CENG 384 - Signals and Systems for Computer Engineers
20212
Written Assignment 1 Solutions

April 8, 2022

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1. (a) (4 pts) Given a complex number in Cartesian coordinate system, $z = x + jy$ and $2z - 9 = 4j - \bar{z}$, find $|z|^2$ and plot z on the complex plane

$$2x + 2yj - 9 = 4j - x + yj \tag{1}$$
$$3x + yj = 9 + 4j \tag{2}$$
$$x = 3, y = 4 \tag{3}$$
$$z = 3 + 4j \tag{4}$$
$$|z|^2 = z\bar{z} = (3 + 4j)(3 - 4j) = 25 \tag{5}$$

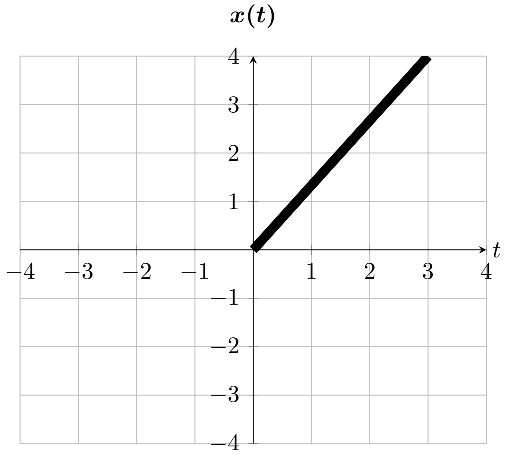

Figure 1: z

(b) (4 pts) Given $z = re^{j\theta}$ and $z^3 = -27j$, find z in polar form.

$$z^3 = r^3 e^{3j\theta} = -27j \tag{6}$$
$$r^3 e^{3j\theta} = 27e^{j\frac{3\pi}{2}} \tag{7}$$

$$r = 3, 3\theta = \frac{3\pi}{2} + 2\pi k, k \in Z \tag{8}$$
$$k = -1, \theta = -\frac{\pi}{6} = -30 \tag{9}$$
$$k = 0, \theta = \frac{\pi}{2} = 90 \tag{10}$$
$$k = 1, \theta = \frac{7\pi}{6} = 210 \tag{11}$$
$$z_1 = 3e^{-j\frac{\pi}{6}}, z_2 = 3e^{j\frac{\pi}{2}}, z_3 = 3e^{j\frac{7\pi}{6}} \tag{12}$$
$$\tag{13}$$

(c) (4 pts) Find the magnitude and angle of $z = \frac{(1+j)(\sqrt{3}-j)}{(\sqrt{3}+j)}$

$$z_1 = (1+j) = \sqrt{2}e^{j\frac{\pi}{4}} \tag{14}$$

$$z_2 = (\sqrt{3}-j) = 2e^{-j\frac{\pi}{6}} \tag{15}$$

$$z_3 = (\sqrt{3}+j) = 2e^{j\frac{\pi}{6}} \tag{16}$$

$$|z| = \frac{z_1 z_2}{z_3} = \sqrt{2}e^{j\frac{\pi}{4}} \tag{17}$$

$$|z| = \sqrt{2}, \theta = -\frac{\pi}{12} \tag{18}$$

(d) (4 pts) Write z in polar form where $z = -(1+j)^8 e^{j\pi/2}$.

$$-(1+j)^8 e^{j\pi/2} = e^{j\pi}(\sqrt{2}e^{j\frac{\pi}{4}})^8 e^{j\pi/2} \tag{19}$$

$$z = 16e^{j\frac{7\pi}{2}} = 16e^{j\frac{3\pi}{2}} \tag{20}$$

2. (a) $x[n] = nu[n]$

$$E_\infty = \sum_{-\infty}^{n=\infty} |x[n]|^2 = \sum_{n=0}^{\infty} n^2 = \infty \tag{21}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} n^2 = \infty \tag{22}$$

This signal is neither energy nor power signal.

