ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

Department of Computer Engineering

CENG 384 Homework Exam 2 Key

10 June 2022

METU Honor Code and Pledge

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*"I have read and understood the implications of the METU Honor Code. To be precise, I understand that this is a **open** formula-sheet exam, and I am forbidden to access any other source of information other than provided within the exam. I will TURN OFF all my electronic equipment (phones, smart watches, etc.) and put it off the table along with other notes and materials that I may have with me. I understand that leaving electronic devices on during the exam is strictly forbidden. I understand and accept to obey all the rules announced by the course staff, and that failure to obey these will result in disciplinary action."*

Name, SURNAME: **Signature:**
ID:

Specifications About the Exam

- **Duration:** **110** minutes.
- **Notes:**
  - One A4-size double-sided formula-sheet is allowed.
  - Each answer should contain a justifying explanation. However, give shortest possible answers. Unnecessary or irrelevant wordings will reduce your grades.
  - If you use the back of a page for extra space, please mark down a proper notice on the relevant pages.

Grade

Q1:

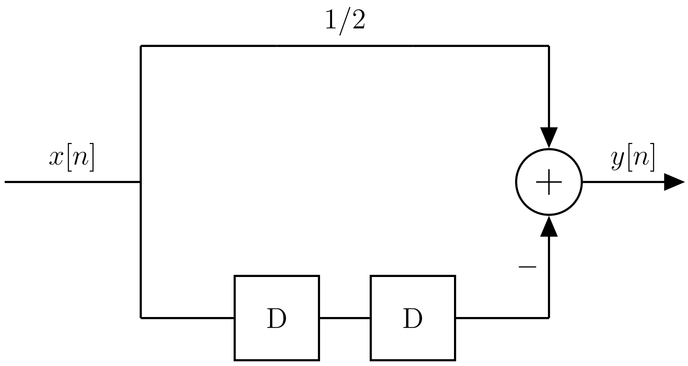
Q2:

Q3:

Q4:

Total:

1 (25 pts, 5-5-7-8 pts respectively) Consider the following block diagram representation of a discrete time LTI system, where D represents the unit delay operator.



a) Find the difference equation which represents this system.

b) Find the frequency response, $H(e^{j\omega})$.

c) Find and plot the impulse response, $h[n]$, using $H(e^{j\omega})$ you found in part b.

d) Using Fourier transform, find and plot the output, $y[n]$, when the input is

$$x[n] = \delta[n] + \frac{1}{2}\delta[n-1].$$

a)

$$\frac{1}{2}x[n] - x[n-2] = y[n].$$

b)

$$\frac{1}{2}X(e^{j\omega}) - e^{-2j\omega}X(e^{j\omega}) = Y(e^{j\omega}),$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{2} - e^{-2j\omega}.$$

c) Using table 5.2 we can take the IFT of $H(e^{j\omega})$ and reach $h[n]$:

$$h[n] = \frac{1}{2}\delta[n] - \delta[n-2].$$

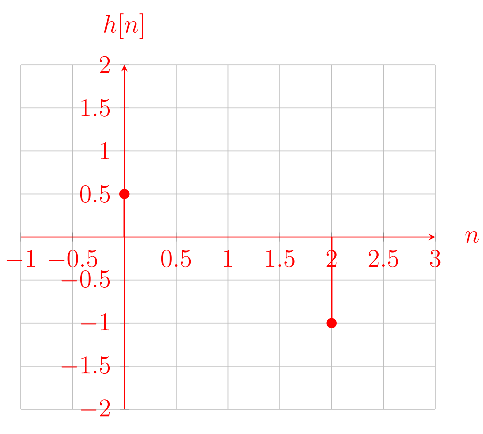


Figure 1: n vs. $h[n]$.

d) Using table 5.2 we can take the FT of $x[n]$ and reach $X(e^{j\omega})$:

$$X(e^{j\omega}) = 1 + \frac{1}{2}e^{-j\omega},$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{2} - e^{-2j\omega} + \frac{1}{4}e^{-j\omega} - \frac{1}{2}e^{-3j\omega},$$

Using table 5.2 we can take the IFT of $Y(e^{j\omega})$ and reach $y[n]$:

$$y[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1] - \delta[n-2] - \frac{1}{2}\delta[n-3].$$

