Optimal Allocation of Digital Marketing Budget: The Empirical Bayes Approach

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Abstract

The paper outlines a framework for online advertising budget allocation. First, it explores the empirical Bayes methodology for learning the effectiveness of different online ad placements - from historical data of varying quality. Second, it describes an analytical procedure for optimal budget allocation, which builds on risk management and reinforcement learning techniques.

Keywords: digital marketing, online advertising, budget allocation, spend optimization, empirical Bayes

1. Introduction

Allocation of advertising spend in the digital space is a challenge frequently faced by corporate marketers. Unfortunately, most of the research that addresses this issue focuses primarily on search advertising, approaches the process from the ad platform's side, and demands an unrealistic quality of data.

In the meantime, many companies - especially in the non-ecommerce settings - do not have access to the rich data on the effects of their online ads and have no ways of identifying the exact monetary value asscociated with the visitors coming in through online advertising.

Frequently, (a) # of times an ad is shown on a particular website though a specific media channel, and (b) # of clicks (and thus traffic to the website) such an ad attracts are the primary metrics available to the corporate decision maker.

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In an attempt to provide marketers with an instrument suited for the harsh realities of corporate data, the paper outlines how a decision maker could estimate the future expected effectiveness of different ad placements in terms of the above metrics, based on historical data, and then use the estimates to optimally allocate marketing budget.

The paper is structured as follows.

First, we introduce a Bayesian framework for learning - from historical data - the probabilistic models of (a) ad impression count per $1,000 of spend, and (b) click-through rate (CTR) for specific "*ad placements*" - combination of websites, where ads are placed, and media channels, through which ads are delivered (e.g. standard banners, smartphone banners, pushdown, video etc.)

CTR is defined as # of clicks on the ad/ # of ad impressions.

Second, we propose an analytical procedure for optimal digital budget allocation, which builds on risk management and reinforcement learning techniques.

Note, one implicit assumption underlies all subsequent discussion - it is the assumption that # of ad impressions and # of ad clicks per $1,000 of spend are valid criteria for judging the effectivenes of ad placements (when no other information is available). This assumption is not that unreasonable, given that marketers are typically interested in maximizing the visibility of their ads (ad impressions) and the amount of traffic driven to the company's website (ad clicks).

2. Empirical Bayes Modeling

The first challenge one faces when trying to come up with an ad budget allocation is a need to know how well each ad placement will perform in the future periods. One obvious way to get such estimates is to take simple averages from historical data (e.g. average # of ad impressions per $1,000 or an average CTR).

However, there are several issues with this approach.

First, such numbers may frequently exhibit outlier behavior, which could lead to a skew in the derived estimates. E.g. if only 1 impression is shown and 1 click comes through that impression for a particular ad placement, it would result in an extremely high CTR estimate - equal to 1, which is understandably misleading. This is primarily an issue when the available data covers just a few observation periods.

Second, one may be in a situation when no historical data at all is available for a specific website-channel combination. In this case, we may still be interested in making a guess about how well an untested placement would work, based on the data we have available for other placements.

Empirical Bayes framework is a natural way of dealing with these issues. The broad idea behind it is that one may more accurately predict the metric of interest for the considered ad placement (especially if data for it is scarce) by using historical data for all other ad placements.

Following empirical Bayes approach, in order to derive an estimate of the ad metric, we need to first learn the probabilistic model of how such metric is generated, based on all historical data available.

Once we have this information, empirical Bayes methodology provides us with a tool to incorporate our knowledge of how well all different ad placements have done into calculation of effectiveness estimate for the ad placement of interest. The extent to which we rely on the data for the considered ad placement vs. on data for all other ad placements will depend on how much data we have for the former.

Given that we deal with 2 types of data - namely (a) counts (# of ad impressions per $1,000) and (b) proportions (CTR) - 2 types of models will be necessary.

We begin with a model for count data.

