ECE 4560
Assignment 4

Due: September 20th, 11:59pm

Note: Please denote the number of hours you spent on this homework (Feel free to also throw in a rating from 0-10). Please separate your time into homework vs. lab hours. I am *not* keeping track of effort per student, I just want to know if the homeworks are a reasonable length on average.

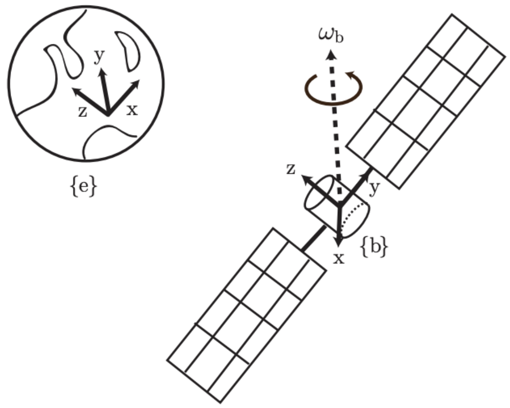
1. The purpose of this exercise is to practice our understanding of how to represent ξ in both vector notation as well as in homogeneous representation.
   (a) (1 point) Given that $\xi \in \mathfrak{se}(2)$ and in particular, $\xi = (10, 2, -\frac{\pi}{2})^T$, what is $\hat{\xi}$?
   (b) (1 point) Given that $\xi \in \mathfrak{se}(3)$ and in particular, $\xi = (4, 2, 3, \pi, \frac{\pi}{4}, \frac{\pi}{2})^T$, what is $\hat{\xi}$?
   (c) (1 point) Given
   $$\hat{\xi} = \begin{bmatrix} 0 & -\frac{\pi}{8} & 7 \\ \frac{\pi}{8} & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix},$$
   what is $\xi \in \mathfrak{se}(2)$?

2. The purpose of this second exercise is to practice our understanding of skew-symmetric matrices.
   (a) (1 point) Given a vector $v = (1, 2, 3)^T$, what is $[v]_\times$?
   (b) (1 point) Given a skew-symmetric matrix $[v]_\times = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$, what is v?

3. Let $g_{sb} \in SE(2)$ represent the configuration of the frame $\{b\}$ relative to $\{s\}$. For simplicity, this is typically written as just g. If $\{b\}$ moves over time, you could represent its velocity as $\dot{g}_{sb}$ (or simply $\dot{g}$), the time derivative of g_{sb}.
   (a) (1 point) What do we call the quantity $\dot{g}g^{-1}$? What does it represent? And how many values are needed to uniquely specify it (in $SE(2)$ and $SE(3)$)?
   (b) (1 point) What do we call the quantity $g^{-1}\dot{g}$? What does it represent? And how many values are needed to uniquely specify it (in $SE(2)$ and $SE(3)$)?

4. (1 point) Let the orientation of $\{b\}$ relative to $\{a\}$ be $R_{ab} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ and a point p be represented in $\{a\}$ as $p_a = (1, 2, 3)$. What is p_b? (Give a numeric 3-vector.) Hint: Use the subscript cancellation rule.

5. Consider the satellite and Earth shown below



Let $\omega_b = (0, 1, 1)$ be the angular velocity of the satellite expressed in the satellite body frame $\{b\}$. Assume a fixed Earth frame $\{e\}$ (a geocentric view of the universe like the ancient Greeks had).

(a) (2 points) Solve for the coordinate axis velocities of $\{b\}$ $(\dot{\hat{x}}_b, \dot{\hat{y}}_b, \dot{\hat{z}}_b)$ represented in the $\{b\}$ frame. Sketch the velocity vectors on the figure above to confirm that your solutions make sense.

(b) (2 points) The orientation of the $\{b\}$ frame is equivalent to the $\{e\}$ frame after it has been rotated 90 degrees about its $\hat{z}_e$-axis (i.e., $R_{be} = R(\pi/2)$). Solve for ω_e, the satellite angular velocity represented in $\{e\}$. Sketch the velocity vectors on the figure above to confirm that your solution makes sense.

6. (3 points) **LAB COMPONENT:** For lab this week, please start brainstorming with your group possible goals for the end-of-year project. In addition, all groups will be asked to complete this week's assignment for the SO101 robot track. This will introduce the MuJoCo simulation environment, which may also be useful for future homework assignments. The instructions for this lab assignment can be found on the course website.