Dynamics

Introduction

While kinematics allows us to describe motion, dynamics lets us determine how that motion will change. Dynamics is grounded in our intuition of how the world works, but reaches beyond that intuition. It represents an approach that turns out to be so valuable that it can be described as the very foundation of physics. It is the first step in our journey to describe nature in a way that reaches beyond our common sense.

We all developed our common sense about how the world works based on the instincts and understandings that have allowed humans to survive on this planet for so many years. Those understandings have served us well throughout that time and they are correct within their context. If they were not, we could not have relied on them to survive. However, as we strive to extend our understanding beyond what is necessary to survive, we will find that we need to question some of the very ideas that we have relied upon.

Concepts that seem very familiar, such as force, mass, etc. will be revisited and defined in very precise ways that allow them to be used to understand our universe in a new way. While doing this, you will still need to maintain your intuition about how things work. You will not be asked to discard those intuitions. In fact, we will rely on your making use of them. But you will need to flexible enough to reach beyond them to grasp their deeper meaning.

Inertial Reference Frames

The framework that will be developed in this chapter is built on Newton's three laws of motion. Those laws were developed to work in **inertial reference frames**; the same reference frames that were discussed in the last chapter. An inertial reference frame is anyplace that is not accelerating. For instance, if you are sitting in a room reading this book, you can consider yourself in an inertial reference frame. Similarly, if you are in a car driving at a constant velocity on a smooth road, you are also in an inertial reference frame. The laws of motion that we will be developing in this chapter apply to you. On the other hand, if you are in a car that is accelerating onto the highway, you are not in an inertial reference frame. The system of dynamics being developed in this chapter has to be modified to explain the motion of objects in that car.

A good test to see if you are in an inertial reference frame will turn out to be whether the laws that we are developing in this chapter work for you. If you do an experiment and find that Newton's laws of motion are obeyed, then you are in an inertial reference frame. If they are not obeyed, then you are not in one. That's a bit circular, but it works within the self-consistent system that we will be developing.

It turns out that there are no completely valid inertial reference frames. The room that you are sitting in is spinning once around the axis of the earth every 24 hours. And the earth is spinning around the sun once every 365 days. And the sun is spinning around the center of the galaxy once every __ million years. And even our galaxy is moving with respect to the Local Group of galaxies. As we will learn later, in our study of circular motion, all of these rotations involve accelerations, since the direction of our motion keeps changing. As a result, there are no pure inertial reference frames easily available to us. However, these effects turn out to be very small compared to the problems that we will be solving so we can treat the room that you are sitting in an inertial reference frame for most purposes. However, it is worth taking a moment to consider the wild path that you and your room are taking through space as you read this.

Force

To a great extent your intuitive understanding of **force** will work in dynamics. Whenever you push on something you are exerting a force on it. Or when something pushes on you, it is exerting a force on you. When you drop an object, you can think of the earth as pulling it down, exerting a force on that object. These are a few examples of forces. You'll be learning about a number of forces in this book. Forces like friction, electricity, magnetism, gravity, etc. However, there are some common features of all forces that we will introduce here and develop further as they become necessary.

A very important property of a force is that it always involves two interacting objects. You can exert a force by pushing on something, but it wouldn't mean anything to say that you are exerting a force by pushing on nothing. Similarly, something can push on you, but it's meaningless to say that you are experiencing a force because "nothing" is pushing on you. For a force to exist there must be something pushing and something being pushed. A force always involves the interaction between two different things. So when we describe a force we will always need to identify the two objects that are involved in that force.

As problems get more complicated we will have to keep track of numerous forces. We will use subscripts to do that. The first subscript will represent the object being pushed while the second will indicate the object doing the pushing. So, for example, if I am pushing an object with my hand, I would describe the force on the object by: $\mathbf{F_{object\ hand}}$ or $\mathbf{F_{oh}}$. Or if I want to describe the force on a book due to earth pulling down on it I would write $\mathbf{F_{book\ earth}}$ or $\mathbf{F_{be}}$. By reversing the subscripts I can describe the force on my hand due to the object as $\mathbf{F_{ho}}$ or the force on the earth due to the book as $\mathbf{F_{eb}}$. **The subscript for the object experiencing the force is always written first while the object exerting the force is written second.** The relationship between these pairs of forces, for instance $\mathbf{F_{ho}}$ and $\mathbf{F_{oh}}$, will be discussed when you are introduced to Newton's Third Law a bit later in this chapter.

The SI unit of force is the Newton (N). When a force is being specified its size, or magnitude, is given in Newtons. However, **a critical property of forces is that they are vectors**. As in the case of the vectors discussed in the prior chapter, a force must be described by giving both its magnitude and its direction. So it is not sufficient to say, for example, that $\mathbf{F_{be}}$ (the force on a book due to the earth) is 20 N. You need to say that F_{be} is 20 N towards the center of the earth. You need to specify both the size of the force (20N) and its direction (towards the center of the earth).

When more than one force acts on an object those forces must be added together as vectors. One implication of this is that forces that act in the same direction will add together, while forces that act in opposite directions will reduce each other. The critical determiner of how an object moves will not be any one force that acts on it, but rather, the sum of all the forces that act on it. **The vector sum of all the forces that act on an object is called the net force on the object.** This will be discussed further as part of Newton's Second Law.

Newton's First Law

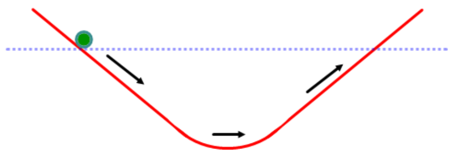
Galileo was actually the developer of the principal of **inertia** that was to become Newton's first law. Newton used that principal as the first of the three pillars that was to support his theory of dynamics, which is how it came to be known as Newton's First Law. However, the thinking that developed the underlying principal came from Galileo. His thinking represents an excellent example of how physics both uses human intuition and moves beyond it.

Our common sense tells us that an object will naturally stay at rest unless something, or someone, is pushing it. If a book were seen to be sliding along the floor, we would wonder why. Who pushed it? However, if a book were to remain still on the floor, we would not wonder why. It would seem perfectly reasonable that it remain in one place. In fact, if we came back to the room, and it wasn't there, we would reasonably wonder who moved it.

