CENG 222

Statistical Methods for Computer Engineering

Spring '2016-2017

Assignment 4

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Answer 9.8

a)

Confidence interval for the mean when σ is known : $\overline{X} \pm$ z $_{\alpha/2}$ $\frac{\sigma}{\sqrt{n}}$

$$\sigma = 5 \text{ , n} = 64$$

$$1 - \alpha = 0.95$$
$$\alpha/2 = 0.025$$

$$\begin{aligned}\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} &= 42 \pm z_{0.025}\frac{5}{\sqrt{64}} \\ &= 42 \pm 1.96\frac{5}{\sqrt{64}} \\ &= [\mathbf{40.775{,}43.225}]\end{aligned}$$

b)

Standardize and use Table A4.For a Normal($\mu = 900, \sigma = 200$) variable X,

$$
\begin{aligned}
P(40.775 \le X \le 43.225) &= P(\frac{40.775-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{43.225-\mu}{\sigma}) \\
&= P(\frac{40.775-40}{5} \le Z \le \frac{43.225-40}{5} \\
&= P(0.155 \le Z \le 0.645) \\
&= \Phi(0.645) - \Phi(0.155) = 0.7406 - 0.5616 = \mathbf{0.1790}
\end{aligned}
$$

Answer 9.16

a)

We have $\eta_1 = 250$, $\eta_2 = 300$, $\hat{p}_1 = 10/250 = 0.04$ and $\hat{p}_2 = 18/300 = 0.06$.For the confidence interval,we have

$$\text{center} = \hat{p}_1\text{- } \hat{p}_2 = \text{-0.02}$$

and

$$
\begin{aligned}
margin &= z_{0.02/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{\eta_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{\eta_2}} \\
&= (2.326)\sqrt{\frac{(0.04)(0.96)}{(250)} + \frac{(0.06)(0.94)}{(300)}} \\
&= 0.043
\end{aligned}
$$

Then

$$\text{-0.02} \pm 0.043 = [\textbf{-0.063,0.023}]$$

b)

We are 98% sure that the real value of the difference of proportions p_1 and p_2 is between -0.063 and 0.023.There is **no** significant difference between the quality of the two lots.

Answer 10.3

I chose to divide the observations into 10 bins, as follows:

Bin	Observed Counts
(<-2.0)	4
(-2.0, -1.5)	4
(-1.5, -1.0)	15
(-1.0, -0.5)	9
(-0.5, 0.0)	22
(0.0, 0.5)	15
(0.5, 1.0)	12
(1.0, 1.5)	11
(1.5, 2.0)	7
(>2.0)	1

a)

The corresponding standard normal probabilities and the expected number of observations (with n=100) are the following:

Bin	Observed Counts	Standart Normal Probability	Expected Counts	Observed-Expected	Chi-Square
(<-2.0)	4	0.023	2.3	1.7	1.26
(-2.0, -1.5)	4	0.044	4.4	-0.4	0.04
(-1.5, -1.0)	15	0.092	9.2	5.8	3.66
(-1.0, -0.5)	9	0.150	15.0	-6.0	2.40
(-0.5, 0.0)	22	0.191	19.1	2.9	0.44
(0.0, 0.5)	15	0.191	19.1	-4.1	0.88
(0.5, 1.0)	12	0.150	15.0	-3.0	0.60
(1.0, 1.5)	11	0.092	9.2	1.8	0.36
(1.5, 2.0)	7	0.044	4.4	2.6	1.53
(>2.0)	1	0.023	2.3	-1.3	0.73

The chi-square statistic is the sum of of the values in the last column, and is equal to **11.89**.Since the data are divided into 10 bins and we have estimated two parameters,the calculated value may be tested against the chi-square distribution with 10 -1 -2 = 7 degrees of freedom.

$$\boldsymbol{P} = \boldsymbol{P}\{\chi^2 \geq 11.89\} = \textbf{\textit{between 0.1 and 0.2}}$$

We conclude that there is no evidence against a Standard Normal distribution of collected sample.

b)

The corresponding uniform probabilities and the expected number of observations (with n=100) are the following:

Bin	**Observed Counts**	**Uniform Probability**	**Expected Counts**	**Observed - Expected**	**Chi-Square**
(<-2.0)	4	0.166	16.6	-12.6	9.56
(-2.0, -1.5)	4	0.083	8.3	-4.3	2.23
(-1.5, -1.0)	15	0.083	8.3	6.7	5.40
(-1.0, -0.5)	9	0.083	8.3	0.7	0.06
(-0.5, 0.0)	22	0.083	8.3	13.7	22.61
(0.0, 0.5)	15	0.083	8.3	6.7	5.40
(0.5, 1.0)	12	0.083	8.3	3.7	1.65
(1.0, 1.5)	11	0.083	8.3	2.7	0.88
(1.5, 2.0)	7	0.083	8.3	-1.3	0.20
(>2.0)	1	0.166	16.6	-15.6	14.67

The chi-square statistic is the sum of of the values in the last column, and is equal to **62.66**.Since the data are divided into 10 bins,the calculated value may be tested against the chi-square distribution with 10 -1 = 9 degrees of freedom.

There is significant evidence that the distribution is **not** Uniform.

c)

Yes,accepting both null hypotheses in (a) and (b) is theoretically possible although they are contradicting to each other.Because, when we apply the χ^2 goodness-of-fit test , we can only show that the hypotheses is wrong.If we can't find significant evidence to reject the hypotheses , this means that the hypotheses may be true.