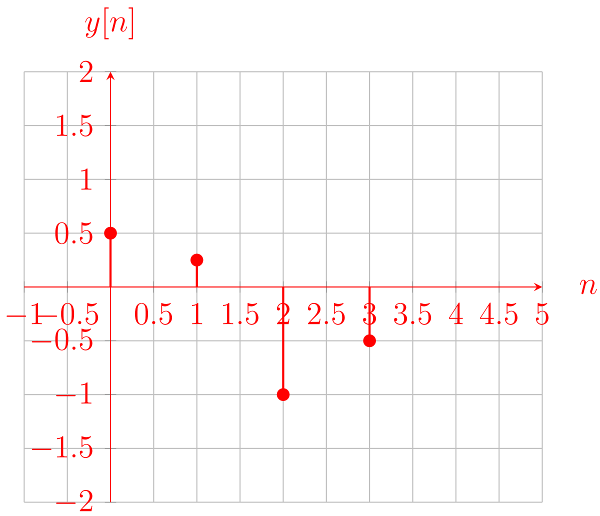


Figure 2: n vs. $y[n]$.

$\boxed{\textbf{2}}$ (25 pts, 5-10-10 pts respectively) Consider a continuous time LTI system represented by the following frequency response,

$$H(j\omega) = \frac{j\omega}{2 + 3j\omega - \omega^2}.$$

a) Find the differential equation which represents this system.

b) Find a block diagram representation of this system using minimum number of adders, integrators and differentiators.

c) Find and plot the impulse response, $h(t)$.

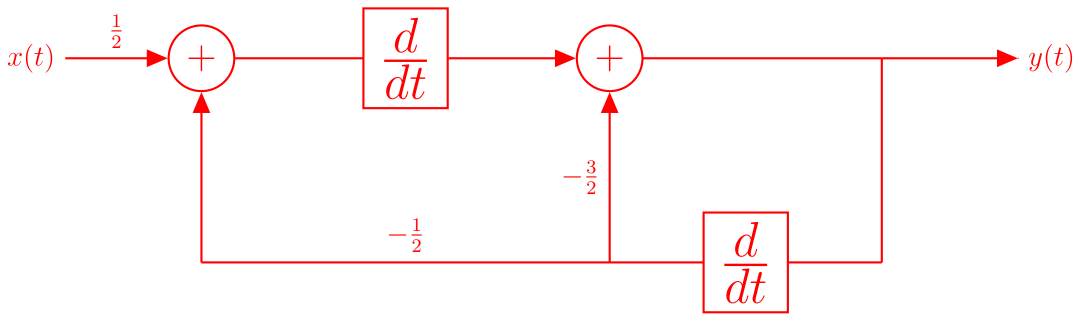
a)

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega}{2 + 3j\omega - \omega^2},$$

$$((j\omega)^2 + 3j\omega + 2)Y(j\omega) = j\omega X(j\omega),$$

$$y''(t) + 3y'(t) + 2y(t) = x'(t).$$

b) There are many alternatives. One such alternative is as follows:



c) By partial fraction we get the following representation for $H(j\omega)$:

$$H(j\omega) = \frac{-1}{j\omega + 1} + \frac{2}{j\omega + 2},$$

Using table 4.2 we can take the IFT of $H(j\omega)$ and reach $h(t)$:

$$h(t) = (2e^{-2t} - e^{-t})u(t).$$

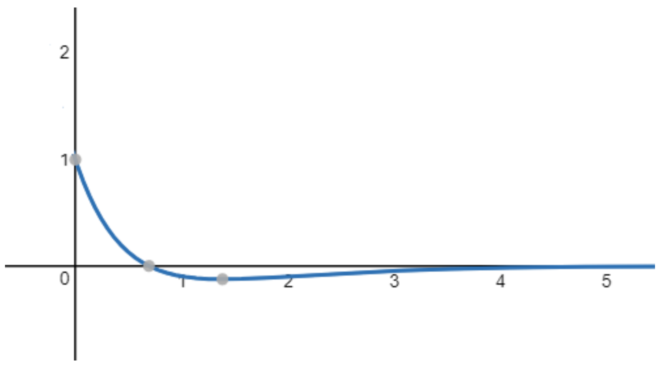


Figure 3: t vs. $h(t)$.