Our intuition tells us that the natural state of an object is at rest. Even if something is moving, we expect that it will eventually come to a stop. If you give that book, which had been sitting on the floor, a quick shove it will move for some distance and then come to a stop. If you want to keep it moving you know that you'll have to push it again.

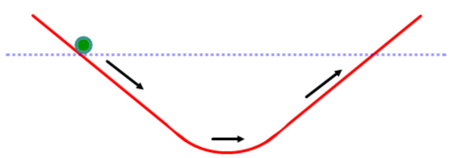
Lift the book up off the floor and then let go of it. It'll fall to ground as our intuition tells us is natural for unsupported objects. But upon reaching the floor it will once again come to a stop. Moving objects just naturally seem to come to rest and, once at rest, they don't start moving again on their own. Galileo was the first person that we know of to question this idea and demonstrate that it is limited to the context of our everyday experience. He recognized that moving beyond that intuition would lead to a much deeper understanding of how the world works. Let's follow his general approach to this question.

Figure 1:

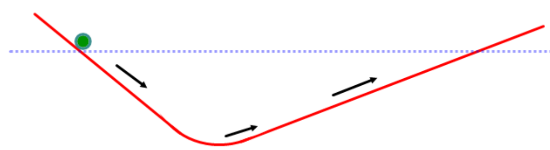


First, **imagine** a device that consists of two ramps that are connected together (Figure 1). If we were to release a ball on the first ramp it would roll down that ramp and then up the second ramp to a height that is less than the original height from which it was released. If we make the ball and the ramp smoother and increase the density of the ball, making it heavier for its size, we will find that it gets closer and closer to its original height. The more **ideal** we make the ramps and the ball **-** the closer the ball gets to reaching its original height on the second ramp. We can now imagine the ideal case; if we were to make the ball and the ramp perfectly smooth and eliminate all air resistance that the ball would make it back to its original height. This step of doing practical experiments but then extending the results by imagining a more ideal set of circumstances was a critical element of Galileo's genius. The critical step here is to recognize that in the ideal case the ball will keep moving until it reaches its original height.

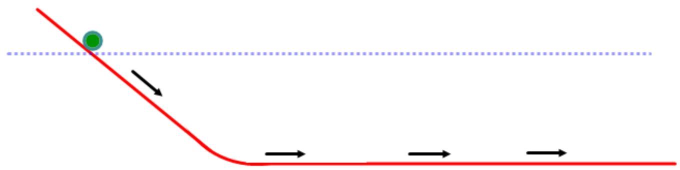
Figure 2:



a



b



c

Now examine the Devices “a” and “b” in Figure 2. While the “launching” ramp has the same slope, the “target” ramp has less of a slope in each device. If the ball is to reach the same height, it must move further to the right in each succeeding case. Now, consider Device “c” in Figure 2, where the slope of the second ramp is zero. Instead of a ramp, it is just a continuation of the straight part of the track. The ball could never get back to its original height, so it would keep traveling to the right. If we imagine the ideal case, it would keep moving forever, without anything pushing it. This directly opposes the idea that an object must be pushed by something to keep moving. In fact, this would indicate that if nothing is pushing an object its velocity will not change. **Galileo was perhaps the first to recognize that constant velocity is the natural state of an object.**

How do we reconcile this with our common sense perspective that something comes to a stop unless it is being pushed? In our everyday experience there are a number of forces that affect an object that we don’t think about because they’re always there. One of these is friction. When you gave the book a shove across the floor, it came to a stop because the friction between the floor and the book pushed on the book. Imagine now that instead of the floor, we were to give an object a shove on a sheet of ice. It will move much farther than it did on the floor. But it will eventually come to stop. Now use your imagination to picture a surface with no friction. Can you imagine that once you shove the book on that surface, it will simply keep moving without coming to a stop…ever. This turns out to be a core concept to physics, so if you can keep that image in your mind it will prove very helpful to you.

Newton adopted and formalized this principal of Galileo’s and it is now known as Newton’s First Law, or the Law of Inertia. **An object will maintain a constant velocity, both speed and direction, unless another object exerts a net force on it.** Please note that only a non-zero <u>net force</u> will result in a change of velocity. That means that if the vector sum of all the forces acting on an object is zero, it will maintain a constant velocity. That could be the case because there are no forces acting on an object or because there a number of forces that are adding up to zero, canceling each other out. In this circumstance the object is said to be in **equilibrium**.

How does this allow us to reconnect back to our intuition? Let’s look at a different case, one where I push a book across the floor with a constant velocity. If the book is moving with a constant velocity that means that there must be zero net force acting on it. That means that if I add up all the forces acting on the book, they must add up to zero. I know that I’m pushing the book in one direction. There must be an equal and opposite force, friction, pointed in the opposite direction. The book maintains a constant velocity not because there are no forces acting on it, but rather because the sum of all the forces acting on it is equal to zero. Our intuition says that I have to keep pushing something to keep it moving. But it turns out that that’s only true because friction is pushing the other way. If there were no friction, I wouldn’t have to keep pushing. But we live in a world where friction is everywhere around us. So our intuition works, but only in this context.

<u>Newton’s Second Law</u>

We now know that an object maintains a constant velocity unless a net force acts on it. But how does it behave if a net force does act on it? The answer is that **an object subject to a net force accelerates**. If I push on an object I can make it start moving, thus changing its velocity from nothing to something. Acceleration is just the change in velocity over a period of time. So if pushing something can make it start moving then a net force must result in acceleration. Note, however, that **only a net force results in acceleration**. If I push on an object in one direction and something else pushes that object with an equal force in the opposite direction, no acceleration will result. **So the first part of Newton's second law is that the acceleration of an object is proportional to the net force acting on it.**

But not all objects will accelerate the same amount when subject to the same net force. An object's acceleration also depends on its **mass**. Mass is a fundamental property of matter. Understanding what mass is and where it comes from is a very complex issue and would involve understanding Einstein's General Theory of Relativity (his Special Theory of Relativity won't do it!). That is well beyond the bounds of this book. However, there are some properties of mass that you will need to understand.

1. Mass is intrinsic to an object. It does not depend on where the object is located. No matter whether it is; on earth, in outer space or deep under the ocean, the mass of an object will not change.

2. An object's acceleration is inversely proportional to its mass. The more mass an object has, the less it will accelerate when subject to a given net force. If you have a certain amount of net force, you can compare the masses of different objects by observing how they accelerate. The more massive an object is, the less it will accelerate. If you double the mass of an object, it will accelerate half as much when the same net force acts on it.

