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Progressive Science Initiative

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Electric Field, Potential Energy and Voltage

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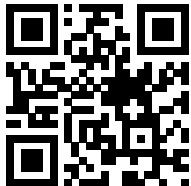
How to Use this File

- Each topic is composed of brief direct instruction
- There are formative assessment questions after every topic denoted by black text and a number in the upper left.
 - > Students work in groups to solve these problems but use student responders to enter their own answers.
 - > Designed for SMART Response PE student response systems.
 - > Use only as many questions as necessary for a sufficient number of students to learn a topic.
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Electric Field, Potential Energy and Voltage

[Click on the topic to go to that section](#)

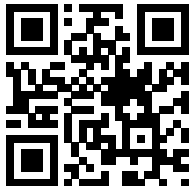
- **Electric Field**
- ***Electric Field relationship to Gravitational Field**
- **Electric Field of Multiple Charges**
- ****The Net Electric Field**
- **Electric Potential Energy**
- **Electric Potential (Voltage)**
- **Uniform Electric Field**



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Electric Field

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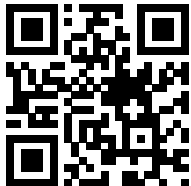
Electric Field

The Electric Field starts with Coulomb's Law:

$$F_E = \frac{k|q_1||q_2|}{r_{12}^2}$$

This gives the force between two charges, q_1 and q_2 . Similar to the gravitational force, no contact is needed between the two charges for them to feel a force from the other charge.

This "action at a distance" is best understood by assuming that each charge has a field surrounding it that affects other charges - this is called the Electric Field.



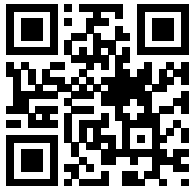
Electric Field

Let's find the Electric Field due to one charge. The notation in Coulomb's Law will be modified slightly - assuming that one charge is very large - and the other charge is a small, positive test charge that will have a negligible Electric Field due to its size.

The large charge will be labeled, Q , and the small charge, q , and the distance between them is r .

The absolute value signs will be removed, as we will now consider the vector quality of the Force (note the arrow on the top of the F - that means that F is a vector - it has magnitude and direction).

$$\vec{F}_E = \frac{kQq}{r^2}$$



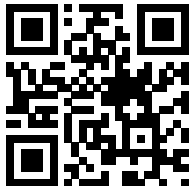
Electric Field

$$\vec{F}_E = \frac{kQq}{r^2}$$

To find the Force that the large charge exerts on the little charge, the above equation will be divided by q , and this will be defined as the Electric Field.

$$\vec{E} = \frac{\vec{F}_E}{q} = \frac{kQ}{r^2}$$

The Electric Field now shows both the magnitude and direction of the force exerted by Q on any charge. To find the force, the Electric Field is multiplied by the charge that is being considered.



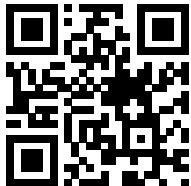
Electric Field

Q creates the electric field. The size of charge Q and the distance to a point determine the strength of the electric field (E) at that point.

E is measured in N/C (*Newtons per Coulomb*).

The Electric Field is represented as a group of lines that show its direction and strength - which is the Force that it would exert on a positive charge within its field.

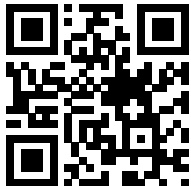
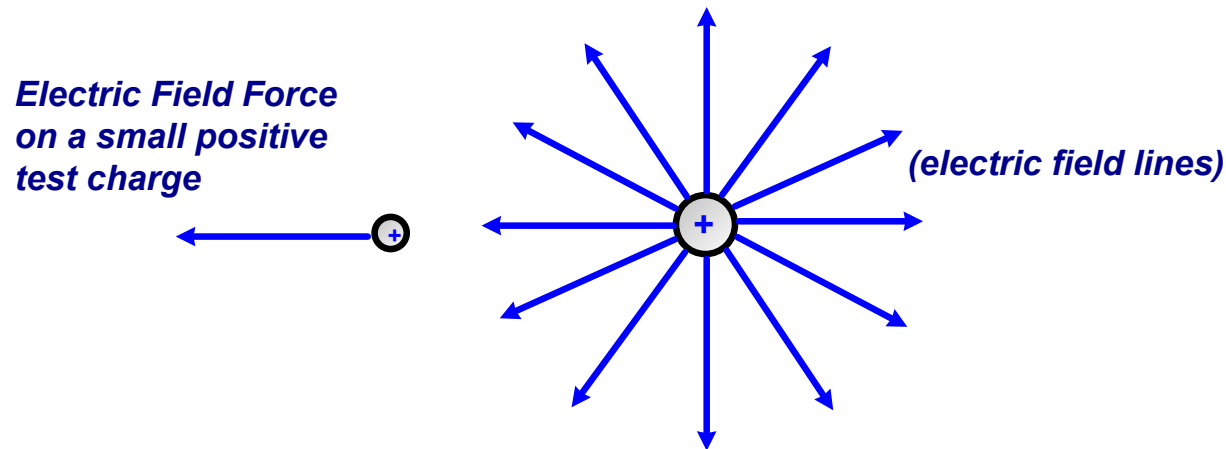
Hence, these Electric Field lines (which are imaginary, but help us visualize what is happening) originate on positive charges and end on negative charges.



<http://njc.tl/fv>

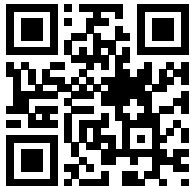
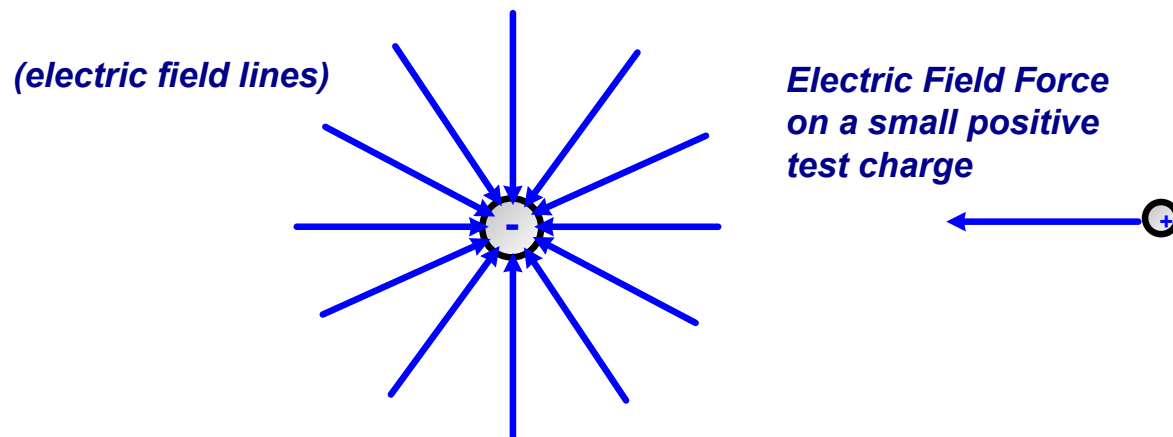
Electric Field due to a Positive Charge

If there is an isolated positive charge, it will create an Electric Field that points radially away from it in all directions, since a positive test charge in the field will be repelled by this charge.

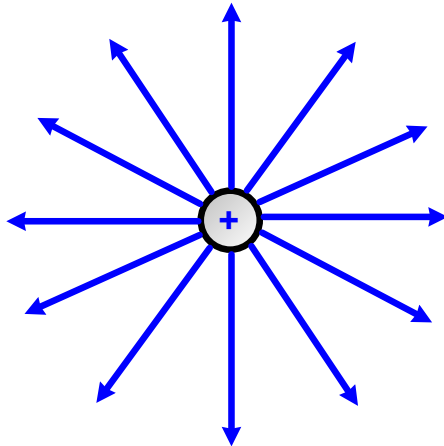


Electric Field due to a Negative Charge

If there is an isolated negative charge, it will create an Electric Field that points radially towards it in all directions, since a positive test charge in the field will be attracted by this charge.



Electric Field Direction and Magnitude

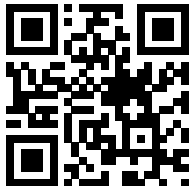


The definition of the Electric Field shows that the strength of the field decreases as distance increases

$$F \propto E \propto \frac{1}{r^2}$$

This can be seen by looking at the density of the field lines.

Note that the Electric Field lines are closer together (more dense) when they are closer to the charge that is generating the Field. This indicates the Electric Field is greater nearer the charge.



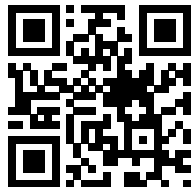
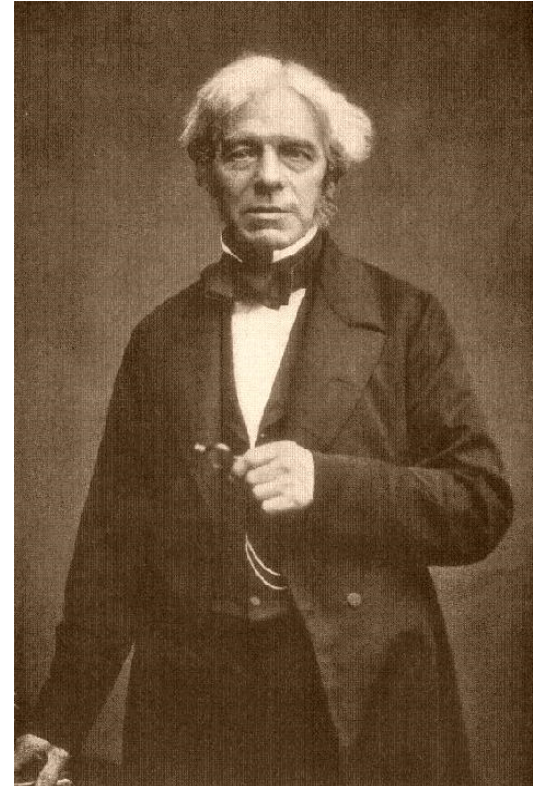
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Michael Faraday

The electric field is attributed to Michael Faraday. Faraday was born in London in 1791. He came from a poor family. At 13, he apprenticed as a book seller and binder while also attending local lectures on philosophical and scientific topics.

A member of the Royal Institute took notice of Faraday and bought him tickets to several Royal Institute lectures.

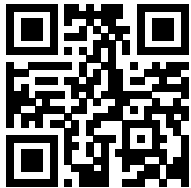


<http://njc.tl/fv>

, he was invited to work at the Royal Institute where he made
ous contributions to physics and chemistry.

- 1 Find the magnitude of the electric field for a charge of 5.6 nC at a distance of 3.0 m.

Answer



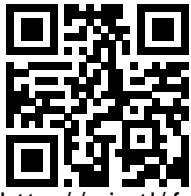
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- 1 Find the magnitude of the electric field for a charge of 5.6 nC at a distance of 3.0 m.

Answer

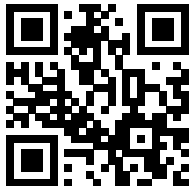
$$E = \frac{kQ}{r^2} = \frac{\left(9 \times 10^9 \frac{Nm^2}{C^2}\right)(5.6 \times 10^{-9} C)}{(3.0m)^2}$$

$$E = 5.6 N/C$$



2 A 4.5 mC charge experiences an electrical force of 9.0 mN in the presence of an electric field. What is the magnitude of the electric field?

Answer



<http://njc.tl/fy>

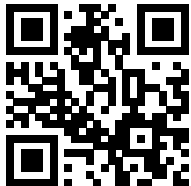
- 2 A 4.5 mC charge experiences an electrical force of 9.0 mN in the presence of an electric field. What is the magnitude of the electric field?

Answer

$$F = qE$$

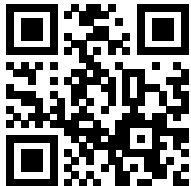
$$E = \frac{F}{q} = \frac{9.0 \times 10^{-3} \text{ N}}{4.5 \times 10^{-3} \text{ C}}$$

$$E = 2.0 \text{ N/C}$$



3 If E_0 is the Electric Field generated at a distance r from a charge Q , what is the Electric Field at a distance $2r$?

Answer



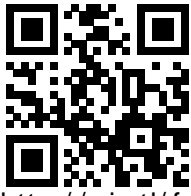
<http://njc.tl/fz>

3 If E_0 is the Electric Field generated at a distance r from a charge Q , what is the Electric Field at a distance $2r$?

Answer

$$E_0 = \frac{kQ}{r^2}$$

$$E_f = \frac{kQ}{(2r)^2} = \frac{1}{4} \frac{kQ}{r^2} = \frac{1}{4} E_0$$



4 The direction of the Electric Field can be found by using:

- ☐ A the direction of the gravitational force.
- ☐ B the direction that a positive test charge would accelerate.
- ☐ C the direction that a negative test charge would accelerate.

