



## **New Jersey Center for Teaching and Learning**

### **Progressive Science Initiative**

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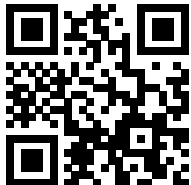
# Momentum

[www.njctl.org](http://www.njctl.org)

# Momentum

*Click on the topic to go to that section*

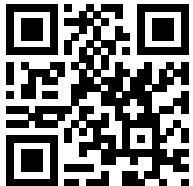
- **Momentum**
- **Impulse**
- **Momentum of a System of Objects**
- **Conservation of Momentum**
- **Inelastic Collisions and Explosions**
- **Elastic Collisions**



<http://njc.tl/ko>

# Momentum

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<http://njc.tl/kp>

# Momentum Defined

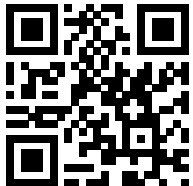
Newton's First Law tells us that – objects remain in motion with a constant velocity unless acted upon by a force.

In our experience:

- When two objects of different masses travel with the same velocity, the one with more mass is harder to stop.
- When objects of the equal masses travel with different speeds, the faster one is harder to stop.
- Define a new quantity, **momentum** ( $p$ ), that takes these observations into account:

$$\text{momentum} = \text{mass} \times \text{velocity}$$

$$p = mv$$



<http://njc.tl/kp>

click here for a introductory video on  
momentum from Bill Nye!

# Momentum is a Vector Quantity

Recall that:

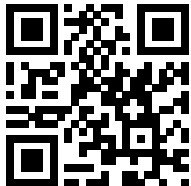
mass is a scalar quantity  
and  
velocity is a vector quantity

Since:

$$\textit{momentum} = \textit{mass} \times \textit{velocity}$$

momentum must be a vector quantity

$$\vec{p} = m\vec{v}$$

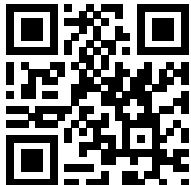


# SI Unit for Momentum

There no specially named unit for momentum.  
We just use the product of the units of mass and velocity...

**mass x velocity**


$$\text{kg}\cdot\text{m/s}$$

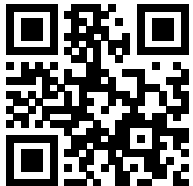


<http://njc.tl/kp>

1 Which has more momentum?

- ☐ A A large truck moving at 30 m/s
- ☐ B A small car moving at 30 m/s
- ☐ C Both have the same momentum.

**Answer**



<http://njc.tl/kq>



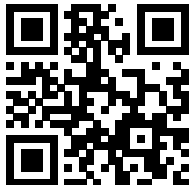
1 Which has more momentum?

- ☒ A A large truck moving at 30 m/s
- ☐ B A small car moving at 30 m/s
- ☐ C Both have the same momentum

**Answer**

**A**

Both have the same speed, but the truck has the larger mass



<http://njc.tl/kq>

2 What is the momentum of a 20 kg object with a velocity of +5.0 m/s?

**Answer**



<http://njc.tl/kr>

- 2 What is the momentum of a 20 kg object with a velocity of +5.0 m/s?

**Answer**

$$m = 20 \text{ kg}$$

$$v = 5 \text{ m/s}$$

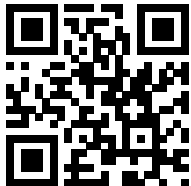
$$p = mv = (20 \text{ kg})(5 \text{ m/s})$$

$$= 100 \text{ kgm/s}$$



- 3 What is the momentum of a 20kg object with a velocity of  $-5.0\text{m/s}$ ?

**Answer**



<http://njc.tl/ks>

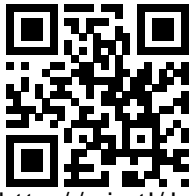
- 3 What is the momentum of a 20kg object with a velocity of  $-5.0\text{m/s}$ ?

**Answer**

$$m = 20 \text{ kg}$$

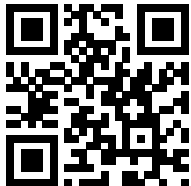
$$v = -5 \text{ m/s}$$

$$\begin{aligned} p &= mv = (20 \text{ kg})(-5 \text{ m/s}) \\ &= -100 \text{ kgm/s} \end{aligned}$$



- 4 What is the velocity of a 5.0kg object whose momentum is  $-15.0 \text{ kg}\cdot\text{m/s}$ ?

**Answer**



<http://njc.tl/kt>

- 4 What is the velocity of a 5.0kg object whose momentum is  $-15.0 \text{ kg}\cancel{\text{m}}/\text{s}$ ?

**Answer**

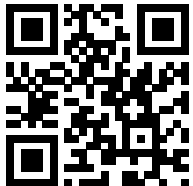
$$m = 5 \text{ kg}$$

$$p = -15 \text{ kg}\cancel{\text{m}}/\text{s}$$

$$p = mv$$

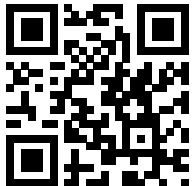
divide both sides by  $m$

$$v = p/m = (-15 \text{ kg}\cancel{\text{m}}/\text{s})/(\cancel{\text{m}})$$



- 5 What is the mass of an object whose momentum is  $35 \text{ kg}\cdot\text{m/s}$  when its velocity is  $7.0 \text{ m/s}$ ?

**Answer**



<http://njc.tl/ku>



- 5 What is the mass of an object whose momentum is 35 kg·m/s when its velocity is 7.0 m/s?

**Answer**

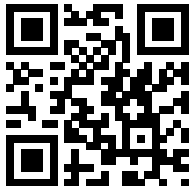
$$v = 7 \text{ m/s}$$

$$p = 35 \text{ kgm/s}$$

$$p = mv$$

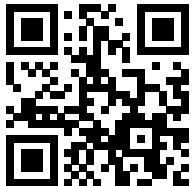
divide both sides by  $v$

$$m = p/v = (35 \text{ kgm/s})/(7 \text{ m/s}) \\ = 5$$



# Momentum Change & Impulse

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<http://njc.tl/kv>

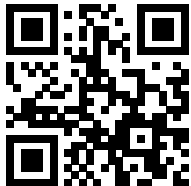
# Change in Momentum

Suppose that there is an event that changes an object's momentum.

