Progress of Theoretical Physics, Vol. 86, No. 5, November 1991

Effect of Rayleigh Scattering on Cosmic Microwave Background Anisotropies

Fumio TAKAHARA and Shin SASAKI

Department of Physics, Tokyo Metropolitan University, Hachioji 192-03

(Received July 5, 1991)

We study the effect of Rayleigh scattering off neutral hydrogens on CMB anisotropies. We numerically solved the evolution of frequency dependent radiation perturbations coupled with matter perturbations for baryon dominated $\Omega=1$ universe in the synchronous gauge. It is found that at wavelengths shorter than 0.05 cm radiation perturbations are significantly damped and that small scale CMB anisotropies are smaller by 20% than those obtained when only Thomson scattering is taken into account. Thus the effect of Rayleigh scattering is milder than suggested by La. Rayleigh scattering has little effect on matter perturbations and radiation perturbations at longer wavelengths.

§ 1. Introduction

In the standard Big Bang cosmology, galaxies and clusters of galaxies are assumed to be originated from infinitesimal perturbations in the early universe. Such perturbations at the recombination epoch should appear as CMB (cosmic microwave background) anisotropies. In spite of extensive observational studies, no CMB anisotropies have been detected except for the dipole anisotropy due to the motion of the Local group and their upper limits are lower than predicted values for most scenarios of galaxy formation (see, e.g., Suto et al.[1]). This discrepancy between theory and observation is one of the serious problems of cosmology and requires close examinations of physical processes in the universe at and after the recombination epoch. One of the possibilities to solve this discrepancy is that radiation perturbations have been damped by some processes after the recombination.

In this paper, we study the effect of Rayleigh scattering off neutral hydrogens on CMB anisotropies. Rayleigh scattering has been neglected in most works for CMB anisotropies since it is only related to very short wavelength regions and has only a minor effect on the evolution of perturbations. It is a correct assumption if we discuss the radiation perturbations integrated over frequency. However, Rayleigh scattering can influence the fluctuations at high frequencies. The cross section of Rayleigh scattering is frequency dependent and at short wavelengths the effect is stronger than or comparable to that of the Thomson scattering. Thus, anisotropies of CMB should depend on frequency. The frequency region of submillimeter wavelengths is potentially important because of relatively strong intensity fluctuations expected and little contamination components. La[2] pointed out that radiation perturbations at wavelengths shorter than 1 mm are damped by Rayleigh scattering. However, his treatment did not take account of the effects of matter perturbation, and quantitative assessment remains to be done. In this paper we numerically solve the evolution of frequency dependent radiation perturbations coupled with matter perturbations for baryon dominated $\Omega=1$ universe in the synchronous gauge.

Section 2 presents the evolution equations of perturbations with Rayleigh scattering. Numerical results and discussion are contained in §§ 3 and 4, respectively.

§ 2. Basic equations

Since the cross section of Rayleigh scattering depends on frequency, we extend the method of Silk and Wilson[3] (see also Wilson and Silk[4]) to incorporate the frequency dependence of radiation perturbations. We assume that radiation and matter interact through both Thomson scattering and Rayleigh scattering. For simplicity, we assume that the universe consists of photons and hydrogens and that the unperturbed universe is $K=0$ Friedmann universe with the Hubble constant of $H_0=50$km/sec/Mpc. We study adiabatic perturbations in the synchronous gauge

$$g_{00}=-1\,, \quad g_{0i}=0\,, \quad g_{ij}=T^{-2}(\delta_{ij}-h_{ij})\,, \tag{1}$$

where the scale factor R is expressed with the temperature T as $R=1/T$.

