CENG 384 - Signals and Systems for Computer Engineers
20222
Written Assignment 2 Solutions

April 20, 2022

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1. (a)

$$x(t) - 2x'(t) + 3y(t) - 2\int_{-\infty}^{t} y(\tau)d\tau = y'(t)$$
$$x'(t) - 2x''(t) + 3y't - 2y(t) = y''(t)$$
$$y''(t) - 3y'(t) + 2y(t) = x' - 2x''(t)$$

(b) char eqn. : $r^2 - 3r + 2 = 0 \quad \Rightarrow \quad r_1 = 2, r_2 = 1 \quad \Rightarrow \quad y_h(t) = A \cdot e^{2t} + B \cdot e^t$
$y_p(t) = C \cdot e^{-t} + D \cdot e^{-2t}$
$y_p'(t) = -C \cdot e^{-t} - 2D \cdot e^{-2t}$
$y_p''(t) = C \cdot e^{-t} + 4D \cdot e^{-2t}$
$x'(t) = -e^{-t} - 2e^{-2t}$
$x''(t) = e^{-t} + 4e^{-2t}$

$$C \cdot e^{-t} + 4D \cdot e^{-2t} + 3C \cdot e^{-t} + 6D \cdot e^{-2t} + 2C \cdot e^{-t} + 2D \cdot e^{-2t} = (-e^{-t} - 2e^{-2t}) - 2(e^{-t} + 4e^{-2t})$$
$$6C \cdot e^{-t} + 12D \cdot e^{-2t} = -3e^{-t} - 10e^{-2t}$$

$6C = -3 \quad \Rightarrow \quad C = -1/2$
$12D = -10 \quad \Rightarrow \quad D = -5/6$

$$y(t) = y_h(t) + y_p(t) = (A \cdot e^{2t} + B \cdot e^t - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t})u(t)$$
$$y(0) = A + B - \frac{1}{2} - \frac{5}{6} = 0 \quad \Rightarrow \quad A + B = \frac{8}{6}$$
$$y'(t) = 2A \cdot e^{2t} + B \cdot e^t + \frac{1}{2}e^{-t} + \frac{5}{3}e^{-2t}$$
$$y'(0) = 2A + B + \frac{1}{2} + \frac{5}{3} = 0 \quad \Rightarrow \quad 2A + B = -\frac{13}{6} \quad \Rightarrow \quad A = -\frac{7}{2}, B = \frac{29}{6}$$
$$y(t) = (-\frac{7}{2}e^{2t} + \frac{29}{6}e^t - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t})u(t)$$

2. (a)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= x[-2]h[n+2] + x[1]h[n-1]$$
$$= 3(2\delta[n+4] - \delta[n+3]) + 2\delta[n+1] - \delta[n]$$
$$= -\delta[n] + 2\delta[n+1] - 3\delta[n+3] + 6\delta[n+4]$$

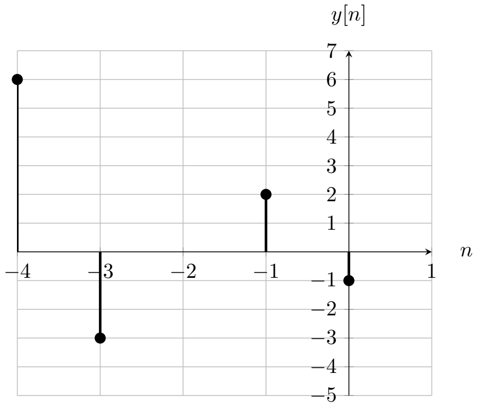

Figure 1: n vs. $y[n]$.

