New Jersey Center for Teaching and Learning

Progressive Science Initiative

This material is made freely available at www.njctl.org and is intended for the non-commercial use of students and teachers. These materials may not be used for any commercial purpose without the written permission of the owners. NJCTL maintains its website for the convenience of teachers who wish to make their work available to other teachers, participate in a virtual professional learning community, and/or provide access to course materials to parents, students and others.

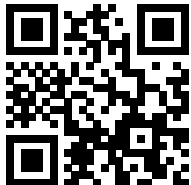**Click to go to website:**
www.njctl.org

Momentum

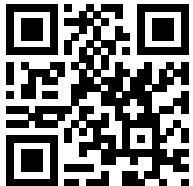www.njctl.org

Momentum

Click on the topic to go to that section

http://njc.tl/ko

Momentum

Return to Table of Contents

http://njc.tl/kp

Momentum Defined

Newton's First Law tells us that – objects remain in motion with a constant velocity unless acted upon by a force.

In our experience:

- When two objects of different masses travel with the same velocity, the one with more mass is harder to stop.

- When objects of the equal masses travel with different speeds, the faster one is harder to stop.

- Define a new quantity, **momentum** (p), that takes these observations into account:

$$momentum = mass \times velocity$$

$$p = mv$$
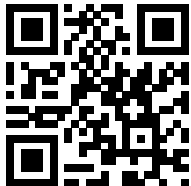
click here for a introductory video on momentum from Bill Nye!
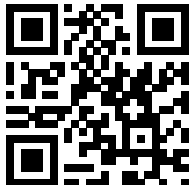
http://njc.tl/kp

Momentum is a Vector Quantity

Recall that:

mass is a scalar quantity
and
velocity is a vector quantity

Since: $momentum = mass \times velocity$

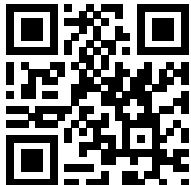momentum must be a vector quantity

$$\vec{p} = m\vec{v}$$

http://njc.tl/kp

SI Unit for Momentum

There no specially named unit for momentum.
We just use the product of the units of mass and velocity...
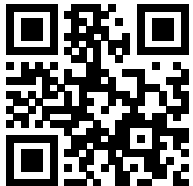
mass x velocity

kg·m/s

1   Which has more momentum?

○ A   A large truck moving at 30 m/s

○ B   A small car moving at 30 m/s

○ C   Both have the same momentum.

Answer

http://njc.tl/kq

2   What is the momentum of a 20 kg object with a velocity of +5.0 m/s?
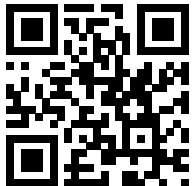
Answer

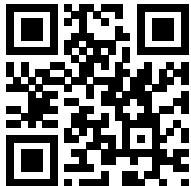3   What is the momentum of a 20kg object with a velocity of −5.0m/s?

Answer

http://njc.tl/ks

4   What is the velocity of a 5.0kg object whose momentum is −15.0 kg#m/s?

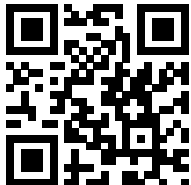**Answer**

5   What is the mass of an object whose momentum is 35 kg·m/s when its velocity is 7.0 m/s?
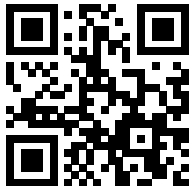
Answer

Momentum Change & Impulse

Return to
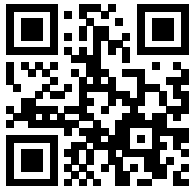Table of
Contents

http://njc.tl/kv

Change in Momentum

Suppose that there is an event that changes an object's momentum.

- from p_0 - the initial momentum (just before the event)

- by Δp - the change in momentum

- to p_f - the final momentum (just after the event)

The equation for momentum change is:

$$p_0 + \Delta p = p_f$$

http://njc.tl/kv

Momentum Change = Impulse

Momentum change equation:
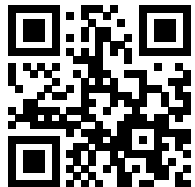
Newton's First Law tells us that the velocity (and so the momentum) of an object won't change unless the object is affected by an external force.