Answer



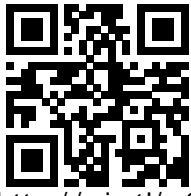
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4 The direction of the Electric Field can be found by using:

- ☐ A the direction of the gravitational force.
- ☐ B the direction that a positive charge would accelerate.
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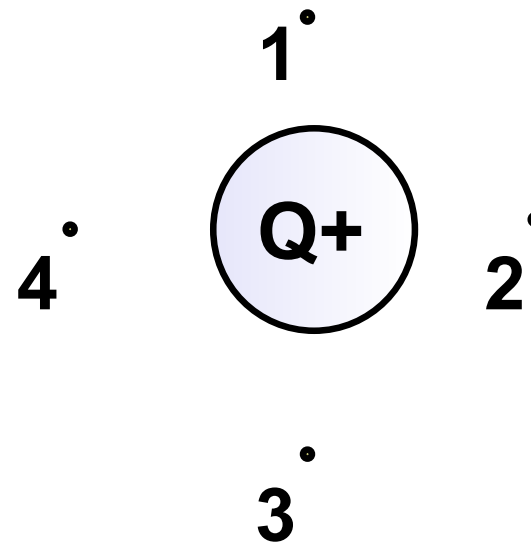
Answer

B

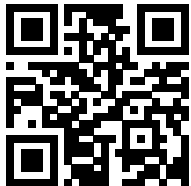


5 What is the direction of the Electric Field at points 1, 2, 3 and 4?

- ☐ A up, right, down, left.
- ☐ B up, left, down, right.
- ☐ C down, right, up, left.
- ☐ D down, left, up, right.



Answer

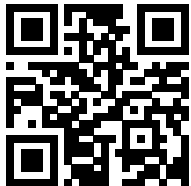


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- ☐ D down, left, up, right

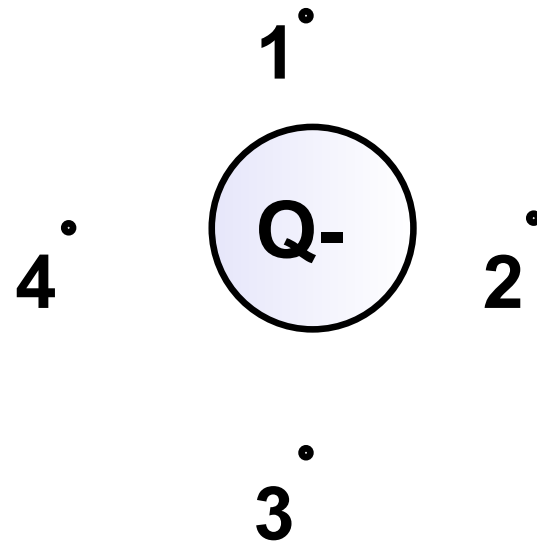
Answer

A

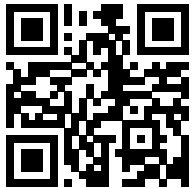


6 What is the direction of the Electric Field at points 1, 2, 3 and 4?

- ☐ A up, right, down, left.
- ☐ B up, left, down, right.
- ☐ C down, right, up, left.
- ☐ D down, left, up, right.



Answer

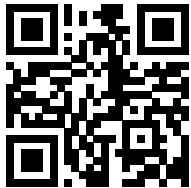


6 What is the direction of the Electric Field at points 1, 2, 3 and 4?

- ☐ A up, right, down, left
- ☐ B up, left, down, right
- ☐ C down, right, up, left
- ☐ D down, left, up, right

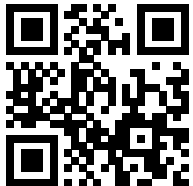
Answer

D



7 What is the magnitude and direction of the electric field at a distance of 2.3 m due to a charge of $-4.9 \text{ } \mu\text{C}$?

Answer



<http://njc.tl/g3>

- 7 What is the magnitude and direction of the electric field at a distance of 2.3 m due to a charge of -4.9 μC ?

Answer

$$E = \frac{kQ}{r^2} = \frac{\left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right)(4.9 \times 10^{-6} \text{C})}{(2.3\text{m})^2}$$

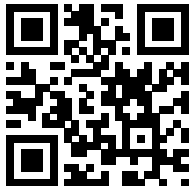
$$E = 8300 \text{ N/C}$$

Towards the charge



***Electric Field relationship to the Gravitational Field**

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<http://njc.tl/lp>

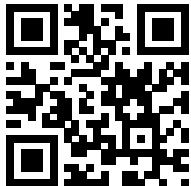
***Electric Field relationship to Gravitational Field**

In the chapter on Electric Charge and Force, the similarity between the electric force and the gravitational force was noted.

There is a similar relationship between the Electric Field and the Gravitational Field.

The reason for this is that the two forces are both central forces in that they act along the line connecting objects.

There is a key difference between the two fields and forces. Mass, which is the source of the gravitational field is always positive, and the force is always attractive. Charge, the source of the Electric Field can be negative or positive and the force is either attractive or repulsive.



<http://njc.tl/lp>

*Electric Field relationship to Gravitational Field

Given that a mass m is located at the surface of the planet with a mass of M and radius R , Newton's Law of Universal Gravitation is used to determine the gravitational force, F_G , between the planet and mass m :

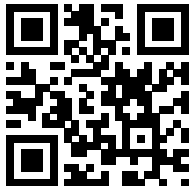
$$F_G = \frac{GMm}{r^2}$$

Divide this expression by m (where $m \ll M$) - similar to what was done with the small positive test charge, q , and call this " g ", the Gravitational Field:

$$g = \frac{F_G}{m} = \frac{GM}{r^2}$$

used to express the "weight" of the mass m on the planet:

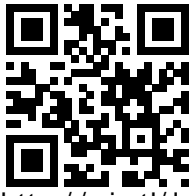
$$W = F_G = mg$$



<http://njc.tl/lp>

*Electric Field relationship to Gravitational Field

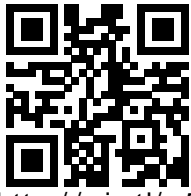
Gravity	Electric Fields
Newton's Law of Universal Gravitation	Coulomb's Law
$F_G = \frac{GMm}{r^2}$	$F_E = \frac{kQq}{r^2}$
$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$	$k = 8.99 \times 10^9 \frac{Nm^2}{C^2}$
mass (kg)	charge (Coulombs)
distance, r, between centers of mass	distance, r, between centers of charge
Gravitational Field	Electric Field
$g = \frac{GM}{r^2}$	$E = \frac{kQ}{r^2}$



8 How are Gravitational and Electric Fields similar?

- ☐ A They both increase the further away you get from the source.
- ☐ B They both decrease as a factor of the square of the distance between the two masses or charges.
- ☐ C The fields decrease as a factor of the distance between the masses or charges.
- ☐ D The fields are constant throughout space.

Answer

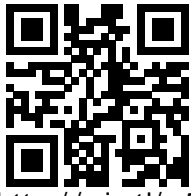


8 How are Gravitational and Electric Fields similar?

- ☐ A They both increase the further away you get from the source.
- ☐ B They both decrease the further away you get from the source.
- ☐ C The fields decrease the further away you get from the source.
- ☐ D The fields are constant.

Answer

B



9 How are Gravitational and Electric Fields different?

- ☐ A The Gravitational Field can exert a repulsive force on a mass, where an Electric Field always attracts charges independent of their polarity (positive or negative).
- ☐ B The Gravitational Field always exerts a repulsive force on masses, where the Electric Field always exerts an attractive force.
- ☐ C Masses in a Gravitational Field always feel an attractive force, where an Electric Field can either repel or attract a charge depending on its polarity.
- ☐ D There are no differences.

Answer

9 How are Gravitational and Electric Fields different?

- ☐ A The Gravitational Field can exert a repulsive force on a mass, where an Electric Field always attracts charges independent of whether they are positive or negative).
- ☐ B The Gravitational force on masses always exerts an attractive force. Masses in a Gravitational field always attract each other.
- ☐ C Masses in a Gravitational field always repel or attract each other depending on whether they are positive or negative.
- ☐ D There are no differences.

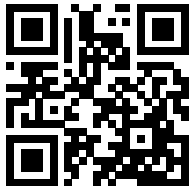
Answer

C



Electric Field of Multiple Charges

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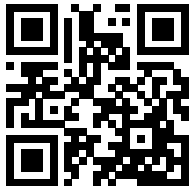


<http://njc.tl/g4>

Electric Field of Multiple Charges

Since the Electric Field of a single charge is a vector, the Electric Field of multiple charges may be calculated by adding, point by point, the individual Electric Fields.

The methodology for adding Electric Fields will be covered in the section entitled "***The Net Electric Field."

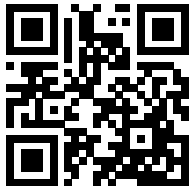


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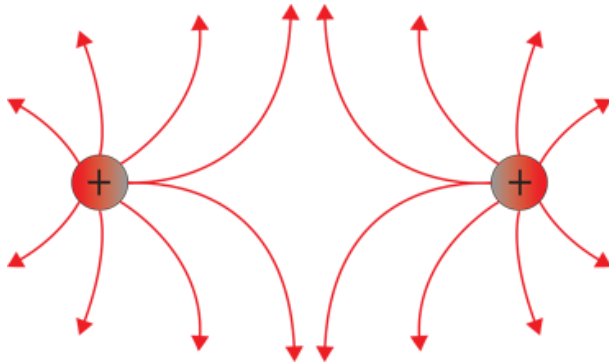
Electric Field of Multiple Charges

Adding individual electric fields will give lines of Electric Force that obey 4 rules:

1. Electric Field Lines begin on a positive charge and end on a negative charge.
2. The density of magnetic field lines distribution is proportional to the size of the charges.
3. The lines never cross (or else there would be multiple values of electric force at the crossing point).
4. The lines are continuous.

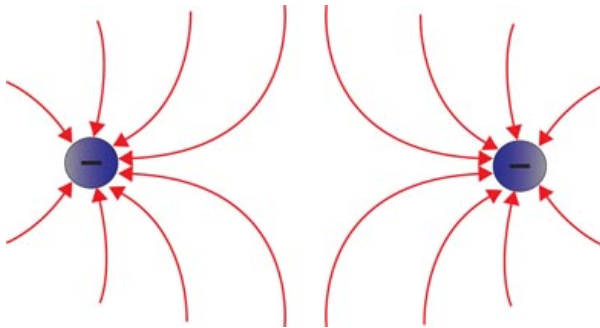


Electric Field of Multiple Charges

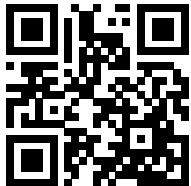


This is the electric field configuration due to two like charges.

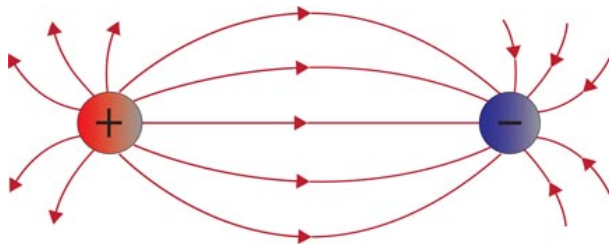
There is no electric field midway between the two like charges - the individual electric field vectors cancel out.



The shape of the field is the same for both positive and negative charges - only the field direction is different.

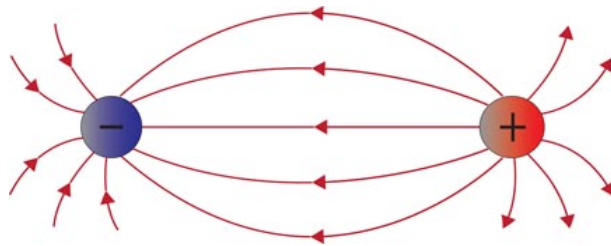


Electric Field of Multiple Charges



This is the electric dipole configuration, consisting of two unlike charges.

There are no places where the electric field is zero.

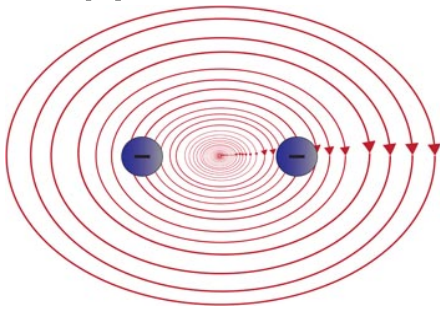


Again, the shape of the field is the same for both positive and negative charges - only the field direction is different.

