Omega} + \frac{1}{12\Omega}$$

Use a common denominator of 12Ω

$$\frac{1}{R_{12}} = \frac{2}{12\Omega} + \frac{1}{12\Omega}$$

$$\frac{1}{R_{12}} = \frac{3}{12\Omega}$$

Invert our preliminary answer to find R_{12}, not $\frac{1}{R_{12}}$

$R_{12} = \frac{12\Omega}{3}$

$R_{12} = 4\Omega$

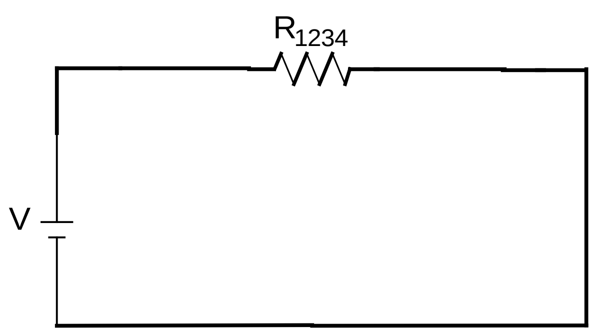
Since R_3 and R_4 are in series we use the formula $R_{eq} = R_3 + R_4$ to determine the value of R_{34}.

$R_{34} = R_3 + R_4$

$R_{34} = 3\Omega + 7\Omega$

$R_{34} = 10\Omega$

So we now have the above circuit and we know the values of the resistors: $R_{12} = 4\Omega$ and $R_{34} = 10\Omega$. The next step is to combine them into a circuit that looks like this:



While R_{12} and R_{34} are not geometrically parallel to one another (they are in fact perpendicular) that is not the meaning of parallel in the context of electrical circuits. In circuit analysis, parallel means that charges can go through one or the other but not both. That is clearly the case here, so R_{12} is parallel to R_{34}. That being the case, we combine them using the formula: $\frac{1}{R_{eq}} = \frac{1}{R_{12}} + \frac{1}{R_{34}}$

$\frac{1}{R_{1234}} = \frac{1}{R_{12}} + \frac{1}{R_{34}}$

$\frac{1}{R_{1234}} = \frac{1}{4\Omega} + \frac{1}{10\Omega}$

Let's do this directly, without using a common denominator

$\frac{1}{R_{1234}} = 0.25\Omega + 0.10\Omega$

Note that the units are then Ω^{-1}, $1/\Omega$, not Ω

$\frac{1}{R_{1234}} = 0.35\Omega$

Inverting to get the final answer

$R_{1234} = 2.9\,\Omega$

Since this combines all the resistors, it is R_{eq} which equals: **2.9 Ω**

This means that from the perspective of the battery, the four resistors in the original circuit could all be replaced by one 2.9 Ω resistor, and nothing would change. Of course, it would be a different circuit, but not in terms of the current from the battery or the total power consumed.

We can now find both of those. Ohm's law will give us the current leaving the battery:

$$I_1 = \frac{V}{R}$$

$$I_1 = \frac{24V}{2.9\Omega}$$

$$\boldsymbol{I_1 = 8.3A}$$

The power supplied by the battery can be obtained a number of ways including:

$P = IV$

$P = (8.3A)(24V)$

$P = 200\ W$

While this approach allowed us to determine the total current, resistance and power consumption of the circuit, it also allows us to determine what is going on with any of the individual components.

Example 12: What is the current through, voltage drop across and power consumption of R_1 and R_3 in the Example 11?

First, let's analyze R_1, as this is the simpler of the two. The voltage V_1 can be seen directly from the original diagram, R_1 is connected directly across the battery. So that is the voltage drop across it.

$\boldsymbol{V_1 = 24V}$

We can obtain the current passing through it directly by using Ohm's law. The other elements of the circuit have not affect on this result. Once you know the voltage across a resistor and its resistance, you can directly determine the current passing through it.

$I_1 = \frac{V}{R}$

$I_1 = \frac{24V}{6\Omega}$

$\boldsymbol{I_1 = 4A}$

Now that we have the current, resistance and voltage across R_1, we can find its power consumption any of the three ways.

$P_1 = I_1V_1$

$P_1 = (4A)(24V)$

$P_1 = 96W$

Now let's answer those same questions for R_3. This requires a little more thought since R_3 is not connected directly across the battery; one side is connected to the battery but the other side is connected to R_4. Rather than going back to the original diagram, we're better off going to the second diagram and find the current passing through R_{34} since we know that'll be the same current passing through R_3; the current is the same for all resistors in a series branch of a circuit.

In the second diagram, R_{34} is connected across the battery, so the voltage across it is 24V; V_{34}=24V. Additionally, we found that R_{34} has a resistance of 10 Ω; together these two facts let us determine I_{34}; which equals I_3.

