MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

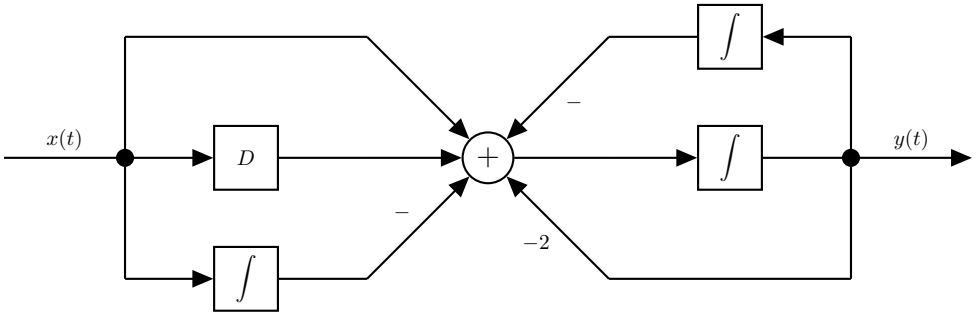
CENG 384

Spring 2022
Homework 4

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Submission:** You need to submit a pdf file named 'hw4.pdf' to the odtuclass page of the course. You need to use the given template 'hw4.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 6 June, 2022 (Monday).
- **Late Submission:** Not allowed.

1. (30 pts) Consider an LTI system given by the following block diagram:



where D is the differentiator.

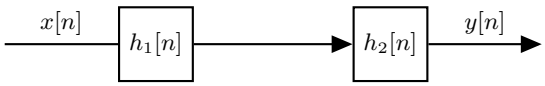
   (a) (5 pts) Find the differential equation which represents this system.
   (b) (5 pts) Find the frequency response of this system.
   (c) (10 pts) Find the impulse response of this system from its frequency response.
   (d) (10 pts) Find the output $y(t)$ for the input $x(t) = e^{-t}u(t)$ using the frequency response.

2. (20 pts) Consider a continuous-time system represented by the following equation:

$$\frac{dy(t)}{dt} = x(t+1) - x(t-1)$$

   (a) (10 pts) Find the impulse response of this system.
   (b) (10 pts) Find the frequency response of this system.

3. (30 pts) Consider the following discrete-time LTI system:



where $x[n] = \sin(\frac{\pi}{3}n + \frac{\pi}{4})$ and $h_1[n] = h_2[n] = (\frac{1}{2})^n u[n]$.

   (a) (10 pts) Find the overall frequency response of this system.
   (b) (10 pts) Find the Fourier Transform of the input, $x[n]$.
   (c) (10 pts) Find the Fourier Transform of the output, $y[n]$.

4. (20 pts) Consider an LTI system with the impulse response: $h[n] = 2\delta[n] + 2^{-n}u[n]$.

   (a) (5 pts) Find $H(e^{j\omega})$, the Fourier Transform of $h[n]$.
   (b) (5 pts) Find the difference equation describing this system.
   (c) (10 pts) For $x[n] = (-1)^n$ as the input, find $Y(e^{j\omega}) = \mathcal{F}\{y[n]\}$, i.e., the Fourier Transform of the output. Hint: $(-1)^n$ can be represented with a complex exponential.