(b) $x(t) = e^{-2t}u(t)$

$$E_\infty = \int_{-\infty}^{\infty} |e^{-2t}u(t)|^2 dt = \int_0^\infty e^{-4t} dt = \frac{1}{4} \tag{23}$$

$$P_\infty = \lim_{T\to\infty} \int_0^T e^{-4t} dt = -\frac{1}{4}e^{-4t}\Big|_0^T = 0 \tag{24}$$

Energy signal because E is finite and P is 0

3. i. flip the plot horizontally because we multiply t by negative 1
   ii. time scale, expand by 3
   iii. time shift, shift by 6 to right 2
   iv. scale y axis values by $\frac{1}{2}$

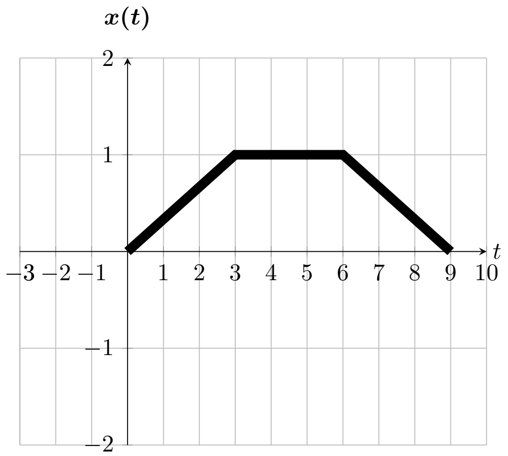

Figure 2: solplot q2

4. (a) $x[-2n]$ is obtained by shrinking $x[n]$ by 2 and reflecting it along y-axis. and take the integer values
   $x[n+2]$: we shift to the left by 2.
   At the end we sum $x[-2n]$ and $x[n+2]$.

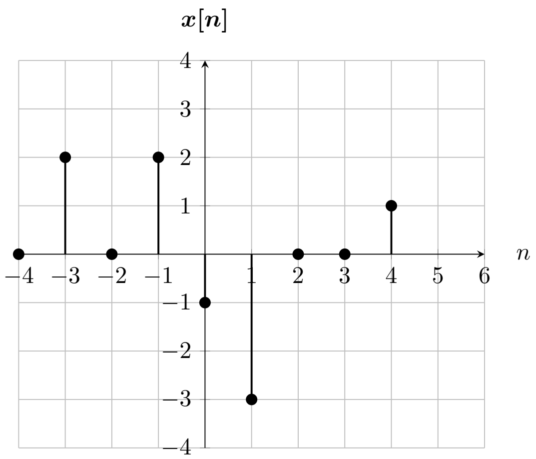

Figure 3: n vs. $x[-2n] + x[n+2]$.

   (b) $x[-2n] + x[n+2] = 2\delta[n+3] + 2\delta[n+1] - \delta[n] - 3\delta[n-1] + \delta[n-4]$

5. (a) $x(t) = \frac{e^{j3t}}{-j}$

$$\frac{e^{j3t}}{-j} = e^{3t+\frac{\pi}{2}} \tag{25}$$

$$T_0 = \frac{2\pi}{3} \tag{26}$$

   (b) $x[n] = \frac{1}{2}sin[\frac{7\pi}{8}n] + 4cos[\frac{3\pi}{4}n - \frac{\pi}{2}]$
   for x_1

$$\omega_0 = \frac{7\pi}{8}, N_0 = \frac{2\pi}{\omega_0}m = \frac{16m}{7}, m = 7, N_0 = 16 \tag{27}$$

   for x_2

$$\omega_0 = \frac{3\pi}{4}, N_0 = \frac{2\pi}{\omega_0}m = \frac{8m}{3}, m = 3, N_0 = 8 \tag{28}$$

   LCM is 16

6. (a) Signal is not symmetric on y-axis, this means that it is not even. It is also not symmetric w.r.t origin, so it is not odd.
   (b) $Ev\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$ and $Odd\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$