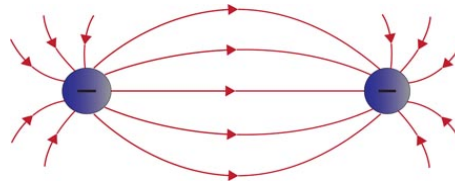


10 Which of the following represents the electric field map due to a combination of two negative charges?

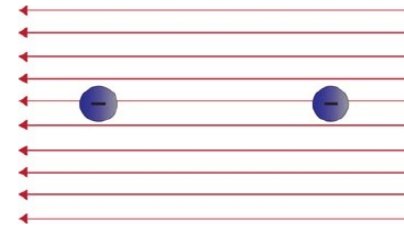
☐ A



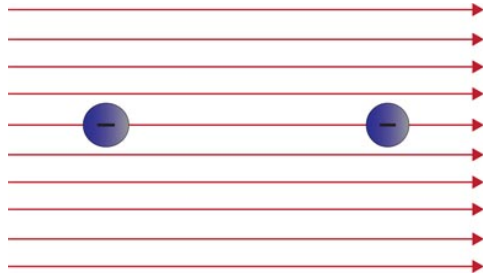
☐ B



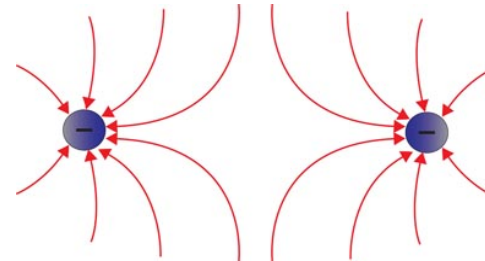
☐ C



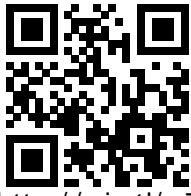
☐ D



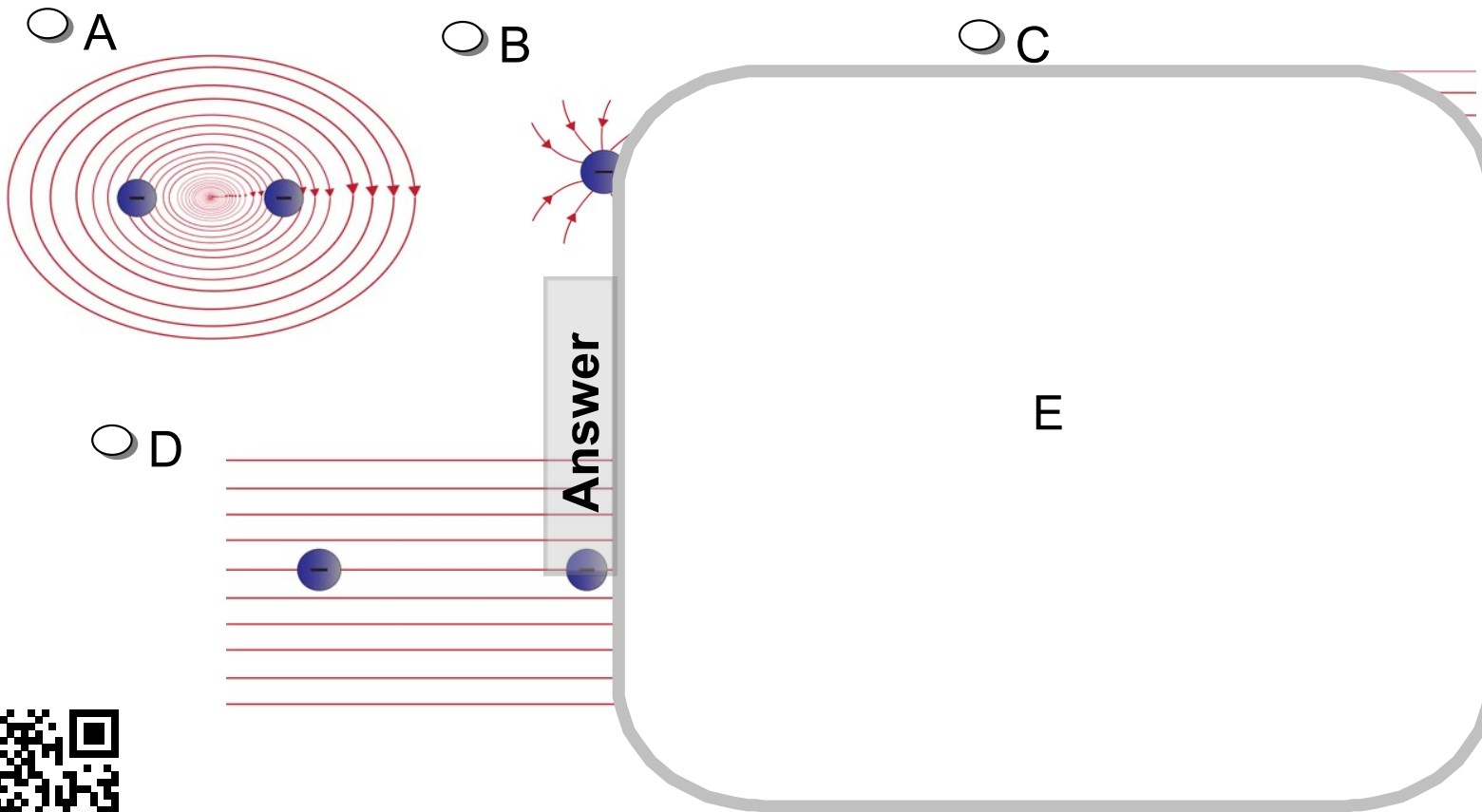
☐ E



Answer

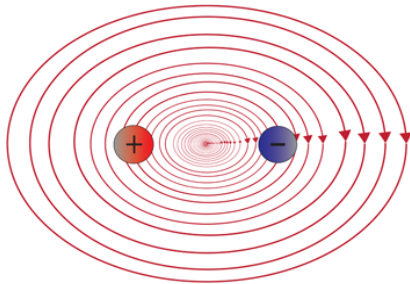


10 Which of the following represents the electric field map due to a combination of two negative charges?

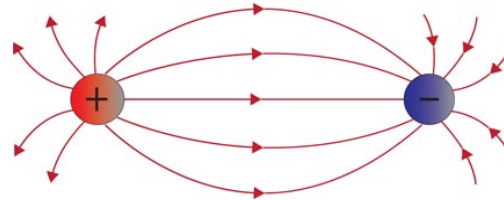


11 Which of the following represents the electric field map due to a combination of a positive and a negative charge?

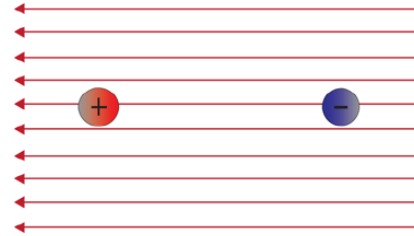
☐ A



☐ B



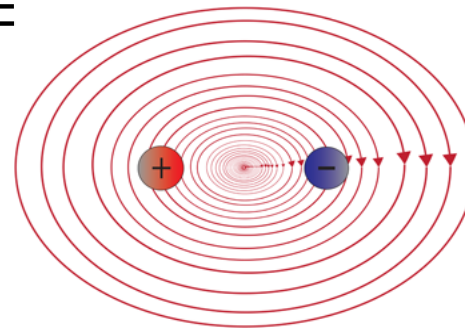
☐ C



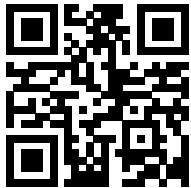
☐ D



☐ E

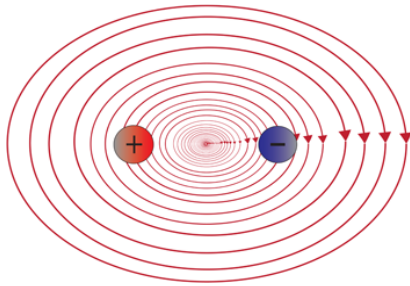


Answer

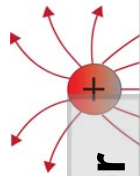


11 Which of the following represents the electric field map due to a combination of a positive and a negative charge?

☐ A



☐ B

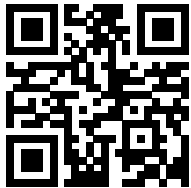


☐ D



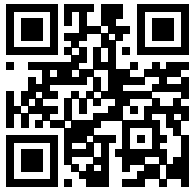
Answer

B



****The Net Electric Field**

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<http://njc.tl/g9>

****The Net Electric Field**

Since the Electric Field is represented by vectors, the net Electric Field at a location due to multiple charges is calculated by adding each of the vectors together.

$$E_{\text{net}} = \Sigma E$$

$$E_{\text{net}} = E_1 + E_2 + E_3 + \dots + E_n$$

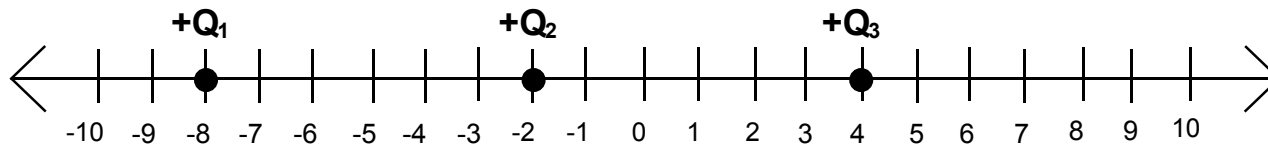
Where n is the total number of fields acting on a location

The direction of each electric field determines the sign used.



****The Net Electric Field**

Objective: Find the net electric field at the origin.



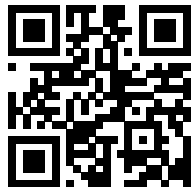
Strategy:

1. Mark the point on the drawing where the Electric Field is to be calculated (the point is at $x=0$ for this example).
2. Draw the electric fields acting at that point.



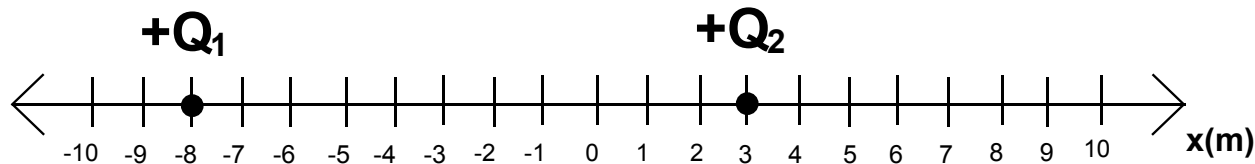
3. Calculate E_1 , E_2 , and E_3 (assign negative values to fields pointing left, and positive values to fields pointing right).
Combine the electric fields.

$$E_{\text{net}} = \sum E_n = E_1 + E_2 - E_3$$



****The Net Electric Field Example**

Find the net electric field at the origin.



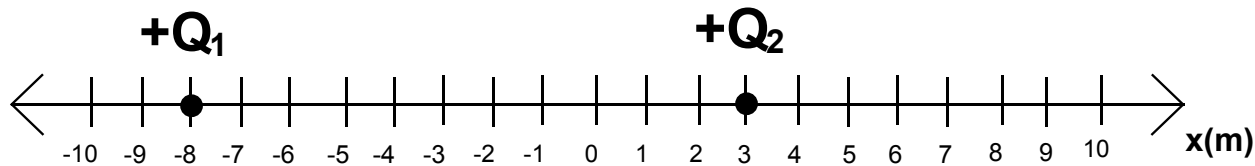
A positive charge, $Q_1 = +9.1 \mu\text{C}$ is located at $x_1 = -8.0 \text{ m}$, and another positive charge, $Q_2 = +3.0 \mu\text{C}$ is located at $x_2 = +3.0 \text{ m}$.

- Draw the electric fields acting on $x=0$
 - Find the magnitude and direction of the electric field at the origin due to charge Q_1 .
 - Find the magnitude and direction of the electric field at the origin due to charge Q_2 .
- Find the magnitude and direction of the net electric field at origin by adding the results from a. and b. (with proper signs).



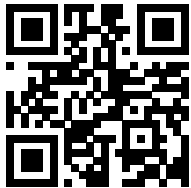
****The Net Electric Field Example**

Find the net electric field at the origin.



a. Draw the electric fields acting at $x=0$

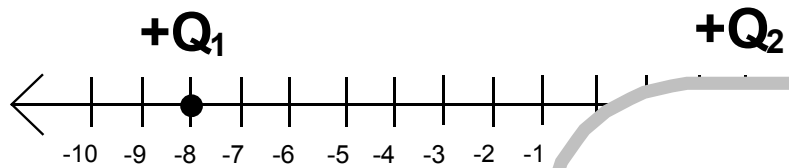
Answer



<http://njc.tl/g9>

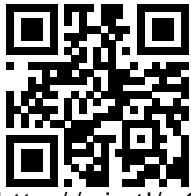
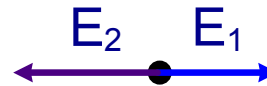
****The Net Electric Field Example**

Find the net electric field at the origin.



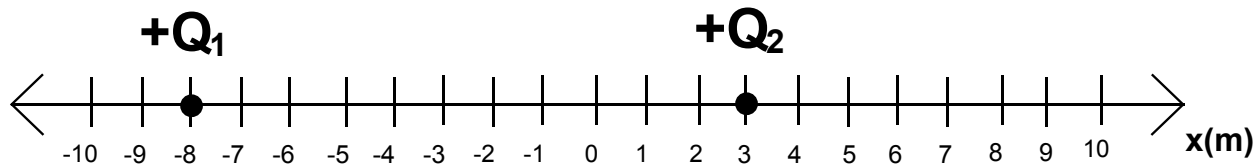
a. Draw the electric fields

Answer



****The Net Electric Field Example**

Find the net electric field at the origin.



b. Find the magnitude and direction of the electric field at the origin due to charge Q_1 .

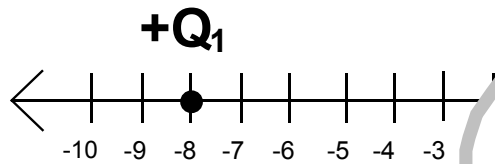
Answer



<http://njc.tl/g9>

****The Net Electric Field Example**

Find the net electric field at the origin.



b. Find the magnitude of the electric field at the origin due to charge Q_1 .

Answer

$$E_1 = \frac{kQ_1}{r^2} = \frac{(9 \times 10^9 \frac{Nm^2}{C^2})(9.1 \times 10^{-6} C)}{(8.0m)^2}$$

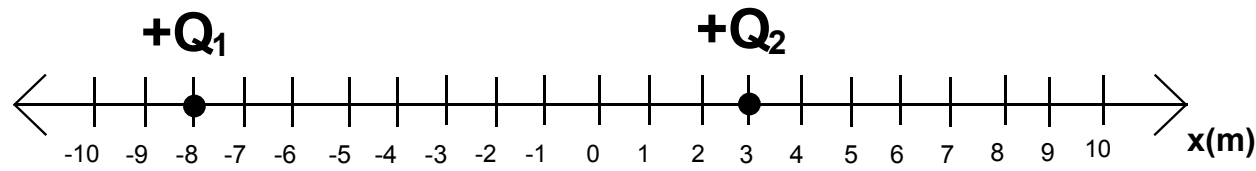
$$E_1 = 1.3 \times 10^3 \frac{N}{C}$$

directed towards the right (positive).



****The Net Electric Field Example**

Find the net electric field at the origin.



c. Find the magnitude and direction of the electric field at the origin due to charge Q_1 .