- from  $p_0$  - the initial momentum (just before the event)
- by  $\Delta p$  - the change in momentum
- to  $p_f$  - the final momentum (just after the event)

The equation for momentum change is:

$$p_0 + \Delta p = p_f$$



# Momentum Change = Impulse

*Momentum change equation:*

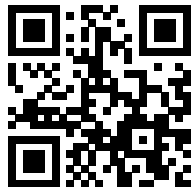
**Newton's First Law** tells us that the velocity (and so the momentum) of an object won't change unless the object is affected by an external force.

When an outside force ***F*** acts on the object for a time ***Δt***, it delivers an ***impulse I*** to the object that ***changes its momentum***:

$$p_0 + \Delta p = p_f$$

$$p_0 + I = p_f$$

$$I = F \Delta t$$

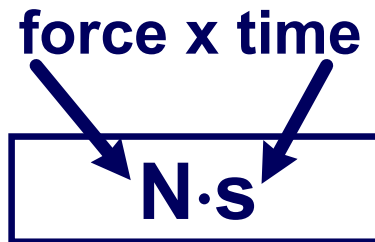


<http://njc.tl/kv>

the ***impulse*** is:

# SI Unit for Impulse

There no specially named unit for impulse.  
We just use the product of the units of force and time...

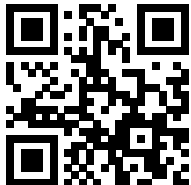


Recall that  $N = \text{kg} \cdot \text{m} / \text{s}^2$ , so

$$\mathbf{N \cdot s = kg \cdot m / \cancel{s^2} \times \cancel{s}}$$

$$= \boxed{\mathbf{kg \cdot m / s}}$$

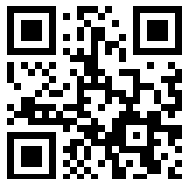
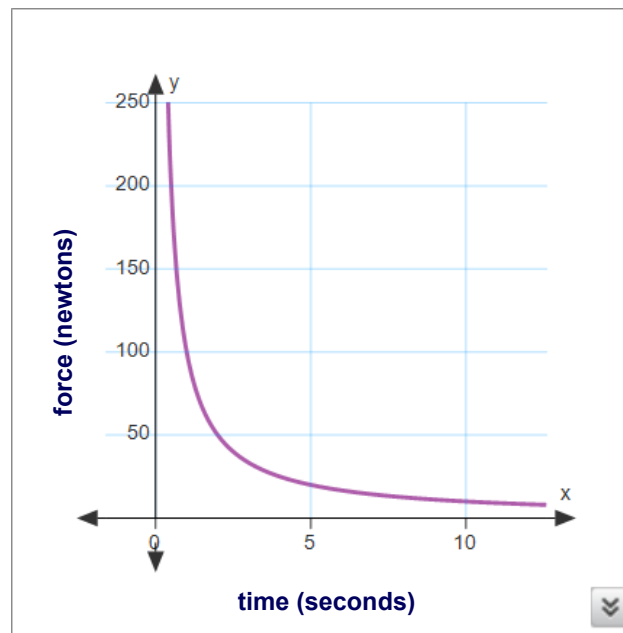
- the same as momentum!



<http://njc.tl/kv>

# Effect of Collision Time on Force

$$\text{Impulse} = F_t = F \Delta t \text{ change in momentum}$$



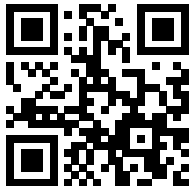
<http://njc.tl/kv>

Changing the duration ( $t$ ) of an impulse by a small amount can greatly reduce the force on an object

# Real World Applications

$$\text{Impulse} = \mathbf{F}_t = F \mathbf{t} \text{ change in momentum}$$

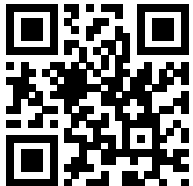
- Car Design / Accidents
  - > airbags
  - > collisions head-on vs walls
  - > crush zones
- Jumping / Landing
- Boxing / Martial Arts
- Hitting Balls - Golf, Baseball...
- Catching Balls



<http://njc.tl/kv>

- 6 An external force of 25N acts on a system for 10s. How big is the impulse delivered to the system?

**Answer**



<http://njc.tl/kw>



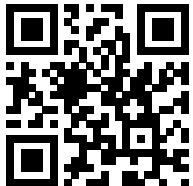
- 6 An external force of 25N acts on a system for 10s. How big is the impulse delivered to the system?

**Answer**

$$F = 25 \text{ N}$$

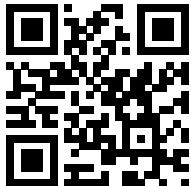
$$\Delta t = 10 \text{ s}$$

$$I = F \Delta t$$



- 7 In the previous problem, an external force of 25N acted on a system for 10s. We found that the impulse delivered was 250 N-s. What is the magnitude of the change in momentum of the system?

**Answer**



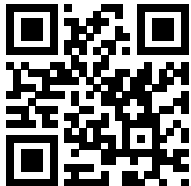
<http://njc.tl/kx>

- 7 In the previous problem, an external force of 25N acted on a system for 10s. We found that the impulse delivered was 250 N-s. What is the magnitude of the change in momentum of the system?