When an outside force **F** acts on the object for a time **Δt**, it delivers an **impulse I** to the object that **changes its momentum**:

the **impulse** is:

$$p_0 + \Delta p = p_f$$

$$p_0 + I = p_f$$

$$I = F \Delta t$$

http://njc.tl/kv
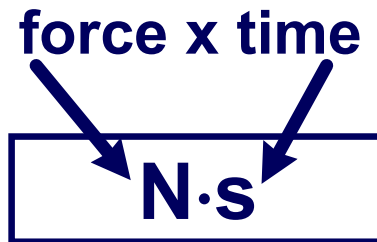
SI Unit for Impulse

There no specially named unit for impulse.
We just use the product of the units of force and time...

force x time

$$\boxed{\textbf{N·s}}$$

Recall that $N=kg·m/s^2$, so

$$\textbf{N·s}=\textbf{kg}·m/\cancel{s^2} \times \cancel{s}$$

$$=\boxed{\textbf{kg·m/s}}$$

- the same as momentum!

http://njc.tl/kv
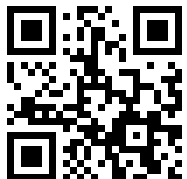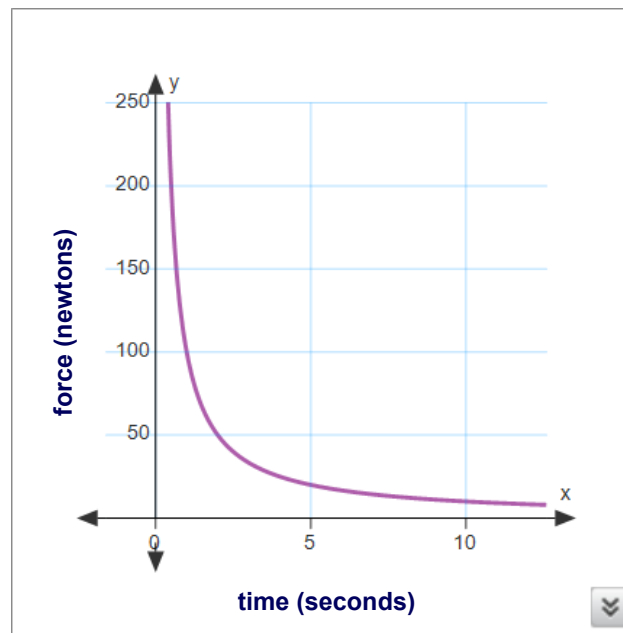
Effect of Collision Time on Force

$$\text{Impulse} = F_t = _F t \text{ change in momentum}$$

Changing the duration (t) of an impulse by a small amount can greatly reduce the force on an object

http://njc.tl/kv

Real World Applications

$$\boxed{\text{Impulse} = F_t = {}_F t \text{ change in momentum}}$$

- Car Design / Accidents

  > airbags

  > collisions head-on vs walls

  > crush zones

- Jumping / Landing

- Boxing / Martial Arts

- Hitting Balls - Golf, Baseball...

- Catching Balls

http://njc.tl/kv
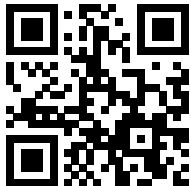
6 An external force of 25N acts on a system for 10s. How big is the impulse delivered to the system?

Answer
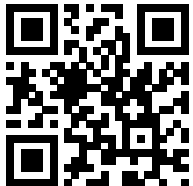
7 In the previous problem, an external force of 25N acted on a system for 10s. We found that the impulse delivered was 250 N-s. What is the magnitude of the change in momentum of the system?