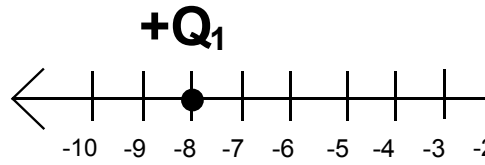
Answer



<http://njc.tl/g9>

****The Net Electric Field Example**

Find the net electric field at the origin.



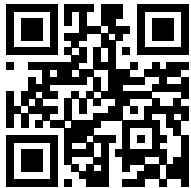
c. Find the magnitude of the net electric field at the origin due to charge $+Q_1$.

Answer

$$E_2 = \frac{kQ_2}{r^2} = \frac{(9 \times 10^9 \frac{Nm^2}{C^2})(3.0 \times 10^{-6} C)}{(3.0m)^2}$$

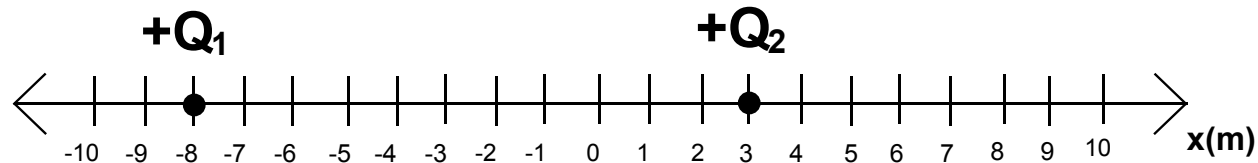
$$E_2 = -3.0 \times 10^3 \frac{N}{C}$$

directed towards the left (negative).



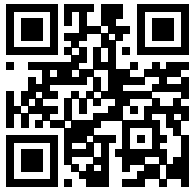
****The Net Electric Field Example**

Find the net electric field at the origin.



d. Find the magnitude and direction of the net electric field at the origin.

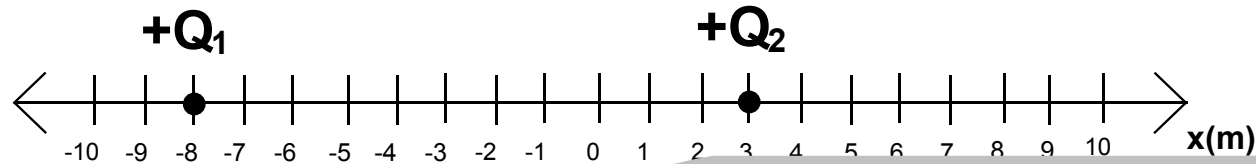
Answer



<http://njc.tl/g9>

****The Net Electric Field Example**

Find the net electric field at the origin.

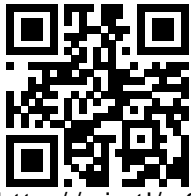


d. Find the magnitude and direction of the net electric field at the origin.

Answer

$$\begin{aligned}
 E_{net} &= E_1 + E_2 \\
 &= 1.3 \times 10^3 \frac{N}{C} - 3.0 \times 10^3 \frac{N}{C} \\
 E_{net} &= -1.7 \times 10^3 \frac{N}{C}
 \end{aligned}$$

directed towards the left



Electric Potential Energy

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<http://njc.tl/lq>

Electric Potential Energy

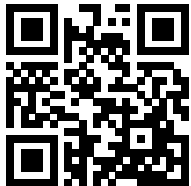


Start with two like charges initially at rest, with Q at the origin, and q at infinity.

In order to move q towards Q, a force opposite to the Coulomb repulsive force (like charges repel) needs to be applied.

$$\vec{F}_E = \frac{kQq}{r^2}$$

Note that this force is constantly increasing as q gets closer to Q, since it depends on the distance between the charges, r, and r is decreasing.



Work and Potential Energy

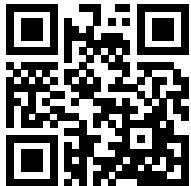


Recall that Work is defined as: $W = F \bullet r_{parallel}$

To calculate the work needed to bring q from infinity, until it is a distance r from Q , we need to use calculus, because of the non constant force. Then, use the relationship: $\Delta U_E = -W$

Assume that the potential energy of the Q - q system is zero at infinity, and adding up the incremental force times the distance between the charges at each point, we find that the Electric Potential Energy, U_E is:

$$U_E = \frac{kQq}{r}$$



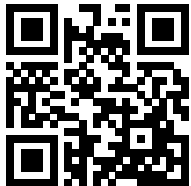
Electric Potential Energy

This is the equation for the potential energy due to two point charges separated by a distance r .

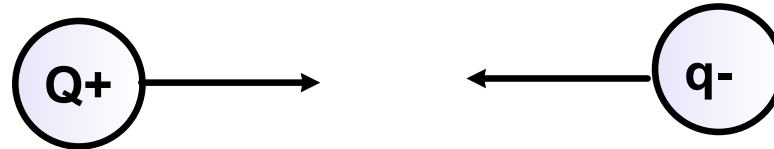
$$U_E = \frac{kQq}{r}$$

This process summarized on the previous page is similar to how Gravitational Potential Energy was developed.

The benefit of using Electric Potential Energy instead of the Electrical Force is that energy is a scalar, and calculations are much simpler. There is no direction, but the sign matters.



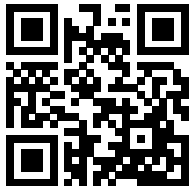
Electric Potential Energy



If you have a positive charge and a negative charge near each other, you will have a negative potential energy.

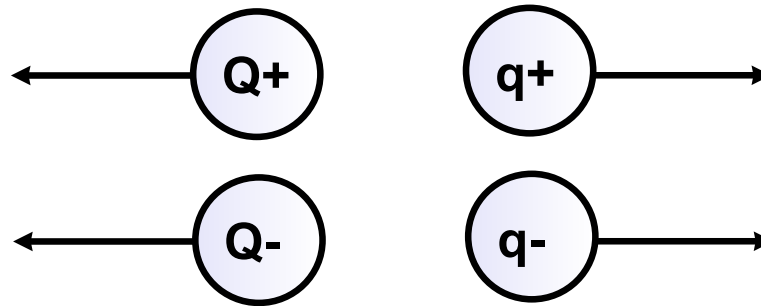
$$U_E = \frac{k(Q)(-q)}{r} = -\frac{kQq}{r}$$

This means that it takes work by an external agent to keep them from getting closer together.



<http://njc.tl/lq>

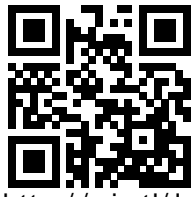
Electric Potential Energy



If you have two positive charges or two negative charges, there will be a positive potential energy.

$$U_E = \frac{k(Q)(q)}{r} = \frac{kQq}{r}$$

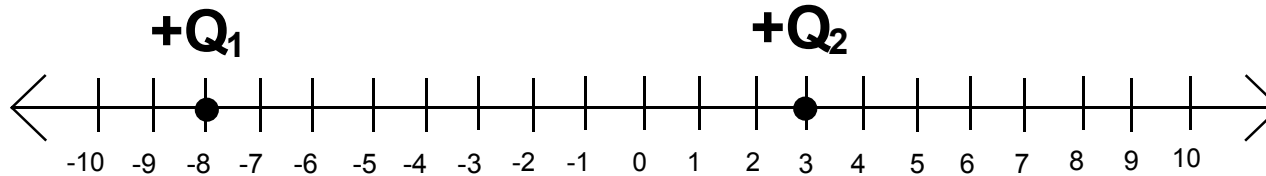
$$U_E = \frac{k(-Q)(-q)}{r} = \frac{kQq}{r}$$



<http://njc.tl/lq>

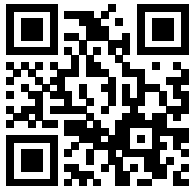
means that it takes work by an external agent to keep them flying apart.

12 Compute the potential energy of the two charges in the following configuration:

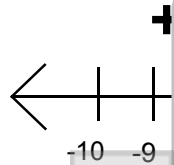


A positive charge, $Q_1 = 5.00 \text{ mC}$ is located at $x_1 = -8.00 \text{ m}$, and a positive charge $Q_2 = 2.50 \text{ mC}$ is located at $x_2 = 3.00 \text{ m}$.

Answer



12 Compute the electric potential in the following

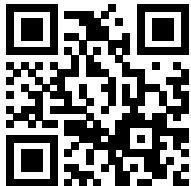


A positive charge is placed at a positive position

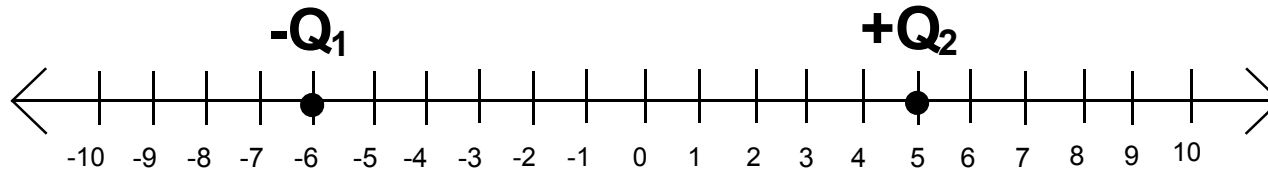
Answer

$$U_E = \frac{kQ_1Q_2}{r_{12}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(5 \times 10^{-3} \text{ C})(2.5 \times 10^{-3} \text{ C})}{11 \text{ m}}$$

$$= 1.02 \times 10^4 \text{ J}$$

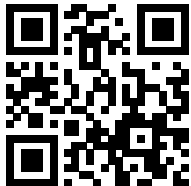


13 Compute the potential energy of the two charges in the following configuration:

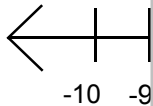


A negative charge, $Q_1 = -3.00$ mC is located at $x_1 = -6.00$ m, and a positive charge $Q_2 = 4.50$ mC is located at $x_2 = 5.00$ m.

Answer



13 Compute the potential energy of the two charges in the following

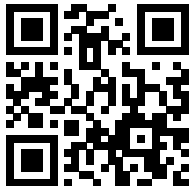


A negative
and a positive

Answer

$$U_E = \frac{kQ_1Q_2}{r_{12}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(-3 \times 10^{-3} \text{ C})(4.5 \times 10^{-3} \text{ C})}{11 \text{ m}}$$

$$= -1.10 \times 10^4 \text{ J}$$

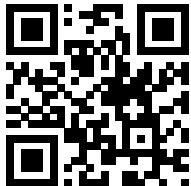


14 Compute the potential energy of the two charges in the following configuration:

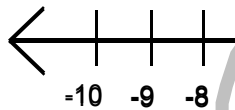


A negative charge, $Q_1 = -3.00$ mC is located at $x_1 = -6.00$ m, and a negative charge $Q_2 = -2.50$ mC is located at $x_2 = 7.00$ m.

Answer



14 Compute the potential energy of the two charges in the following configuration:

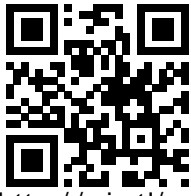


A negative
and a negative

Answer

$$U_E = \frac{kQ_1Q_2}{r_{12}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(-3 \times 10^{-3} \text{ C})(-2.5 \times 10^{-3} \text{ C})}{13 \text{ m}}$$

$$= 6.92 \times 10^3 \text{ J}$$



Electric Potential Energy of Multiple Charges

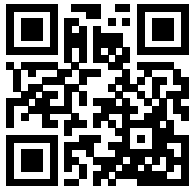
To get the total energy for multiple charges, you must first find the energy due to each pair of charges.

Then, you can add these energies together. Since energy is a scalar, there is no direction involved - but, there is a positive or negative sign associated with each energy pair.

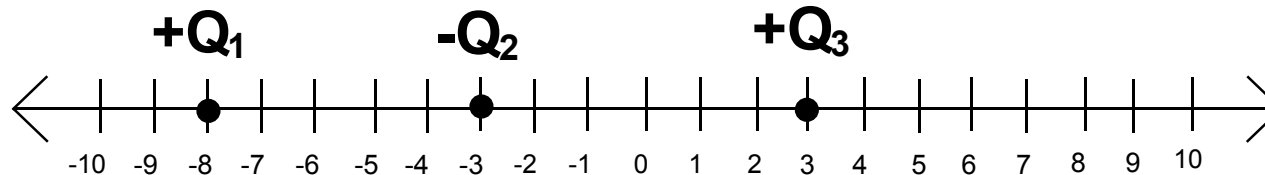
For example, if there are three charges, the total potential energy is:

$$U_T = U_{12} + U_{23} + U_{13}$$

... e U_{xy} is the Potential Energy of charges x and y.



15 Compute the potential energy of the three charges in the following configuration:

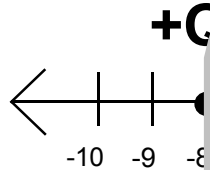


A positive charge, $Q_1 = 5.00 \text{ mC}$ is located at $x_1 = -8.00 \text{ m}$, a negative charge $Q_2 = -4.5 \text{ mC}$ is located at $x_2 = -3.00 \text{ m}$, and a positive charge $Q_3 = 2.50 \text{ mC}$ is located at $x_3 = 3.00 \text{ m}$.