**Answer**

$$I =$$

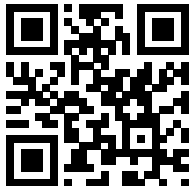
$$I = \Delta p$$



8 The momentum change of an object is equal to the \_\_\_\_\_.

- ☐ A force acting on it
- ☐ B impulse acting on it
- ☐ C velocity change of the object
- ☐ D object's mass times the force acting on it

**Answer**

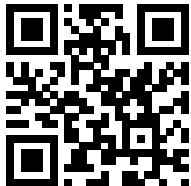


<http://njc.tl/ky>

8 The momentum change of an object is equal to the \_\_\_\_\_.

- ☐ A force acting on it
- ☐ B impulse acting on it
- ☐ C velocity change of the
- ☐ D object's mass times t

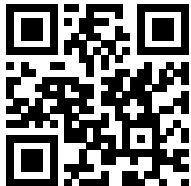
**Answer**



9 Air bags are use in cars because they:

- ☐ A increase the force with which you hit the dashboard
- ☐ B increase the duration (time) of impact in a collision
- ☐ C decrease the momentum of a collision
- ☐ D decrease the impulse in a collision

**Answer**



<http://njc.tl/kz>

9 Air bags are use in cars because they:

- ☐ A increase the force with which you hit the dashboard
- ☐ B increase the duration (time) of impact in a collision
- ☐ C decrease the mo
- ☐ D decrease the i

**Answer**

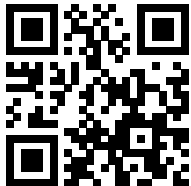
By increasing the amount of time during the collision, the force required to reduce the passenger's momentum is reduced. This in turn reduces or prevents injury.



10 One car crashes into a concrete barrier. Another car crashes into a collapsible barrier at the same speed. What is the difference between the 2 crashes?

- ☐ A change in momentum
- ☐ B force on the car
- ☐ C impact time
- ☐ D both B & C are true

**Answer**



<http://njc.tl/10>

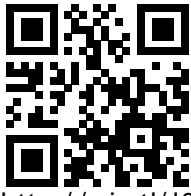


10 One car crashes into a concrete barrier. Another car crashes into a collapsible barrier at the same speed. What is the difference between the 2 crashes?

- ☐ A change in momentum
- ☐ B force on the car
- ☐ C impact time
- ☐ D both B & C are

**Answer**

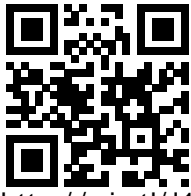
Whether the wall is padded or not, the car experiences the same change in momentum, and so the same impulse. The only difference is that the impact time has increased (which in turn reduces the force experienced by the car).



11 In order to increase the final momentum of a golf ball, we could:

- ☐ A not change the speed of the golf club after the collision
- ☐ B increase the force acting on it
- ☐ C increase the time of contact between the club and ball
- ☐ D all of the above

**Answer**



<http://njc.tl/l1>

11 In order to increase the final momentum of a golf ball, we could:

- ☐ A not change the speed of the golf club after the collision
- ☐ B increase the force acting on it
- ☐ C increase the time of contact between the club and ball
- ☐ D all of the above

**Answer**

**D**

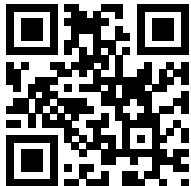
Following through, hitting the golf ball harder, and/or increasing the impact time will all result in an increase in the final momentum of the golf ball.



<http://njc.tl/l1>

- 12 An external force acts on an object for 0.0020 s. During that time the object's momentum increases by 400 kg-m/s. What was the magnitude of the force?

**Answer**



<http://njc.tl/l2>

- 12 An external force acts on an object for 0.0020 s. During that time the object's momentum increases by 400 kg·m/s. What was the magnitude of the force?

**Answer**

$$\Delta t = 0.002 \text{ s}$$

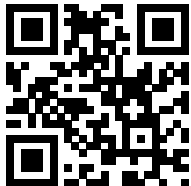
$$\Delta p = 400 \text{ kg}\cdot\text{m/s}$$

$$I = F\Delta t = \Delta p$$

$$F = \Delta p / \Delta t$$

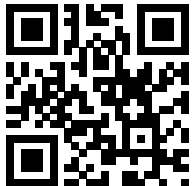
$$= (400 \text{ kg}\cdot\text{m/s}) / (0.002 \text{ s})$$

$$= 200,000 \text{ N}$$



\* 13 A 50,000 N force acts for 0.030 s on a 2.5 kg object that was initially at rest. What is its final velocity?

**Answer**



<http://njc.tl/ls>

- \* 13 A 50,000 N force acts for 0.030 s on a 2.5 kg object that was initially at rest. What is its final velocity?

**Answer**

$$F = 50,000 \text{ N}$$

$$\Delta t = 0.03 \text{ s}$$

$$m = 2.5 \text{ kg}$$

$$v_0 = 0$$

$$F\Delta t = \Delta p ; \Delta p = m\Delta v$$

$$F\Delta t = m\Delta v ; \Delta v = (v_f - v_0)$$

$$F\Delta t = m(v_f - v_0) = mv_f \text{ (since } v_0 = 0)$$

$$v_f = (F/m) \Delta t$$

$$= (50,000 \text{ N}) / (2.5 \text{ kg}) \times (0.03 \text{ s}) = 600 \text{ m/s}$$



# The Momentum of a System of Objects

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<http://njc.tl/l3>



# The Momentum of a System of Objects

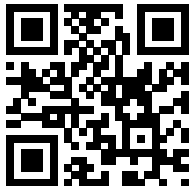
If a system contains more than one object, its total momentum is the vector sum of the momenta of those objects.

$$\mathbf{p}_{\text{system}} = \sum \mathbf{p}$$

$$\mathbf{p}_{\text{system}} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \dots$$

$$\mathbf{p}_{\text{system}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 + \dots$$

*It's critically important to note that  
momenta add as vectors,  
not as scalars.*



# The Momentum of a System of Objects

$$p_{\text{system}} = m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots$$

