CENG 384 - Signals and Systems for Computer Engineers
20212
Written Assignment 3 Solutions

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1. (a) $\sin(\frac{\pi}{5}t) \rightarrow T = 10$ and $\cos(\frac{\pi}{4}t) \rightarrow T = 8$, so overall T is 40 and $\omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$.

   By Euler's Equation:
   $$x(t) = \frac{1}{2j}e^{j\frac{\pi}{5}} - \frac{1}{2j}e^{-j\frac{\pi}{5}} + \frac{1}{2}e^{j\frac{\pi}{4}} + \frac{1}{2}e^{-j\frac{\pi}{4}}$$
   $a_4 = \frac{1}{2j} = \frac{-j}{2}, \qquad a_{-4} = -\frac{1}{2j} = \frac{j}{2}, \qquad a_5 = \frac{1}{2}, \qquad a_{-5} = \frac{1}{2}.$

   (b)
   $$x[n] = \frac{1}{2} + e^{j\pi n} + \sin(4\pi n) + \cos(2\pi n)$$
   $cos(2\pi n)$ is always equal to 1 and $\sin(4\pi n)$ are always equal to 0.

   Therefore,
   $$x[n] = \frac{3}{2} + e^{j\pi n},$$
   where $\omega_0 = \pi$ and $N = 2$.
   $a_0 = \frac{3}{2}, \qquad a_1 = 1.$

   a_k is periodic with 2.

2. $\omega_0 = \frac{2\pi}{7}$ and $x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$ so
   $$x[n] = 2je^{j\frac{2\pi}{7}n} - 2je^{-j\frac{2\pi}{7}n} + 2e^{j\frac{4\pi}{7}n} + 2e^{-j\frac{4\pi}{7}n} + 2je^{j\frac{6\pi}{7}n} - 2je^{-j\frac{6\pi}{7}n}.$$
   Using Euler's Equation we get:
   $$x[n] = -4\sin\left(\frac{2\pi}{7}n\right) + 4\cos\left(\frac{4\pi}{7}n\right) - 4\sin\left(\frac{6\pi}{7}n\right).$$

3. (a)
   $$x(t) = \sin\left(\frac{\pi}{8}t\right) = \frac{1}{2j}e^{j\frac{\pi}{8}t} - \frac{1}{2j}e^{-j\frac{\pi}{8}t}$$
   $a_1 = \frac{1}{2j} = \frac{-j}{2}, \qquad a_{-1} = -\frac{1}{2j} = \frac{j}{2}.$

   (b)
   $$y(t) = \cos\left(\frac{\pi}{8}t\right) = \frac{1}{2}e^{j\frac{\pi}{8}t} + \frac{1}{2}e^{-j\frac{\pi}{8}t}$$
   $b_1 = \frac{1}{2}, \qquad b_{-1} = \frac{1}{2}.$

   (c)
   $$x(t)y(t) \longleftrightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$
   $c_2 = a_1 b_1 = \frac{1}{4j} = \frac{-j}{4}, \qquad c_{-2} = a_{-1}b_{-1} = -\frac{1}{4j} = \frac{j}{4}.$

4. $x(t)$ is odd, so are its spectral coefficients, $a_{-k} = -a_k$. From Parseval's relation we get:

$$\sum_{k=-2}^{2} |a_k|^2 = |a_{-2}|^2 + |a_{-1}|^2 + |a_0|^2 + |a_1|^2 + |a_2|^2 = 18,$$

We know that $a_2 = 3j$ and $a_{-2} = -3j$, so:

$$9 + |a_{-1}|^2 + |a_0|^2 + |a_1|^2 + 9 = 18,$$

Therefore, we conclude that a_0, a_1 and a_{-1} are equal to 0.

Now we can write $x(t)$ using $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$:

$$x(t) = 3je^{j\pi t} - 3je^{-j\pi t} = -6\sin(\pi t).$$

5. For discrete time periodic square wave $x[n]$,

$$x[n] = \begin{cases} 1, & \text{for} \quad 0 \leq n < N_p \\ 0, & \text{for} \quad N_p \leq n < N \end{cases}$$

where N is the period,

We have the following Fourier series coefficients:

$a_0 = \frac{N_p}{N}$ and $a_k = \frac{1}{N} \frac{\sin\left(k \frac{N_p \pi}{N}\right)}{\sin\left(k \frac{\pi}{N}\right)} e^{-jk(N_p - 1)\frac{\pi}{N}}$ for $k \neq 0$, (see reference Example 3.12(p218) in the textbook).

(a) $N_p = 5$, $N = 9$
$a_0 = \frac{5}{9}$, $a_k = \frac{1}{9} \frac{\sin\left(k \frac{5\pi}{9}\right)}{\sin\left(k \frac{\pi}{9}\right)} e^{-jk\left(\frac{4\pi}{9}\right)}$ for $k \neq 0$.

(b) $N_p = 4$, $N = 9$
$b_0 = \frac{4}{9}$, $b_k = \frac{1}{9} \frac{\sin\left(k \frac{4\pi}{9}\right)}{\sin\left(k \frac{\pi}{9}\right)} e^{-jk\left(\frac{\pi}{3}\right)}$ for $k \neq 0$.

(c) $b_k = H(e^{j\omega_0 k}) a_k \rightarrow H(e^{j\omega_0 k}) = \frac{b_k}{a_k}$, setting $\omega = \omega_0 k$, we obtain $H(e^{j\omega})$.
$\frac{b_k}{a_k} = \frac{\sin\left(k \frac{4\pi}{9}\right)}{\sin\left(k \frac{5\pi}{9}\right)} e^{-jk\left(\frac{\pi}{3} - \frac{4\pi}{9}\right)}$ setting $\omega_0 = \frac{\pi}{9}$ and $\omega = \omega_0 k$, we obtain

$$H(e^{j\omega}) = \begin{cases} \frac{\sin(4\omega)}{\sin(5\omega)} e^{j\omega}, & \text{for} \quad \omega \neq 0 \\ \frac{4}{5}, & \text{for} \quad \omega = 0 \end{cases}$$