$\boxed{\mathbf{3}}$ (30 pts, 10-5-5-10 pts respectively) Consider a discrete-time LTI system, represented by the following impulse response:

$$h[n] = (n-1)(0.3)^n u[n]$$

a) Find the frequency response, $H(e^{j\omega})$, of this system.

b) Find the spectral coefficients, a_k, of the input

$$x[n] = \sin(\omega_0 n) + \sin(4\omega_0 n).$$

c) Find the spectral coefficients, b_k, of the output for the input given in part (b).

d) Find the difference equation which represents this system.

a)

$$h[n] = (n+1)(0.3)^n u[n] - 2(0.3)^n u[n]$$

Using table 5.2 we can take the FT of $h[n]$ and reach $H(e^{j\omega})$:

$$H(e^{j\omega}) = \frac{1}{(1-0.3e^{-j\omega})^2} - \frac{2}{1-0.3e^{-j\omega}}.$$

b) From Euler's Formula we get the following expansion for $x[n]$:

$$x[n] = \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} + \frac{e^{4j\omega_0 n} - e^{-4j\omega_0 n}}{2j},$$

$$a_1 = \tfrac{1}{2j} = \tfrac{-j}{2}, \qquad a_{-1} = -\tfrac{1}{2j} = \tfrac{j}{2}, \qquad a_4 = \tfrac{1}{2j} = \tfrac{-j}{2}, \qquad a_{-4} = -\tfrac{1}{2j} = \tfrac{j}{2}.$$

c) $b_1 = a_1 H(e^{j\omega_0}) = \frac{1}{2j} H(e^{j\omega_0}), \qquad b_{-1} = a_{-1} H(e^{-j\omega_0}) = \frac{-1}{2j} H(e^{-j\omega_0}),$

$b_4 = a_4 H(e^{4j\omega_0}) = \frac{1}{2j} H(e^{4j\omega_0}), \qquad b_{-4} = a_{-4} H(e^{-4j\omega_0}) = \frac{-1}{2j} H(e^{-4j\omega_0}).$

d)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{0.6e^{-j\omega} - 1}{0.09e^{-2j\omega} - 0.6e^{-j\omega} + 1},$$

$$(0.09e^{-2j\omega} - 0.6e^{-j\omega} + 1)Y(e^{j\omega}) = (0.6e^{-j\omega} - 1)X(e^{j\omega}),$$

$$0.09y[n-2] - 0.6y[n-1] + y[n] = 0.6x[n-1] - x[n].$$

$\boxed{4}$ (20 pts, 10 pts each) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation:

$$\frac{dy(t)}{dt} + 9y(t) = \int_{-\infty}^{+\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where $z(t) = e^{-t}u(t) + 2\delta(t)$.

a) Find the frequency response, $H(j\omega)$, of this system.

b) Determine the impulse response, $h(t)$, of the system.

a)

$$y'(t) + 9y(t) = x(t) * z(t) - x(t),$$
$$j\omega Y(j\omega) + 9Y(j\omega) = X(j\omega)Z(j\omega) - X(j\omega),$$
$$Z(j\omega) = \frac{1}{1+j\omega} + 2 = \frac{3+2j\omega}{1+j\omega},$$

Now plug $Z(j\omega)$ into the equation above:

$$(j\omega+9)Y(j\omega) = \left(\frac{3+2j\omega}{1+j\omega} - 1\right) X(j\omega),$$

$$(j\omega+9)Y(j\omega) = \left(\frac{2+j\omega}{1+j\omega}\right) X(j\omega),$$
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2+j\omega}{(1+j\omega)(9+j\omega)}.$$

b) By partial fraction we get the following representation for $H(j\omega)$:

$$H(j\omega) = \frac{1/8}{j\omega+1} + \frac{7/8}{j\omega+9},$$

Using table 4.2 we can take the IFT of $H(j\omega)$ and reach $h(t)$:

$$h(t) = (\frac{1}{8}e^{-t} + \frac{7}{8}e^{-9t})u(t).$$