Answer



15 Compute the potential energy of the three charges in the following



A positive
negative
positive

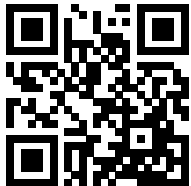
Answer

$$U_{12} = \frac{kQ_1Q_2}{r_{12}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(5 \times 10^{-3} \text{ C})(-4.5 \times 10^{-3} \text{ C})}{5 \text{ m}} = -4.05 \times 10^4 \text{ J}$$

$$U_{13} = \frac{kQ_1Q_3}{r_{13}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(5 \times 10^{-3} \text{ C})(2.5 \times 10^{-3} \text{ C})}{11 \text{ m}} = 1.02 \times 10^4 \text{ J}$$

$$U_{23} = \frac{kQ_2Q_3}{r_{13}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(-4.5 \times 10^{-3} \text{ C})(2.5 \times 10^{-3} \text{ C})}{6 \text{ m}} = -1.69 \times 10^4 \text{ J}$$

$$U_T = U_{12} + U_{23} + U_{13} = (-4.05 + 1.02 - 1.69) \times 10^4 \text{ J} = -4.72 \times 10^4 \text{ J}$$



Electric Potential (Voltage)

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<http://njc.tl/gf>

Electric Potential or Voltage

Our study of electricity began with Coulomb's Law which calculated the electric force between two charges, Q and q . By assuming q was a small positive charge, and dividing F by q , the electric field E due to the charge Q was defined.

$$F = \frac{kQq}{r^2} \qquad E = \frac{F}{q} = \frac{kQ}{r^2}$$

The same process will be used to define the Electric Potential, or V , from the Electric Potential Energy, where V is a property of the space surrounding the charge Q :

$$U_E = \frac{kQq}{r} \qquad V = \frac{U_E}{q} = \frac{kQ}{r}$$



<http://njc.tl/gf>

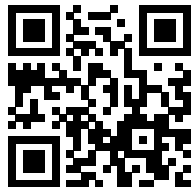
V is also called the voltage and is measured in volts.

Electric Potential or Voltage

Voltage is the Electric Potential Energy per charge, which is expressed as Joules/Coulomb. Hence:

$$V = \frac{J}{C}$$

To make this more understandable, a Volt is visualized as a battery adding 1 Joule of Energy to every Coulomb of Charge that goes through the battery.



<http://njc.tl/gf>

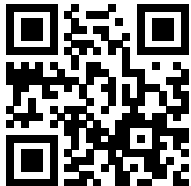
Electric Potential or Voltage

Another helpful equation can be found from $V = \frac{U_E}{q}$ by realizing

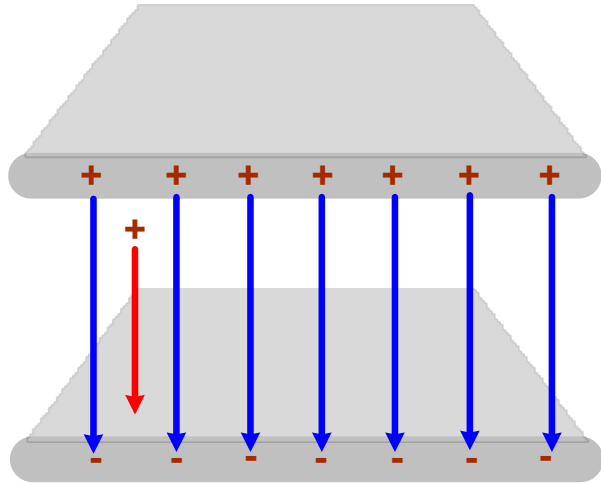
that the work done on a positive charge by an external force (a force that is external to the force generated by the electric field) will increase the potential energy of the charge, so that:

$$W = U_E = qV$$

Note, that the work done on a negative charge will be negative - the sign of the charge counts!

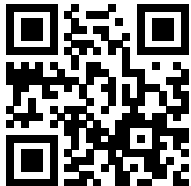


Electric Potential or Voltage



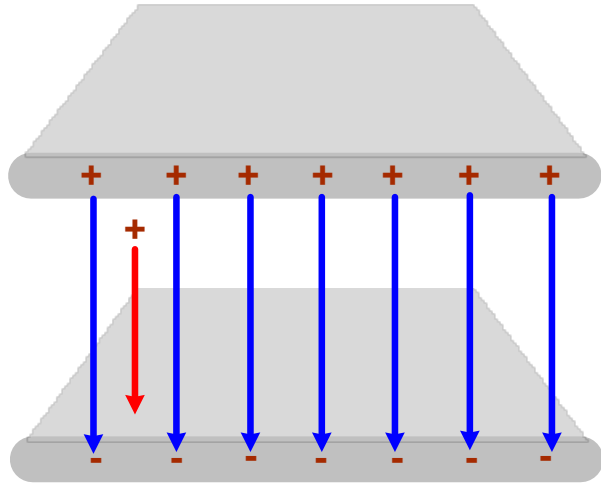
Consider two parallel plates that are oppositely charged. This will generate a uniform Electric Field pointing from top to bottom (which will be described in the next section).

A positive charge placed within the field will move from top to bottom. In this case, the Work done by the Electric Field is positive (the field is in the same direction as the charge's motion). The potential energy of the system will decrease - this is directly analogous to the movement of a mass within a gravitational Field.

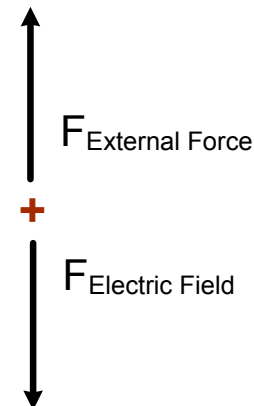


<http://njc.tl/gf>

Electric Potential or Voltage



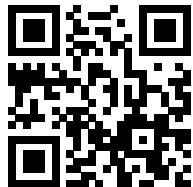
If there is no other force present, then the charge will accelerate to the bottom by Newton's Second Law.



But, if we want the charge to move with a constant velocity, then an external force must act opposite to the Electric Field force.

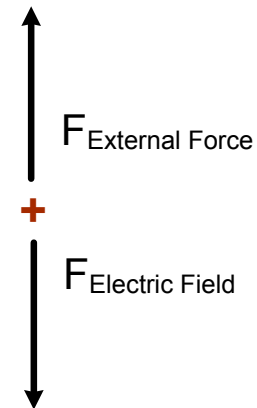
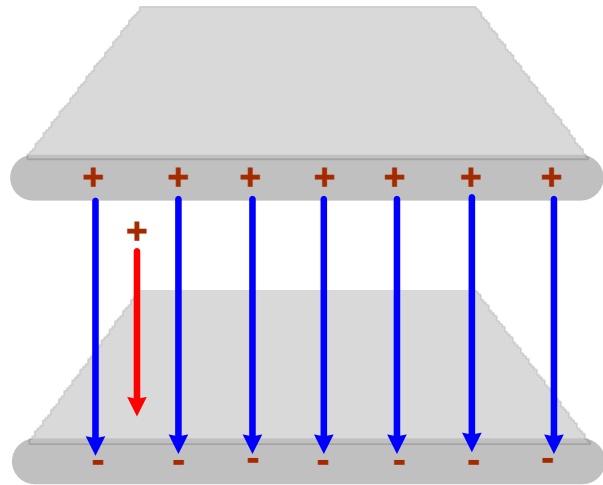
This external force is directed upwards. Since the charge is still

moving down (but not accelerating), the Work done by the external force is negative.

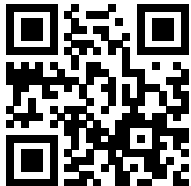


<http://njc.tl/gf>

Electric Potential or Voltage

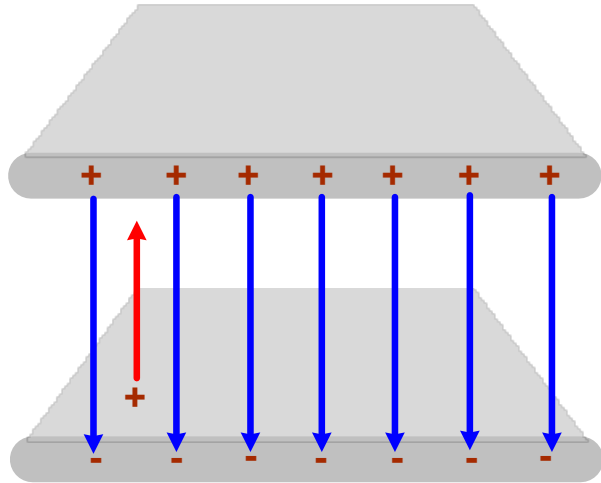


The Work done by the external force is negative.
The Work done by the Electric Field is positive.
The Net force, and hence, the Net Work, is zero.
The Potential Energy of the system decreases.



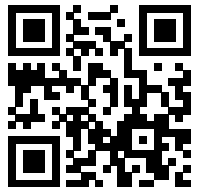
<http://njc.tl/gf>

Electric Potential or Voltage



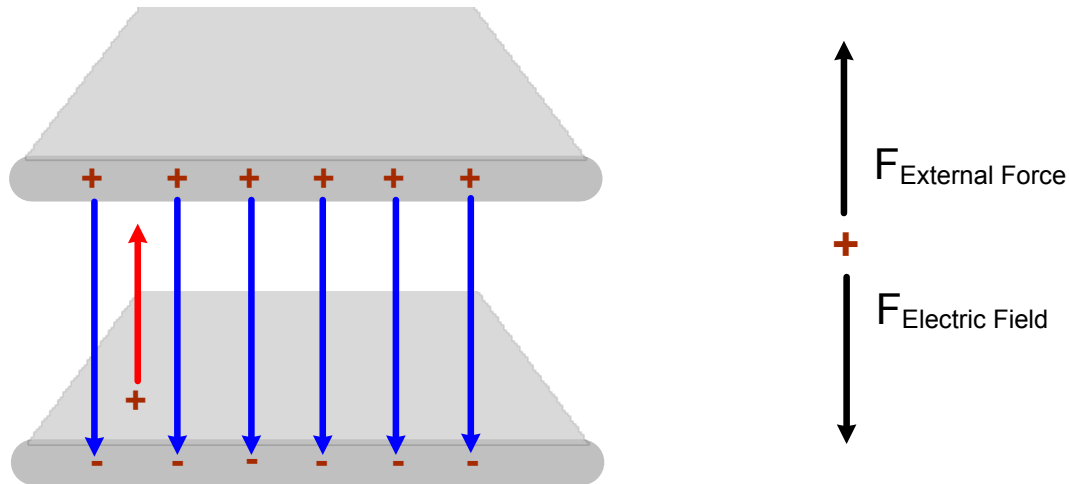
Now consider the case where we have a positive charge at the bottom, and we want to move it to the top.

In order to move the charge to the top, an external force must act in the up direction to oppose the Electric Field force which is directed down. In this case, the Work done by the Electric Field is negative (the field is opposite the direction of the charge's motion). The potential energy of the system will increase - again, is directly analogous to the movement of a mass within a gravitational field.

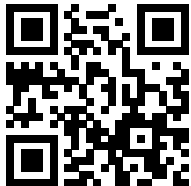


<http://njc.tl/gf>

Electric Potential or Voltage

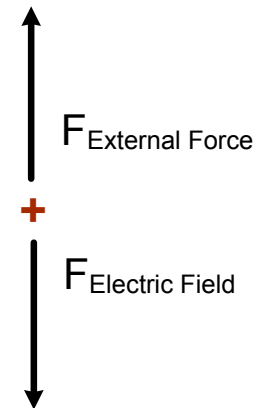
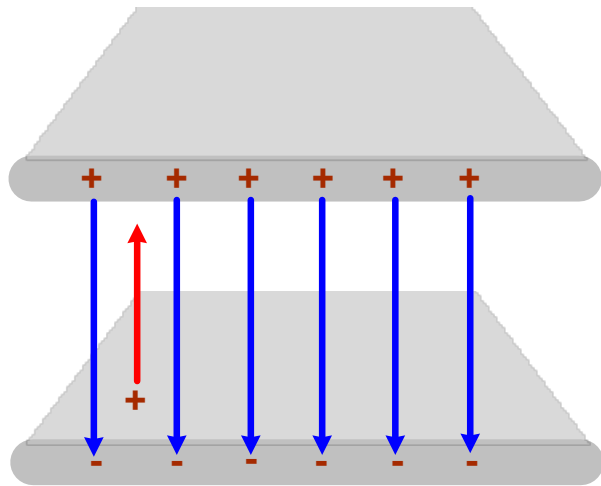


If the charge moves with a constant velocity, then the external force is equal to the Electric Field force. Since the charge is moving up (but not accelerating), the Work done by the external force is positive.

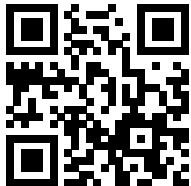


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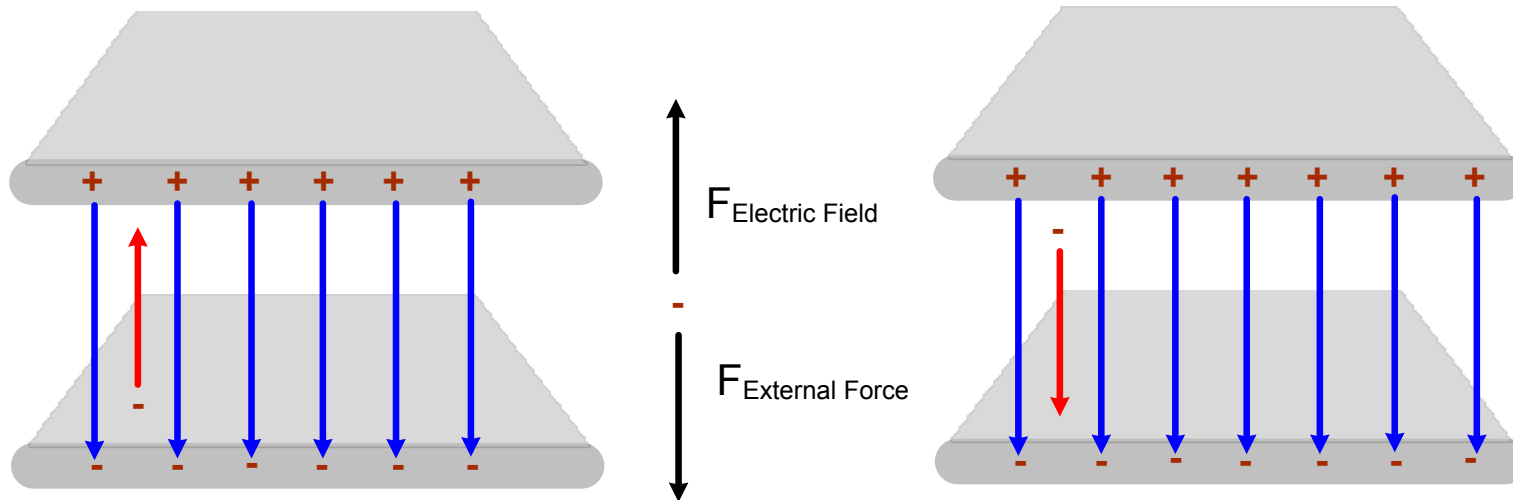
Electric Potential or Voltage



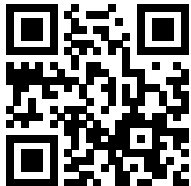
The Work done by the external force is positive.
The Work done by the Electric Field is negative.
The Net force, and hence, the Net Work, is zero.
The Potential Energy of the system increases.