Answer

8 The momentum change of an object is equal to the

　　_____.

　　○ A force acting on it

　　○ B impulse acting on it
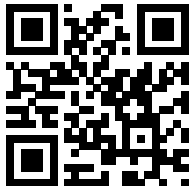
　　○ C velocity change of the object

　　○ D object's mass times the force acting on it

Answer
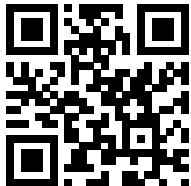
9 Air bags are use in cars because they:

○ A increase the force with which you hit the dashboard

○ B increase the duration (time) of impact in a collision

○ C decrease the momentum of a collision

○ D decrease the impulse in a collision

Answer

http://njc.tl/kz
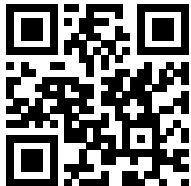
10 One car crashes into a concrete barrier. Another car crashes into a collapsible barrier at the same speed. What is the difference between the 2 crashes?

○ A change in momentum

○ B force on the car

○ C impact time

○ D both B & C are true

Answer
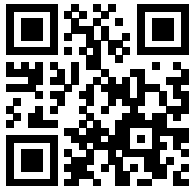
11 In order to increase the final momentum of a golf ball, we could:

○ A   not change the speed of the golf club after the collision

○ B   increase the force acting on it

○ C   increase the time of contact between the club and ball

○ D   all of the above

Answer

http://njc.tl/l1
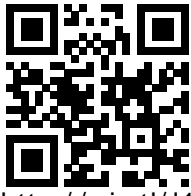
12 An external force acts on an object for 0.0020 s. During that time the object's momentum increases by 400 kg-m/s. What was the magnitude of the force?

Answer
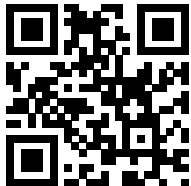
* 13 A 50,000 N force acts for 0.030 s on a 2.5 kg object that was initially at rest. What is its final velocity?

Answer

http://njc.tl/ls
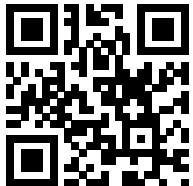
The Momentum of
a System of Objects

Return to
Table of
Contents

http://njc.tl/l3

The Momentum of a System of Objects

If a system contains more than one object, it's total momentum is the _vector sum_ of the momenta of those objects.

$$p_{system} = \sum p$$

$$p_{system} = p_1 + p_2 + p_3 + ...$$

$$p_{system} = m_1 v_1 + m_2 v_2 + m_3 v_3 + ...$$

It's critically important to note that

momenta add as vectors,

not as scalars.
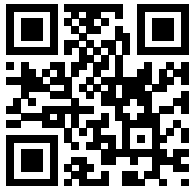
The Momentum of a System of Objects

$$p_{system} = m_1v_1 + m_2v_2 + m_3v_3 + ...$$

In order to determine the total momentum of a system,

First:
- Determine a direction to be considered positive
- Assign positive values to momenta in that direction
- Assign negative values to momenta in the opposite direction

Then:

