Solving Stable Marriage Problems using Answer Set Programming

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Abstract

Matching problems are generally about markets where individuals are matched with firms, items or other individuals. There is a variety of real life applications of matching problems, such as university entrance (e.g., which students go to which school) and kidney donation (e.g., who receives which transplantable organ). In this project, we study matching problems called stable marriage problem, using answer set programming.

Keywords: stable marriage problem, answer set programming

1 Introduction

Matching theory is a subfield of economics that deals with a common problem of real life: matching. Matching problems aim to construct a pairwise relations between the elements of the same set or two different sets. We call such a relation a match, and the resulting relationship a matching.

There are a variety of matching problems. In the roommate problem, the aim is to find roommates by matching people with other people. In this problem, a match is between two people. On the other hand, the university entrance problem aims to match students with universities according to the preference list of students and the preference list of universities (e.g., exam scores of the students), and the quotas. In this problem, a match is between a student and a university.

In this project, we have studied the stable marriage problem by Gale and Shapley (1962), one of the earliest problems in matching theory. In this problem, we are given a set of men with their preference lists and a set of women with their preference lists. The objective of this problem is to find a matching between men and women such that all of the marriages are stable.

In stable marriage problem, a marriage is stable if and only if there is no blocking pair. A blocking pair is a pair of man and woman that are not married together, but they both prefer each other to their spouses. In case there exists a blocking pair, this pair of man and woman would divorce from their spouses to marry with each other, therefore the matching wouldn't be stable.

In this project, we have introduced a novel method for solving the stable marriage problem and some of its variants using answer set programming. Answer set programming is a form of declarative programming oriented towards difficult combinatorial search problems. ASP is based on the concept of answer sets described by Gelfond and Lifschitz (1988). We have used `CLINGO` (Gebser, Kaminski, Kaufmann, & Schaub, 2014), an answer set solver developed by Torsten Schaub's group at the University of Potsdam.

2 Solving Stable Marriage Problem using ASP

The stable marriage problem (SMP) is defined as follows:

An instance I of the SMP consists of n men and n women, where each person has a preference list that strictly orders all members of the opposite sex. If a man m prefers w_1 to w_2, we write $w_1 \succ_m w_2$; similarly, if a woman w preferes m_1 to m_2, we write $m_1 \succ_w m_2$.

A perfect matching M on I is a set of disjoint man-woman pairs on I. When a man m and a woman w are matched to each other in M, we write $M(m) = w$ and $M(w) = m$.

A man m and a woman w are said to form a blocking pair for M (or to block M) when: (i) $M(m) \neq w$; (ii) $w \succ_m M(m)$; and (iii) $m \succ_w M(w)$.

A matching M is unstable if a blocking pair exists for M, and stable otherwise. The SMP asks for finding a stable matching M.

We model SMP in ASP, and present it in `CLINGO` as follows:

```
% GENERATE -- generates a matching

{pair(M, F) : woman(F)} = 1 :- man(M).
:- {pair(M, F) : man(M)} > 1, woman(F).

% DEFINE -- definitions to use

% M prefers DF to CF.
mprefer(M, DF, CF) :- mpref(M, CF, CUR), mpref(M, DF, DEV), DEV < CUR.

% F prefers DM to CM.
wprefer(F, DM, CM) :- wpref(F, CM, CUR), wpref(F, DM, DEV), DEV < CUR.

% TEST -- constraints

:- man(M1; M2), woman(F1; F2), pair(M1, F1), pair(M2, F2),
   mprefer(M1, F2, F1), wprefer(F2, M1, M2).
```

2.1 Stable Marriage Problem with Unacceptability and Ties (SMPTI)

The stable marriage problem with unacceptability and ties (SMPTI) is a variant of the stable marriage problem with a few differences.

On the contrary to an SMP instance with n men and n women, SMPTI can have different numbers of men and women. In addition to that, in SMPTI, preferences of men and women do not have to be complete and may include ties.

As a result of the change in the input, in SMPTI, not all men and women have to be matched with each other, but they may be single.

A man m and a woman w are said to form a blocking pair for M (or to block M) when: (i) $M(m) \neq w$; (ii) both m and w are either single and find the other one acceptable, or prefers the other one to his/her actual partner.

According to this new definition, one of the four following cases applies:

1. m and w are both single. In this case, $\langle m, w\rangle$ is a blocking pair if m finds w acceptable and w finds m acceptable.
2. m and w are both married, but $M(m) \neq w$. In this case, $\langle m, w\rangle$ is a blocking pair if $w \succ_m M(m)$ and $m \succ_w M(w)$.
3. m is single and w is married. In this case, $\langle m, w\rangle$ is a blocking pair if m finds w acceptable and $m \succ_w M(w)$.
4. w is single and m is married. This case is similar to the third case above.

A matching M is unstable if a blocking pair exists for M, and stable otherwise. The SMPTI asks for finding a stable matching M.

The solve SMPTI, we slightly modify our ASP program and present to CLINGO as follows:

```
% GENERATE -- generates a matching

msingle(M) :- man(M).
wsingle(F) :- woman(F).

{pair(M, F) : woman(F)} = 1 :- man(M), not msingle(M).
:- {pair(M, F) : man(M)} > 1, woman(F).

marriedwoman(F) :- man(M), woman(F), pair(M, F).
:- marriedwoman(F), wsingle(F), woman(F).
:- not wsingle(F), not marriedwoman(F), woman(F).

% DEFINE -- definitions to use

% M prefers F to single
m2single(M, F) :- man(M), woman(F), mpref(M, F, X).

bfseries % F prefers M to single
w2single(F, M) :- man(M), woman(F), wpref(F, M, X).

% M prefers DF to CF.
mprefer(M, DF, CF) :- mpref(M, CF, CUR), mpref(M, DF, DEV), DEV < CUR.