Electric Potential or Voltage

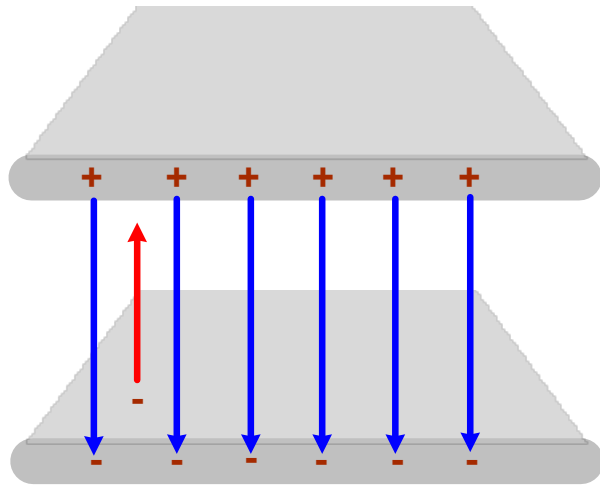


Similar logic works for a negative charge in the same Electric Field. But, the directions of the Electric Field force and the external force are reversed, which will change their signs, and the potential energy as summarized on the next slide.

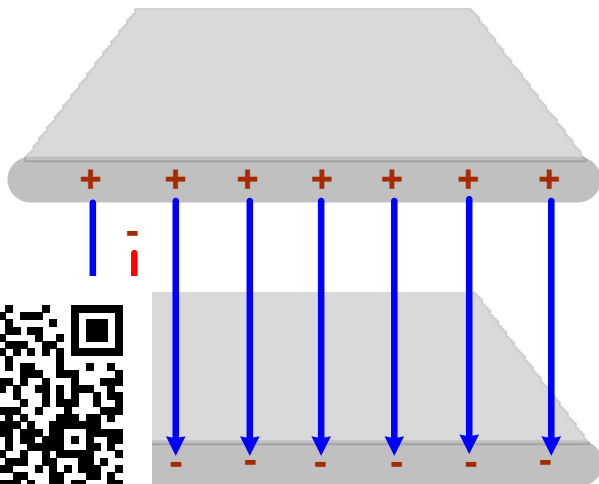
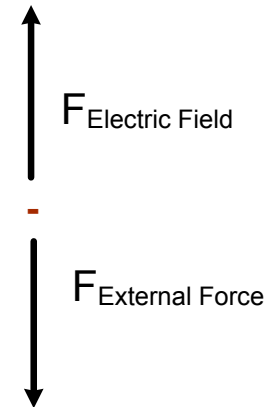


<http://njc.tl/gf>

Electric Potential or Voltage



Work done by the external force is negative.
 Work done by the Electric Field is positive.
 Net force, and hence, the Net Work, is zero.
 Potential Energy of the system decreases.



Work done by the external force is positive.
 Work done by the Electric Field is negative.
 Net force, and hence, the Net Work, is zero.
 Potential Energy of the system increases.



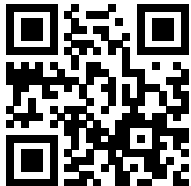
<http://njc.tl/gf>

Electric Potential or Voltage

Like Electric Potential Energy, Voltage is NOT a vector, so multiple voltages can be added directly, taking into account the positive or negative sign.

Like Gravitational Potential Energy, Voltage is not an absolute value - it is compared to a reference level. By assuming a reference level where $V=0$ (as we do when the distance from the charge generating the voltage is infinity), it is allowable to assign a specific value to V in calculations.

The next slide will continue the gravitational analogy to help understand this concept.

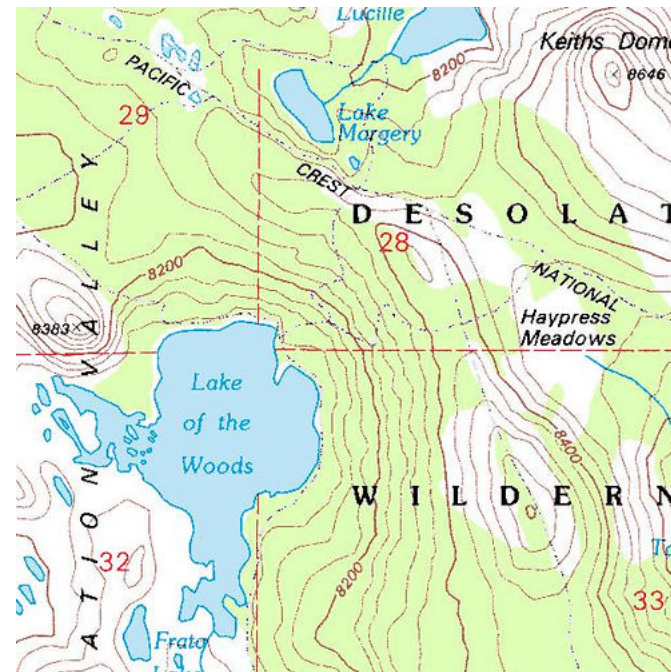


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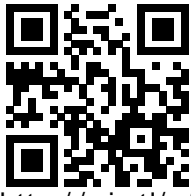
Topographic Maps

Each line represents the same height value. The area between lines represents the change between lines.

A big space between lines indicates a slow change in height. A little space between lines means there is a very quick change in height.

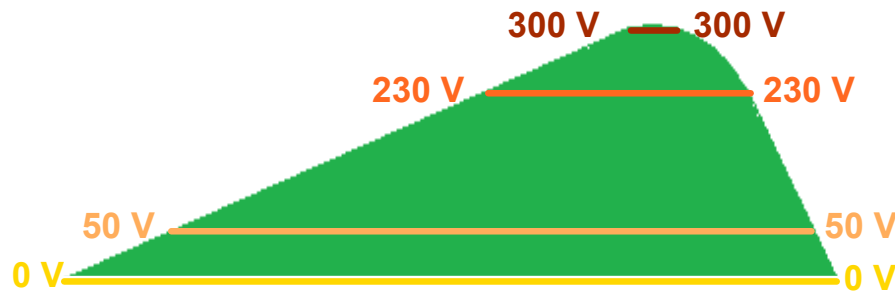


Where in this picture is the steepest incline?

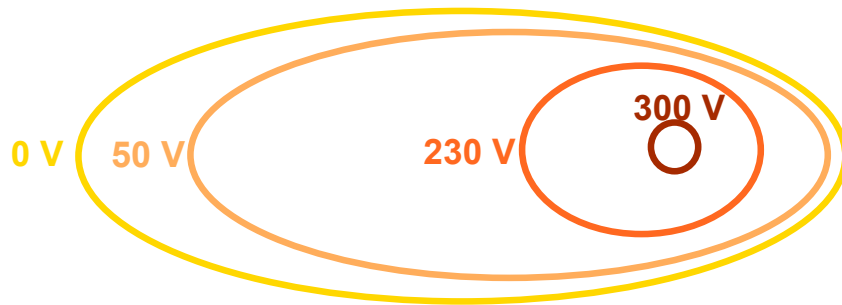


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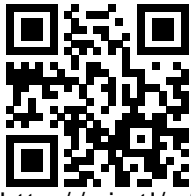
Equipotential Lines



These "topography" lines are called "Equipotential Lines" when we use them to represent the Electric Potential - they represent lines where the Electric Potential is the same.



The closer the lines, the faster the change in voltage.... the bigger the change in Voltage, the larger the Electric Field.

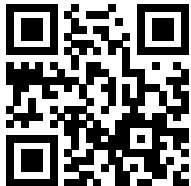
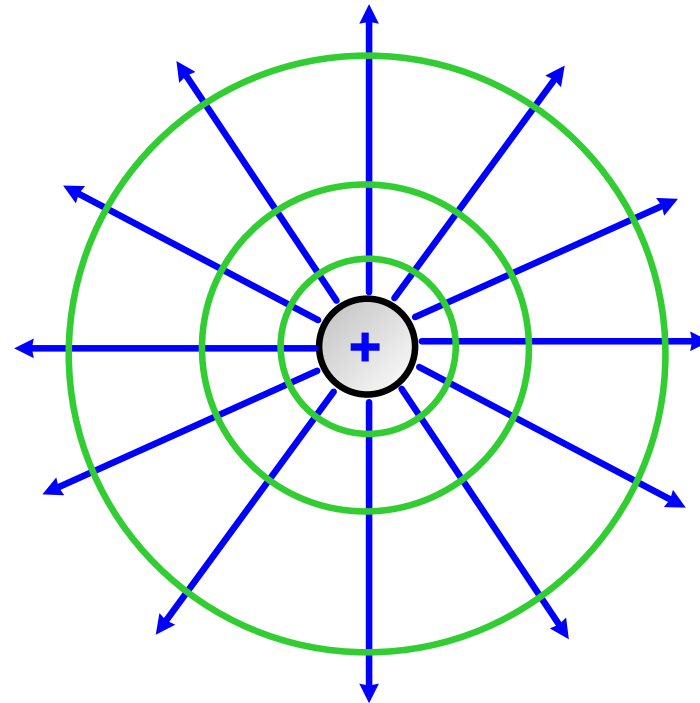


Equipotential Lines

The direction of the Electric Field lines are always perpendicular to the Equipotential lines.

The Electric Field lines are farther apart when the Equipotential lines are farther apart.

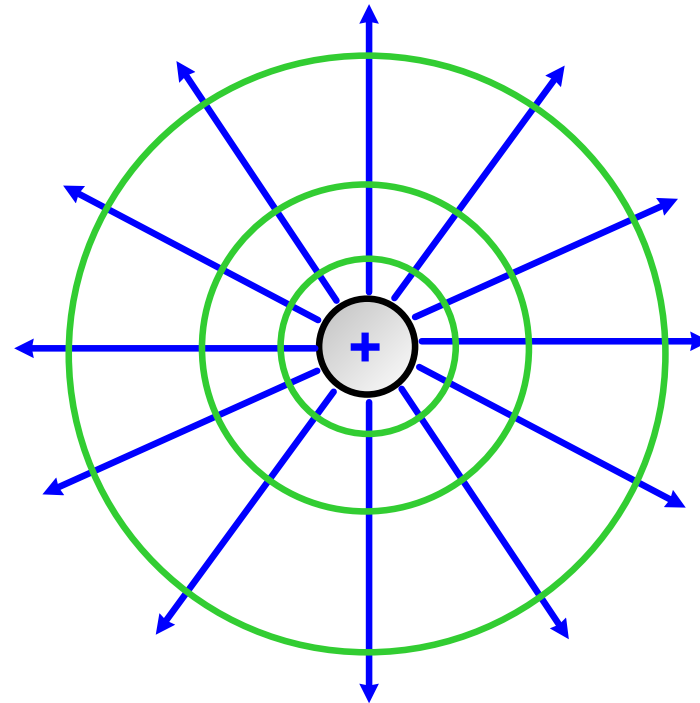
The Electric Field goes from high to low potential (just like a positive charge).



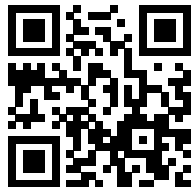
<http://njc.tl/gf>

Equipotential Lines

For a positive charge like this one the equipotential lines are positive, and decrease to zero at infinity. A negative charge would be surrounded by negative equipotential lines, which would also go to zero at infinity.

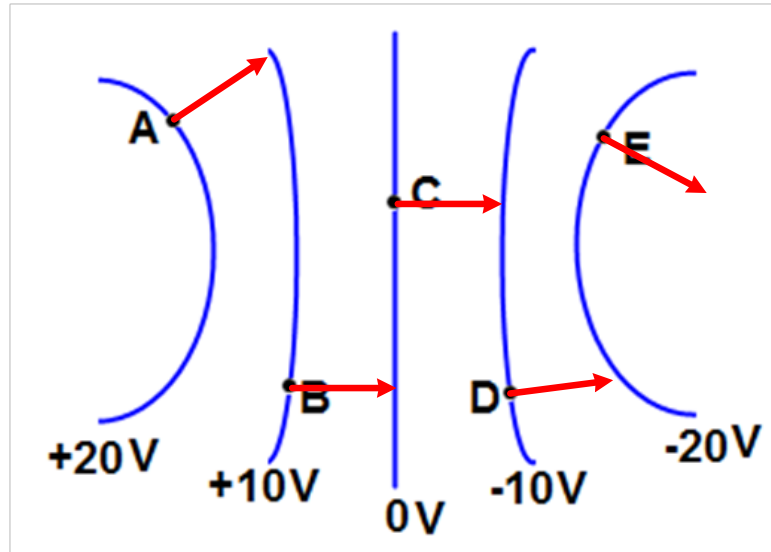


More interesting equipotential lines (like the topographic lines on a map) are generated by more complex charge configurations.