momenta to get a total.
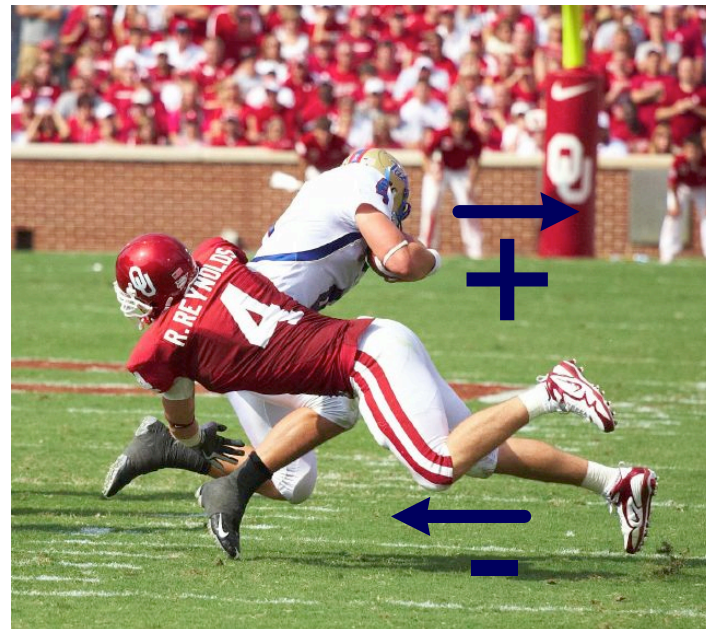
Example

Determine the momentum of a system of two objects: m_1, has a mass of 6 kg and a velocity of 13 m/s towards the east and m_2, has a mass of 14 kg and a velocity of 7 m/s towards the west.

(Choose east as positive)

$m_1 = 6$ kg
$v_1 = 13$ m/s

$m_2 = 14$ kg
$v_2 = -7$ m/s

$p_{system} = p_1 + p_2$

$= m_1 v_1 + m_2 v_2$

Answer

http://njc.tl/l3
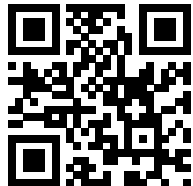
14 Determine the magnitude of the momentum of a system of two objects: m_1, has a mass of 6.0kg and a velocity of 20m/s north and m_2, has a mass of 3kg and a velocity 20m/s south.

Answer

http://njc.tl/l4
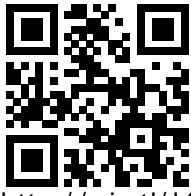
15 Determine the momentum of a system of two objects: the first has a mass of 8 kg and a velocity of 8 m/s to the east while the second has a mass of 5 kg and a velocity of 15 m/s to the west.

Answer

16 Determine the momentum of a system of 3 objects:
The first has a mass of 7.0 kg and a velocity of 23 m/s north; the
second has a mass of 9.0 kg and a velocity of 7 m/s north; and
the third has a mass of 5.0 kg and a velocity of 42 m/s south.

Answer

http://njc.tl/l6
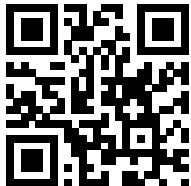
Conservation of Momentum

Return to
Table of
Contents

http://njc.tl/l7
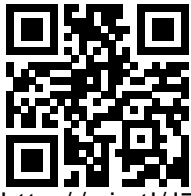
Conservation Laws

Some of the most powerful concepts in science are called "conservation laws".

Conservation laws:

- apply to closed systems - where the objects only interact with each other and nothing else.

- enable us to solve problems without worrying about the details of an event.

http://njc.tl/l7
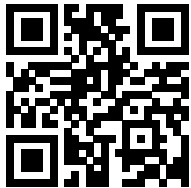
Momentum is Conserved

In the last unit we learned that energy is conserved.

Like energy, momentum is a conserved property of nature. It is not created or destroyed;

So in a closed system we will always have the same amount of momentum.

The only way the momentum of a system can change is if momentum is added or taken away by an outside force.

http://njc.tl/l7

Conservation of Momentum

To apply Conservation of Momentum,
Take snapshots of a system just <u>before</u> and <u>after</u> an event.
By comparing these two snapshots we can learn a lot.

We'll explore this more a little later.

http://njc.tl/l7
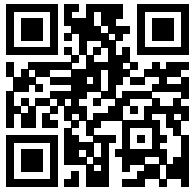
Conservation of Momentum and Impulse

Recall from our discussion of change of momentum and impulse:

$$p_0 + I = p_f$$

When a net external force acts on an object, it imparts an impulse I to the object, changing its momentum.