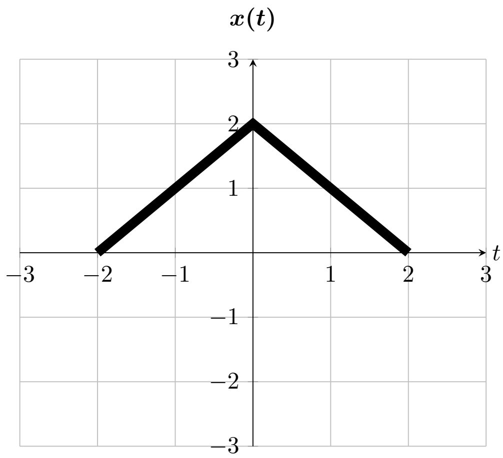

Figure 4: $Ev\{x(t)\}$.

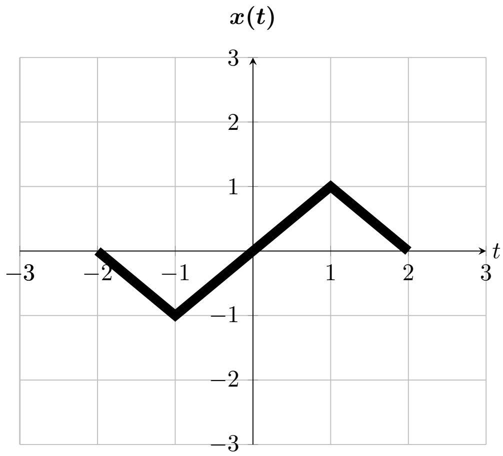

Figure 5: $Odd\{x(t)\}$.

7. (a) $x(t) = 3u(t+3) - 3u(t+1) + 2u(t-2) - 4u(t-4) + 3u(t-6)$
   (b) $\frac{dx(t)}{dt} = 3\delta(t+3) - 3\delta(t+1) + 2\delta(t-2) - 4\delta(t-4) + 3\delta(t-6)$

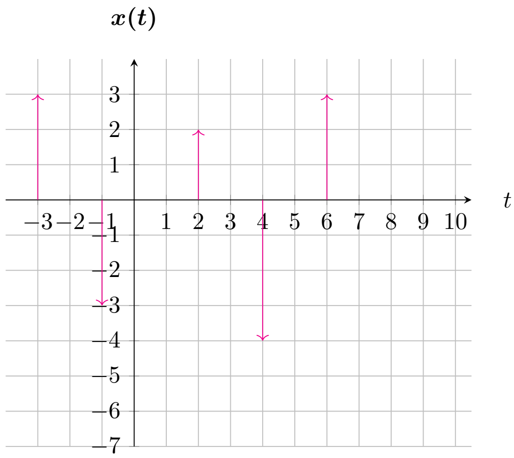

Figure 6: t vs. $\frac{dx(t)}{dt}$.

8. (a) $y[n] = x[2n-2]$
   i. **Memory** Has memory,$y[1] = x[0]$
   ii. **Stability** Stable, all bounded inputs result in bounded outputs
   iii. **Causality** Not Causal, $y[5] = x[8]$ output depends on future input values.
   iv. **Linearity** Superposition property hold, therefore linear
   v. **Invertibility** Not invertible, because $x[n] = y[\frac{n+2}{2}]$ is not defined for all n values
   vi. **Time Invariance** Time varying, $x[2n - 2n_0 - 2] \neq x[2n - n_0 - 2]$

(b) $y(t) = tx(\frac{t}{2} - 1)$

i. **Memory** Has memory,y(1) = x(0)

ii. **Stability** Not stable, t is unbounded.

iii. **Causality** Not Causal y(2) = x(4), y depends on inputs in the future

iv. **Linearity** Superposition property holds, therefore linear

v. **Invertibility** Not Invertible, $x(t) = \frac{y(2t+2)}{2t+2}$ this is not defined when t = -1

vi. **Time Invariance** Time varying, $tx(\frac{t-t_0}{2} - 1) \neq (t - t_0)x(\frac{t-t_0}{2} - 1)$