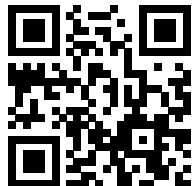
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Equipotential Lines



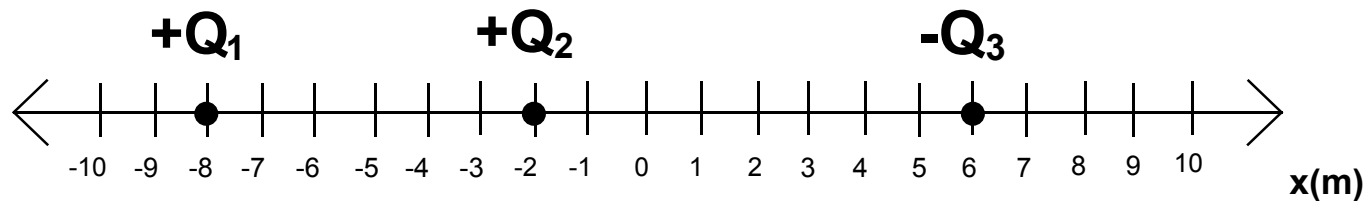
This configuration is created by a positive charge to the left of the +20 V line and a negative charge to the right of the -20 V line.

Note the signs of the Equipotential lines, and the directions Electric Field vectors (in red) which are perpendicular to the lines tangent to the Equipotential lines.



<http://njc.tl/gf>

16 Compute the electric potential of three charges at the origin in the following configuration:



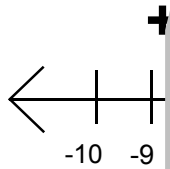
A positive charge, $Q_1 = 5.00$ nC is located at $x_1 = -8.00$ m, a positive charge $Q_2 = 3.00$ nC is located at $x_2 = -2.00$ m, and a negative charge $Q_3 = -9.00$ nC is located at $x_3 = 6.00$ m.

Answer



<http://njc.tl/gg>

16 Compute the electric potential of three charges at the origin in V



A positive
positive
negative

Answer

$$V_1 = \frac{kQ_1}{r_{12}} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(5 \times 10^{-9} \text{ C})}{8 \text{ m}} = 5.63 \text{ V}$$

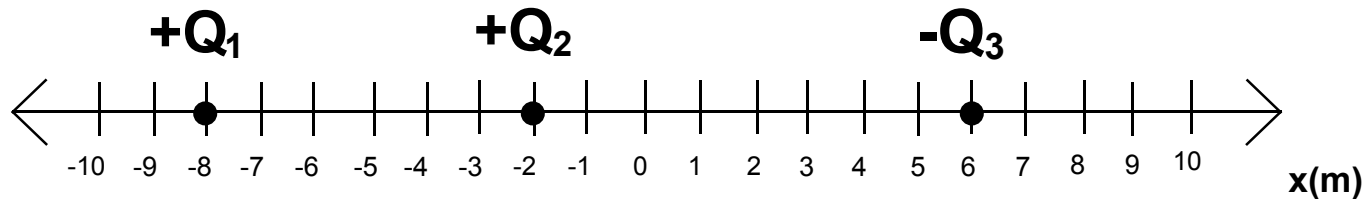
$$V_2 = \frac{kQ_2}{r_2} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(3 \times 10^{-9} \text{ C})}{2 \text{ m}} = 13.5 \text{ V}$$

$$V_3 = \frac{kQ_3}{r_3} = \frac{(9 \times 10^9 \text{ Nm}^2 / \text{C}^2)(-9 \times 10^{-9} \text{ C})}{6 \text{ m}} = -13.5 \text{ V}$$

$$V_T = V_1 + V_2 + V_3 = 5.63 \text{ V} + 13.5 \text{ V} - 13.5 \text{ V} = 5.63 \text{ V}$$

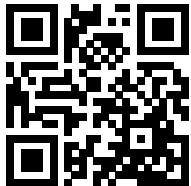


17 How much work must be done by an external force to bring a 1×10^{-6} C charge from infinity to the origin of the following configuration?



A positive charge, $Q_1 = 5.00$ nC is located at $x_1 = -8.00$ m, a positive charge $Q_2 = 3.00$ nC is located at $x_2 = -2.00$ m, and a negative charge $Q_3 = -9.00$ nC is located at $x_3 = 6.00$ m.

Answer



17 How much work must be done by an external force to
 origin of the

Answer

The electric potential of this configuration was found to equal 5.63 V. Positive work must be done to move a positive charge towards an area of positive Electric Potential:

$$W = qV = (1 \times 10^{-6} \text{ C}) \times 5.63 \text{ V}$$

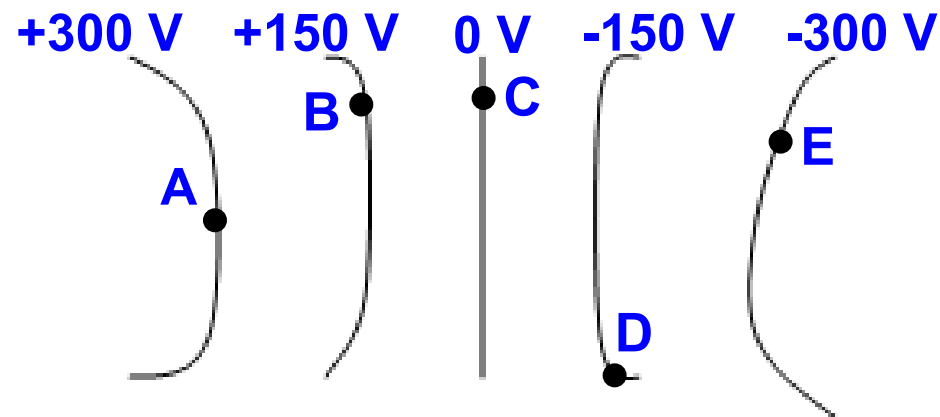
$$= 5.63 \times 10^{-6} \text{ J}$$



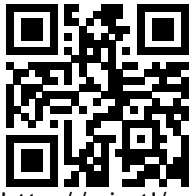
<http://njc.tl/gh>

18 At point A in the diagram, what is the direction of the Electric Field?

- ☐ A Up
- ☐ B Down
- ☐ C Left
- ☐ D Right

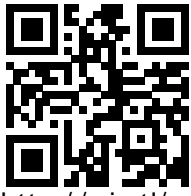
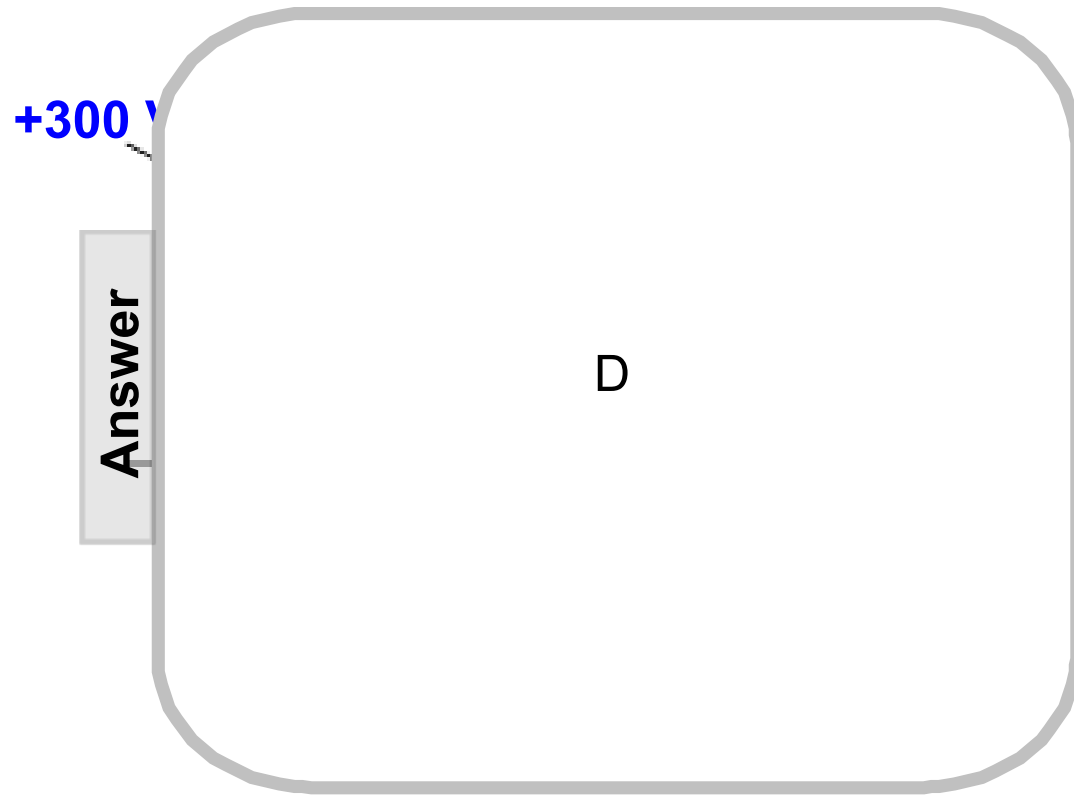


Answer

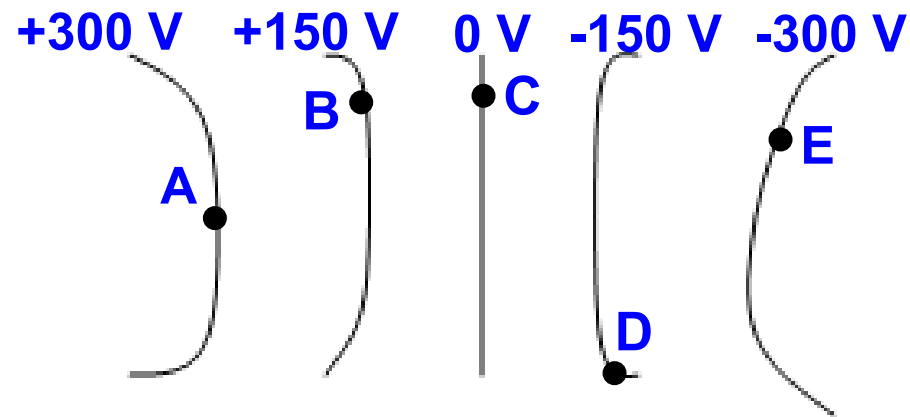


18 At point A in the diagram, what is the direction of the Electric Field?

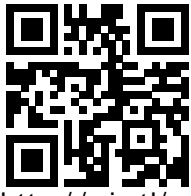
- ☐ A Up
- ☐ B Down
- ☐ C Left
- ☐ D Right



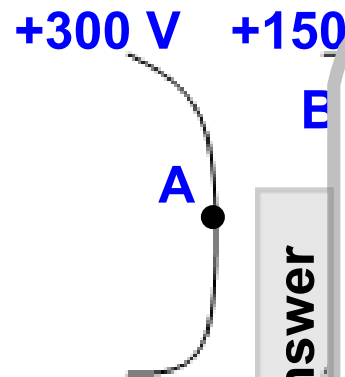
19 How much work is done by an external force on a $10\mu\text{C}$ charge that moves from point C to B?



Answer



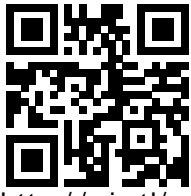
19 How much work is done by an external force on a $10\mu\text{C}$ charge that moves from point C to B?



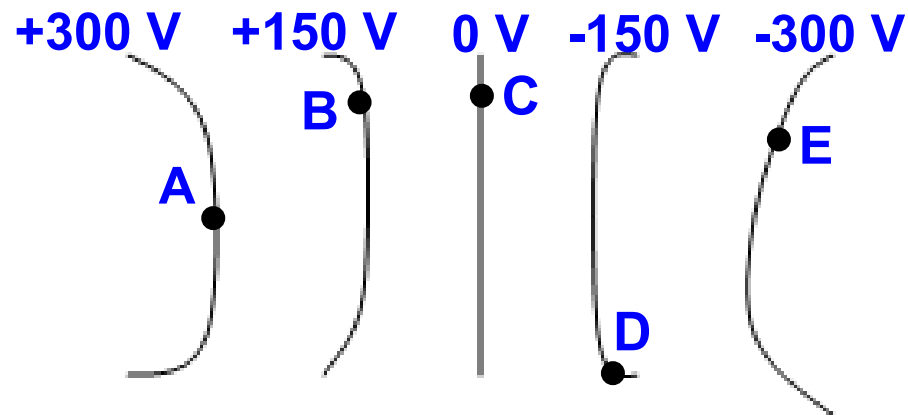
$$W = q\Delta V$$

$$W = (10 \times 10^{-6} \text{ C})(150 \text{ V} - 0 \text{ V})$$

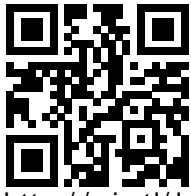
$$W = 1.5 \times 10^{-3} \text{ J}$$



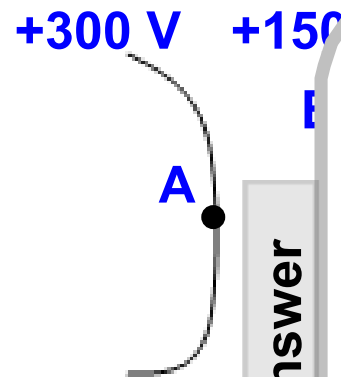
20 How much work is done by an external force on a $-10\mu\text{C}$ charge that moves from point C to B?



