CEND 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 1

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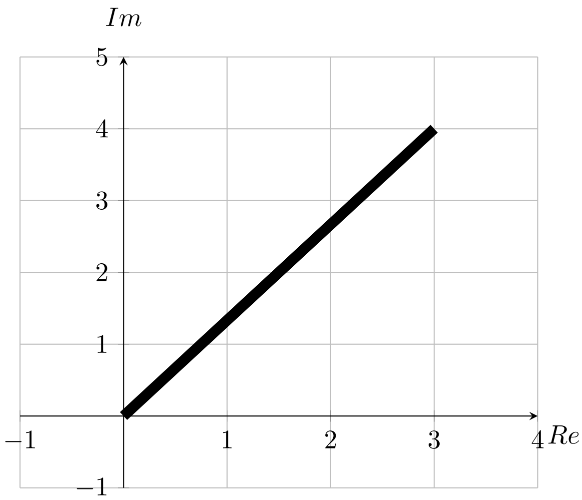
1. (a) **(I)** We are given $z = x + jy$ and $2z - 9{=}4j - x + jy$, if we solve those equations together;
$2x + 2jy - 9{=}4j - x + jy \Rightarrow 3x + jy{=}4j + 9$; $x = 3$, $y = 4$ and $z = 3 + 4$j.
$|z|^2{=}(3 + 4j)*(3 - 4j) = \mathbf{25}$.
(II) $z = 3 + 4$j



(b) $z^3 = -27j \Rightarrow z = (-27j)^{\frac{1}{3}}$, $j = e^{j\frac{\pi}{2}} \Rightarrow z = \left(-27e^{j\frac{\pi}{2}}\right)^{\frac{1}{3}} \Rightarrow z = -3e^{j\frac{\pi}{6}}$ in polar form.

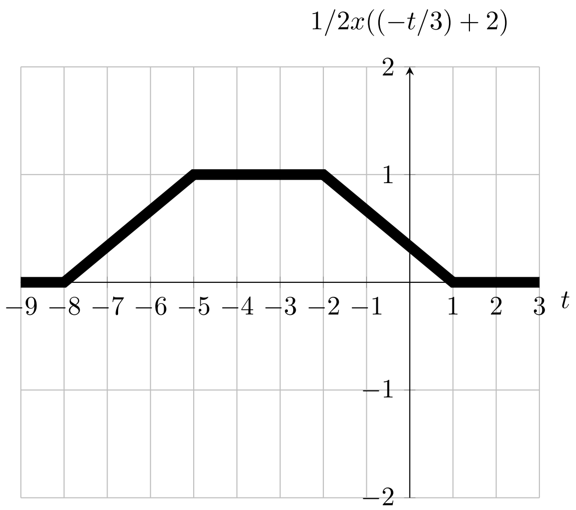
(c) $z = \frac{(1+j)\left(\sqrt{3}-j\right)^2}{\left(\sqrt{3+j\left(\sqrt{3}-j\right)}\right)} = \frac{\left(3-2\sqrt{3}j-1\right)(1+j)}{3+1} = \frac{\left(2+2\sqrt{3}\right)}{4} + j\frac{\left(2-2\sqrt{3}\right)}{4}$
$a = \frac{\left(2+2\sqrt{3}\right)}{4}$, $b = \frac{\left(2-2\sqrt{3}\right)}{4}$
Angle of z: $tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-2+\sqrt{3}}{2}\right)$
Magnitude of z: $|z|^2 = 4, |z| = 2$

(d) $1 + j = 2e^{jtan^{-1}\left(\frac{b}{a}\right)} = 2e^{\frac{j\pi}{4}}$
$z = -\left(1+j\right)^8 e^{\frac{j\pi}{2}} = -256e^{j2\pi}e^{\frac{j\pi}{2}} = -256e^{\frac{j5\pi}{2}}$

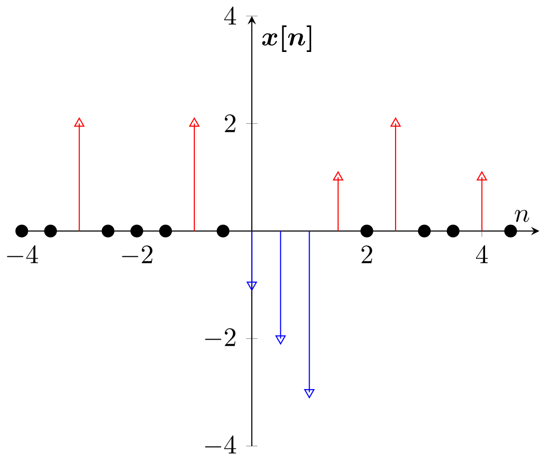
2. (a) -Energy of $x[n] = nu[n]$ is $E_x = \sum_{n=-\infty}^{\infty} |nu\left[n\right]|^2 = \sum_{n=-\infty}^{0} 0 + \sum_{n=0}^{\infty} |nu\left[n\right]|^2 = \sum_{n=0}^{\infty} |n|^2 = \infty$
Therefore, $E_x = \infty$.Hence $x[n] = nu[n]$ is not an energy signal.
-Power of $x[n] = nu[n]$ is $P_x = \lim_{n\to\infty}\left(\frac{1}{2N+1}\right)\sum_{n=-N}^{N} |n|^2 = \infty$
Therefore, this is not a power signal either.