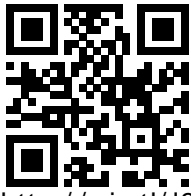
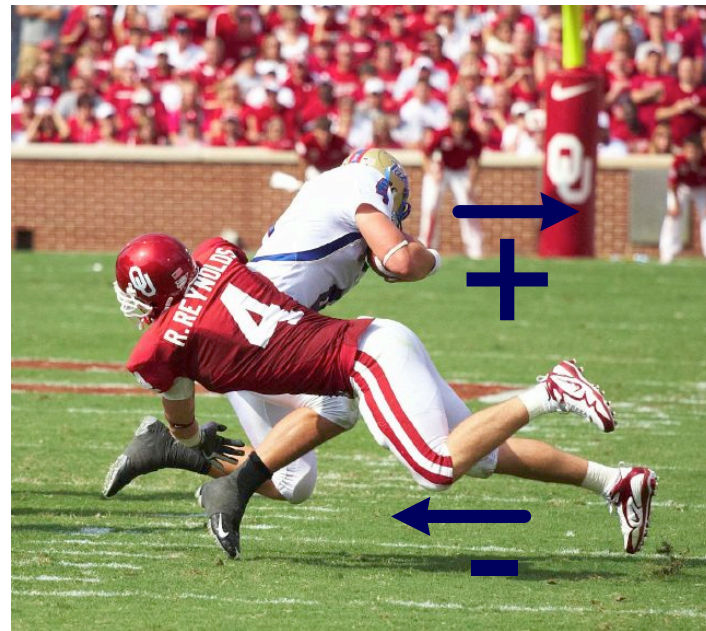
In order to determine the total momentum of a system,

## First:

- Determine a direction to be considered positive
- Assign positive values to momenta in that direction
- Assign negative values to momenta in the opposite direction

## Then:

• momenta to get a total.



<http://njc.tl/I3>

## Example

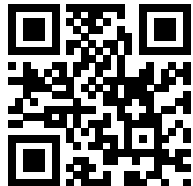
Determine the momentum of a system of two objects:  $m_1$ , has a mass of 6 kg and a velocity of 13 m/s towards the east and  $m_2$ , has a mass of 14 kg and a velocity of 7 m/s towards the west.

*(Choose east as positive)*

$$m_1 = 6 \text{ kg}$$
$$v_1 = 13 \text{ m/s}$$

$$m_2 = 14 \text{ kg}$$
$$v_2 = -7 \text{ m/s}$$

$$p_{\text{system}} = p_1 + p_2$$



$$= m_1 v_1 + m_2 v_2$$

<http://njc.tl/l3>

Answer

## Example

Determine the momentum of a system of two objects:  $m_1$ , has a mass of 6 kg and a velocity of 13 m/s towards the east and  $m_2$ , has a mass of 14 kg and a velocity of 7 m/s towards the west.

*(Choose east as positive)*

$$m_1 = 6 \text{ kg}$$

$$v_1 = 13 \text{ m/s}$$

$$m_2 = 14 \text{ kg}$$

$$v_2 = -7 \text{ m/s}$$

$$p_{\text{system}} = p_1 + p_2$$



<http://njc.tl/l3>

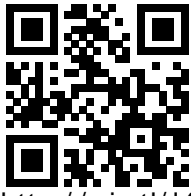
**Answer**

$$\begin{aligned} p_{\text{system}} &= m_1 v_1 + m_2 v_2 \\ &= (6 \text{ kg})(13 \text{ m/s}) + (14 \text{ kg})(-7 \text{ m/s}) \\ &= (78 \text{ kg}\cdot\text{m/s}) + (-98 \text{ kg}\cdot\text{m/s}) \\ &= -20 \text{ kg}\cdot\text{m/s} \end{aligned}$$

- 14 Determine the magnitude of the momentum of a system of two objects:  $m_1$ , has a mass of 6.0kg and a velocity of 20m/s north and  $m_2$ , has a mass of 3kg and a velocity 20m/s south.



**Answer**



<http://njc.tl/l4>

- 14 Determine the magnitude of the momentum of a system of two objects:  $m_1$ , has a mass of 6.0kg and a velocity of 20m/s north and  $m_2$ , has a mass of 3kg and a velocity 20m/s south.



$$m_1 = 6 \text{ kg}$$

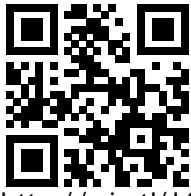
$$v_1 = +20 \text{ m/s (North)}$$

$$m_2 = 3 \text{ kg}$$

$$v_2 = -20 \text{ m/s (South)}$$

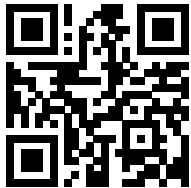
$$\begin{aligned} p_{\text{system}} &= m_1 v_1 + m_2 v_2 \\ &= (6 \text{ kg})(20 \text{ m/s}) + (3 \text{ kg})(-20 \text{ m/s}) \\ &= (120 \text{ kg}\cdot\text{m/s}) + (-60 \text{ kg}\cdot\text{m/s}) \\ &= 60 \text{ kg}\cdot\text{m/s (magnitude)} \end{aligned}$$

direction is North



- 15 Determine the momentum of a system of two objects: the first has a mass of 8 kg and a velocity of 8 m/s to the east while the second has a mass of 5 kg and a velocity of 15 m/s to the west.

**Answer**



<http://njc.tl/I5>

- 15 Determine the momentum of a system of two objects: the first has a mass of 8 kg and a velocity of 8 m/s to the east while the second has a mass of 5 kg and a velocity of 15 m/s to the west.

**Answer**

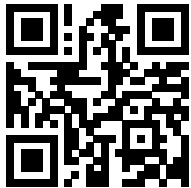
$$m_1 = 8 \text{ kg}$$

$$v_1 = +8 \text{ m/s (East)}$$

$$m_2 = 5 \text{ kg}$$

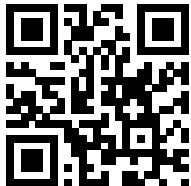
$$v_2 = -15 \text{ m/s (West)}$$

$$\begin{aligned} p_{\text{system}} &= m_1 v_1 + m_2 v_2 \\ &= (8 \text{ kg})(8 \text{ m/s}) + (5 \text{ kg})(-15 \text{ m/s}) \\ &= (64 \text{ kg}\cdot\text{m/s}) + (-75 \text{ kg}\cdot\text{m/s}) \\ &= -11 \text{ kg}\cdot\text{m/s (West)} \end{aligned}$$





- 16 Determine the momentum of a system of 3 objects:  
The first has a mass of 7.0 kg and a velocity of 23 m/s north; the second has a mass of 9.0 kg and a velocity of 7 m/s north; and the third has a mass of 5.0 kg and a velocity of 42 m/s south.