2.1. Modeling Count Data with Gamma-Poisson (a.k.a. Negative Binomial) Distribution: Ad Impressions per $1,000 (Fader and Hardie, 2009; Fader et al., 2005; Wang et al., 2011; Richardson et al., 2007)

In order to predict the count of ad impressions per $1,000 we need to first determine how the count is generated. Poisson distribution is perfectly suited for this purpose. It describes the probability of x events, given parameter λ (which itself constitutes the mean expected # of events). Here x will denote the # of impressions generated by $1,000 spend.

$$P(X = x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{1}$$

This model is not perfect as the distrubution of impression counts (defined by parameter λ) would be different between various channels and websites. To capture this heterogeneity we assume that λ is not just a fixed number we have to estimate, but itself varies according to a Gamma distribution with parameters α & β.

$$g(\lambda|\alpha,\beta) = \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)} \tag{2}$$

Based on this assumption, we are able to derive a Gamma mixture of Poisson distributions, also known as a Negative Binomial distribution (NBD).

$$P(X = x|\alpha,\beta) = \int_0^\infty P(X = x|\lambda) g(\lambda|\alpha,\beta) d\lambda = \frac{\Gamma(\alpha + x)}{\Gamma(\alpha) x!} \left(\frac{\beta}{\beta+1}\right)^\alpha \left(\frac{1}{\beta+1}\right)^x \tag{3}$$

Using the above formulation, we pick NBD parmaters with highest log-likelihood - that is, such parameters of NBD, which maximize the probability of our data under NBD, and thus describe the distibution, which fits our data best. Log-likelihood formula is presented below.

$$LL(\alpha,\beta|data) = \sum_{i=1}^{n} ln\{P(X = x_i|\alpha,\beta)\} \tag{4}$$

The mean and variance of the NBD are calculated as follows:

$$E(x|\alpha,\beta) = \frac{\alpha}{\beta} \tag{5}$$

$$var(x|\alpha,\beta) = \frac{\alpha}{\beta} + \frac{\alpha}{\beta^2} \tag{6}$$

Once the parameters have been learned, to estimate the expected (posterior) count for a particular ad placement, we only need to plug its historically observed # of impressions per $1,000 into the below formula.

$$E(x_{new}|\alpha,\beta,x_{1:n}) = \frac{\alpha + \sum_{i=1}^{n} x_i}{\beta + n} = \frac{\beta}{\beta+n}\frac{\alpha}{\beta} + \frac{n}{\beta+n}\frac{\sum_{i=1}^{n} x_i}{n} \tag{7}$$

(Note: $\alpha + \sum_{i=1}^{n} x_i$, $\beta + n$ are the posterior parameters of NBD.)

It can be seen from the formula that the posterior estimate of impression count for a particular ad placement is fundamentally a weigted average of historical count for that ad placement and the population mean count.

Posterior variance estimate is calculated by plugging the posterior parameters into variance formula of the NBD distribution (6).

$$var(x_{new}|\alpha, \beta, x_{1:n}) = \frac{\alpha + \sum_{i=1}^{n} x_i}{\beta + n} + \frac{\alpha + \sum_{i=1}^{n} x_i}{(\beta + n)^2} \tag{8}$$

2.2. Modeling Proportions with Beta-Binomial Distribution: Click-through Rates (Fader and Hardie, 2009; Fader et al., 2005; Wang et al., 2011; Richardson et al., 2007)

As in the case of impression count, we need to come up with a model of how CTR is generated. The metric represents a proportion - and thus lies in the inteval between 0 and 1.

The simplest distribution for proportions is a Binomial distribution,

$$\Pr(x = k|m, p) = \left(\begin{array}{c} m \\ k \end{array} \right) p^k (1 - p)^{m-k} \tag{9}$$

where

$$\left(\begin{array}{c} m \\ k \end{array} \right) = \frac{m!}{(m - k)!k!} \tag{10}$$

The formula represents the probability of k successes in m trials, provided that the probability of success is equal to p. If we put it in marketing lingo, it could be interpreted as the probability of k clicks generated by m impressions, when the true CTR is equal to p.