Evolution of the distribution function of photons f is described by the Boltzmann equation

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}}-\Gamma^{\lambda}_{\mu\nu}p^{\mu}p^{\nu}\frac{\partial f}{\partial p^{\lambda}}=-(p^{\mu}u_{\mu})[n_e\sigma_T+n_H\sigma_R](\bar{f}-f)\,, \tag{2}$$

where

$$\bar{f}(x^i, E, \hat{\gamma}, t)=\frac{3}{16\pi}\int[1+(\hat{\gamma}\cdot\hat{\gamma}')^2]f(x^i, E', \hat{\gamma}', t)d\Omega'\,. \tag{3}$$

Here $\hat{\gamma}$ is a unit vector indicating photon direction, $E'=E(1-\gamma^i v_i)$ and u^{μ} is the four velocity of matter with $v^i=u^i/T$. The number density of free electrons and hydrogen atoms are denoted by n_e and n_H, respectively, and σ_T and σ_R are the cross sections of Thomson and Rayleigh scattering, respectively, which are given by

$$\sigma_T=6.65\times10^{-25}\ \mathrm{cm}^2 \tag{4}$$

and

$$\sigma_R(\nu)=\sigma_T\sum_{k=2}\left(\frac{f_{1k}\nu^2}{\nu_{1k}^2-\nu^2}\right)^2, \tag{5}$$

where f_{1k} is the oscillator strength, and ν_{1k} is the frequency corresponding to the energy difference between the ground state and the k-th level. When evaluating the cross section of Rayleigh scattering, summation is taken up to $k=5$. Defining $f=f_0(E, t)(1+\delta_r(x^i, E, \hat{\gamma}, t))$ with f_0 being a Planck function, perturbation equation for f becomes

$$\begin{aligned}&\dot{\delta}_r+\frac{\dot{T}}{T}E\frac{\partial\delta_r}{\partial E}+T\hat{\gamma}^i\frac{\partial\delta_r}{\partial x^i}+\frac{1}{2}\dot{h}_{ij}\hat{\gamma}^i\hat{\gamma}^j\frac{E}{f_0}\frac{\partial f_0}{\partial E}\\&=\frac{1}{f_0}[n_e\sigma_T+n_H\sigma_R]\left\{-E\frac{\partial f_0}{\partial E}v^i\hat{\gamma}^i+f_0(E)\left[\frac{3}{4}\bar{\delta}_r+\frac{1}{4}f_{ij}^{(r)}\hat{\gamma}^i\hat{\gamma}^j-\delta_r\right]\right\},\end{aligned} \tag{6}$$

where $f_{ij}^{(r)}\equiv(3/4\pi)\int\delta_r\hat{\gamma}^i\hat{\gamma}^j d\Omega$. Notations and other equations are the same as those for frequency integrated case given by Wilson and Silk[4] except the equation of velocity, which is given by

$$\dot{v}^i-\frac{\dot{T}}{T}v^i=\left(\frac{aT^4}{4\pi}\right)^{-1}\frac{aT}{b}\left\{\frac{1}{3}\int E^4[n_e\sigma_T+n_H\sigma_R]\frac{\partial f_0}{\partial E}v^i dE\right.$$
$$\left.+\int E^3[n_e\sigma_T+n_H\sigma_R]f_0(E)f^{(r)i}dE\right\} \tag{7}$$

with $\rho_m=bT^3$, $\rho_r=aT^4$ and $f^{(r)}\equiv(1/4\pi)\int\delta_r\hat{\gamma}^i d\Omega$. We expand perturbed quantities with plane waves and use Legendre polynomial expansion for both δ_r and perturbed energy density of photons δ as

$$\delta_r(k,t,\mu)=\sum_{l=0}^{\infty}\delta_l^{(r)}P_l(\mu)\,, \tag{8}$$

where $\mu=\hat{\gamma}\cdot\hat{k}$ and the sum is truncated at $l=99$. Partial differential equation (6) can be reduced to an ordinary differential equation by transforming the independent variables (t,E) to (t,E_0) as

$$E(t)=\frac{T(t)}{T_0}E_0\,, \tag{9}$$

where E_0 is the energy of a photon at the present epoch. Thus we should solve a set of ordinary differential equations instead of a set including a partial differential equation. We finally obtain the following set of ordinary differential equations for perturbations of matter density δ_m, velocity v, time derivative of the trace of perturbed metric $\dot{h}$ and $\delta_l^{(r)}$,