This is exactly the same for a system of objects.

$$p_{system,0} + I = p_{system,f}$$

If there is no net external force on the system, the momentum of the system is conserved.

$$p_{system,0} = p_{system,f}$$

http://njc.tl/l7

*Conservation of Momentum & Impulse Proof

Both the <u>Conservation of Momentum</u> and the concept
of <u>Impulse</u> follow directly from Newton's Second Law:

$F = ma$ where F is the net external force

$F = m(\Delta v / \Delta t)$ since $a = \Delta v / \Delta t$

$F\Delta t = m\Delta v$ after multiplying both sides by Δt

$F\Delta t = \Delta(mv)$ since m is constant

$\boxed{I = F\Delta t}$ - the definition of impulse I

$I = \Delta p$ substituting I for $F\Delta t$, and p for mv

$I = p_f - p_0$ since $\Delta p = p_f - p_0$

$I = p_f$

p_f *when there no net external force (F=0), I=0 so...*
momentum is conserved
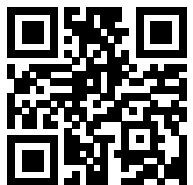
Conservation Laws, Collisions and Explosions

Objects in an isolated system can interact with each other in a number of ways...

- They can <u>collide</u>

- If they are stuck together, they can <u>explode</u> (push apart)
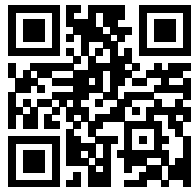
In an isolated system both momentum and total energy are conserved.

But the energy can change from one form to another.

<u>Conservation of momentum</u> and <u>change in kinetic energy</u> help us ＿ t what happened or what will happen in one of these ＿ ＿.

http://njc.tl/l7

Collisions and Explosions

We differentiate collisions and explosions by the way the energy changes or does not change form.

- **explosions**: an object or objects break apart because potential energy stored in one or more of the objects is transformed into kinetic energy

- **inelastic collisions:** two objects collide and stick together converting some kinetic energy into bonding energy, heat, sound...

- **elastic collisions**: two objects collide and bounce off each other while conserving kinetic energy

http://njc.tl/l7

Collisions and Explosions - Summarized

| Event	Description	Momentum Conserved?	Kinetic Energy Conserved?
Inelastic Collision	General collision: Objects bounce off each other	Yes	No. Some kinetic energy is converted to heat, sound... energy
Inelastic Collision	Objects stick together	Yes	No. Kinetic energy is converted to potential energy, bonding, or heat, sound...energy
Elastic Collision	Objects bounce off each other	Yes	Yes
Explosion	Object or objects break apart	Yes	No. Release of potential energy increases kintetic energy

http://njc.tl/l7

17 In _____ collisions momentum is conserved.

○ A Elastic

○ B Inelastic

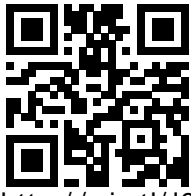○ C All

Answer

http://njc.tl/l8
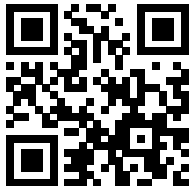
18 In _____ collisions kinetic energy is conserved.

○ A Elastic

○ B Inelastic

○ C All

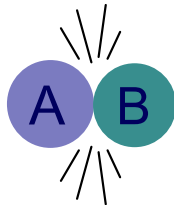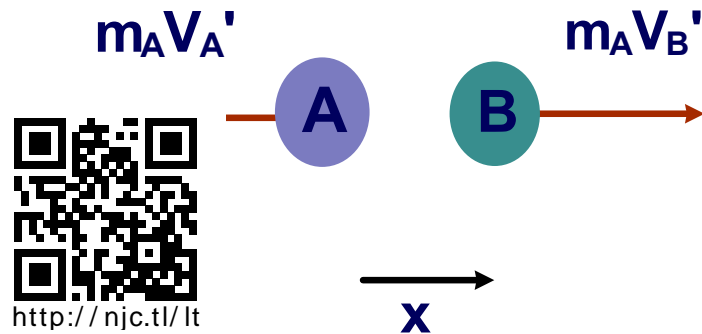**Answer**

Conservation of Momentum

During a collision or an explosion, measurements show that
the total momentum does not change:

$m_A V_A$ $m_B V_B$

A →   ← **B**

the prime means "after"

A B

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$m_A V_A'$ $m_A V_B'$

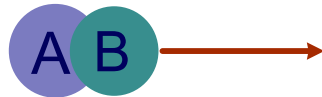— **A**   **B** →

http://njc.tl/lt

X →

Explosions

In an explosion, one object breaks apart into two or more pieces (or coupled objects break apart) moving afterwards as separate objects.