This model is not perfect as CTR (i.e. parameter p) would vary between different channels, websites and time periods. To capture this heterogeneity we assume that p is not just a fixed number we have to estimate, but itself varies according to a 2-parameter Beta distribution,

$$g(p|\alpha, \beta) = \frac{p^{\alpha-1}(1 - p)^{\beta-1}}{B(\alpha, \beta)} \tag{11}$$

where $\alpha > 0$, $\beta > 0$ and $B(\alpha, \beta)$ is a beta function.
(See Appendix A)
The mean and variance of Beta distribution are expressed as follows:

$$E(p) = \frac{\alpha}{\alpha + \beta} \tag{12}$$

$$var(p) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{13}$$

Beta-Binomial distribution takes the following form:

$$\Pr(x = k|m, \alpha, \beta) = \int_0^1 \Pr(x = k|m, p) g(p|\alpha, \beta) dp = \binom{m}{k} \frac{B(\alpha + k, \beta + m - k)}{B(\alpha, \beta)} \tag{14}$$

As in the previous section, we can use the above formulation to pick parameters, which maximize the probability of our data under Beta-Binomial distribution (i.e. the parameters with the highest log-likelihood).

$$LL(\alpha, \beta|data) = \sum_{i=1}^{n} ln\{P(X = k_i|m_i, \alpha, \beta)\} \tag{15}$$

Once the parameters have been learned, and we want to estimate the posterior CTR for a particular ad placement, we only need to to plug the observed # of impressions and the # of clicks for the particular placement into the below formula (16), where $\alpha + x$ and $\beta + m - k$ are the posterior parameters of Beta-Binomial distribution. The formula is simply an expression for mean of Beta distribution (12), using the posterior parameters.

$$E(p|k, m, \alpha, \beta) = \frac{\alpha + k}{\alpha + \beta + m} = \left(\frac{\alpha + \beta}{\alpha + \beta + m}\right) \frac{\alpha}{\alpha + \beta} + \left(\frac{m}{\alpha + \beta + m}\right) \frac{k}{m} \tag{16}$$

The above formula essentially produces a weighted average between the population-wide CTR estimate ($\alpha/(\alpha + \beta)$ - a population mean for Beta distribution) and the historical CTR for a particular ad placement (k/m).

Posterior variance estimate is calculated by plugging posterior parameters into variance formula of the Beta distribution (13).

$$var(p|k, m, \alpha, \beta) = \frac{(\alpha + k)(\beta + m - k)}{(\alpha + \beta + m)^2(\alpha + \beta + m + 1)} \tag{17}$$

2.3. Adding Extra Flexibility: Including Covariates in Parameter Estimation Process (Lora and Singer, 2011; Lesnoff and Lancelot, 2012)

To build a truly flexible model, one might want to learn how hyperparameters vary between websites and ad channels. The effects of placement on hyperparameters can be estimated within the framework of generalized linear models for NBD and Beta-Binomial distributions - so that each placement gets its own sets of hyperparameters (and thus its own distributions - NBD and BB). The procedure is implemented in R package ”aod”.

The specification for both models is briefly covered below.

2.3.1. Re-parametrization to Accomodate Covariates: NBD

NBD is re-parametrized as follows

$$\mu = \frac{\alpha}{\beta} = exp(Xb) \tag{18}$$

$$\phi = \frac{1}{\alpha} \tag{19}$$

X is a covariates matrix, b is a fixed-effects vector. The parameters b and ϕ are estimated by maximizing the log-likelihood of data under the NBD.

Under this model, estimated hyperparameter α is common to all ad placements, whereas β estimate is re-calculated for each combination of websites and media channels. The fixed effects of specific websites and ad channels on β are captured by their respective dummy variables in the covariates matrix X.

2.3.2. Re-parametrization to Accomodate Covariates: Beta-Binomial Distribution

Beta-binomial distribution is re-parametrized as follows

$$r = \frac{\alpha}{\alpha + \beta} = exp(Xb) \tag{20}$$

$$\phi = \frac{1}{\alpha + \beta + 1} \tag{21}$$

X is a covariates matrix, b is a fixed-effects vector. The parameters b and ϕ are estimated by maximizing the log-likelihood of data under the beta-binomial distribution.

