CENG 384 - Signals and Systems for Computer Engineers
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Homework 4

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1. (a) $y(t) = \int \left(-\int x(t) + x(t-1) + x(t) - \int y(t) - 2y(t)\right)$
$y'(t) = -\int x(t) + x(t-1) + x(t) - \int y(t) - 2y(t)$
$y''(t) = -x(t) + x'(t-1) + x'(t) - y(t) - 2y'(t)$

   (b) $y''(t) + 2y'(t) + y(t) = -x(t) + x'(t-1) + x'(t)$
$H(jw) = \frac{jw - jw - 1}{(jw)^2 + 2jw + 1} = \frac{-1}{(jw+1)^2}$

   (c) $H(jw) = \frac{-1}{(jw+1)^2} = \frac{1}{jw+1} - \frac{2}{jw+1}$
$h(t) = e^{-t}u(t) - 2e^{-t}u(t) = -e^{-t}u(t)$

   (d) $Y(jw) = X(jw)\,H(jw)$
$Y(jw) = X(jw) \cdot \frac{-1}{(jw+1)^2}$
If we take $x(t) = e^{-t}u(t)$, then we get $X(jw) = \frac{1}{jw+1}$
$Y(jw) = \frac{1}{jw+1} \cdot \frac{-1}{(jw+1)^2} = \frac{-1}{(jw+1)^3}$
Hence, $y(t) = -e^{-3t}u(t)$.

2. (a) $\frac{Y(jw)}{X(jw)} = H(jw) = \frac{e^{jw} - e^{-jw}}{jw} = \frac{2sinw}{w}$
$h(t) = 1$ for $t < 1$ and $h(t) = 0$ for $t > 1$.

   (b) $H(jw) = \frac{e^{jw} - e^{-jw}}{jw} = \frac{2sinw}{w}$

3. (a) $H\left(e^{jw}\right) = H_1\left(e^{jw}\right) H_2\left(e^{jw}\right) = \frac{1}{\left(1 - \frac{e^{-jw}}{2}\right)^2}$

   (b)

   (c)

4. (a) $H\left(e^{jw}\right) = \frac{Y\left(e^{jw}\right)}{X\left(e^{jw}\right)} = 2 - \frac{1}{1 - \frac{1}{2}e^{-jw}}$

   (b) $H\left(e^{jw}\right) = \frac{Y\left(e^{jw}\right)}{X\left(e^{jw}\right)} = 2 - \frac{1}{1 - \frac{1}{2}e^{-jw}} = \frac{1 - e^{-jw}}{1 - \frac{1}{2}e^{-jw}}$
$Y\left(e^{jw}\right) - Y\left(e^{jw}\right)\frac{e^{jw}}{2} = X\left(e^{jw}\right) - e^{-jw}X\left(e^{jw}\right)$
$y[n] - \frac{y[n-1]}{2} = x[n] - x[n-1]$

   (c)