**Answer**<http://njc.tl/l6>

- 16 Determine the momentum of a system of 3 objects:  
The first has a mass of 7.0 kg and a velocity of 23 m/s north; the second has a mass of 9.0 kg and a velocity of 7 m/s north; and the third has a mass of 5.0 kg and a velocity of 42 m/s south.

**Answer**

$$m_1 = 7 \text{ kg}$$

$$v_1 = +23 \text{ m/s (North)}$$

$$m_2 = 9 \text{ kg}$$

$$v_2 = 7 \text{ m/s (North)}$$

$$m_3 = 5 \text{ kg}$$

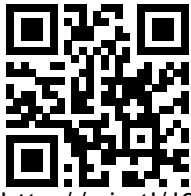
$$v_3 = -42 \text{ m/s (South)}$$

$$p_{\text{system}} = m_1 v_1 + m_2 v_2 + m_3 v_3$$

$$= (7\text{kg})(23\text{m/s}) + (9\text{kg})(7\text{m/s}) + (5 \text{ kg})(-42\text{m/s})$$

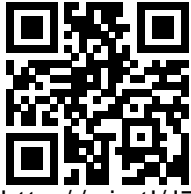
$$= (161 \text{ kg}\cdot\text{m/s}) + (63 \text{ kg}\cdot\text{m/s}) + (-210 \text{ kg}\cdot\text{m/s})$$

$$= 14 \text{ kg}\cdot\text{m/s (North)}$$



# Conservation of Momentum

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<http://njc.tl/l7>

# Conservation Laws

Some of the most powerful concepts in science are called "conservation laws".

Conservation laws:

- apply to closed systems - where the objects only interact with each other and nothing else.
- enable us to solve problems without worrying about the details of an event.



<http://njc.tl/l7>

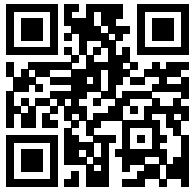
# Momentum is Conserved

In the last unit we learned that energy is conserved.

Like energy, momentum is a conserved property of nature. It is not created or destroyed;

So in a closed system we will always have the same amount of momentum.

*The only way the momentum of a system can change is if momentum is added or taken away by an outside force.*



<http://njc.tl/l7>

# Conservation of Momentum

To apply Conservation of Momentum,  
Take snapshots of a system just before and after an event.  
By comparing these two snapshots we can learn a lot.

We'll explore this more a little later.



<http://njc.tl/17>

# Conservation of Momentum and Impulse

Recall from our discussion of change of momentum and impulse:

$$p_0 + I = p_f$$

When a net external force acts on an object, it imparts an impulse  $I$  to the object, changing its momentum.

This is exactly the same for a system of objects.

$$p_{system,0} + I = p_{system,f}$$

If there is no net external force on the system, the momentum of the system is conserved.

$$p_{system,0} = p_{system,f}$$



# \*Conservation of Momentum & Impulse Proof

Both the Conservation of Momentum and the concept of Impulse follow directly from Newton's Second Law:

$$F = ma \quad \text{where } F \text{ is the net external force}$$

$$F = m(\Delta v / \Delta t) \quad \text{since } a = \Delta v / \Delta t$$

$$F \Delta t = m \Delta v \quad \text{after multiplying both sides by } \Delta t$$

$$F \Delta t = \Delta(mv) \quad \text{since } m \text{ is constant}$$

$$I = F \Delta t \quad \text{- the definition of impulse } I$$

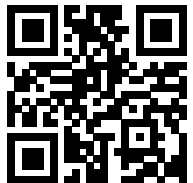
$$I = \Delta p \quad \text{substituting } I \text{ for } F \Delta t, \text{ and } p \text{ for } mv$$

$$I = p_f - p_0 \quad \text{since } \Delta p = p_f - p_0$$

$$I = p_f$$

$$p_f$$

when there no net external force ( $F=0$ ),  $I=0$  so...  
momentum is conserved



<http://njc.tl/17>



# Conservation Laws, Collisions and Explosions

Objects in an isolated system can interact with each other in a number of ways...

- They can collide
- If they are stuck together, they can explode (push apart)

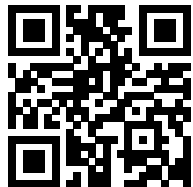
In an isolated system both momentum and total energy are conserved.

But the energy can change from one form to another.

Conservation of momentum and change in kinetic energy help us

t what happened or what will happen in one of these

).

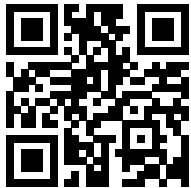


<http://njc.tl/17>

# Collisions and Explosions

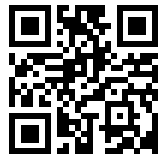
We differentiate collisions and explosions by the way the energy changes or does not change form.

- **explosions:** an object or objects break apart because potential energy stored in one or more of the objects is transformed into kinetic energy
- **inelastic collisions:** two objects collide and stick together converting some kinetic energy into bonding energy, heat, sound...
- **elastic collisions:** two objects collide and bounce off each other while conserving kinetic energy



# Collisions and Explosions - Summarized

Event	Description	Momentum Conserved?	Kinetic Energy Conserved?
Inelastic Collision	General collision: Objects bounce off each other	Yes	No. Some kinetic energy is converted to heat, sound... energy
Inelastic Collision	Objects stick together	Yes	No. Kinetic energy is converted to potential energy, bonding, or heat, sound...energy
Elastic Collision	Objects bounce off each other	Yes	Yes
Explosion	Object or objects break apart	Yes	No. Release of potential energy increases kinetic energy



<http://njc.tl/17>

17 In \_\_\_\_\_ collisions momentum is conserved.

- ☐ A Elastic
- ☐ B Inelastic
- ☐ C All

**Answer**



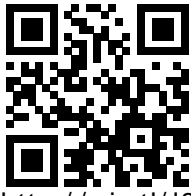
<http://njc.tl/l8>

17 In \_\_\_\_\_ collisions momentum is conserved.

- ☐ A Elastic
- ☐ B Inelastic
- ☐ C All

**Answer**

**C**



<http://njc.tl/l8>

18 In \_\_\_\_\_ collisions kinetic energy is conserved.

- ☐ A Elastic
- ☐ B Inelastic
- ☐ C All

**Answer**



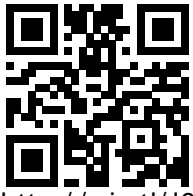
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18 In \_\_\_\_\_ collisions kinetic energy is conserved.