```

```
% F prefers DM to CM.
wprefer(F, DM, CM) :- wpref(F, CM, CUR), wpref(F, DM, DEV),  DEV < CUR.

% TEST -- constraints

% an individual deviates -- acceptability (2 cases)

% m-f -- m deviates to single
:- man(M), woman(F), pair(M, F), not m2single(M, F).

% m-f -- f deviates to single
:- man(M), woman(F), pair(M, F), not w2single(F, M).

% a pair deviates -- blocking pair (4 cases)

% m, f single -- m and f deviate to m-f
:- man(M), woman(F), msingle(M), wsingle(F), mpref(M, F, X), wpref(F, M, Y).

% m1-f1, m2-f2 -- m1 and f2 deviate to m1-f2
:- man(M1; M2), woman(F1; F2), pair(M1, F1), pair(M2, F2),
     mprefer(M1, F2, F1), wprefer(F2, M1, M2).

% m1-f, m2 single -- m2 and f deviate to m2-f
:- man(M1; M2), woman(F), pair(M1, F), msingle(M2),
     wprefer(F, M2, M1), mpref(M2, F, X).

% m-f1, f2 single -- m and f2 deviate to m-f2
:- man(M), woman(F1; F2), pair(M, F1), wsingle(F2),
     mprefer(M, F2, F1), wpref(F2, M, X).
```

2.2 Sex Equal SMPTI

Sex equal SMPTI, is a variant of SMPTI that maximizes sex equality of a stable marriage S, defined by the following cost function C:

$$C(S) = \left| \sum_{i \in M} C_i(S) - \sum_{i \in W} C_i(S) \right|$$

where $C_i(S) = k$ if i is matched to his/her k^{th} preferred partner in S.

To solve the optimization variant of SMPTI, we simply add some weak constraints to the ASP formulation of SMPTI as follows:

```
% DEFINITION -- additional definitions

% pair rank

mrank(M, R) :- pair(M, F), mpref(M, F, R).
wrank(F, R) :- pair(M, F), wpref(F, M, R).

% total rank

mtot(T) :- T = #sum{R, M: mrank(M, R)}.
wtot(T) :- T = #sum{R, F: wrank(F, R)}.

% OPTIMIZATION -- to optimize

% weak constraint

:~ mtot(M), wtot(F). [|M-F|]
```

2.3 Egalitarian SMPTI

Egalitarian SMPTI, is a variant of SMPTI that maximizes equality of a stable marriage S, defined by the following cost function C:

$$C(S) = \sum_{i \in M} C_i(S) + \sum_{i \in W} C_i(S)$$

where $C_i(S) = k$ if i is matched to his/her k^{th} preferred partner in S.

To solve the optimization variant of SMPTI, we simply add some weak constraints to the ASP formulation of SMPTI as follows:

```
% DEFINITION -- additional definitions

% pair rank

mrank(M, R) :- pair(M, F), mpref(M, F, R).
wrank(F, R) :- pair(M, F), wpref(F, M, R).

% total rank

mtot(T) :- T = #sum{R, M: mrank(M, R)}.
wtot(T) :- T = #sum{R, F: wrank(F, R)}.

% OPTIMIZAION -- to optimize

% weak constraint

:~ mtot(M), wtot(F). [M+F]
```

2.4 Minimum Regret SMPTI

Minimum regret SMPTI, is a variant of SMPTI that minimizes the maximum regret of a stable marriage S, defined by the following cost function C:

$$C(S) = max\{C_i(S)\}_{i \in M \cup W}$$

where $C_i(S) = k$ if i is matched to his/her k^{th} preferred partner in S.

To solve the optimization variant of SMPTI, we simply add some weak constraints to the ASP formulation of SMPTI as follows:

```
% DEFINITION -- additional definitions

% pair rank

mrank(M, R) :- pair(M, F), mpref(M, F, R).
wrank(F, R) :- pair(M, F), wpref(F, M, R).

% max regret

regret(man, T) :- T = #max{R, M: mrank(M, R)}.
regret(woman, T) :- T = #max{R, F: wrank(F, R)}.

```

```
maxregret(T) :- T = #max{R: regret(X, R)}.

% OPTIMIZATION -- to optimize

% weak constraint

:~ maxregret(R). [R]
```

3 Conclusion

Matching is an area with lots of applications in real world. In this project, we have studied one of the earliest problem in the area of matching theory, stable marriage problem. First, we have solved the stable marriage problem and its variant, stable marriage problem with unacceptability and ties using answer set programming. Then we have performed some optimizations using different cost formulas: sex-equality, egalitarian and minimum regret.

Although matching is a common problem in our lives, there are very few algorithms and implementations about this problem. In this project, we have introduced a new method to solve matching problems using answer set programming.

In future, we plan to solve some other matching problems such as roommate problem or university entrance problem using answer set programming. We believe that, by solving these problem, we will contribute to the usage of answer set programming in matching theory.

References

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