Based on this model, individual α and β estimates are calculated for each combination of websites and media channels. The fixed effects of specific websites and ad channels on α and β are captured by dummy variables in the covariates matrix X.

3. Optimization Procedure

Optimization procedure should take into consideration the following:

- We have different confidence in effectiveness of various ad placements. This confidence is captured by the posterior variance of the respective estimates - the greater the variance, the less confidence we have, and vice versa.
- We would want to spend more on the more effective placements, while balancing such choices based on the confidence we have in the estimates.
- Spend should be distributed among different placements - even if we are rather confident that some of the placements are relatively more effective - to mitigate the risk in case some placement shows unexpected disappointing results.
- Even if we are very confident about the estimates, we may still want to spend at least a small portion of the budget on the less effective placements - to detect a possible improvement in effectiveness.

Based on these considerations, the optimal allocation procedure could look as follows.

First, during the starting period, the decision maker allocates the spend randomly or in line with his expert knowledge.

Second, based on the outcome data, decision maker estimates hyperparameters of Negative-Binomial and Beta-Binomial distributions for each combination of websites and ad channels.

Third, decision maker allocates spend so as to maximize the posterior expected returns for the next period in terms of the said metrics, while accounting for the risk.

3.1. Risk-adjusted Expected Reward (Sahai and Khurshid, 1993)

The risk is accounted for by allocating spend to placements in proportion to their minimum expected return - e.g. at the lower mark of a 95% confidence interval - to penalize estimates with high variance. (We define this measure as a risk-adjusted expected reward, which can be viewed as an application of MaxMin principle, where we attempt to maximize the minimum expected reward).

The posterior confidence interval is calculated based on a normal approximation.

For NBD we assume the posterior mean x is approximately normally distributed, and an approximate $100(1-\alpha)\%$ confidence interval for x is given by:

$$x \pm z_{1-\alpha/2}\sqrt{var(x)} \tag{22}$$

Similarly - for the Beta-Binomial distribution:

$$p \pm z_{1-\alpha/2}\sqrt{var(p)} \tag{23}$$

Where $z_{1-\alpha/2}$ is the $100(1-\alpha/2)^{th}$ percentile of the standard normal distribution. (1.96 - for the 95% confidence interval).

Posterior variance equations (8 and 17) are provided in the previous sections.

Note: Strictly speaking, this confidence interval is only accurate for a sufficiently large # of observations, where one can apply the central limit theorem and assume that the sample mean is approximately normally distributed. However, for the purposes of this optimization procedure such precision is not necessary, and, it will be shown, the adjustment works well as is.

3.2. Determining the Proportion of Spend on Ad Placements: Soft Max Function (Barto, 1998)

The proportion, in which budget is allocated to each placement, is determined via a soft max function, frequently used in reinforcement learning sub-field of machine learning.

$$P_t(a) = \frac{exp(q_t(a)/\tau)}{\sum_{i=1}^{n} exp(q_t(i)/\tau)} \tag{24}$$

Action value $q_t(a)$ corresponds to expected reward (e.g. # of clicks on the ad), resulting from spend on ad placement a. τ is called a temperature parameter.

For high temperatures ($\tau \rightarrow \infty$), all the spend is distributed nearly equally between different placements, and the lower the temperature, the more expected rewards affect the spend allocation. For a low temperature ($\tau \to 0^+$), the proportion of spend on the placement with the highest expected reward tends to 1.

Temperature parameter is chosen in a way to strike a balance between maximizing the reward over the observed past periods, and making sure a satisfying degree of spend diversification is achieved.

The choice of τ should be approached carefully. If the parameter is too low, more spend will be allocated to the top performing ad placements, and the short-term reward may increase. However, because the effectiveness of any single ad placement may unexpectedly plunge, such over-concentration of spend inevitably carries a risk.