- ☐ A Elastic
- ☐ B Inelastic
- ☐ C All

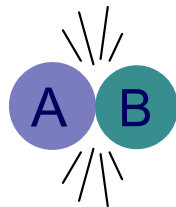
**Answer**

**A**



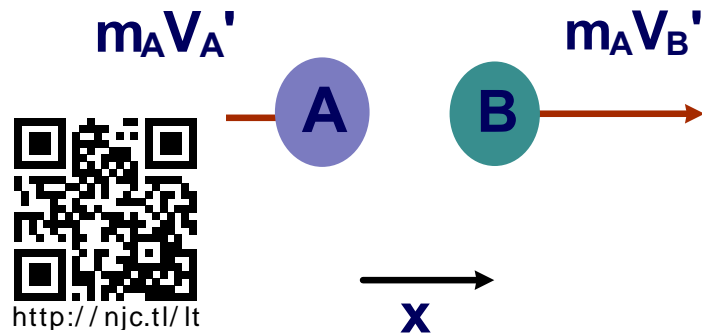
# Conservation of Momentum

During a collision or an explosion, measurements show that the total momentum does not change:



*the prime means "after"*

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$



<http://njc.tl/lt>



# Explosions

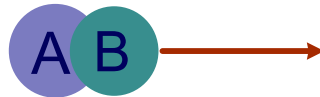
In an explosion, one object breaks apart into two or more pieces (or coupled objects break apart) moving afterwards as separate objects.

We will assume:

- the object (or a coupled pair of objects) breaks into two pieces
- explosion is along the same line as the initial velocity

**Before (moving together)**

$$p_A + p_B = (m_A + m_B)v$$



**After (moving apart)**

$$p_A' = m_A v_A'$$

$$p_B' = m_B v_B'$$



$$p_A + p_B = p_A' + p_B'$$

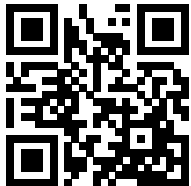
$$(m_A + m_B)v = m_A v_A' + m_B v_B'$$



<http://njc.tl/lt>

19 A 5 kg cannon ball is loaded into a 300 kg cannon. When the cannon is fired, it recoils at 5 m/s. What is the cannon balls's velocity after the explosion?

**Answer**



<http://njc.tl/la>

19 A 5 kg cannon ball is loaded into a 300 kg cannon. When the cannon is fired, it recoils at 5 m/s. What is the cannon balls's velocity after the explosion?

**Answer**

$$m_1 = 5\text{kg}$$

$$m_2 = 300\text{kg}$$

$$v_1 = v_2 = v = 0$$

$$v_2' = 5\text{m/s}$$

$$(m_1 + m_2) v = m_1 v_1' + m_2 v_2'$$

$$0 = m_1 v_1' + m_2 v_2'$$

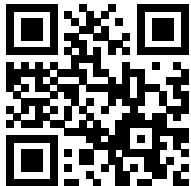
$$m_1 v_1' = -m_2 v_2'$$

$$\begin{aligned} v_1' &= -m_2 v_2' / m_1 \\ &= -(300\text{kg})(5\text{m/s}) / (5\text{kg}) \\ &= -300\text{m/s} \end{aligned}$$



20 Two railcars, one with a mass of 4000 kg and the other with a mass of 6000 kg, are at rest and stuck together. To separate them a small explosive is set off between them. The 4000 kg car is measured travelling 6 m/s. How fast is the 6000 kg car going?

Answer



<http://njc.tl/lb>

- 20 Two railcars, one with a mass of 4000 kg and the other with a mass of 6000 kg, are at rest and stuck together. To separate them a small explosive is set off between them. The 4000 kg car is measured travelling 6 m/s. How fast is the 6000 kg car going?

**Answer**

$$m_1 = 6000\text{kg}$$

$$m_2 = 4000\text{kg}$$

$$v_1 = v_2 = v = 0$$

$$v_2' = 6\text{m/s}$$

$$(m_1 + m_2) v = m_1 v_1' + m_2 v_2'$$

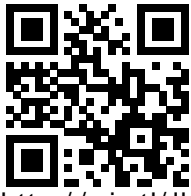
$$0 = m_1 v_1' + m_2 v_2'$$

$$m_1 v_1' = -m_2 v_2'$$

$$v_1' = -m_2 v_2' / m_1$$

$$= -(4000\text{kg})(6\text{m/s}) / (6000\text{kg})$$

$$= -4\text{m/s}$$



# Inelastic Collisions

In an inelastic collision, two objects collide and stick together, moving afterwards as one object.

Before (moving towards the other)

$$p_A = m_A v_A$$



$$p_B = m_B v_B$$



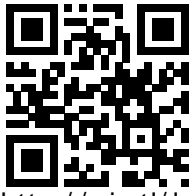
After (moving together)

$$p_A' + p_B' = (m_A + m_B) v'$$

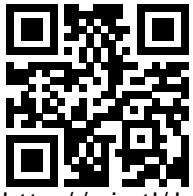


$$p_A + p_B = p_A' + p_B'$$

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$



21



<http://njc.tl/lc>

**Answer**

21

**Answer**

$$m_1 = 13,500\text{kg}$$

$$m_2 = 25,000\text{kg}$$

$$v_1 = 4.5\text{m/s}$$

$$v_2 = 0 \text{ m/s}$$

$$v_1' = v_2' = v'$$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$m_1v_1 + 0 = (m_1 + m_2) v'$$