We will assume:

· the object (or a coupled pair of objects) breaks into two pieces

· explosion is along the same line as the initial velocity

Before (moving together)   **After (moving apart)**

$p_A+p_B=(m_A+ m_B)v$ $p_A'=m_Av_A'$ $p_B'=m_Bv_B'$

http://njc.tl/lt

$$p_A + p_B = p_A' + p_B'$$

$$(m_A + m_B)v = m_Av_A' + m_Bv_B'$$

19 A 5 kg cannon ball is loaded into a 300 kg cannon. When the cannon is fired, it recoils at 5 m/s. What is the cannon balls's velocity after the explosion?
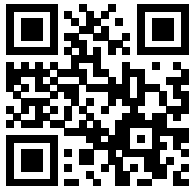
Answer

20 Two railcars, one with a mass of 4000 kg and the other with a mass of 6000 kg, are at rest and stuck together. To separate them a small explosive is set off between them. The 4000 kg car is measured travelling 6 m/s. How fast is the 6000 kg car going?

Answer

http://njc.tl/lb

Inelastic Collisions

In an inelastic collision, two objects collide and stick together, moving afterwards as one object.
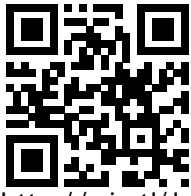
Before (moving towards the other)   **After (moving together)**

$p_A = m_A v_A$ $p_B = m_B v_B$ $p_A' + p_B' = (m_A + m_B)v'$

A   B   A B

$$p_A + p_B = p_A' + p_B'$$

$$m_A v_A + m_B v_B = (m_A + m_B)v'$$
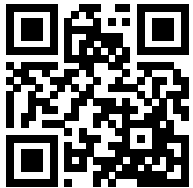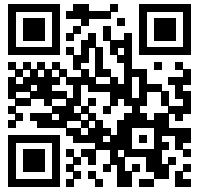
http://njc.tl/lu

21
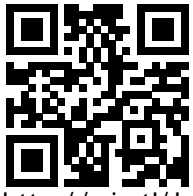
Answer

22

Answer

23

Answer

Elastic Collisions

In a elastic collision, two objects collide and bounce off each other.

Both momentum and kinetic energy are conserved.

If we know the masses and any two of the velocities, these two conservation equations enable us to calculate the other two velocities.

Before (moving towards)   **After (moving apart)**

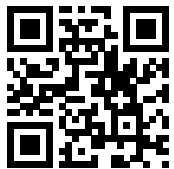$p_A=m_Av_A$ $p_B=m_Bv_B$ $p_A'=m_Av_A'$ $p_B'=m_Bv_B'$

A ⟶ ⟵ B   ⟵ A   B ⟶

$$p_A + p_B = p_A' + p_B'$$

$$m_Av_A + m_Bv_B = m_Av_A' + m_Bv_B'$$

$$KE_A + KE_B = KE_A' + KE_B'$$

$$\frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2 = \frac{1}{2}m_Av_A'^2 + \frac{1}{2}m_Bv_B'^2$$

http://njc.tl/lf

**Derivation of Elastic Collision Condition

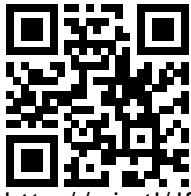| Conservation of Momentum	Conservation of Kinetic Energy
$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$	$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$
	$m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$
$m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$	$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2 - m_2v_2^2$
$m_1(v_1 - v_1') = m_2(v_2' - v_2)$	$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$
	$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$

$$\frac{m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)}{m_1(v_1 - v_1') = m_2(v_2' - v_2)}$$

$$v_1 + v_1' = v_2' + v_2$$

$$\boxed{v_1 - v_2 = -(v_1' - v_2')}$$

$$a^2 - b^2$$
$$= (a+b)(a-b)$$

Properties of Elastic Collisions

1. **For all elastic collisions**, regardless of the masses of the objects, the objects separate after the collision with the same relative speed that they collided with.

