CENG 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 2

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April 19, 2022

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1. (a) $2y(t) - 3y'(t) + y''(t) = x'(t) - 2''(t)$

   (b) $2y(t) - 3y'(t) + y''(t) = x'(t) - 2''(t)$
   Homogeneous Solution: $y_h = ce^{at}$, $y_h'(y) = cae^{at}$, $y_h''(t) = ca^2e^{at}$
   $\left(a^2 - 3a + 2\right)Ce^{at} = 0$; $a_1 = 1$, $a_2 = 2$; $Thus\, y_h(t) = C_1e^t + C_2e^{2t}$
   Particular Solution: $y_p(t) = Kx(t)\ where\, x(t) = \left(e^{-t} + e^{-2t}\right)u(t)\,;\ y_p(t) = \left(ae^{-t} + be^{-2t}\right)u(t)$
   $y_p'(t) = \left(-ae^{-t} - 2be^{-2t}\right)u(t)$
   $y_p''(t) = \left(ae^{-t} + 4be^{-2t}\right)u(t)$
   The we write the informations above to the equations and then we get;
   $6ae^{-t} + 12be^{-2t} = -3e^{-t} - 10e^{-2t},\ therefore\, a = -\frac{1}{2},\ b = -\frac{5}{6}$
   $y_p(t) = -\frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t}\ for\, t > 0$
   $y(t) = C_1e^t + C_2e^{2t} - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t}$
   Moreover, we know that $y(0) = y'(0) = 0$
   $Therefore\, C_1 = \frac{29}{6},\ C_2 = -\frac{21}{6}$
   Hence, $y(t) = \frac{29}{6}e^t - \frac{21}{6}e^{2t} - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t}$.

2. (a) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$
   $y[n] = x[1]\,h[n-1] + x[-2]\,h[n+2]$
   $= 2\delta[n+1] - \delta\ [n] + 3\left(2\delta\ [n+4] - \delta\ [n+3]\right)$
   $= 2\delta[n+1] - \delta\ [n] + 6\delta\ [n+4] - 3\delta\ [n+3]$

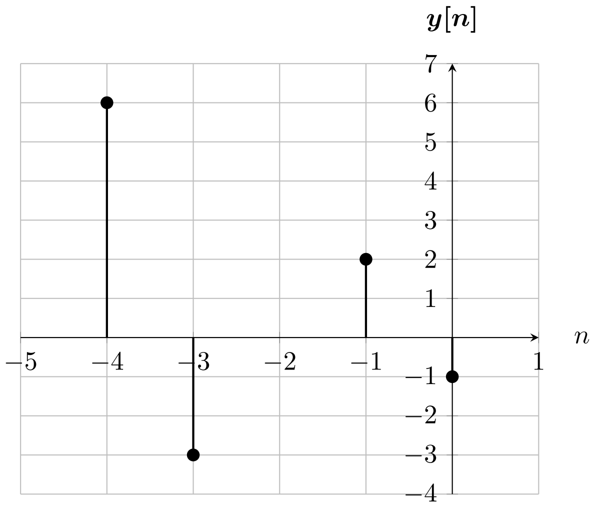

Figure 1: n vs. $y[n]$.

   (b) $x[n] = \delta[n+1] + \delta\ [n] + \delta\ [n-1]$
   $h[n] = \delta[n-4] + \delta\ [n-5]$
   $y[n] = \delta[n-3] + \delta[n-4] + \delta\ [n-4] + \delta[n-5] + \delta[n-5] + \delta[n-6]$
   $y[n] = \delta[n-3] + 2\delta[n-4] + 2\delta[n-5] + \delta[n-6]$

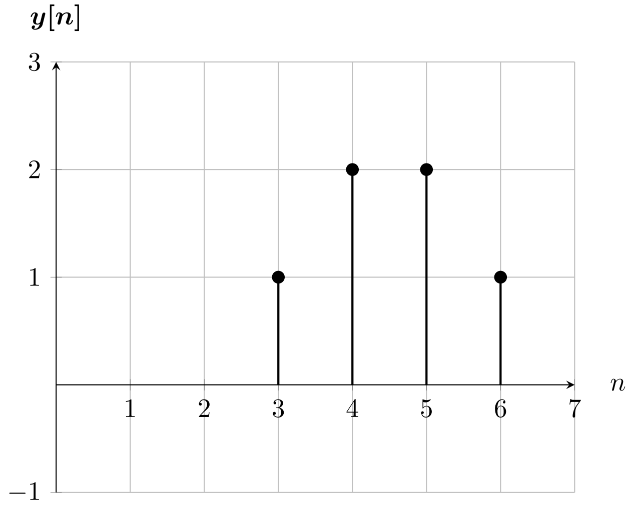

Figure 2: n vs. $y[n]$.

3. (a) $y(t) = \int_{\infty}^{-\infty} x(\tau)\, h(t-\tau)\, d\tau$
   $= \int_{t}^{0} e^{-\tau}\, e^{-\frac{1}{2}t+\frac{1}{2}\tau} = e^{-\frac{1}{2}t} \int_{t}^{0} e^{-\frac{\tau}{2}}\, d\tau$
   $= -2e^{-\frac{1}{2}t} - 2e^{-t}$
   Thus, $y(t) = \left[-2e^{-\frac{1}{2}t} - 2e^{-t}\right] u(t)$.

   (b) $y(t) = \int_{\infty}^{-\infty} x(\tau)\, h(t-\tau)\, d\tau$
   $= \int_{0}^{\infty} e^{-3\tau}\, (u(t-\tau) - u(t-\tau-4))\, d\tau$
   $for\, t\, \leq 0\, the\, convolution\, evaluates\, to\, ZERO$
   $for\, 0 < \ t\ \leq\ 4;\ \int_{0}^{t}\ e^{-3(t-\tau)}\, d\tau\ \ =\ -\frac{1}{3}e^{-3t} + \frac{1}{3}\,(u(t) - u(t-4))$
   $for\, 4 <\ t;\ \int_{0}^{4}\ e^{-3(t-\tau)}\, d\tau\ \ =\ \frac{1}{3}e^{-3t+12} - \frac{1}{3}e^{-3t}\,(u(t-4))$

4. (a)
   (b)

5. (a)
   (b)
   (c)