$I_{34} = \frac{V_{34}}{R_{34}}$

$I_{34} = \frac{24V}{10\Omega}$

$I_{34} = 2.4A$

$\boldsymbol{I_3 = 2.4A}$

Once we have the current through R3, and we already knew its resistance, we can use Ohm's law to find the voltage across it and then any of the three power equations to find the power used by it.

$V_3=I_3R_3$

$V_3=(2.4A)(3\Omega)$

Note that to find the current in the branch we needed the total resistance of the branch, R_{34}, but to find the voltage drop across R3, we only use its resistance.

$\mathbf{V_3=7.2V}$

Just as a check we can also find V4 easily enough. If we're correct $V_3 + V_4 = V$. The current through R_4 is the same as that through R_3, but its resistance is 7Ω.

$V_4=I_4R_4$

$V_4=(2.4A)(7\Omega)$

$V_4=16.8V$

Since 7.2V + 16.8V = 24V, this confirms our previous result

Finally, we can determine the power consumed by that R_3 by using any of the three equations to determine power.

$$P_3 = \frac{{V_3}^2}{R_3}$$

$$P_3 = \frac{(7.2v)^2}{3\Omega}$$

Note that we use V_3 and R_3, not V_{34} or R_{34}.

$P_3 = 17W$

This approach can be used to find the detailed information for any component in a circuit. It's important when doing these problems to keep all your intermediate diagrams and results legible, as you will need to use those in different ways depending on the question you are answering.

Ammeters and Voltmeters

When analyzing an actual circuit it is necessary to be able to measure the voltage across different parts of the circuit as well as the current passing through different branches. Voltages are measured using voltmeters; currents are measured using ammeters.

Voltage is only meaningful when the difference in voltage between two locations in the circuit is being measured. Only voltage differences have physical meaning. In order to measure voltage, the circuit must be kept intact and the probes of the voltmeter are connected by wires to the two locations being compared. **A voltmeter is always connected in parallel to the part of the circuit being measured.** The ideal voltmeter has a very high resistance so, while it is an additional branch to the circuit, very little current passes through it; it does not affect the performance of the circuit. But since it is in parallel with the part of the circuit being analyzed, the voltage across the voltmeter will be identical to that part of the circuit; parallel branches always have the same voltage across them.

Current is measured by the flow of charge passing through a point in the circuit. In order to measure that, the ammeter must be put in series with the circuit at that location. That is because in a series connection, all the components have the same current passing through them. By putting the ammeter in series with that part of the circuit, it will read the same current as is passing through the other components in that branch.

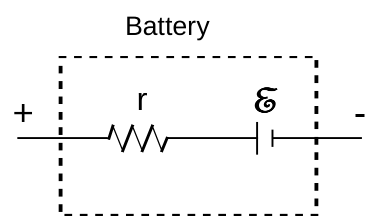
The ideal ammeter has very low resistance. By having such a low resistance, the insertion of the ammeter into the circuit will not increase the resistance of the branch and will not change the current being measured.

There is a key difference between the process of connecting a voltmeter and an ammeter to a portion of a circuit. Since the voltmeter is connected in parallel to the existing components, those components do not have to be disconnected to make the measurement. On the other hand, the only possible way to put an ammeter in series with a circuit is to disconnect a component from the circuit and then add the ammeter as an additional series component.

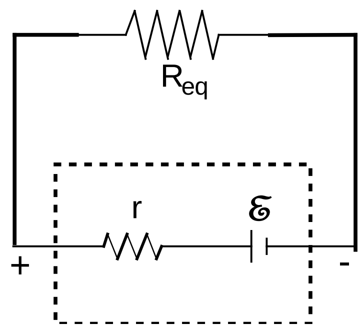
Electromotive Force and Terminal Voltage

When we described how a battery works at the beginning of this chapter we made one simplification that we will now correct. We described that a chemical reaction carries the charge from the negative to the positive terminal, but we did not address the fact that there is a resistance associated with that process as well. We need to think of a battery as a combination of a voltage source and a resistor. Till now, we have neglected that internal resistance.

In the below diagram, the dotted rectangular box represents a battery; its terminals are labeled "+" and "-". Those are the terminals to which we have been connecting our circuits; those are the only locations that we have access to in a real battery. However, we can look inside the battery to see that it consists of two parts: an **internal resistance** and a source of **electromotive force**. Electromotive force, or EMF, is the process that carries charge from a low to a high voltage; in a battery it is a result of a chemical process; in a solar cell it the result of light interacting with a semiconductor; in a generator it is the result of electromagnetic induction. The internal resistance of the battery is indicated by the resistor symbol and the label "r" and its electromotive force is indicated by the battery symbol and the label "$\mathcal{E}$".