$$v_1 - v_2 = -(v_1' - v_2')$$

2. **In an elastic collision where one object is much more massive** than the other, the velocity of the smaller mass after the collision will be about twice that of the projectile while the more massive object's velocity will be almost unchanged.

m_1

m_2

v_1

$v_2 = 0$

m_1

m_2

$v_1' \approx v_1$

$v_2' \approx 2v_1$

$$v_1' = v_1 \ and \ v_2' = 2v_1$$

http://njc.tl/lf
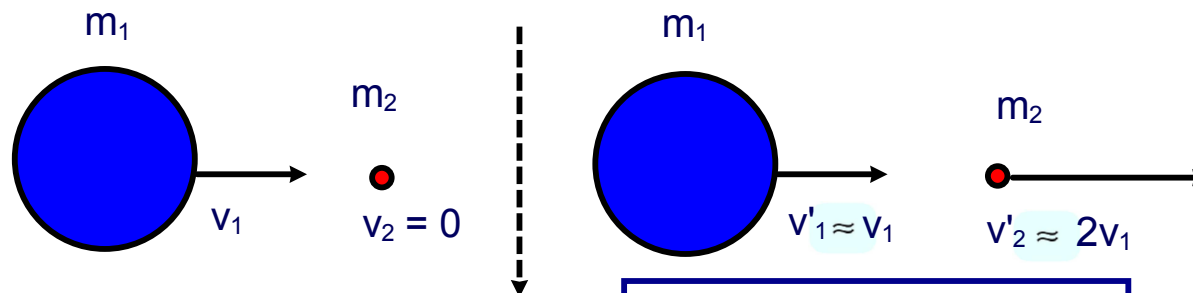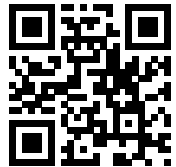
In an elastic collision between two objects of identical masses, the two objects exchange velocities.

$$v_1' = v_2 \ and \ v_2' = v_1$$

24 Two objects have an elastic collision. Before they collide they are approaching each other with a velocity of 4m/s relative to each other. With what velocity do they go apart from one another?
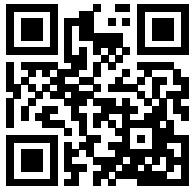
Answer

25　Two objects have an elastic collision. One object, m_1, has an initial velocity of +4.0 m/s and m_2 has a velocity of -3.0 m/s. After the collision, m_1 has a velocity of 1.0 m/s. What is the velocity of m_2?

Answer

26   A bowling ball has a velocity of +v when it collides with a ping pong ball that is at rest. The velocity of the bowling ball is virtually unaffected by the collision. What will be the speed of the ping pong ball?
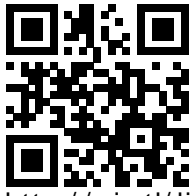
Answer

27 A baseball bat has a velocity of +v when it collides with a baseball that has a velocity of -2v. The bat barely changes velocity during the collision. How fast is the baseball going after it's hit?
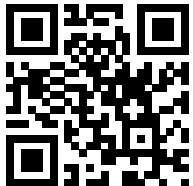
Answer

28 Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +6.0m/s and m_2 is -3.0m/s. What is the velocity of m_1 after the collision?

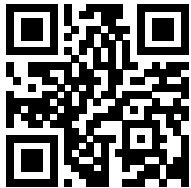**Answer**

29 Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +6.0m/s and m_2 is -3.0m/s. What is the velocity of m_2 after the collision?

Answer

30 Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +3.0m/s and m_2 is +2.0m/s. What is the velocity of m_1 after the collision?

Answer

http://njc.tl/lm

31 Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +3.0m/s and m_2 is +2.0m/s. What is the velocity of m_2 after the collision?

Answer