$$\frac{d\delta_m}{dT}=\frac{1}{\dot{T}}\left[-ikTv+\frac{1}{2}\dot{h}\right], \tag{10}$$

$$\frac{dv}{dT}=\frac{v}{T}+\frac{1}{\dot{T}}\frac{4\pi}{3}\frac{1}{bT^3}\left\{v\int E^4(n_e\sigma_T+n_H\sigma_R)\frac{df_0}{dE}dE\right.$$
$$\left.+\int E^3(n_e\sigma_T+n_H\sigma_R)f_0\delta_1^{(r)}dE\right\}, \tag{11}$$

$$\frac{d\dot{h}}{dT}=\frac{2}{T}\dot{h}+\frac{1}{\dot{T}}8\pi G(bT^3\delta_m+2aT^4\delta_0)\,, \tag{12}$$

$$\frac{\partial\delta_0^{(r)}}{\partial T}=-\frac{1}{\dot{T}}\frac{1}{3}ikT\delta_1^{(r)}-\frac{1}{6}\frac{E_0}{f_0}\frac{df_0}{dE_0}\frac{\dot{h}}{\dot{T}}\,, \tag{13}$$

$$\frac{\partial\delta_1^{(r)}}{\partial T}=-\frac{1}{\dot{T}}[n_e\sigma_T+n_H\sigma_R]\left[\delta_1^{(r)}+\frac{E_0}{f_0}\frac{df_0}{dE_0}v\right]-\frac{1}{\dot{T}}ikT\left(\delta_0^{(r)}+\frac{2}{5}\delta_2^{(r)}\right), \tag{14}$$

$$\frac{\partial\delta_2^{(r)}}{\partial T}=-\frac{1}{\dot{T}}\frac{9}{10}[n_e\sigma_T+n_H\sigma_R]\delta_2^{(r)}-\frac{1}{\dot{T}}\frac{1}{3}\frac{E_0}{f_0}\frac{df_0}{dE_0}\dot{h}$$
$$+\frac{1}{\dot{T}}\frac{E_0}{f_0}\frac{df_0}{dE_0}\frac{8\pi Gi}{kT}\left(bT^3v+\frac{1}{3}aT^4\delta_1\right)-\frac{1}{\dot{T}}ikT\left(\frac{2}{3}\delta_1^{(r)}+\frac{3}{7}\delta_3^{(r)}\right), \tag{15}$$

$$\frac{\partial \delta_l^{(r)}}{\partial T} = -\frac{1}{\dot{T}}[n_e\sigma_T + n_H\sigma_R]\delta_l^{(r)} - \frac{1}{\dot{T}} ikT\left(\frac{l}{2l-1}\delta_{l-1}^{(r)} + \frac{l+1}{2l+3}\delta_{l+1}^{(r)}\right) \quad (l>2) \tag{16}$$

and

$$\dot{T} = -T^3\left(\frac{8\pi G}{3}\right)^{1/2}(a+b/T)^{1/2}. \tag{17}$$

Since we treat only adiabatic perturbations, initial conditions are taken as

$$\delta_0 = \frac{4}{3}\delta_m, \quad \dot{h} = \frac{2\delta_m}{t}, \quad v=0, \quad \delta_1 = 4v \quad \text{and} \quad \delta_l = 0 \quad (l>1) \tag{18}$$

at $T=10^8$K. At the early time well before the recombination epoch, we can approximate the system of radiation and matter as a one fluid system. Because matter is almost fully ionized, Rayleigh scattering can be neglected compared with Thomson scattering. We evaluate the set of equations with one fluid approximation until $T=8000$K, and neglect Rayleigh scattering until $T=4500$K, where the ionization degree is above 90%. For $4500\text{K} > T > 1000\text{K}$, we evaluate Eqs. (10)~(17) with a full account of both scattering. We compute n_e by the method of Jones and Wyse.[5]

Comoving mass $M(k)$ is defined by

$$\begin{aligned} M(k) &= \frac{1}{6}\pi\rho_0\left(\frac{2\pi}{kT}\right)^3 \\ &= \frac{1}{6}\pi b\left(\frac{2\pi}{k}\right)^3 \\ &= \left(\frac{2.49\times 10^{-21}}{k}\right)^3 M_\odot . \end{aligned} \tag{19}$$

When we compute the integration over k, we assume that the initial power spectrum is given by $|\delta_m|^2 \propto k^n$.