3. Mass is a scalar, it is not a vector. There is no direction associated with a mass, only magnitude.

So Newton's Second Law says that an object's acceleration is proportional to the net force acting on it and is inversely proportional to its mass.

$$a = \frac{F_{net}}{m}$$

This is often written as:

$$\mathbf{F}_{net} = m\mathbf{a} \quad \text{or} \quad \Sigma\mathbf{F} = m\mathbf{a}$$

In the second expression the Greek symbol "Σ", sigma, acts as an instruction to "add up all" the forces acting on the object. That's the same process as is used to find the net force, so $\mathbf{F}_{net}$ is the same thing as $\Sigma\mathbf{F}$. There are two reasons that we will often use $\Sigma\mathbf{F}$ instead of $\mathbf{F}_{net}$. The first reason is that the "Σ" will serve to remind us that we have to find all the forces and then add them together. The second reason is that that leaves us room for subscripts. Remember that there can be multiple forces acting on a single object. We need to add all the forces together that are acting on that object to get $\Sigma\mathbf{F}$. But when we do that we have to remember which object those forces are acting on. So for example, let's say that there are three objects that are interacting. For simplicity let's name them by giving them each a number. Then to evaluate the motion of object #1 we would use the second law as follows.

$$\Sigma\mathbf{F}_1 = m_1\mathbf{a}_1$$

which in this case, with a total of three objects, means that

$\mathbf{F_{12}} + \mathbf{F_{13}} = m_1\mathbf{a_1}$

While this would allow us to determine the acceleration of the first object, we could equally well find the acceleration of the other two objects with the analogous equations. (In general, the acceleration of each if the three object will not be the same.)

$\Sigma\mathbf{F_2} = m_2\mathbf{a_2}$ or $\mathbf{F_{21}} + \mathbf{F_{23}} = m_2\mathbf{a_2}$

$\Sigma\mathbf{F_3} = m_3\mathbf{a_3}$ or $\mathbf{F_{31}} + \mathbf{F_{32}} = m_3\mathbf{a_3}$

Once again, please remember that forces must be added as vectors, not simply as numbers.

Units of Mass and Force

The SI unit of mass is the kilogram (kg). Since it is a fundamental unit of measure its value has been defined by international agreement. As is true for all fundamental units, such as the meter or the second, its value cannot be derived.

The unit of force is the Newton (N). The Newton is not a fundamental unit as it can be derived, using Newton's Second Law, from the fundamental units of meters, kilograms and seconds. The SI units of mass are kilograms (kg) and of acceleration are meters per second squared (m/s^2). Out of respect for Isaac Newton (1642-1727), the unit of force is named the Newton.

Since

$$\Sigma F = ma$$

Then the units of force must be:

$$N = (kg)(m / s^2)$$

$$\boldsymbol{N = kg{\cdot}m/s^2}$$

One other important aspect of Newton's second law is that the acceleration of an object will always be in the same direction as the net force acting on it. That must be true since in the expression $\mathbf{F_{net}} = m\mathbf{a}$, only "$\mathbf{F_{net}}$" and "$\mathbf{a}$" are vectors while "m" is not. That means that "m" will change the size of the acceleration, but not its direction. **The net force and acceleration will always have the same direction.**

Example 1: Determine the acceleration of a ball whose mass is 20 kg and which is subject to a net force of 40N to the right due to a collision with a second object.

First, let us define the +x-direction as being to the right. That will allow us to interpret the phrase "40N to the right" as +40N. Then we can apply Newton's Second Law

$\boldsymbol{\Sigma F_b = m_b a_b}$ *Start with the initial equation.*

$$a_b = \frac{\Sigma F_b}{\mathrm{m_b a_b}}$$

$$a_b = \frac{+40N}{20kg}$$

$$a_b = \frac{+40kg\frac{m}{s^2}}{20kg}$$

$$a_b = +2\frac{m}{s^2}$$

$$a_b = 2\frac{m}{s^2} \text{ to the right}$$

Because we are looking for the acceleration, we need to solve the equation for a_b. To isolate a_b, divide both sides by m_b.

With the found equation, plug in the given values for ΣF_b and m_b.

Because we defined "to the right" as the positive direction.

Example 2: Determine the acceleration of a package whose mass is 20 kg and which is subject to two forces. The first force is 40N to the right while the second force is 20N to the left.

First, let us define the +x-direction as being to the right. That will allow us to interpret the phrase "40N to the right" as +40N and "20N to the left" as -20N. We'll use "p" as the subscript to denote the package and "1" and "2" to label the two forces. Then we can apply Newton's Second Law keeping in mind that we need to add the forces to find $\Sigma\mathbf{F}_p$.

$$\Sigma \mathbf{F}_p = m_p \mathbf{a}_p$$

$$a_p = \frac{\Sigma F_p}{\mathrm{m_p}}$$

$$a_p = \frac{F_{p1}+F_{p2}}{20\mathrm{kg}}$$

$$a_p = \frac{+40N-20N}{20\mathrm{kg}}$$

$$a_p = \frac{+20kg\frac{m}{s^2}}{20\mathrm{kg}}$$

$$\mathrm{a_p} = +1.0\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\mathrm{a_p} = 1.0\,\frac{\mathrm{m}}{\mathrm{s}^2} \text{ to the right}$$

Newton's Second Law in Two Dimensions

When working with vectors, (e.g. displacement, velocity, acceleration and force) it is important to first define a set of axes. We will define the +x axis to the right and the +y axis as up, away from the center of the earth, unless otherwise noted. If the axes are chosen in this way the vector equation $\Sigma\mathbf{F} = m\mathbf{a}$ can be broken into two independent equations, one along the horizontal, x-axis, and the other along the vertical, y-axis.

The vector equation $\Sigma\mathbf{F} = m\mathbf{a}$ becomes $\Sigma\mathbf{F}_x = m\mathbf{a}_x$ and $\Sigma\mathbf{F}_y = m\mathbf{a}_y$. The motion and the forces along the x-axis and the y-axis can then be treated separately. The dynamics and kinematics in the horizontal and vertical directions are independent and unaffected by each other so Newton's second law must be true in both directions.
Example 3: Determine the acceleration of an object whose mass is 20 kg and which is subject to three forces. The first force, F_{o1}, is 80N to the right, the second force, F_{o2}, is 20N upwards and the third force, F_{o3}, is 20N downwards.