(b)

$$
\begin{aligned}
x[n] &= \delta[n+1] + \delta[n] + \delta[n-1] \\
h[n] &= \delta[n-4] + \delta[n-5] \\
y[n] &= \delta[n-3] + \delta[n-4] + \delta[n-4] + \delta[n-5] + \delta[n-5] + \delta[n-6] \\
&= \delta[n-3] + 2\delta[n-4] + 2\delta[n-5] + \delta[n-6]
\end{aligned}
$$

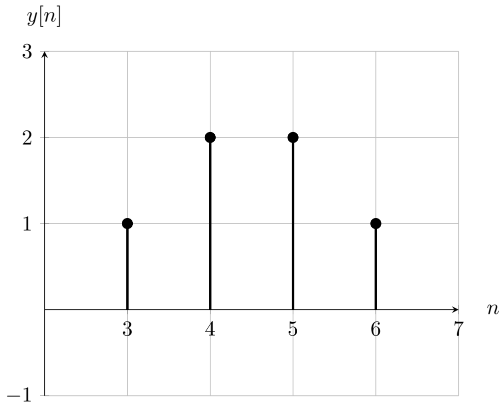

Figure 2: n vs. $y[n]$.

3. (a)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \\
&= \int_0^t e^{-\tau} e^{-\frac{1}{2}(t-\tau)} d\tau \\
&= e^{-\frac{t}{2}} \int_0^t e^{-\frac{\tau}{2}} d\tau \;=\; e^{-\frac{1}{2}t}(-2e^{-\frac{t}{2}} - 2)u(t) \;=\; (-2e^{-t} + 2e^{-\frac{1}{2}t})u(t)
\end{aligned}
$$

(b)

$$
y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau
$$

For $0 < t < 4$:

$$
\int_0^t e^{-3(t-\tau)} d\tau \;=\; e^{-3t} \int_0^t e^{3\tau} d\tau \;=\; \frac{1}{3} e^{-3t}(e^{3t} - 1) \;=\; \frac{1}{3}(1 - e^{-3t})(u(t) - u(t-4))
$$

For $t > 4$:

$$
\int_0^4 e^{-3(t-\tau)} d\tau \;=\; \frac{1}{3} e^{-3t}(e^{12} - 1) \;=\; \frac{1}{3}(e^{12-3t} - e^{-3t})u(t-4)
$$

4. (a) We have $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau-3)d\tau$. We use substitution $\tau' = \tau - 3$ to make the integral look like convolution integral.

$$\tau' = \tau - 3 \tag{1}$$
$$d\tau' = d\tau \tag{2}$$
$$y(t) = \int_{-\infty}^{t-3} e^{-(t-3-\tau')}x(\tau')d\tau' \tag{3}$$

From the last equation, we find $h(t) = e^{-(t-3)}u(t-3)$.

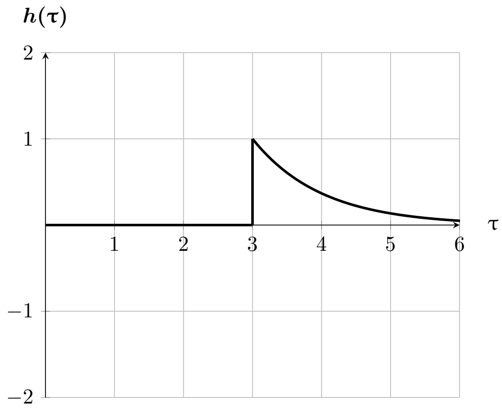

Figure 3: $h(\tau)$

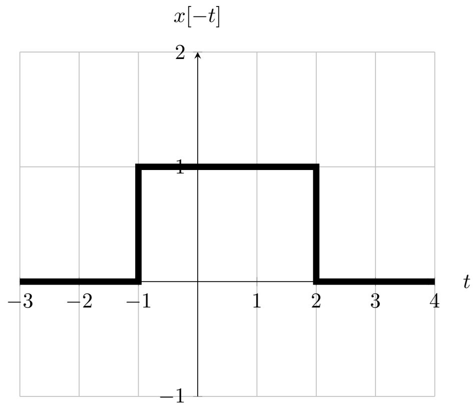

Figure 4: n vs. $y[n]$.