Answer



20 How much work is done by an external force on a $-10\mu\text{C}$ charge that moves from point C to B?



$$W = q\Delta V$$

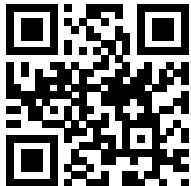
$$W = (-10 \times 10^{-6} \text{ C})(150 \text{ V} - 0 \text{ V})$$

$$W = -1.5 \times 10^{-3} \text{ J}$$



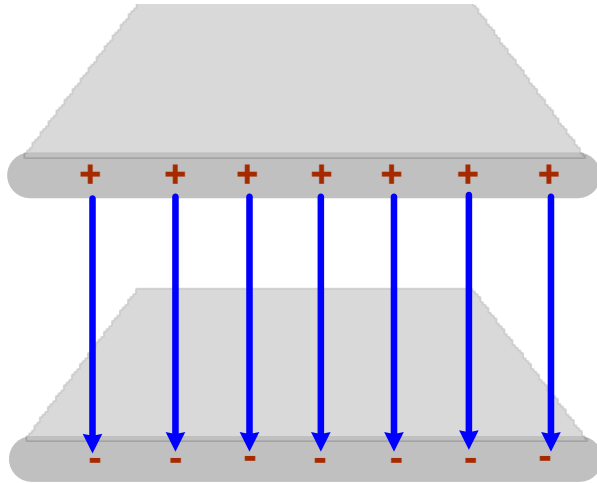
Uniform Electric Field

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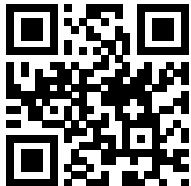
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Uniform Electric Field



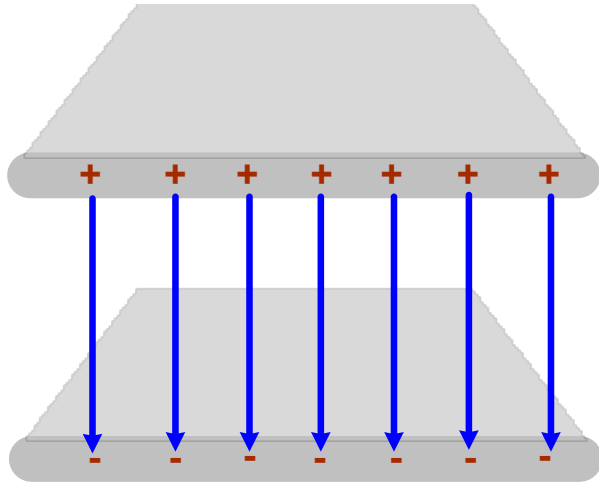
Up until now, we've dealt with Electric Fields and Potentials due to individual charges. What is more interesting, and relates to practical applications is when you have configurations of a massive amount of charges.

Let's begin by examining two infinite planes of charge that are separated by a small distance. The planes have equal amounts of charge, with one plate being charged positively, and the other, negatively. The above is a representation of two infinite planes (rather hard to draw infinity).



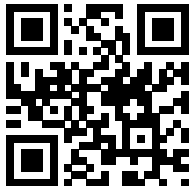
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Uniform Electric Field

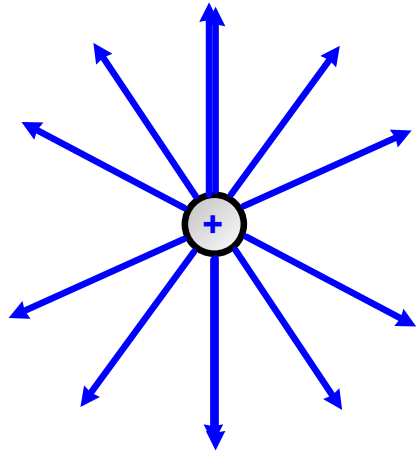


By applying Gauss's Law (a law that will be learned in AP Physics), it is found that the strength of the Electric Field will be uniform between the plates - it will have the same value everywhere between the plates.

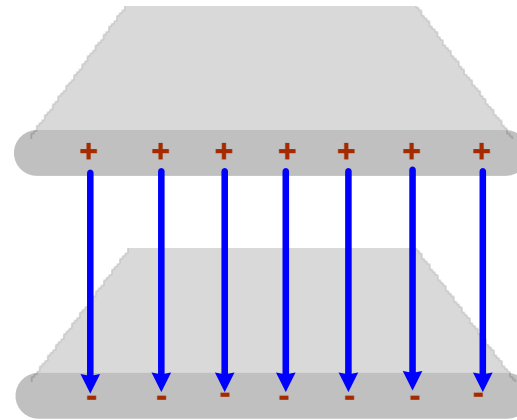
And, the Electric Field outside the two plates will equal zero.



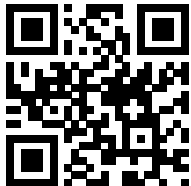
Uniform Electric Field



Point charges have a non-uniform field strength since the field weakens with distance.



Only some equations we have learned will apply to uniform electric fields.

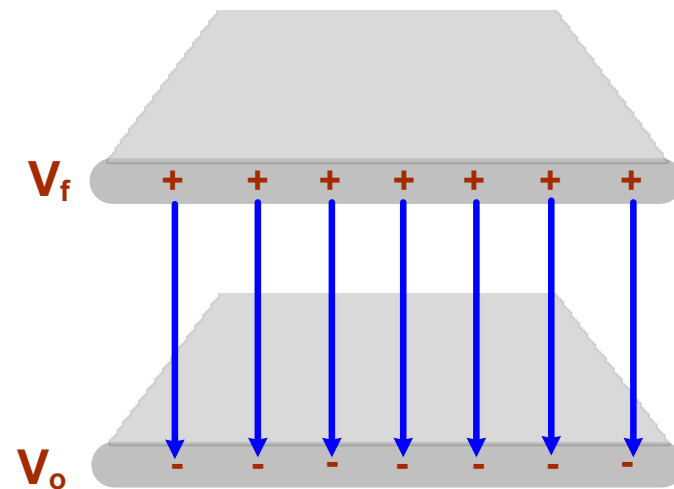


<http://njc.tl/gk>

Uniform Electric Field & Voltage

For the parallel planes, the Electric Field is generated by the separation of charge - with the field lines originating on the positive charges and terminating on the negative charges.

The difference in electric potential (voltage) is responsible for the electric field.

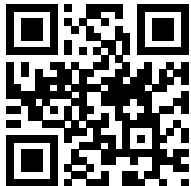
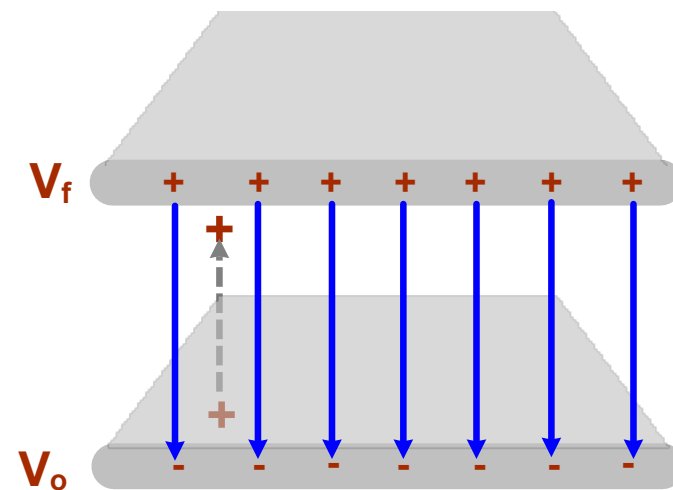


Uniform Electric Field & Voltage

The change in voltage is defined as the work done per unit charge against the electric field.

Therefore energy is being put into the system when a positive charge moves in the opposite direction of the electric field (or when a negative charge moves in the same direction of the electric field).

Positive work is being done by the external force, and since the positive charge is moving opposite the Electric Field, negative work is being done by



<http://njc.tl/gk>

Uniform Electric Field & Voltage

For a field like this, a very interesting equation relating Voltage and the Electric Field can be derived.

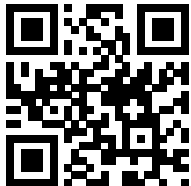
Since the work done by the Electric Field is negative, and the force is constant on the positive charge, the Work-Energy Equation is used:

$$U_E = -W = -F\Delta x = -qE\Delta x \quad \text{where } d \text{ is the distance between the two planes.}$$

Divide both sides by q .

$$\frac{U_E}{q} = -Ed \quad \text{and recognize that} \quad \frac{U_E}{q} = \Delta V$$

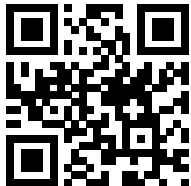
$$\Delta V = -Ed \quad \text{or} \quad E = -\frac{\Delta V}{d}$$



Uniform Electric Field & Voltage

$$E = \frac{-\Delta V}{\Delta x} = \frac{-\Delta V}{d}$$

A more intuitive way to understand the negative sign in the relationship is to consider that just like a mass falls down in a gravitational field, from higher gravitational potential energy to lower, a positive charge "falls down" from a higher electric potential (V) to a lower value.



<http://njc.tl/gk>

Uniform Electric Field & Voltage

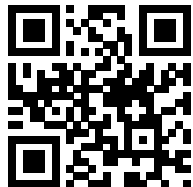
$$E = \frac{-\Delta V}{\Delta x} = \frac{-\Delta V}{d}$$

Since the electric field points in the direction of the force on a hypothetical positive test charge, it must also point from higher to lower potential.

The negative sign just means that objects feel a force from locations with greater potential energy to locations with lower potential energy.

This applies to all forms of potential energy.

This "sign" issue is a little tricky - and will be covered in more depth in AP Physics course. For now, we will just use the magnitude of electric field in the problems (so, no negative sign).



<http://njc.tl/gk>

Uniform Electric Field & Voltage

The equation only applies to uniform electric fields

$$E = \frac{-\Delta V}{\Delta x} = \frac{-\Delta V}{d}$$

It follows that the electric field can also be shown in terms of volts per meter (V/m) in addition to Newtons per Coulomb (N/C).

$$\frac{V}{m} = \frac{J/C}{m}$$

Since $V = J/C$.

$$\frac{J/C}{m} = \frac{N \cdot m/C}{m}$$

Since $J = N \cdot m$.

$$\frac{N \cdot m/C}{m} = \frac{N}{C}$$

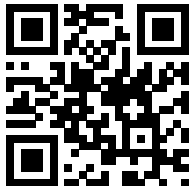
The units are equivalent.



21 In order for a charged object to experience an electric force, there must be a:

- ☐ A large electric potential
- ☐ B small electric potential
- ☐ C the same electric potential everywhere
- ☐ D a difference in electric potential

Answer



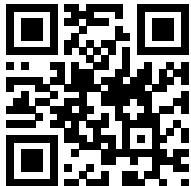
<http://njc.tl/gl>

21 In order for a charged object to experience an electric force, there must be a:

- ☐ A large electric potential
- ☐ B small electric potential
- ☐ C the same electric potential
- ☐ D a difference in electric potential

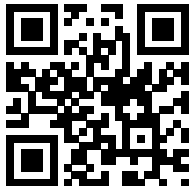
Answer:

D



22 How strong (in V/m) is the electric field between two metal plates 0.25 m apart if the potential difference between them is 100 V?

Answer



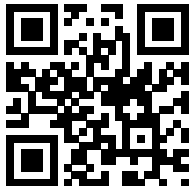
<http://njc.tl/gm>

22 How strong (in V/m) is the electric field between two metal plates 0.25 m apart if the potential difference between them is 100 V?

Answer

$$E = \frac{\Delta V}{d} = \frac{100V}{0.25m}$$

$$E = 400 \frac{V}{m}$$



23 An electric field of 3500 N/C is desired between two plates which are 0.0040 m apart; what Voltage should be applied?

Answer



<http://njc.tl/gn>

23 An electric field of 3500 N/C is desired between two plates which are 0.0040 m apart; what Voltage should be applied?

Answer

$$E = \frac{\Delta V}{d}$$

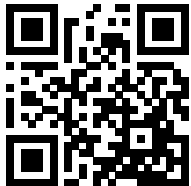
$$\Delta V = Ed = \left(3500 \frac{N}{C} \right) (4 \times 10^{-3} m)$$

$$\Delta V = 14V$$



24 How much Work is done by a uniform 300 N/C Electric Field on a charge of 6.1 mC in accelerating it through a distance of 0.20 m?

Answer



<http://njc.tl/go>

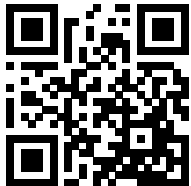
24 How much Work is done by a uniform 300 N/C Electric Field on a charge of 6.1 mC in accelerating it through a distance of 0.20 m?

Answer

$$W = q\Delta V = qEd$$

$$W = \left(300 \frac{N}{C}\right)(6.1 \times 10^{-3} C)(2.0 \times 10^{-1} m)$$

$$W = 0.37 J$$



Uniform Electric Field & Voltage Summary

$$F = \frac{kQq}{r^2}$$

$$E = \frac{kQ}{r^2}$$

$$U_E = \frac{kQq}{r}$$

$$V = \frac{kQ}{r}$$

Use
ONLY
with point
charges.

Equations
with the "k"
are point
charges
ONLY.

$$F = qE$$

$$U_E = qV$$

Use in
ANY
situation.

For point
charges **AND**
uniform electric
fields

$$U_E = -qEd$$

$$E = -\frac{\Delta V}{d}$$

ONLY for
uniform electric
fields