With our convention of defining the +x-direction as being to the right and the +y-direction as up, we can interpret the phrase "80N to the right" as +80N along the x-axis, "20N upwards" as +20N along the y-axis and "20N downwards" as -20N along the y-axis and. Then we can apply Newton's Second Law independently along each axis keeping in mind that we need to add the forces to find $\mathbf{F}_{net}$.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

x-direction	**y-direction**
$\Sigma \mathbf{F}_o = m_o \mathbf{a}_o$	$\Sigma \mathbf{F}_o = m_o \mathbf{a}_o$
$a_o = \frac{\Sigma F_o}{m_o}$	$\mathbf{a}_o = \frac{\Sigma F_o}{m_o}$
$\mathbf{a}_o = \left(\frac{F_{o1}}{m_o}\right)$	$\mathbf{a}_o = \frac{F_{o2}+F_{o3}}{m_o}$
$\mathbf{a}_o = \left(\frac{+80N}{20kg}\right)$	$\mathbf{a}_o = \frac{+20-20N}{20kg}$
$\mathbf{a}_o = +1.0\ m/s^2$	$\mathbf{a}_o = 0$

$$\mathbf{a} = 1.0\ m/s^2 \text{ to the right}$$

In this example, all the vectors (forces and acceleration) were directed either parallel or perpendicular to the x-axis. In nature, forces and accelerations can be directed in any arbitrary direction. That does not represent any new physics, but would require the use of trigonometry. Since this book does not presume a prior background in trigonometry, we will only work with vectors that are either parallel or perpendicular to the x-axis. Dealing with vectors at arbitrary angles will not represent much additional difficulty once you have mastered basic trigonometry.

Newton's Third Law

We indicated earlier that forces can only exist between two objects. A single isolated object cannot experience a force. Newton's third law indicates the very simple relationship between the force that each object exerts on the other; they will always be equal and act in opposite directions. So if I push on a box with a force of 20 N to the right, the box will be pushing on me with a force of 20 N to the left. Using our subscripts this simply says that $\mathbf{F}_{mb} = -\mathbf{F}_{bm}$, with the subscript "m" standing for me and "b" standing for box.

Another way of saying this is that forces always occur in equal and opposite pairs. Some people get confused by this law in that they think that these two forces will cancel out and no acceleration will ever result. However, that is not the case since each force is acting on a different object. Only one force acts on each object so they cannot cancel out. This can most easily be made clear by working through an example.

Example 3: I push on a box whose mass is 30 kg with a force of 210 N to the right. Both the box and I are on a frictionless surface so there are no other forces acting on either the box or me. My mass is 90 kg. Determine the acceleration of both the box and me? Are they the same?

There are no forces in the y-direction so I only need to use one axis. First, let's solve for the acceleration of the box.

$$\mathbf{\Sigma F_b = m_b a_b}$$
$$\mathbf{a_b = \frac{\Sigma F_b}{m_b}}$$
$$\mathbf{a_b = \frac{Fm_b}{m_b}}$$
$$\mathbf{a_b = \frac{+210N}{30kg}}$$
$$\mathbf{a_b = \frac{+210kg\frac{m}{s^2}}{30kg}}$$

a_b = +7.0 m/s^2
a_b = 7.0 m/s^2 to the right

Now let's solve for my acceleration.

$$\mathbf{\Sigma F_m = m_m a_m}$$
$$\mathbf{a_m = \frac{\Sigma F_m}{m_b}}$$

but $\boldsymbol{F_{mb}} = \boldsymbol{-F_{bm}}$

$$\mathbf{a_m = \frac{-F_{bm}}{m_b}}$$

(Note the change in the order of the subscripts)

$$\mathbf{a_m = \frac{-210N}{90kg}}$$
$$\mathbf{a_m = \frac{-210kg\frac{m}{s^2}}{30kg}}$$

a_m = -2.3 m/s^2
a_m = 2.3 m/s^2 to the left

So as I push the box to the right, the box pushes me to the left. Since my mass is greater, my acceleration is less. The accelerations are not the same. Nor is either of them equal to zero.

Sketches and Free Body Diagrams

I. The first step in solving a problem is to make a quick sketch, unless one is provided. Without having a rough sketch, it's easy to go in the wrong direction or not know how to get started. Often, a sketch is provided, but if isn't make this your first step.   After you've done that, you'll need to **draw a free body diagram for each of the objects** in your sketch.

II. Before we get started with that, it's important to define what we mean by an object. Obviously any single entity, like a wooden block, ball, book, car, person, etc. represents an object. But also, if two blocks of wood are connected together so that if one moves, the other must move, those two blocks together can be treated as an object. In this case, you can generate three free body diagrams, one for each block of wood separately and a third for the combination of the two blocks. Anytime two objects are locked together so that they have the same acceleration, they can be treated as a single object. This technique may give you the crucial bit of information that you need to solve a problem.

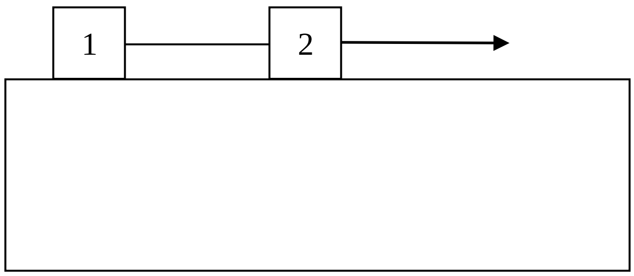
III. Now let's go through how to draw a free body diagram.

1. Draw and label a dot to represent the first object.
2. Draw an arrow from the dot pointing in the direction of one of the forces that is acting on that object. Clearly label that arrow with the name of the force, for instance F_{c1}.
3. Repeat for every force that is acting on the object. Try to draw each of the arrows to roughly the same scale, bigger forces getting bigger arrows.
4. Once you have finished your free body diagram, recheck it to make sure that you have drawn and labeled an arrow for every force. This is no time to forget a force.
5. Draw a separate arrow next to your free body diagram indicating the likely direction of the acceleration of the object. This will help you use your free body diagram effectively.
6. Repeat this process for every object in your sketch.

