MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

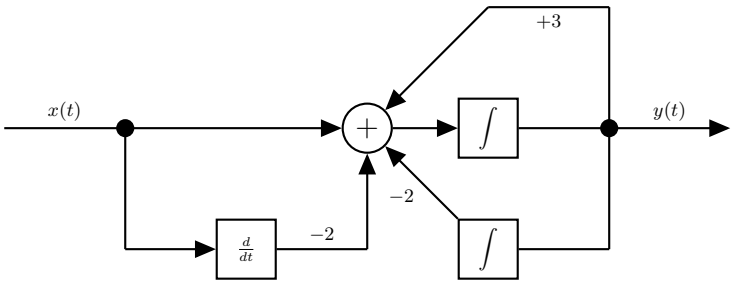
CENG 384

Spring 2022
Homework 2

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw2.pdf' to the odtuclass page of the course. You need to use the given template 'hw2.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 17 April, 2022 (Sunday).
- **Late Submission:** Not allowed.

1. (30 pts) Consider an LTI system given by the following block diagram:



   (a) (15 pts) Find the differential equation which represents this system.

   (b) (15 pts) Find the output $y(t)$, when the input $x(t) = (e^{-t} + e^{-2t})u(t)$. Assume that the system is initially at rest.

2. (10 pts) Evaluate the following convolutions.

   (a) (5 pts) Given $x[n] = \delta[n-1] + 3\delta[n+2]$ and $h[n] = 2\delta[n+2] - \delta[n+1]$, compute and draw $y[n] = x[n] * h[n]$.

   (b) (5 pts) Given $x[n] = u[n+1] - u[n-2]$ and $h[n] = u[n-4] - u[n-6]$, compute and draw $y[n] = x[n] * h[n]$.

3. (10 pts) Evaluate the following convolutions.

   (a) (5 pts) Given $h(t) = e^{-\frac{1}{2}t}u(t)$ and $x(t) = e^{-t}u(t)$, find $y(t) = x(t) * h(t)$.

   (b) (5 pts) Given $h(t) = e^{-3t}u(t)$ and $x(t) = u(t) - u(t-4)$, find $y(t) = x(t) * h(t)$.

4. (20 pts) Consider a continuous LTI system, initially at rest, given by the following input and output equation:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 3) d\tau \tag{1}$$

   (a) (10 pts) Find and plot the impulse response $h(t)$.

   (b) (10 pts) Find the response of the system when the input $x(t)$ is shown as:

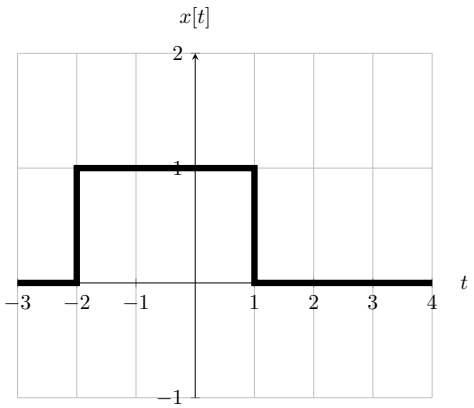
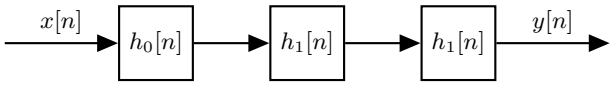


Figure 1: t vs. $x(t)$.

5. (30 pts) Consider a discrete LTI system represented by the following block diagram which is initially at rest:



where

$$h_1^{-1}[n] = (\frac{1}{2})^n u[n] \tag{2}$$

and the overall impulse response of the system is:

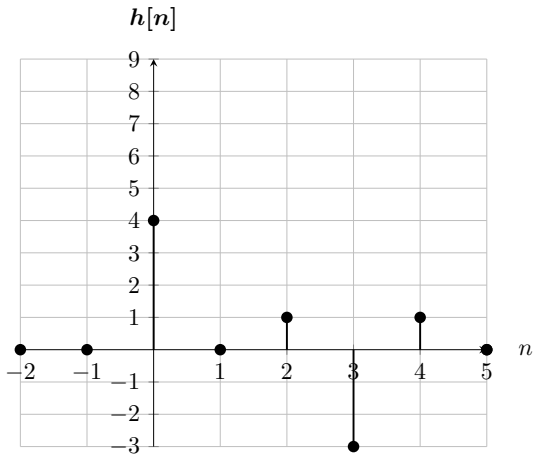
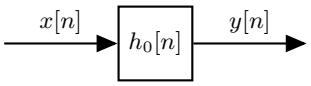


Figure 2: n vs. $h[n]$.

(a) (10 pts) Find $h_1[n] * h_1[n]$.

(b) (10 pts) Find the impulse response $h_0[n]$.

(c) (10 pts) Consider a discrete LTI system represented by the following block diagram which is initially at rest:



What is the response of this system when $x[n] = \delta[n] + \delta[n-2]$?