In addition to that, some minimal amount of spend on an ad placement may be necessary to elicit its true effectiveness, and thus lack of spend diversification may impede one's ability to pick up changes in effectivens of the underperforming ad placements, when the spend on them is minute.

On the other hand, very high τ would lead one to allocate spend almost uniformly across all of the ad placements, preventing one from capitalizing on known differences in ad placements' performance, and thus also resulting in sub-optimal payoff.

However, one could choose to set τ to a higher value at the early stages - to ensure a more equal representation of various ad placements in the early spend mix, and thus to enable a more uniform exploration of the ad placements' performance. This may in turn allow for more informed ad placement choices in the future.

During the initial iterations, when no historical data is available, the parameter τ could be set to an arbitrary value. The rule of thumb would be that no more than 50% of spend should be allocated to any single ad placement - or a similar cut-off threshold suited for the specific situation.

In the experimental results section we provide some insight into what τ values may lead to a more optimal outcome.

Before we conclude with this subsection, it is worth elaborating more on the reasoning behind the use of soft max function in spend allocation procedure. We single out 2 arguments for such use.

First, as noted previously, soft max framework is extremely flexible in allowing one to easily vary the levels of spend diversification by tweaking temperature parameter τ. This dependence of spend diversification on just one parameter makes possible a simple optimization routine to infer the optimal level of spend diversification (optimal τ) from historical data - i.e. such spend diversification that maximizes the specified success metric - whatever this metric may be. This feature also renders the framework very well suited as a basis for automation of the spend allocation process.

Second, soft max framework fits well some desiderata we outlined earlier. Due to exponents in the expression, it complies with the principle of increased spend on the more cost-effective ad placements, while also allowing for some minor spend on the less effective ad placements - for monitoring and information collection purposes.

Note, however, that regardless of the motivations outlined above, other allocation schemes may be more practical/relevant in specific corporate circumstances, and, in that sense, the proposed allocation scheme is not necesserily the "ultimate truth".

3.3. Optimization Objective

The proposed optimization procedure may seem to suggest the presence of a single ubiquitos objective, towards which we optimize our spend - namely, expected reward $q_t(a)$ from spend on an ad placement a - as in equation (24).

Indeed, in the experimental results section we optimize towards a single metric - expected # of clicks per \$1,000 of spend (# of impressions * CTR).

However, the proposed framework can easily accomodate optimization against several metrics. To accomplish this we can define the expected reward $q_t(a)$ for a specified ad placement a during period t as a weighted function (with weights w_i) of different expected rewards/metrics $u_{it}(a)$ corresponding to n different objectives (e.g # of impressions per \$1,000 of spend as a proxy for ad's impact on awareness, # of clicks generated per \$1,000 of spend as a measure of how well an ad drives traffic):

$$q_t(a) = \sum_{i=1}^{n} w_i u_{it}(a)^{norm}, \quad where \;\; 0 \leq w_i \leq 1 \;\; and \;\; \sum_{i=1}^{n} w_i = 1 \tag{25}$$

The expected rewards/metrics $u_{it}(a)$ should be first normalized to a common scale before inclusion in the above formula - e.g. through 0-1 normalization:

$$u_{it}(a)^{norm} = \frac{u_{it}(a) - u_{it}(a)_{min}}{u_{it}(a)_{max} - u_{it}(a)_{min}} \tag{26}$$

The weights w_i would capture the relative importance of different objectives to the decision maker. Quite frequently choice of weights is subjective, rather than based on some objective measure. Exact procedure to determine such weights is a subject of studies in the field of decision theory and is beyond the scope of this work.

4. Experimental Results

4.1. Overview of the Experiment

The data used in the experiment covers weekly # of ad impressions and # of ad clicks per $1,000 of spend for specific media channels (video, banners etc.) on specific websites during a 6-week period.

For each separate media channel, ads displayed on the targeted websites are all of the same type (design, message). However, the framework can be easily extended to accomodate several types of ads for each channel. This is accomplished by including the ad type dummy variables in the covariates matrix. Estimated fixed-effect coefficients would capture how a particular ad type impacts the hyperparameters of the Negative-Binomial and Beta-Binomial distributions.