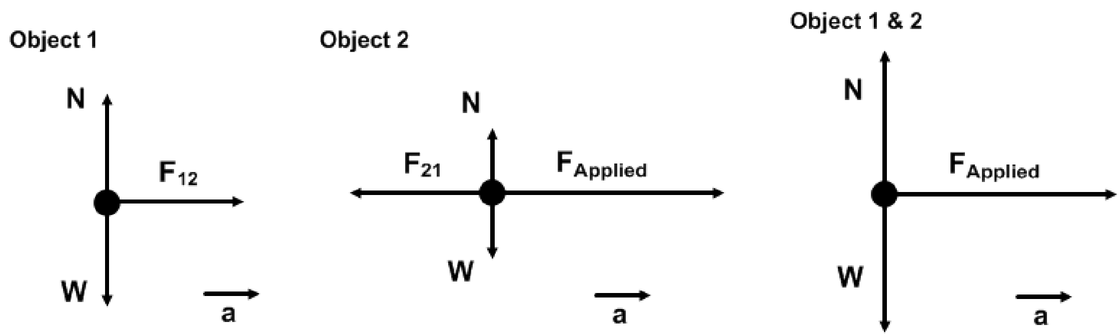
IV. Now let's use your free body diagram to apply Newton's Second Law. One of the wonderful things about this law is that it will work for each object independently. That means that you can apply it to any of your free body diagrams and proceed to analyze it.

1. Write the law in its general vector form using a subscript to indicate the object being analyzed, $\Sigma \mathbf{F_o} = m_o \boldsymbol{a_o}$.
2. If there are forces acting along more than one axis, break this general equation into its x and y components.
3. Now read your free body diagram to write down all the forces on the right side of the equation. Using whatever convention you wish, as long as you are consistent, indicate the direction of each force by a + or – sign. For instance if the forces were in the vertical direction, you might use + for up and – for down.
4. Now's when you can make use of the acceleration vector that you drew next to your free body diagram. Be consistent. If you used a negative sign for a downward or leftward directed force and the acceleration vector is also pointing down or to the left, you need to put a negative sign in front of the "ma" on the right side of the equation. On the other hand, if the acceleration vector is pointed upwards or to the right, it should have a positive sign.
5. You can now repeat this process for as many of your other free body diagrams as is necessary to solve the problem.

Let's do a couple of examples.
Example 4: Two objects are connected together by a string. Object 1 has a mass of 20 kg while object 2 has a mass of 10 kg. Object 2 is being pulled to the right by a force of 60 N. Determine the acceleration of the objects and the tension in the string that connects them. .



First, note that there are three possible objects, 1, 2 and 1+2. The combined object 1+2 exists because the string connecting objects 1 and 2 insures that they will always move together and have the same acceleration.
Here are the three free body diagrams.



Note that in the diagrams, F_{12}=- F_{21} *so the arrows are the same size but pointed in opposite directions.* $F_{applied}$ *is the same size for both objects 2 and (1+2) so those arrows are the same size. Since the acceleration of all three objects is the same and must be to the right, all three of the arrows labeled "a" are the same size and directed to the right.*

First, let's solve for the acceleration of the combined object (1+2). That's the easiest to solve since I know the combined mass and the one force that acts on it.

$\Sigma \mathbf{F}_{(1+2)} = m_{(1+2)} \mathbf{a}_{(1+2)}$

$a_{(1+2)} = \frac{\Sigma F_{(1+2)}}{m_{(1+2)}}$

$a_{(1+2)} = \frac{F_{applied}}{m_{(1+2)}}$

$a_{(1+2)} = \frac{+60N}{20kg+10kg}$

$a_{(1+2)} = \frac{+60kg\frac{m}{s^2}}{30kg}$

$\mathbf{a}_{(1+2)}$ = +2.0 m/s^2

$\mathbf{a}_{(1+2)}$ = 2.0 m/s^2 to the right

Now let's solve for tension in the string, F_{12}*, by using the free body diagram for object 1. This will be simplest since I now know the acceleration and the mass of object 1 and* F_{12} *is the only force acting on it.*

$\Sigma F_1 = m_1 a_1$
$F_{12} = +m_1 a_1$
$F_{12} = (20\text{ kg})(+\ 2.0\text{ m/s}^2)$
$F_{12} = +\ 40\text{ kg}\cdot\text{m/s}^2$
$F_{12} = 40$ N to the right

Just to show that all three free body diagrams would work we can also solve it using the free body diagram for Object 2. This can be used to check our work but will not give us any extra information.

$\Sigma F_2 = m_2 a_2$
$F_{applied} + F_{21} = m_1 a_1$
$F_{applied} - F_{21} = +m_1 a_1$
$F_{21} = -\ m_1 a_1 + F_{applied}$
$F_{21} = -(10\text{ kg})(+\ 2.0\text{ m/s}^2) + 60\text{N}$
$F_{21} = +\ 40\text{ kg}\cdot\text{m/s}^2$
$F_{21} = 40$ N to the left

Some Examples of Forces

There are a wide variety of forces that we experience every day. One of the goals of physics is to show the source of these forces and how they are connected to one another. That theme will present itself as you continue your study of physics. However, in order to reach that stage of understanding you will first need to master the skills that are based on the everyday, not the fundamental, forces of nature. In this section of the book, we will present four of those everyday forces. Their connection to more fundamental forces will be touched upon, but a full exploration of those connections will have to wait until later in this book. A complete understanding is not yet available in physics. There is much that remains to be understood.

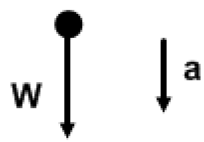
Weight

Everyone has seen that if you drop an object, like a book or a pen, it falls downwards. Since the object wasn't moving when you dropped it (that's the meaning of the word "dropped") and is moving after you let go of it that must mean the object accelerated upon being released. In general, unsupported objects near the surface of the earth accelerate downwards. (I say "in general" since this won't work if you "drop" a helium balloon, but we'll deal with that later in the book when we talk about buoyancy.) This attraction of objects towards the center of the earth is a result of an important fundamental force, **gravity**. A fuller discussion of gravity in general requires its own chapter. However, we will now explore its consequences near the surface of the earth.