(b) $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$:

$$y(t) = \int_{2}^{\infty} e^{-(\tau-3)}(u(t-\tau+2) - u(t-\tau-1))d\tau \tag{4}$$

$h(\tau)$ and $x(t-\tau)$ can be derived from figures above. Finally, by analizing the figures, we can conclude:

$$y(t) = \begin{cases} 0, & t < 1 \\ \int_{3}^{t+2} e^{-(\tau-3)}d\tau = 1 - e^{-(t-1)}, & 1 < t < 4 \\ \int_{t-1}^{t+2} e^{-(\tau-3)}d\tau = e^{-(t-4)}(1-e^{-3}), & t > 4 \end{cases} \tag{5}$$

5. (a) We have $h_1^{-1}[n] = (\frac{1}{2})^n u[n]$ and $h_1^{-1}[n] * h_1[n] = \delta[n]$:
We first need to find $h_1[n]$. We see that $h_1^{-1}[n]$ is actually a unit step function multiplied by powers of $\frac{1}{2}$. This means that if we perform the substraction $h_1^{-1}[n] - A \cdot h1^{-1}[n-1]$, we will have $\delta[n]$.

$$h_1^{-1}[n] - A \cdot h_1^{-1}[n-1] = \delta[n] \tag{6}$$

We already know $h_1^{-1}[0] = 1$ and $h_1^{-1}[1] = \frac{1}{2}$, so

$$h_1^{-1}[1] - A \cdot h_1^{-1}[0] = \delta[1] = 0 \tag{7}$$

From the equation above, we get that $A = \frac{1}{2}$. So we have the equation:

$$h_1^{-1}[n] - \frac{1}{2} \cdot h_1^{-1}[n-1] = \delta[n] \tag{8}$$

We can arrange the equation above using properties of convolution;

$$h_1^{-1}[n] * (\delta[n] - \frac{1}{2} \cdot \delta[n-1]) = \delta[n] \tag{9}$$

In the equation above, we can see that:

$$h_1[n] = \delta[n] - \frac{1}{2}\delta[n-1] \tag{10}$$

$$\begin{align} h_1[n] &= \delta[n] - \frac{1}{2}\delta[n-1] \tag{11} \\ h_1[n] * h_1[n] &= h_1[n] - \frac{1}{2}h_[n-1] \tag{12} \\ h_1[n] * h_1[n] &= \delta[n] - \delta[n-1] + \frac{1}{4}\delta[n-2] \tag{13} \end{align}$$

(b) By analizing the overall impulse response of the system $h[n]$, we can see that:

$$\begin{align} h[0] &= h_0[0], & h_0[0] &= 4 \tag{14} \\ h[1] &= h_0[1] - h_0[0] & h_0[1] &= 4 \tag{15} \\ h[2] &= h_0[2] - h_0[1] + \frac{1}{4}h_0[0] & h_0[2] &= 4 \tag{16} \\ h[3] &= h_0[3] - h_0[2] + \frac{1}{4}h_0[1] & h_0[3] &= 0 \tag{17} \end{align}$$

When $n < 0$ or $n > 2$, $h_0[n] = 0$. So;

$$h_0[n] = 4\delta[n] + 4\delta[n-1] + 4\delta[n-2] \tag{18}$$

(c) We have $x[n] = \delta[n] + \delta[n-2]$. Response of this system is:

$$\begin{align} h[n] &= h_0[n] + h_0[n-2] \tag{19} \\ h[n] &= 4\delta[n] + 4\delta[n-1] + 4\delta[n-2] + 4\delta[n-2] + 4\delta[n-3] + 4\delta[n-4] \tag{20} \\ h[n] &= 4\delta[n] + 4\delta[n-1] + 8\delta[n-2] + 4\delta[n-3] + 4\delta[n-4] \tag{21} \end{align}$$