§ 3. Numerical results

Hereafter we refer to the case when Rayleigh scattering is also taken into account as Case A. For comparison, we made numerical calculations for the case when only Thomson scattering is taken into account, which is referred to as Case B. For both cases, δ_m and v are not different from each other irrespective of a scale of fluctuation. Thus Rayleigh scattering does not affect the evolution of matter fluctuations. This is quite reasonable since Rayleigh scattering affects only high frequency photons with a relatively small energy content. Thus, we can safely use results of previous calculations without Rayleigh scattering, when we normalize the amplitude of the initial fluctuations by the observed galaxy correlation function. Rayleigh scattering is more effective than Thomson scattering when

$$\sigma_R(\lambda_{\gamma,i})n_H \geq \sigma_T n_e , \tag{20}$$

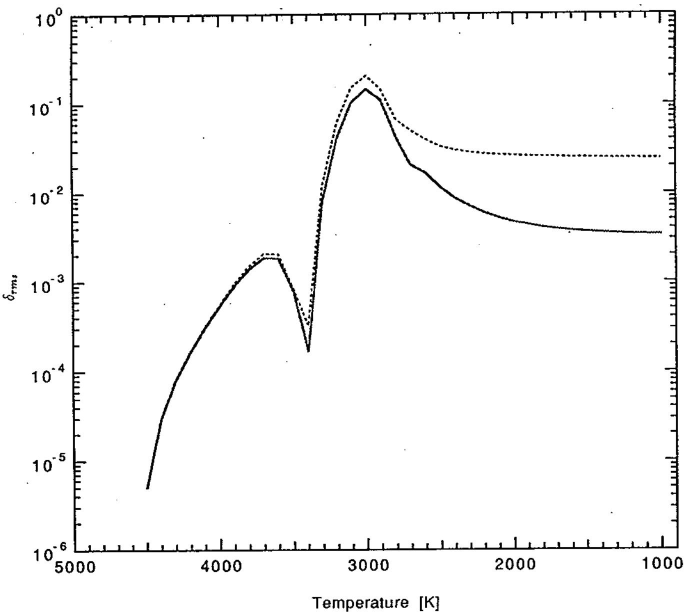

Fig. 1. Time evolution of δ_{rms} for the present wavelength of $\lambda=0.03$cm and the mass scale of 3.0×10^{14} $M_\odot$. For Case A, both Thomson and Rayleigh scattering are taken into account, while for Case B only Thomson scattering is included. Case A is shown by a solid line, and Case B is shown by a dashed line. Effect of Rayleigh scattering after the recombination epoch is seen. The unit of δ_{rms} is arbitrary.

where $\lambda_{r,i}$ is the wavelength of a photon at $T=T_i$. Since this inequality does not depend on a scale of fluctuations, the critical frequency at which both contributions are equal is determined by the ionization degree. For example, at $T=3000$K when the ionization degree is about 0.01, this critical wavelength corresponds to the present wavelength of $\lambda=0.03$cm.