From Newton's Second Law we know that if an object accelerates there must be a net force acting on it. In this case, the force being exerted on the object is due to the earth so we can call it, F_{oe}. However, this force is so universally felt by all of us on the planet that it has received its own name and is often referred to as the weight (W) of the object. It turns out that the weight, W or F_{oe}, of an object is proportional to its mass. The more mass the object has - the greater its weight. The constant of proportionality between an objects weight and its mass has been given its own symbol, "g", and is equal to about 9.8 m/s^2. This is expressed by the following formula.

$$\mathbf{W = F_{oe} = mg}$$

A surprising result of the fact that the weight of an object is proportional to its mass is that all unsupported objects near the surface of the earth fall with the same acceleration, 9.8 m/s^2, as long as no other force acts on them. We can see how this would be true if we use the idea that W = mg and combine that with Newton's Second Law. If an object only has the force of its weight acting on it, it's free body diagram would show only one force, W, pointing downwards. Its acceleration would also point downwards. So…



$$\Sigma \boldsymbol{F_o} = m_o \boldsymbol{a_o}$$
$$W = m_0 a_o$$
$$mg = ma$$
$$a = g$$
$$a = 9.8\ \text{m/s}^2$$

On the one hand this seems like a reasonable result. A more massive object is heavier, so it has more force pulling it towards the earth. However, it's also more massive in exactly the same proportion, and the more mass an object has the more force it takes to give it the same acceleration. These two effect, more weight but also more mass, exactly cancel out and all objects fall with the same acceleration.

On the one hand, this is a surprising result since it seems like heavy objects should fall faster than light objects. And that is often the case. If you drop a feather and a book at the same time its clear that the feather falls slower than the book. However, we have to recall the condition we put in our statement about falling objects having the same acceleration. We said that this would be true if "no other forces are acting on the object". This is much like the case we discussed earlier where the force of friction made it hard for us to see that an object in motion will continue to move at a constant velocity if no net force acts on it. In this case, it's the resistance of the air to the motion of the feather that serves as a sort of "friction" which opposes the weight of the feather. If you remove the air and repeat the experiment, the feather and book will fall with the same acceleration. Since we not only live on the earth, we also live in the earth's atmosphere, it's reasonable that our intuition would tell us that light objects don't fall to the ground as quickly as heavy objects. But it's important that we go beyond our intuition to try to separate out the two different effects, gravity and air resistance, and study each on its own.

One way you can prove this to yourself is to first drop a piece of 8 ½ x 11 paper at the same time as a pen. You'll see that the pen falls much faster than the paper. However, if you then tightly crumple the paper into a ball and repeat that experiment, you'll see that they fall at about the same rate. Now if you **imagine**, as Galileo did in his experiments, that you could make that ball of paper small enough, then it would fall at the exact same rate as the pen.

<u>Surface Forces</u>

Whenever the surfaces of two objects come in contact there are two types of forces that can result.

- The Normal Force – which always acts perpendicularly to the surfaces
- Friction - which always acts parallel to the surfaces

Both of these forces are due to microscopic effects within the objects. They are both very complicated to understand at that level and require a foundation in electrostatics to get any reasonable idea of how they operate.

Understanding these forces in detail requires a grasp of very advanced physics. However, as we will often find to the be the case, the characteristics of how these forces work can be understood without fully understanding the underlying mechanism that makes them work.

The Normal Force is a result of the atoms in an object trying to avoid being pushed closer together. When they are pushed closer together by another object, they resist that by pushing back on that object. The harder they are pushed together the harder they push back.

Friction is a result of two surfaces rubbing together as they slide, or try to slide, by each other. Each of the surfaces has a texture as well as surface atoms that have electrostatic properties. The interaction of the textures and atoms on the two surfaces determines how slippery they feel to one another.

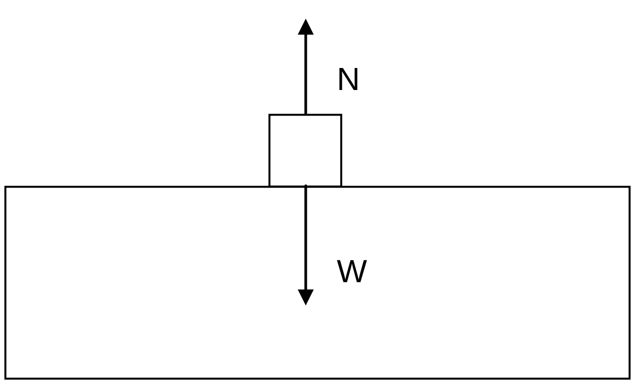
Normal Force

If you put a book on a flat table and let go of it you will see that it does not move, its acceleration is zero. We know that there is at least one force acting on the book, its weight. We also know that if there were a net force acting on the book it would accelerate, and in this case it is not accelerating. The only way that the total force acting on the book could be zero, if we know that there is a least one force acting on the book, is if a second force is acting on the book in the opposite direction (there could be more than one extra force but let's keep this simple for now).

We also know that a force requires the interaction of two objects. In this case, one of the objects is the book. The other object must be the table. This becomes clear if you imagine what would happen if the table suddenly disappeared (or if you try to put the book down next to, but not on, the table). The book will accelerate without the table to support it. We can conclude from this that the table must be furnishing the force that is canceling the effect of the earth on the book.

Using our convention for naming the force between two objects, let's call that force F_{bt}, the force on the book due to the table. This force must be equal and opposite to the force of the earth on the book, in order to cancel out the pull of the earth. Since this direction is perpendicular, or normal, to the surface of the table this force is also called the Normal Force, F_n. You can use either F_{bt} or F_n to name this force.

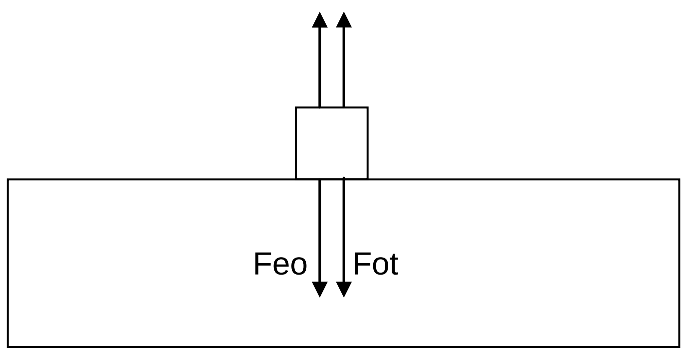
Let's now analyze our book sitting on a table using a free body diagram and Newton's Second Law to determine the normal force exerted on a 10 kg book by a table.