The experiment goes as follows. First, we learn the hyperparameters of Negative Binomial and Beta-Binomial distibutions, based on 2 first weeks of data. We also estimate how the parameters depend on specific websites and media channels, through which an ad is delivered.

Then we itiratively calculate posterior estimates for # of impressions per $1,000 and CTR, based on available history, and make a decision on spend allocation for the next period, using the resulting estimates. We allocate spend in proportion to the expected # of clicks per $1,000 of spend (# of impressions * CTR).

2 success metrics we track are (a) the # of ad impressions and (b) the # of ad clicks we are able to generate by spending $1,000 in line with the derived spend allocation proportions.

We run this process using (a) risk-adjusted posterior estimates, (b) posterior estimates without risk adjustment, and (c) simple historical averages, while varying τ - temperature parameter, which controls how concetrated the spend allocation is on the higher rated ad placements.

Additionally, we re-run this process by plugging in actual future ad placement effectiveness - instead of the estimates. This way we are able to determine the boundary of best results we could possibly achieve within soft max spend allocation scheme - that is, if we knew what exactly is going to happen

Table 1: Success metrics generated per $1,000 of spend, depending on the type of ad effectiveness estimates used in budget allocation procedure

		τ value (spend diversification levels)*					
		5		25		50	
Impressions (#)	True future ad effectiveness	399,760	+93%	327,678	+38%	291,660	+18%
	Bayesian estimates with risk-adjustment	296,209	+43%	297,837	+25%	281,103	+14%
	Bayesian estimates without risk-adjustment	260,376	+26%	266,452	+12%	276,236	+12%
	Simple averages (baseline)	206,752	-	237,757	-	246,770	-
Clicks (#)	True future ad effectiveness	861	+43%	745	+54%	577	+42%
	Bayesian estimates with risk-adjustment	803	+34%	571	+18%	447	+10%
	Bayesian estimates without risk-adjustment	603	+0.%	533	+10%	459	+13%
	Simple averages (baseline)	601	-	484	-	406	-

* Spend on the most cost-effective placement by τ value: ∼85% for $\tau = 5$; ∼16% for $\tau = 25$; ∼7% for $\tau = 50$.

in the future. How close we can get to this boundary when using estimates reflects the quality of those estimates.

4.2. Results

Here we show that spend allocation, based on empirical Bayes estimates, leads to improved results, compared to the approach that uses simple averages. We then show that an adjustment for risk leads to further improvement in the results, compared to use of unadjusted posterior estimates, bringing us closest to the boundary, which demarks the results we could get if we - by magic - knew the true future ad effectiveness. The results are measured in terms of achieved # of impressions and # of clicks per $1,000 of spend.

See Table 1, Fig. 1 & 2.

