ECE 4560
Assignment 8

Due: October 31st 🎃, 11:59pm

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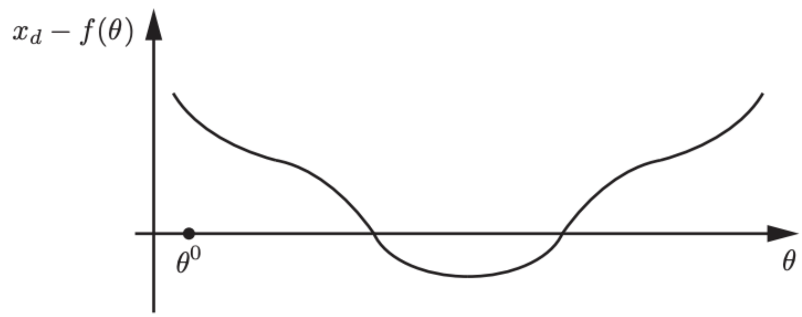
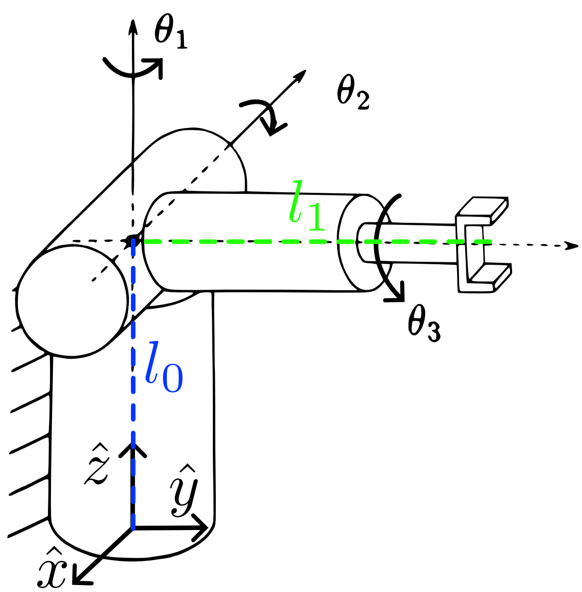
1. (2 points) Perform three iterations of (approximate) iterative Newton-Raphson root finding on the scalar function $x_d - f(\theta)$ in the figure below, starting from θ_0 (shown in the figure). (A general vector function $f(\theta)$ could represent the forward kinematics of a robot, and x_d could represent the desired configuration in coordinates. The roots of $x_d - f(\theta)$ are the joint vectors θ satisfying $x_d - f(\theta) = 0$ i.e., solutions to the inverse kinematics problem.) Draw the iterates $\theta_1, \theta_2, \theta_3$ on the θ axis and illustrate clearly how you obtain these points.



2. Consider the following manipulator:



(a) (2 points) Find the forward kinematics map, expressed in terms of the joint angles. I recommend using the Product of Exponentials since the twists will be used in part b and c. If you've derived the expression correctly, you should arrive at the following configuration for the coordinates $\theta = [\pi/3, -\pi/4, -\pi/2]^\top$ and $l_0, l_1 = 1$:

$$g_e(\theta) = \begin{bmatrix} 0.6124 & -0.6124 & -0.5 & -0.6124 \\ -0.3536 & 0.3536 & -0.8660 & 0.3536 \\ 0.7071 & 0.7071 & 0 & 1.7071 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) (2 points) Derive the Spatial Manipulator Jacobian for this manipulator by directly computing ω_i' and q_i' for each joint. Then use these terms to compute ξ_i' for each joint. If you've derived the expression correctly, you should arrive at the following Jacobian for the same coordinates as before ($\theta = [\pi/3, -\pi/4, -\pi/2]^\top$ and $l_0, l_1 = 1$):

$$J^s = \begin{bmatrix} 0 & 0.8660 & -0.3536 \\ 0 & -0.5000 & -0.6124 \\ 0 & 0 & 0 \\ 0 & -0.5000 & -0.6124 \\ 0 & -0.8660 & 0.3536 \\ 1 & 0 & 0.7071 \end{bmatrix}$$

(c) (2 points) Instead of your approach to b, how else could you have solved for the Spatial Manipulator Jacobian using the adjoint of the matrix exponential of the individual twists (i.e., $\mathrm{Ad}_{e^{\hat{\xi}_i \theta_i}}$)? Please write the full formula for the Spatial Manipulator Jacobian using this method.

(d) (2 points) Write code to solve for this Spatial Manipulator Jacobian using your expression from part c. Verify that it matches your solution for part b. Note that you should be able to now compute the Spatial Manipulator Jacobian for any set of joint angles using your code.

(e) (2 points) Let's assume that we want to control the end-effector to have an angular velocity of $\omega_s = (\pi/2, 0, \pi)^\top$ rad/s, what joint velocity $\dot{\theta}$ would achieve this? What about an angular velocity of $\omega_b = (\pi/2, 0, \pi)^\top$. For both, the initial starting configuration is $\theta = [\pi/3, -\pi/4, -\pi/2]^\top$.

Hint: Use your code from part d to compute J_s along with your Spatial Manipulator Jacobian to solve for the joint velocities $\dot{\theta}$ that would achieve this ω_s. Notice that you will need to use the pseudoinverse since the manipulator is underactuated.

Challenge: As an extra challenge, try coding up a simple "simulation" in MATLAB/Python to show the path of the manipulator arm for this $\dot{\theta}$. You can do this using the update:

$$\theta_{k+1} = \theta_k + \dot{\theta}\Delta t$$

where Δt is a small time-step (e.g., 0.1s). You can then use your forward kinematics from part a to compute the position of the end-effector at each time step. Plot the path of the end-effector in 3D space.

3. **LAB COMPONENT:** This week we will be implementing the inverse kinematics (that you computed last week) on hardware. Additional details are provided online

   (a) (3 points) Please show your simulation to one of the TAs during lab office hours. The TAs will test your implementation by giving you random initial block locations and a random final target location (all within the workspace). If this is not possible, please include in your video submission yourself entering various target block locations to show that the inverse kinematics is working accurately. For full credit, each group member must also submit a writeup of the lab this week, how they implemented the code, and any issues encountered. The writeup should be a minimum of one paragraph.