$\Sigma \mathbf{F_o} = m_o \mathbf{a_o}$
$W - F_{bt} = 0$
$W = F_{bt}$
$F_{bt} = mg$
$F_{bt} = (10kg)\ (9.8\frac{m}{s^2})$
$F_{bt} = 98\ N$

While we have only shown two objects, and one pair of forces, in this picture, there are really three objects and two pairs of forces. Newton's Third Law tells that forces only exist between objects and in pairs. The first pair of forces is between the earth and the object, F_{oe} and F_{eo}. The second pair of forces is between the object and the table, F_{ot} and F_{to}. In order to keep the diagram simple, we have not shown the earth. But none of the above would occur if the earth were not present. If you were to put a book on a table in deep space, far from the earth, there would no weight to pull the book against the table; therefore there would be no resulting normal force.

A more comprehensive diagram would show all four forces, as below. However,
in many cases this added complication is not needed and may make drawings harder to read – which would defeat their purpose. This does show, however, that the table is experiencing a normal force downwards, due to the book, that is equal and opposite to the normal force that it is supplying to the book upwards. If that downwards force, were to become too great, the table would break.



While we derived the normal force as a force that opposes gravity in the case of a book resting on a table, it has a much wider range of applications.

- When you stand in an elevator, the floor of the elevator exerts a normal force upwards on your feet. We'll find that you can be accelerated upwards, or downwards, when that normal force does not equal your weight.
- When you lean against a wall, you exert a normal force on that wall. By Newton's Third Law, you know that the wall will exert an equal and opposite normal force on you. Since the wall is vertical, and the normal force is always perpendicular to the surfaces, the provided normal force is in the horizontal direction.
- When a baseball is hit by a bat, the normal force exerted by the bat on the ball is in a direction perpendicular to the surfaces of the bat and ball when they collide. Since the ball is a sphere and the bat is a cylinder, there's a wide range of directions in which that normal force can be directed depending on

where they come in contact. That's why the ball can accelerate in ways that leads to line drives, grounders or pop-ups.

Friction

There are two types of friction that can be generated between a pair of surfaces. The difference between the two types depends on the relative velocity of the surfaces to each other. Please note that that means that if both surfaces are moving along together, their relative velocity is zero.

Kinetic friction is the result of the surfaces of two objects rubbing together as they slide by each other with some relative velocity. In this case, the force of friction will act on each object in a direction opposite to its velocity relative to the other. Kinetic friction acts in a way so as to bring those objects to rest relative to each other.

Static friction results when an object experiences a net force parallel to its contact surface with a second object. The force of static friction is directed opposite to the direction of that net force in order to keep the objects at rest relative to each other.

There are two factors that determine the maximum amount of friction force that can be generated; the slipperiness of the surfaces and the amount of force pushing the surfaces together. This makes sense in that if two surfaces are perfectly slippery, they'll never generate any friction between them no matter how hard you squeeze them together. On the other hand, if two surfaces are not very slippery but are also not actually being pushed together, they also will not generate any friction. They'll just slide by each other, barely touching.

The amount of slipperiness between two surfaces is indicated by the **coefficient of friction** between those surfaces. This is given the symbol "**μ**" which is a Greek letter and is pronounced "mu". Values of μ range from zero to one. Two surfaces that have a coefficient of friction of zero will generate no friction regardless of how hard they are squeezed together; they are perfectly slippery. On the other extreme, two surfaces with a μ of 1 generate a very high level of friction if they are pressed together. Note that values of μ can only be given for pairs of surfaces. There is no way to compute a value for each surface and then combine them together to figure out how the surfaces will behave when placed together.

If you've ever tried to slide something very heavy across the floor you've probably noticed that it's harder to get it started then to keep it going. Once it starts moving, you don't have to push as hard to keep it going. That's a result of the surfaces becoming more slippery once they start sliding by each other. On a microscopic level, the atoms on the surfaces just don't have time to make bonds with each other as they slide by. As a result, there are two different coefficients of friction for each pair of surfaces. The static coefficient of friction, μ_s, gives the measure of slipperiness between the surfaces while they are not moving relative to each other, while the kinetic coefficient of friction, μ_k, gives a measure of that slipperiness when they are. Since higher values of μ result in more friction that means that μ_s is always greater than μ_k. Please note that determining which coefficient to use is based on whether the surfaces are moving relative to one another, sliding past each other, or not. Surfaces that are moving together are treated the same as if they were standing still and you would use μ_s, not μ_k.

The other factor, aside from slipperiness, that determines the force of friction is how hard the surfaces are being squeezed together. A force that squeezes two surfaces together must be perpendicular to them. This is just another way of describing the normal force that we discussed earlier. So we now have enough information to write a formula for the maximum amount of friction that can be generated by two surfaces.

If the two surfaces are gliding by each other than:

$F_{fr\ max} = \mu_k F_n$

If the two surfaces are not moving relative to each other than

$F_{fr\ max} = \mu_s F_n$

We now have just one last step to take and we'll have rounded out our picture of friction. We have to determine when the amount of friction will be at the maximum values given by the above formulas and when that will not be the case. In the latter case, we need to figure out what it will be if it is not at a maximum.
The force of kinetic friction is the simplest because it's always at its maximum value. Once the surfaces are sliding by each other, the force of kinetic friction will be at its maximum value and nothing will affect this value as long as that relative motion continues. So we can replace our maximum formula with:

$F_{fr} = \mu_k F_n$ for the case of kinetic friction

We indicated above that the force of static friction is directed opposite to the direction of an applied force in order to keep the two objects at rest relative to each other. That means that if there is no net force applied to an object parallel to its contact surface with a second object there will not be any static friction. As a result, the minimum static friction must be zero. The maximum amount is given by $F_{fr\ max} = \mu_s F_n$. The formula for static friction becomes.

$F_{fr} \le \mu_s F_n$

There is static friction only if there is a net applied force that would, in the absence of static friction, accelerate the object parallel its contact surface. As that net applied force increases, so does the amount of static friction, always in the direction opposite to the net applied force. However, this static friction cannot increase without limit. Once the maximum is reached, $F_{fr\ max} = \mu_s F_n$, the object will begin to slide. At that point the static friction formula becomes inapplicable and the kinetic friction formula become relevant, $F_{fr} = \mu_k F_n$. This will remain the case until such time as the surfaces once more become at rest with respect to each other.