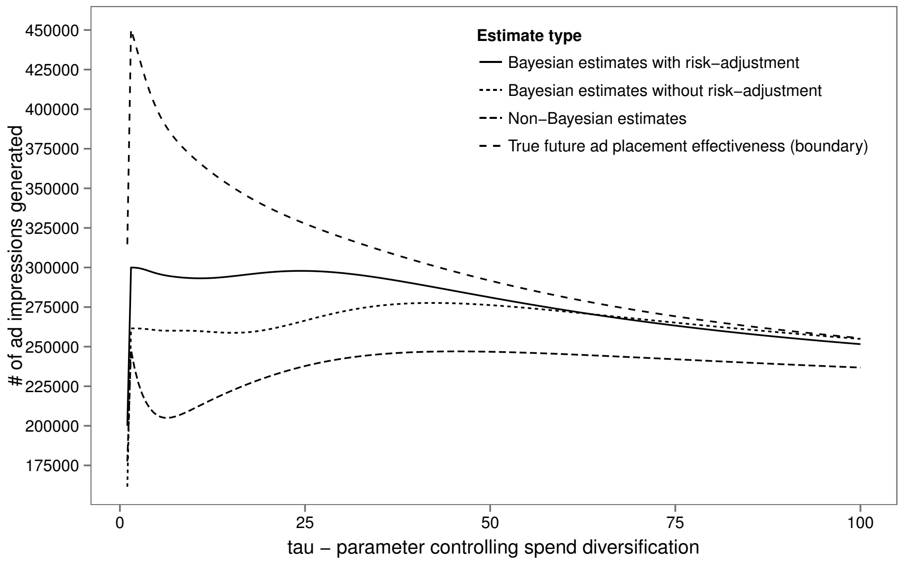

Figure 1: # of ad impresions generated per \$1,000 of spend, based on different estimates of future ad placement effectiveness used when allocating budget

The experimental results also provide some ground for picking τ parameter. As discussed in the previous section, the best long-term results are to be expected when the marketer finds a balancing point between investing solely into ad placements with the highest historical effectiveness and increasing the diversification of the spend and thus stability of the results.

In other words - it is always a trade-off.

For the particular dataset we use, best results - both in terms of # of ad impressions and # of ad clicks per \$1,000 - are achieved when we spend everything on the top performing ad placement.

However, if such risk is unacceptable and there is some minimum required degree of spend diversification (e.g. $\tau >= 10$, which translates roughly into 50% of spend allocated to the most effective ad placement), $\tau = 25$ is where we observe a local maximum for # of impressions per \$1,000 metric, and this value of τ could thus be considered optimal. (This is especially true if the decision maker is interested in maximizing the # of times an ad is shown). # of clicks per \$1,000, however, monotonely decreases with growing τ, and, if it is the target metric, one may choose to go simply with the lowest acceptable value of τ.

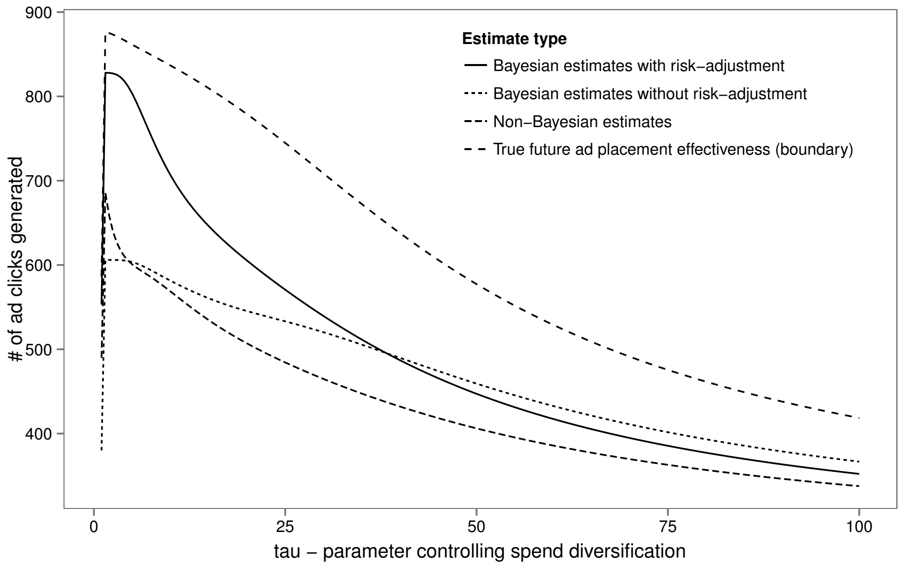

Figure 2: # of ad clicks generated per $1,000 of spend, based on different estimates of future ad placement effectiveness used when allocating budget

Note that when τ value grows beyond a certain point, and we begin to spend almost equal amounts on all ad placements - results from using different kinds of estimates begin to deteriorate, and converge. This is an illustration of how high τ values fundamentally "destroy" extra information carried by the more accurate estimates of ad placment effectiveness, and thus lead to equally sub-optimal results, regardless of what specific type of estimate one uses.