(b) -Energy of $x\left(t\right) = e^{-2}tu\left(t\right)$ is $E_x = \int_{-\infty}^{\infty}\left|e^{-2t}u\left(t\right)\right|^2 = \int_{-\infty}^{0} 0 + \int_{0}^{\infty} \left|e^{-2t}\right|^2 = \int_{0}^{\infty} e^{-4t}$
$=\int_{0}^{\infty} \left(e^t\right)^{-4} = \frac{1}{4}$. Therefore this is an energy signal.
-Power of $x\left(t\right) = e^{-2}tu\left(t\right)$ is $P_x = \lim_{T\to\infty}\left(\frac{E_x}{2T}\right) = \lim_{T\to\infty}\frac{1}{8T} = 0$. Therefore this is not a power signal.

3. .



4. .



(a)

(b) $2\delta\,[n+3]+2\delta\ \,[n+1]-\delta\ \,[n]-2\delta\ \left[n-\frac{1}{2}\right]-3\delta\,[n-1]+\delta\ \left[n-\frac{3}{2}\right]+2\delta\ \left[n-\frac{5}{2}\right]+\delta\ \,[n-4]$

5. (a) $x\,(t)=\frac{e^{j3t}}{-j}=e^{\frac{j\pi}{2}}e^{j3t}=e^{j\left(\frac{\pi}{2}+3t\right)}$
$=cos\left(\frac{\pi}{2}+3t\right)+jsin\left(\frac{\pi}{2}+3t\right)=cos\frac{\pi}{2}\cdot cos3t-sin\frac{\pi}{2}\cdot sin3t+j\left(sin\frac{\pi}{2}\cdot cos3t+sin3t\cdot cos\frac{\pi}{2}\right)$
$=-sin3t+jcos3t$
We know that sin and cos functions are periodic for 2π, so periodicities of sin(3t) and cos(3t) are both $\frac{2\pi}{3}$. If we combine those we get 2π.

(b) ??

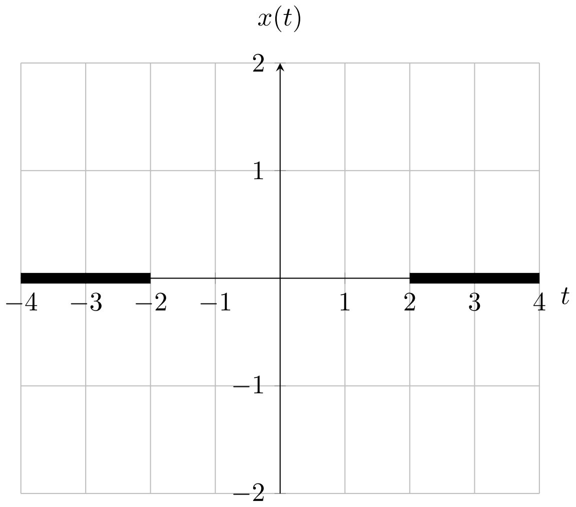
6. (a) For a function to be even it must hold $x(t) = x(-t)$ and to be odd it must hold $x(-t) = -x(t)$. We see that $x(t) \neq x(-t)$ and $-x(t) \neq x(-t)$, so this signal is neither even nor odd.

$$x(t) = \begin{cases} 0 & t \leq -1 \\ 2t+2 & -1 \leq t \leq 0 \\ 2 & 0 < t \leq 1 \\ -2t+4 & 1 < t \leq 2 \\ 0 & 2 < t \end{cases}$$

$$x(-t) = \begin{cases} 0 & 1 \leq t \\ -2t+2 & 0 \leq t \leq 1 \\ 2 & -1 \leq t < 0 \\ 2t+4 & -2 \leq t < 1 \\ 0 & t < -2 \end{cases}$$