Tension Force

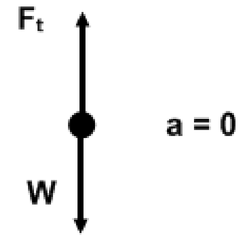
Another way to support an object is to hang it from a string. For instance, imagine hanging a plant from a string that is then connected to the ceiling. The downward force due to the weight of plant will cause the string to stretch a bit and get taut. The tautness in the string results in a tension force that pulls the plant upward and the ceiling downward.

For a tension force to act the string must be connected to objects at both ends. In this example the objects are the plant and the ceiling. Tension always acts on both objects with equal magnitude but in opposite directions. It has the effect of pulling the objects towards one another.

Tension force is not just present in string. It can be found in rope, cable, and metal rods, etc. In fact, any material that resists being stretched can exhibit a tension force when connected between two objects. It can be labeled either as F_t or by the using the subscript system using the object under study first and the material that provides the tension force second. For instance, in the above example the tension force could be named F_t or F_{ps}, where "p" refers to the plant and "s" the string.

Let's work through an example where a 5 kg plant is hung from a string connected to the ceiling. First, let's do a free body diagram for the plant. (Shown to the right.)

Then use Newton's Second Law, keeping mind that the acceleration is zero since the plant is not falling.



$\Sigma \mathbf{F_p} = m_p \mathbf{a_p}$
$W - F_t = 0$
$F_t = W$
$F_t = mg$
$F_t = (5kg)\,(9.8\frac{m}{s^2})$
$F_t = 49\text{ N}$

So far we have discussed tension as a vertical force that offsets the weight of an object when it is hanging. However, tension does not have to act vertically. When an object is pulled across a horizontal surface by a string, the tension force is acting horizontally. The things that are always true about the tension force are:

- Tension force always acts along the direction of the string, or whatever material is under tension. It can never act at an angle to the string.
- Tension force requires that there be two connected objects.
- Tension force acts in opposite directions on those two objects so as to bring them together.

Elastic Force – Hooke's Law

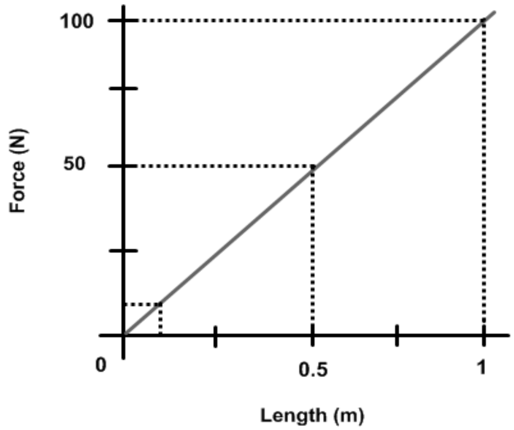
The elastic force, F_e, is the force exerted by a stretched or compressed material. Robert Hooke originally explored the force in the context of springs, but we will see that it has wider applications. It will also take us a step towards better understanding both the tension force and the normal force.

Hooke observed that it takes very little force to stretch a spring a very small amount. However, the further the spring is stretched, the harder it is to stretch it further. The force needed increases in proportion to the amount that it has already been stretched. The same was observed to be true when compressing a spring. This can be stated mathematically as

$$\mathbf{F_{spring} = -\,k\,x}$$

In this equation, k represents the spring constant (a characteristic of the individual spring), and x represents the magnitude of the distance the spring is stretched or compressed from its natural length. The negative sign tell us that the force that the spring exerts is back towards its equilibrium length, its length when it is not being stretched or compressed. Note that x is always either zero or positive since it is a magnitude. The negative sign accounts for the direction of the force.

Therefore, if the spring constant for a particular spring were 100 N/m, I would need to exert a force of 100 N to stretch, or compress, it by a length of 1m. If I were to exert a force of 50N, it would stretch $\frac{1}{2}$ m. A force of 10 N would stretch, or compress, it by a distance of $\frac{1}{10}m$. This is shown graphically below.



The elastic force also underlies the tension and normal forces. In the case of the tension force, the rope or string that is exerting that force actually stretches a bit as that force is exerted, just like a spring. The difference is that in the case of the tension force, the amount of stretch is relatively small and is neglected. We assume that the string does not stretch for the purpose of our calculations. But the string will exert its force so as to go back to its equilibrium length, so it does act just like a spring in this respect.
Another key difference between a string and a spring is that you can't compress a string. Strings, ropes, etc. only exhibit an elastic force when they are stretched, not when they are compressed. Also, they follow Hooke's law for only a small amount of stretch. After that they get a lot more complicated. So while the source of the force is the same as the elastic force, tension is treated differently.

The same holds for the normal force, however, in this case a force is only exhibited upon compression. When you put a book on a table, the table bends just the slightest bit as it compresses in the middle. The bend is so small that it's hard to see. But if you put a lot of books on a bookshelf, you can see that bending take place. Once again, the table or bookshelf wants to go back to its equilibrium position. To do this, it exerts a force in a direction opposite to that which causes it to compress. This is the normal force. It's just like the elastic force due to compression but, as in the case of tension, it follows Hooke's law for only a small distance before getting too complicated. Also, the amount of compression is small enough to be neglected in our calculations. However, that compression is the source of the normal force. Without compression, there is no normal force.

Chapter Questions

1. What is inertia? Give some examples.

2. What is the difference between inertial and non-inertial reference frames?

3. State the relationship between mass and inertia.

4. A boy seems to fall backward in an accelerating bus. What property does this illustrate?

5. A fisherman stands in a boat that is moving forwards towards a beach. What happens to him when the boat hits the beach?

6. A passenger sits in a stationary train. There are some objects on a table: an apple, a box of candy, and a can of soda. What happens to all these objects with respect to the passenger when the train accelerates forward?

7. An object is in equilibrium. Does that mean that no forces act on it?

8. A rock is thrown vertically upward and stops for an instant at its highest point. Is the rock in equilibrium at this point? Are there forces acting on it?

9. Is it possible for an object to have zero acceleration and zero velocity when only one force acts on it?

10. Is it possible for a car to move at a constant speed when its engine is off?

11. Two boys are pulling a spring scale in opposite directions. What is the reading of the spring scale if each boy applies a force of 50 N?

12. Much more damage is done to a car than a truck when the two collide. Is that in agreement with Newton's Third Law?