When selecting τ value, it is also convenient to keep track of a plot, where τ is displayed against a measure of spend diversification, e.g. entropy (Fig. 3 - here entropy is calculated for spend allocation with Bayesian risk-adjusted estimates).

This view helps a decision maker pick the τ value, which strikes a balance between spend diversification (i.e. risk mitigation) and an objective of maximizing success metrics. In particular, we see that increase in spend diversification is very steep to the left of $\tau = 25$ mark, but becomes quite gradual once we move to the right of this point - a case of diminishing returns. Such logic supports our previous conslusion on the appropriate τ value ($\tau = 25$). (See Appendix B for details on entropy calculation).

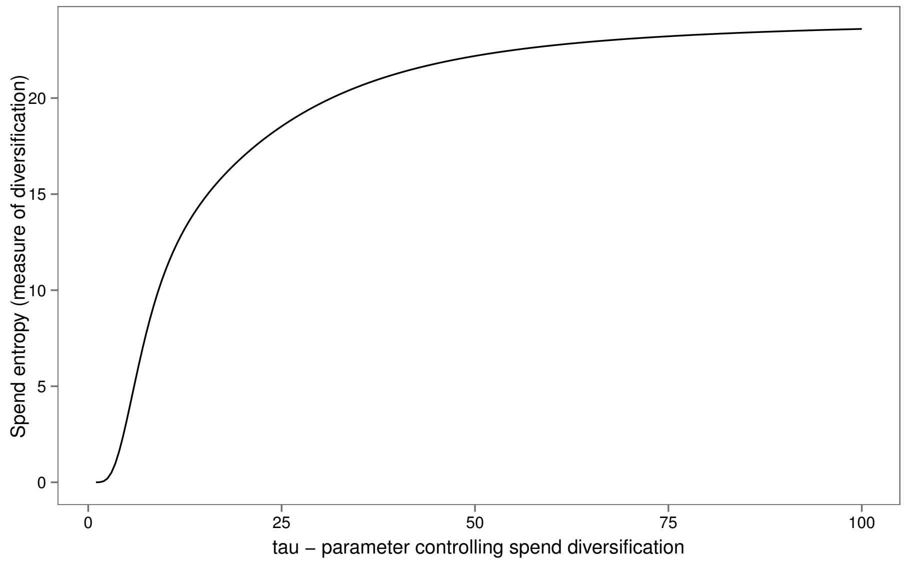

Figure 3: Spend diversification (entropy) vs. τ parameter

5. Conclusion

First, we show that the proposed empirical Bayes method with a risk-adjustment helps a decision maker arrive at estimates of ad placement effectiveness, which lead to a more optimal digital spend allocation - reflected in higher resulting # of ad impressions and # of ad clicks per $1,000.

The approach is well suited for the real-world corporate marketing environment, where high-quality ad effectiveness data is frequently not available.

Second, soft max function is shown to be a flexible budget allocation framework in allowing one to strike a balance between maximizing the expected payoff from the ad spend and diversifying the spend.

We conclude that while concentrated spending on the top performing ad placements may yield better results in the short term, more spend diversification is a better long-term budget allocation strategy, as it allows one to (a) mitigate the risk of negative changes in ad placement performance, and (b) have more opportunity to detect the positive changes. Simultaneously, we show that too much diversification can also lead to sub-optimal results.

Finally, some guidelines are provided as to selection of the optimal level of spend diversification, controlled by temperature parameter τ.

Appendices

Appendix A. Beta function

The Beta function $B(\alpha, \beta)$ is defined by the integral

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt,\ \alpha > 0,\ \beta > 0$$

Alternatively, the beta function can be expressed in terms of gamma functions.

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\Gamma(0) = 1,\ \Gamma(n) = (n-1)! \text{ for positive } n$$

Appendix B. Entropy calculation

We measure spend diversification using the entropy formula from information theory, where $P(x_i)$ indicates a proportion of budget to be spent on a particular ad placement.

$$H(X) = -\sum_i P(x_i) \log_b P(x_i)$$

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