An example of evolution of radiation fluctuation is displayed in Fig. 1, for the scale of fluctuation of 3.0×10^{14} $M_\odot$ and the present wavelength of 0.03 cm. As seen in Fig. 1, radiation fluctuations for Case A damp more strongly than Case B below 3000 K. For example, the amplitude of radiation fluctuations δ_{rms} ($\delta_{rms}^2=\sum_{l=2}^{\infty}|\delta_l|^2/(2l+1)$) for Case A is about 10% of that for Case B at $T=1000$K. The behavior of δ_{rms} strongly depends on both the scale of fluctuations and the wavelength of photon as expected, which is shown in the following. In Fig. 2, the ratio of δ_{rms} for Case A to that for Case B at $T=1000$K is displayed as a function of scale length for the present wavelength of 0.03cm and 0.5cm. The effect of the wavelength dependence of Rayleigh scattering can be seen clearly. At a long wavelength of 0.5 cm, the effect of Rayleigh scattering does not appear for any scale length of the fluctuation. But, at a short wavelength of 0.03cm, a damping effect by Rayleigh scattering appears in all scales of fluctuation. This tendency is explained by the frequency dependence of the cross section of Rayleigh scattering. At short wavelengths, small scale fluctuations are damped by Rayleigh scattering more strongly than large scale fluctuations. This is because the damping occurs when scale of fluctuation becomes shorter than the mean free path of photons and when the scattering time is shorter than the expansion time scale. For example, at $T=2500$K, the mean free path of photons of the present wavelength 0.03cm is 5.8×10^{22}cm which corresponds to the mass scale of about 2×10^{15} $M_{\odot}$, thus fluctuations more massive than 2×10^{15} $M_{\odot}$ do not damp at $T=2500$K while smaller ones should have begun to damp. Since small scale fluctuations begin to damp earlier, small scale fluctuations damp more strongly than large scale ones. For 2×10^{14} $M_{\odot}$ and $\lambda=0.03$cm, the degree of damping of radiation perturbations for Case A is three times stronger than that for Case B. In Fig. 2, the ratio of δ_{rms} for $\lambda=0.03$

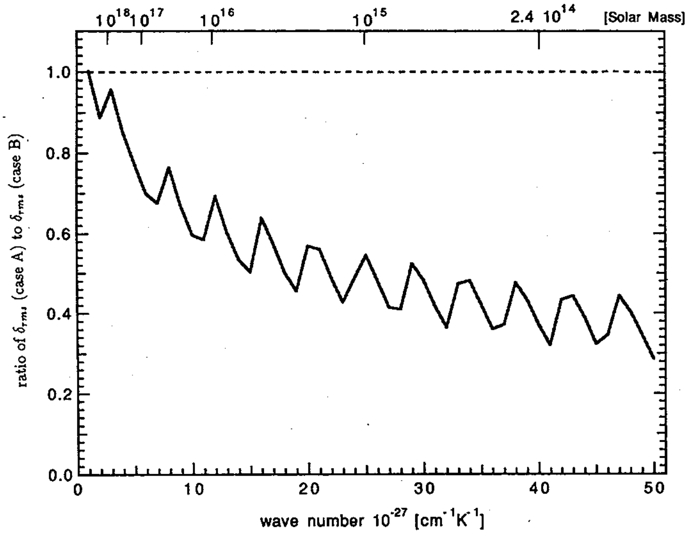

Fig. 2. The ratio of δ_{rms} (Case A) to δ_{rms} (Case B) versus the wavenumber of fluctuation. A dashed line shows the ratio for the present wavelength of 0.5cm, and a solid line shows that for the present wavelength of 0.03cm.