$$v' = m_1v_1 / (m_1 + m_2)$$

$$= (13,500\text{kg})(4.5\text{m/s}) / (13,500 + 25,000)\text{kg}$$

$$= 1.6 \text{ m/s}$$

in same direction as first car's initial velocity





22

**Answer**



<http://njc.tl/ld>

22

**Answer**

$$m_1 = 100\text{kg}$$

$$m_2 = 15,000\text{kg}$$

$$v_1 = 800\text{m/s}$$

$$v_2 = 0 \text{ m/s}$$

$$v_1' = v_2' = v'$$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

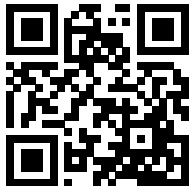
$$m_1v_1 + 0 = (m_1 + m_2) v'$$

$$v' = m_1v_1 / (m_1 + m_2)$$

$$= (100\text{kg})(800\text{m/s}) / (100 + 15,000)\text{kg}$$

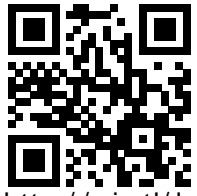
$$= 5.3 \text{ m/s}$$

in same direction as cannon ball's initial velocity



<http://njc.tl/ld>

23



<http://njc.tl/le>

**Answer**

23

**Answer**

$$\begin{aligned}m_1 &= 40\text{kg} \\ m_2 &= 70\text{kg} \\ v_1 &= 5.5\text{m/s} \\ v_2 &= 0 \text{ m/s} \\ v_1' &= v_2' = v'\end{aligned}$$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

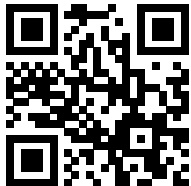
$$m_1v_1 + 0 = (m_1 + m_2) v'$$

$$v' = m_1v_1 / (m_1 + m_2)$$

$$= (40\text{kg})(5.5\text{m/s}) / (40 + 70)\text{kg}$$

$$= 2 \text{ m/s}$$

in same direction as the 40kg girl's initial velocity



# Elastic Collisions

In an elastic collision, two objects collide and bounce off each other.

Both momentum and kinetic energy are conserved.

If we know the masses and any two of the velocities, these two conservation equations enable us to calculate the other two velocities.

**Before (moving towards)**

$$p_A = m_A v_A$$

$$p_B = m_B v_B$$



**After (moving apart)**

$$p_A' = m_A v_A'$$

$$p_B' = m_B v_B'$$



$$p_A + p_B = p_A' + p_B'$$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$KE_A + KE_B = KE_A' + KE_B'$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$



<http://njc.tl/lf>

# \*\*Derivation of Elastic Collision Condition

## Conservation of Momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

## Conservation of Kinetic Energy

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 - m_2 v_2^2$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

$$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$$

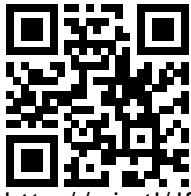
$$\cancel{m_1(v_1 + v_1')(v_1 - v_1')} = \cancel{m_2(v_2' + v_2)(v_2' - v_2)}$$

$$\cancel{m_1(v_1 - v_1')} = \cancel{m_2(v_2' - v_2)}$$

$$v_1 + v_1' = v_2' + v_2$$

$$v_1 - v_2 = -(v_1' - v_2')$$

$$a^2 - b^2 = (a+b)(a-b)$$

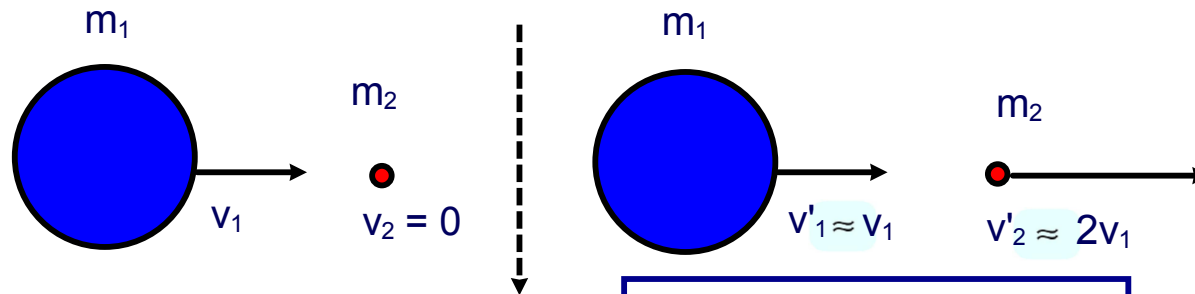


# Properties of Elastic Collisions

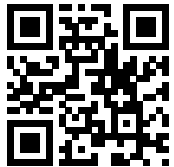
1. **For all elastic collisions**, regardless of the masses of the objects, the objects separate after the collision with the same relative speed that they collided with.

$$v_1 - v_2 = -(v_1' - v_2')$$

2. **In an elastic collision where one object is much more massive** than the other, the velocity of the smaller mass after the collision will be about twice that of the projectile while the more massive object's velocity will be almost unchanged.



$$v_1' = v_1 \text{ and } v_2' = 2v_1$$



<http://njc.tl/lf>

**In an elastic collision between two objects of identical masses**, the two objects exchange velocities.

$$v_1' = v_2 \text{ and } v_2' = v_1$$

24 Two objects have an elastic collision. Before they collide they are approaching each other with a velocity of  $4\text{m/s}$  relative to each other. With what velocity do they go apart from one another?

**Answer**



<http://njc.tl/lq>



- 24 Two objects have an elastic collision. Before they collide they are approaching each other with a velocity of 4m/s relative to each other. With what velocity do they go apart from one another?

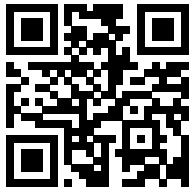
**Answer**

4m/s

Initially they are approaching relative to each other at 4m/s, and afterwards they will move apart relative to each other with a velocity of 4m/s.

$$v_1 + v_1' = v_2 + v_2'$$

The difference in the initial velocities is the same as the difference in the final velocities therefore they will move apart relative to one another at the same velocity



- 25 Two objects have an elastic collision. One object,  $m_1$ , has an initial velocity of +4.0 m/s and  $m_2$  has a velocity of -3.0 m/s. After the collision,  $m_1$  has a velocity of 1.0 m/s. What is the velocity of  $m_2$ ?