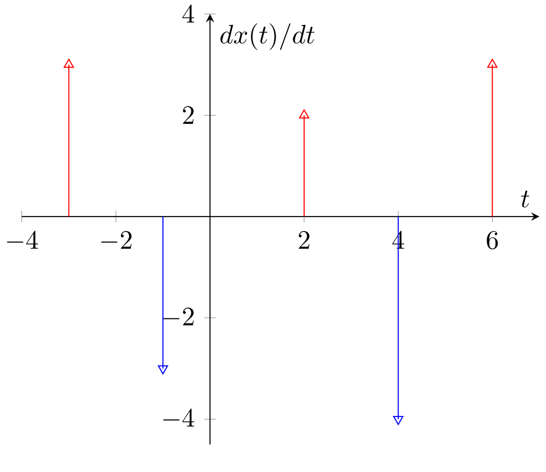
$$-x(t) = \begin{cases} 0 & t \leq -1 \\ -2t-2 & -1 \leq t \leq 0 \\ -2 & 0 < t \leq 1 \\ 2t-4 & 1 < t \leq 2 \\ 0 & 2 < t \end{cases}$$

(b) For $t < -2$ and $t > 2$ this signal is even and odd because $x'(t) = 0$.



7. (a) $x\left(t\right) = 3u\left(t+3\right) - 3u\left(t+1\right) + 2u\left(t-2\right) - 4u\left(t-4\right) + 3u\left(t-6\right)$

(b) $u\left(t\right) = \int_{-\infty}^{t} \delta\left(\tau\right)\, d\tau$. Therefore,
$\frac{dx(t)}{dt} = 3\delta\left(t+3\right) - 3\delta\left(t+1\right) + 2\delta\left(t-2\right) - 4\delta\left(t-4\right) + 3\delta\left(t-6\right)$



8. (a) For $y[n]{=}x[2n-2]$;

-We can say that this system **HAS MEMORY** because if we check for n=1 we see that the equation becomes $y[1]{=}x[0]$ and from that point of view system needs memory to remember $x[0]$ in order to calculate $y[1]$.
-Moreover, if we put some constant 'c' to 'n', equation becomes $x[2c-2]$ and we can now say that output is bounded for input at each and every instant of time. Hence this system is **STABLE**.
-When it comes to causality, if we put '3' to 'n', equation becomes $y[3]{=}x[4]$ and we can say that output depends on future values, therefore this system is **NOT CAUSAL**.
-In order to check this system for linearity, we check for principle of superposition;
(1) $y_1[n]{=}x_1[2n-2]$ and $y_2[n]{=}x_2[2n-2]$, so $y_1[n]{+}y_2[n]{=}x_1[2n-2]{+}x_2[2n-2]$.
(2) $k*y[n]{=}k*x[2n-2]$ and ($k*x[t] \Rightarrow$ (system) $\Rightarrow k*x[2n-2]$), we can see that both equation's solutions are the same. From (1) and (2), we conclude that this system is **LINEAR**.
-Also we can easily see that distinct inputs leads to distinct outputs in this system, hence this system is **INVERTIBLE**.
- If we shift the input and put 'n-3' to 'n', equation becomes $y[n-3]{=}x[2n-8]$. We shifted the input for 3 and our output is shifted for the same amount, hence this system is **TIME-INVARIANT**.

(b) For $y(t){=}t*x\left(\frac{t}{2}-1\right)$;

-We can say that this system **HAS MEMORY** because if we check for t=2 we see that the equation becomes $y(2){=}2x(1)$ and from that point of view system needs memory to remember $x(1)$ in order to calculate $y(2)$.
-Moreover, if we put some constant 'c' to 't', equation becomes $c*x\left(\frac{c}{2}-1\right)$ and we can now say that output is not bounded for input at each and every instant of time. Hence this system is **UNSTABLE**.
-When it comes to causality, if we put '4' to 't', equation becomes $y(4){=}4*x(2)$ and we can say that output does not depend on future values, therefore this system is **CAUSAL**.
-In order to check this system for linearity, we check for principle of superposition;
(1) $y_1(t){=}t*x_1\left(\frac{t}{2}-1\right)$ and $y_2(t){=}t*x_2\left(\frac{t}{2}-1\right)$, so $y_1(t){+}y_2(t){=}t*x_1\left(\frac{t}{2}-1\right){+}t*x_2\left(\frac{t}{2}-1\right)$.
(2) $k*y(t){=}k*t*x\left(\frac{t}{2}-1\right)$ and $k*x(t) \Rightarrow$ (system) $\Rightarrow k*t*x\left(\frac{t}{2}-1\right)$, we can see that both equation's solutions are the same. From (1) and (2), we conclude that this system is **LINEAR**.
-Also we can not easily say that distinct inputs leads to distinct outputs in this system, hence this system is **NON-INVERTIBLE**.
- If we shift the input and put 't-3' to 't', equation becomes $y(t-3){=}(t-3)*x\left(\frac{t-3}{2}-1\right)$. We shifted the input for 3 but our output did not shifted for 3, hence this system is **TIME-VARIANT**.