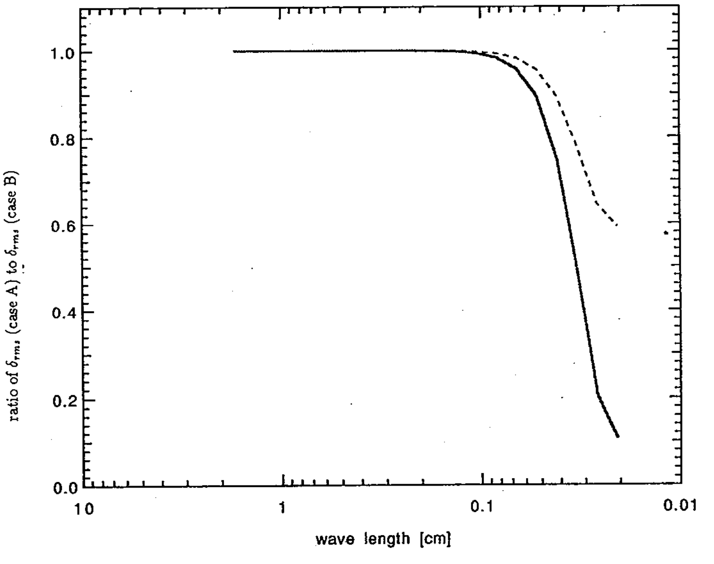

Fig. 3. The ratio of δ_{rms} (Case A) to δ_{rms} (Case B) versus the present wavelength of a photon. A dashed line shows the fluctuation scale of 1.2×10^{17} $M_{\odot}$, and a solid line shows that of 5.7×10^{14} $M_{\odot}$.

cm shows an oscillating behavior with the wavenumber rather than monotonically decreases as the wavenumber increases. This is because Rayleigh scattering is effective in a finite epoch around $T=3000$K: the degree of damping depends on the phase of δ_{rms} at the epoch. If δ_{rms} takes a peak at the epoch, the effect of Rayleigh scattering is strong while it is weak if δ_{rms} takes a minimum at the epoch.

In Fig. 3, we show the wavelength dependence of the ratio of δ_{rms} at $T=1000$K for a fixed mass scale of $1.2\times10^{17}\,M_{\odot}$ and $5.7\times10^{14}\,M_{\odot}$. The effect of Rayleigh scattering appears at wavelengths less than 0.05cm in both scales. Since the critical wavelength where Rayleigh scattering becomes effective compared to Thomson scattering does not depend on the scale of fluctuation, it is expected that the critical wavelength where the ratio becomes smaller than unity is the same for all scales of fluctuations. The critical wavelength turns out to be about 0.08cm. But the strength of damping is more remarkable in small scale fluctuations as mentioned above. At $\lambda=0.03$cm, this ratio turns out to be 0.3 and 0.6 for $M=5.7\times10^{14}\,M_{\odot}$ and $1.2\times10^{17}\,M_{\odot}$, respectively.

Next, we investigate CMB anisotropies at small angular scales. The intensity fluctuation is given by

$$\left(\frac{\delta I}{I}\right)^2(\lambda,\theta)\propto\int_0^{\infty}|\delta_{rms}(\lambda,k)|^2\left(1-\frac{\sin(3ct_0k\theta)}{3ct_0k\theta}\right)k^2dk\,, \tag{21}$$

where θ is a separation angle and λ is photon wavelength at the present epoch. The effect of a finite beam width of antenna is neglected for simplicity. When the wavelength is in the Rayleigh-Jeans region, this quantity is identical with the temperature anisotropies. It is to be noted that it is not the same as the temperature anisotropy when we deal with the Wien region. Since we are interested in the effect of Rayleigh scattering, we consider the ratio of $\delta I/I$ for Case A to that for Case B