The **terminal voltage** (V_T) of a battery is the voltage that is measured when a voltmeter is connected between its terminals. Up until now we have presumed that the terminal voltage of a battery is constant, it does not depend on the circuit to which it is connected. This is correct if the internal resistance of the battery is zero. However, if the internal resistance is not zero, which it never is with actual batteries, then the terminal voltage will vary depending on the circuit connected to the battery. That can be seen by connecting the battery to a circuit whose equivalent resistance is given by R_{eq}, as shown in the below diagram.



Once current begins to flow through this circuit there will be a voltage drop across the internal resistance “r”. From Ohm’s law we can see that that voltage drop will be equal to Ir. When a voltmeter is placed across the terminals, it will not read , it will read ε_0 – Ir.

$$\mathbf{V_T = \varepsilon_0 - Ir}$$

Let’s look at two extreme cases: the case where no external circuit is connected and the case where the terminals are connected together by a shorting wire: a wire with essentially no resistance. If there is no external circuit connected to the battery, then no current will flow; the voltage drop across the internal resistance (Ir) will equal zero. In that case, $V_T = \varepsilon_0$; the EMF can be read directly from the terminals. This is the maximum terminal voltage that can be obtained, and occur when the current equals zero.

On the other hand, if the two terminals are connected together with a wire whose resistance is effectively zero, the only resistance in the circuit will be the internal resistance of the battery. In that case, the current through the battery will be given by $I = \frac{\varepsilon_0}{R}$ This is the maximum current that the battery can supply, and the voltage across the terminals will be zero.

In intermediate cases between these two extremes, you can just treat the internal resistance as one more resistor in the circuit. That will allow you to make predictions about current, power, voltage, etc.

A very important practical application of this understanding becomes clear when the additional fact is added that as batteries age and eventually die, they do so because of an increase in their internal resistance, not because of a decline in their EMF. This means that if you check a nearly dead car battery while it is not connected to a car, it will almost always read the same voltage as it did when it was made; the EMF of the chemical reaction driving it. However, if you try to use it to start your car, it’s terminal voltage will drop dramatically and it won’t work. It is useless, but testing it without first connecting it to an external resistor would fail to show that. When checking a car battery, it must be connected to an external resistor.

Example 13: You have a 12 volt battery, with an internal resistance of 0.5Ω, and a variable resistor that can be adjusted continuously from zero to 10 Ω. Determine the terminal voltage of the battery and the current through the circuit when the external resistor is not connected to the battery and when it is connected and adjusted to 10Ω, 1Ω and 0Ω.

Case 1: No external circuit

This first case is the simplest. Without the external resistor connected, no current will flow, and the terminal voltage will be its maximum, equal to the EMF of the battery.

$\boldsymbol{I = 0}$

$V_T = \varepsilon_0 - Ir$

$V_T = 12V - 0$

$\boldsymbol{V_T = 12V}$

Case 2: External resistor connected and set to 10Ω

In this intermediate case, we must determine the total resistance by combining the internal and external resistances.

$R_T = R_{ext} + R_{int}$

$R_T = 10\Omega + 0.5\Omega$

$R_T = 10.5\Omega$

$I = \frac{V}{R}$

$I = \frac{12v}{10.5\Omega}$

$\boldsymbol{I = 1.14A}$

$V_T = \varepsilon_0 - Ir$

$V_T = 12V - (1.1A)(0.5\Omega)$

$V_T = 12V - 0.55V$

$V_T = 11.4V$

Case 3: External resistor connected and set to 1Ω

Once again, we must determine the total resistance by combining the internal and external resistances.

$R_T = R_{ext} + R_{int}$

$R_T = 1\Omega + 0.5\Omega$

$R_T = 1.5\Omega$

$I = \frac{V}{R}$

$I = \frac{12v}{1.5\Omega}$

$\boldsymbol{I = 8.0A}$

$V_T = \varepsilon_0 - Ir$

$V_T = 12v - (8A)(.5\Omega)$

$V_T = 12v - 4v$

$\boldsymbol{V_T = 8.0V}$

Case 4: External resistor connected and set to 0 ohms

This is the next simplest case. The voltage will be zero, since the wire is shorting the terminals together. The current will be at a maximum and limited only by the internal resistance of the battery, 0.5Ω.

$I = \frac{V}{R}$

$I = \frac{12v}{.5\Omega}$

$\boldsymbol{I = 24A}$

$V_T = 0$

Note that as the externally applied resistance declines in value the current raises to its maximum and the terminal voltage decline from its maximum to zero.