**Answer**



<http://njc.tl/lh>

- 25 Two objects have an elastic collision. One object,  $m_1$ , has an initial velocity of +4.0 m/s and  $m_2$  has a velocity of -3.0 m/s. After the collision,  $m_1$  has a velocity of 1.0 m/s. What is the velocity of  $m_2$ ?

**Answer**

$$v_1 = 4\text{m/s}$$

$$v_2 = -3\text{m/s}$$

$$v_1' = 1\text{m/s}$$

$$v_2' = ?$$

$$v_1 + v_1' = v_2 + v_2'$$

$$v_2' = v_1 + v_1' - v_2$$

$$v_2' = 4\text{m/s} + 1\text{m/s} - (-3\text{m/s})$$

$$v_2' = 8\text{m/s}$$



- 26 A bowling ball has a velocity of  $+v$  when it collides with a ping pong ball that is at rest. The velocity of the bowling ball is virtually unaffected by the collision. What will be the speed of the ping pong ball?

**Answer**



<http://njc.tl/li>

- 26 A bowling ball has a velocity of  $+v$  when it collides with a ping pong ball that is at rest. The velocity of the bowling ball is virtually unaffected by the collision. What will be the speed of the ping pong ball?

**Answer**

$$v_1 = +v$$

$$v_2 = 0$$

$$v_1' = +v$$

$$v_2' = ?$$

$$v_1 + v_1' = v_2 + v_2'$$

$$v_2' = v_1 + v_1' - v_2$$

$$v_2' = v + v - 0$$

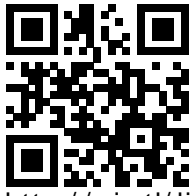
$$v_2' = 2v$$

(ping pong ball's speed is 2x the that of the bowling ball)



- 27 A baseball bat has a velocity of  $+v$  when it collides with a baseball that has a velocity of  $-2v$ . The bat barely changes velocity during the collision. How fast is the baseball going after it's hit?

**Answer**



<http://njc.tl/lj>

- 27 A baseball bat has a velocity of  $+v$  when it collides with a baseball that has a velocity of  $-2v$ . The bat barely changes velocity during the collision. How fast is the baseball going after it's hit?

**Answer**

$$\begin{aligned}v_1 &= +v \\v_2 &= -2v \\v_1' &= +v \\v_2' &= ?\end{aligned}$$

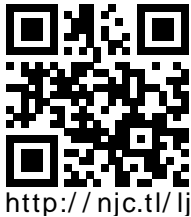
$$v_1 + v_1' = v_2 + v_2'$$

$$v_2' = v_1 + v_1' - v_2$$

$$v_2' = v + v - (-2v)$$

$$v_2' = 4v$$

(baseball's ball's speed doubled after being hit by the bat)



28 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+6.0\text{m/s}$  and  $m_2$  is  $-3.0\text{m/s}$ . What is the velocity of  $m_1$  after the collision?

**Answer**



<http://njc.tl/lk>



28 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+6.0\text{m/s}$  and  $m_2$  is  $-3.0\text{m/s}$ . What is the velocity of  $m_1$  after the collision?

**Answer**

$$v_1 = +6\text{m/s}$$

$$v_2 = -3\text{m/s}$$

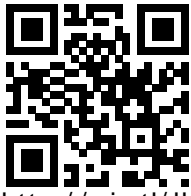
$$v_1' = ?$$

$$v_2' = ?$$

In elastic collisions between objects of identical mass, the velocities are swapped on collision:

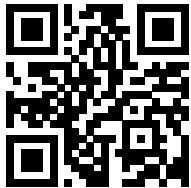
$$v_1' = v_2 = -3\text{m/s}$$

$$v_2' = v_1 = 6\text{m/s}$$



29 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+6.0\text{m/s}$  and  $m_2$  is  $-3.0\text{m/s}$ . What is the velocity of  $m_2$  after the collision?

**Answer**



<http://njc.tl/II>

- 29 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+6.0\text{m/s}$  and  $m_2$  is  $-3.0\text{m/s}$ . What is the velocity of  $m_2$  after the collision?

**Answer**

$$v_1 = +6\text{m/s}$$

$$v_2 = -3\text{m/s}$$

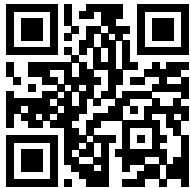
$$v_1' = ?$$

$$v_2' = ?$$

In elastic collisions between objects of identical mass, the velocities are swapped on collision:

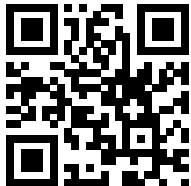
$$v_1' = v_2 = -3\text{m/s}$$

$$v_2' = v_1 = 6\text{m/s}$$



30 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+3.0\text{m/s}$  and  $m_2$  is  $+2.0\text{m/s}$ . What is the velocity of  $m_1$  after the collision?

**Answer**



<http://njc.tl/lm>

- 30 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+3.0\text{m/s}$  and  $m_2$  is  $+2.0\text{m/s}$ . What is the velocity of  $m_1$  after the collision?

**Answer**

$$v_1 = +3\text{m/s}$$

$$v_2 = +2\text{m/s}$$

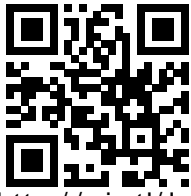
$$v_1' = ?$$

$$v_2' = ?$$

In elastic collisions between objects of identical mass, the velocities are swapped on collision:

$$v_1' = v_2 = +2\text{m/s}$$

$$v_2' = v_1 = +3\text{m/s}$$



- 31 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+3.0\text{m/s}$  and  $m_2$  is  $+2.0\text{m/s}$ . What is the velocity of  $m_2$  after the collision?

**Answer**



<http://njc.tl/ln>

- 31 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is  $+3.0\text{m/s}$  and  $m_2$  is  $+2.0\text{m/s}$ . What is the velocity of  $m_2$  after the collision?

**Answer**

$$v_1 = +3\text{m/s}$$

$$v_2 = +2\text{m/s}$$

$$v_1' = ?$$

$$v_2' = ?$$

In elastic collisions between objects of identical mass, the velocities are swapped on collision:

$$v_1' = v_2 = +2\text{m/s}$$

$$v_2' = v_1 = +3\text{m/s}$$