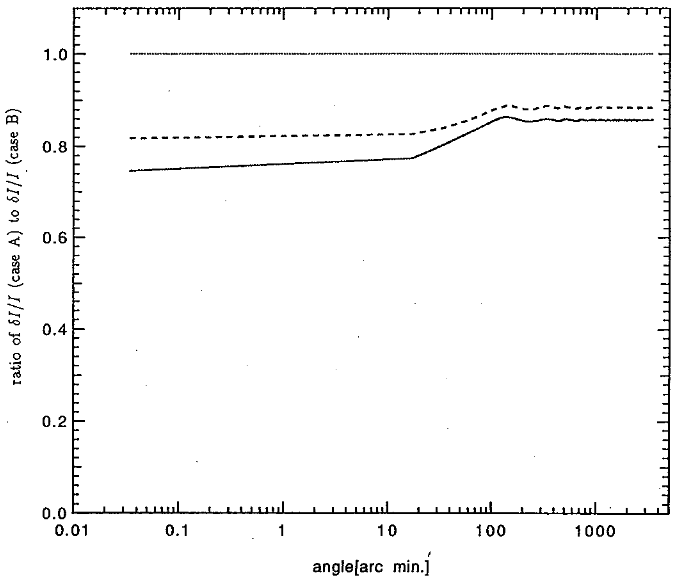

Fig. 4. The ratio of $\delta I/I$ (Case A) to $\delta I/I$ (Case B) versus the angular scale for present wavelength of 0.5 cm and 0.03 cm. A dashed line shows the case of the initial power law index $n=0$ and the present wavelength 0.03 cm. A solid line shows the case of $n=1$ and 0.03 cm. A dotted line shows the case of 0.5 cm for both $n=0$ and $n=1$.

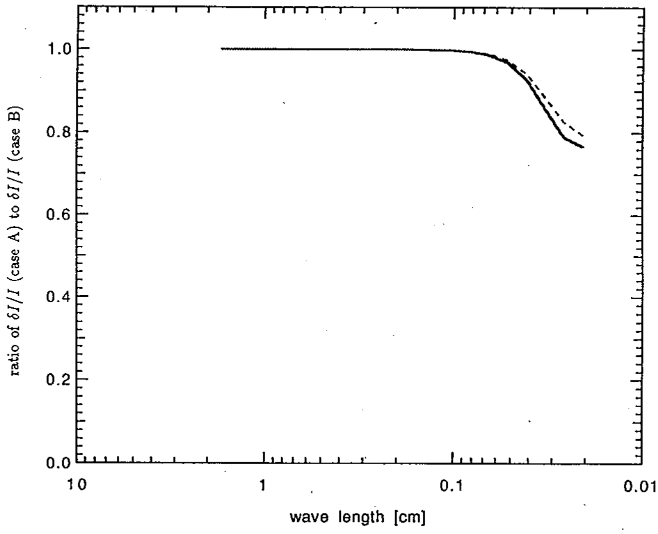

Fig. 5. The wavelength dependence of the ratio of $\delta I/I$ (Case A) to $\delta I/I$ (Case B) at the angular scale of $\theta=108'$. Assumed initial power law index is $n=0$ which is shown by a dashed line and $n=1$ which is shown by a solid line.

instead of $\delta I/I$ itself. In Fig. 4, CMB anisotropies at $\lambda=0.5$cm and 0.03cm are shown for the initial power law index $n=0$ and $n=1$. It is apparent that the effect of Rayleigh scattering does not appear at long wavelengths. Since at long wavelengths δ_{rms} is not affected by Rayleigh scattering as mentioned above, it is clear that $\delta I/I$ is the same for Cases A and B. On the other hand, at short wavelengths we can see the effect of Rayleigh scattering at all angular scales. Damping at short wavelengths is more remarkable for $n=1$ than for $n=0$. This is due to the fact that the model $n=1$ contains more power in small scale fluctuations which are more strongly damped. Although the decrease of anisotropies due to Rayleigh scattering is remarkable in small angles, the amplitude of damping is about 20% for both $n=0$ and $n=1$. This is fairly smaller than the damping factor of δ_{rms}. This is because the anisotropies are determined mainly by large scale fluctuations for which the damping effect is relatively mild since the power of large scale fluctuations is several orders of magnitudes larger than that of smaller scale ones.

Finally, in Fig. 5 we show the wavelength dependence of CMB anisotropies at a fixed angular scale of 108 arcmin. Damping occurs at wavelengths less than 0.05cm for both $n=0$ and $n=1$ as expected from the behavior of δ_{rms}. For $n=1$ the effect of Rayleigh scattering is stronger than $n=0$ as discussed above.

§ 4. Discussion

We compare the above results with those by La.[2] He suggested that CMB anisotropies at the present wavelength shorter than 0.1cm are damped significantly by Rayleigh scattering. In our results, this critical wavelength becomes 0.05cm and the degree of damping turns out to be only 20%. This quantitative difference between La[2] and our results arises because of three reasons. First, he evaluated the cross section of Rayleigh scattering assuming that the oscillator strength is 1. This assumption made his cross section four times as large as our value. Second, he assumed the homogeneous distribution of matter, which is not realistic since there are matter perturbations which grow into galaxies. Matter perturbations create new perturbations of photons or reduce the degree of the damping especially for large scale perturbations. Due to the above two reasons damping effect by Rayleigh scattering in his paper is stronger than ours where realistic oscillator strengths and the effect of perturbations of matter are incorporated. Finally, he assumed damping by Rayleigh scattering begins at $T \approx 4000$K, but in reality, the ionization degree is 0.1 at $T \approx 3500$K where the mean free path of photons by Thomson scattering is smaller than the scale of fluctuations (the mean free path of photons corresponds to about 10^{12} $M_\odot$) and Rayleigh scattering is not effective between $T \approx 4000$K and 3500K. This difference of the epoch when the damping by Rayleigh scattering begins to be effective results in the difference of the critical wavelength at which the effect of damping appears. For these reasons the critical wavelength was somewhat overestimated in La.

In this paper, we calculated only for $\Omega_{\text{baryon}}=1$. If Ω_{baryon} is smaller than 1, the mean free path of photon is longer than for $\Omega_{\text{baryon}}=1$. Thus the scale of fluctuation which is damped by Rayleigh scattering becomes larger than for $\Omega_{\text{baryon}}=1$. At the same time the degree of damping will become milder for $\Omega_{\text{baryon}}<1$ since a photon suffers less Rayleigh scattering after recombination. The effect on CMB anisotropies is determined by an interplay of both effects, although the effect of Rayleigh scattering becomes negligible for $\Omega_{\text{baryon}}< \sim 0.1$.

Recently, de Berrnardis et al.[6] reported the upper limit of CMB anisotropies at wavelength between 0.04cm and 0.2cm as $\Delta T/T \approx 2.2\times 10^{-4}$, for an angular scale of 108′. Their value is averaged with frequency and the equivalent wavelength is ~ 0.1 cm. Thus we cannot determine wavelength dependence of CMB anisotropies from this report. On the other hand, Page et al.[7] made a four-band anisotropy search including 0.044 and 0.063cm; however, up to now these bands have been used to measure the galactic dust emission. Although they have conducted new observations, the results at submillimeter wavelengths are yet to be reported.[8] We expect that future observations of CMB anisotropies in all wavelengths reveal the effect of Rayleigh scattering.

§ 5. Conclusions

It is found that (i) matter perturbations are not influenced by Rayleigh scattering and that (ii) CMB anisotropies depend on frequency since the cross section of Rayleigh scattering depends on frequency. CMB anisotropies at wavelengths longer than 0.05cm are not influenced by Rayleigh scattering. (iii) On the other hand, CMB anisotropies at wavelengths shorter than 0.05cm are damped by about 20%. Thus the effect is shown to be milder than suggested by La.[2] Effect of Rayleigh scattering is potentially important for future observations.

References

1) Y. Suto, N. Gouda and N. Sugiyama, Astrophys. J. Suppl. **74** (1990), 665.
2) D. La, Astrophys. J. **341** (1989), 575.
3) J. Silk and M. L. Wilson, Phys. Scripta **21** (1980), 708.
4) M. L. Wilson and J. Silk, Astrophys. J. **243** (1981), 14.
5) B. J. T. Jones and R. F. G. Wyse, Astron. Astrophys. **249** (1985), 144.
6) P. de Bernardis, L. Amicone, A. De Luca, M. De Petris, M. Epifani, M. Gervasi, G. Guarini, S. Masi and F. Melchiorri, Astrophys. J. Lett. **360** (1990), L31.
7) L. A. Page, E. S. Cheng and S. S. Meyer, Astrophys. J. Lett. **355** (1990), L1.
8) S. S. Meyer, E. S. Cheng and L. A. Page, Astrophys. J. Lett. **371** (1991), L7.