EE 475 HOMEWORK 2

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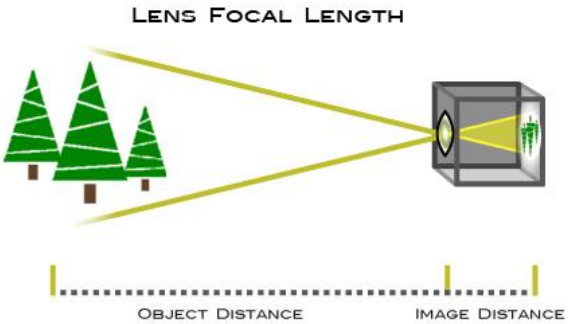
Q1. Cameras and Samplings

A) Camera

The camera system is similar the pinhole camera system.



Object distance is the depth of the image and image distance is focal length. Focal length is f.

-f/p1z = x1/p1x = y1/p1y
-0.01/1=x1/-1=y1/1
X1 = 0.01 m = 1 cm
Y1 = -0.01 m = -1 cm

-f/p2z = x2/p2x = y2/p2y
-0.01/2=x2/4=y2/2
X2 = -0.02 m = -2 cm
Y2 = -0.01 m = -1 cm

-f/p3z = x3/p3x = y3/p3y
-0.01/3=x3/-3=y3/12
X3 = 0.01 m = 1 cm
Y3 = -0.04 m = -4 cm

B) Sampling

(x,y) = 24·cos4πx·cos6πy has sinusoidal patterns:

a) How many horizontal and vertical cycles per meter does the pattern have?

If it is horizontal, the value depends on only x values.

f depends on cos4πx.

f_x= 2π/4π

=1/2 cycles per meter

If it is vertical, the value depends on only y values.

f depends on cos6πy.

f_y = 2 π/6 π

=1/3 cycles per meter

b) What would be the frequency encountered along 0, 30, 45, and 90 degrees?

f along 0-degree equals f_x

$f_0 = f_x$ = 1/2 cycles per meter

f along 30-degree

x = √3y

(x,y) = 24·cos4√3πy·cos6πy

= ½ (cos (4√3-6) πy + cos (6+4√3) πy)

The frequency of (cos (6-4√3) πy is f1= 1/(2√3-3)

The frequency of (cos (6+4√3) πy is f2= 1/(3-+2√3)

The periods are (2√3-3) and (3-+2√3)

Least common multiple of (2√3-3) and (3-+2√3)

is 3. The period is 3 and the frequency is 1/3.

f_{30} = 1/3 cycles per meter

f along 45-degree

x = y

(x,y) = 24·cos4πy·cos6πy

= ½ (cos (2) πy + cos (10) πy)

The frequency of (cos (2) πy is f1= 1

The frequency of (cos (10) πy is f2= 1/(5)

The periods are 1) and (5)

Least common multiple of 1 and 5

is 5. The period is 5 and the frequency is 1/5.

f_{45} = 1/5 cycles per meter

f along 90-degree equals f_y

f_{90} = f_y = 1/3 cycles per meter

c) To represent this image digitally, what should be the Nyquist rate?

fx =1/2, Nyquist rate >= 1

f_y = 1/3, Nyquist rate >= 2/3

then Nyquist rate >= 1 cycles per meter

d) How many samples will 1-meter square of this sampled texture will contain?

Q2. Histogram Equalization

Implement your own histogram equalization function in MATLAB. Try your function on the image "lumberjack" and "moon" images and validate with "histeq.m" MATLAB function. Compare your results with adaptive histogram equalization method. You can use MATLAB embedded function "adapthisteq.m". Do we always obtain better results with adaptive histograms? If so, why do? Comment on the results.

Figure 2.1

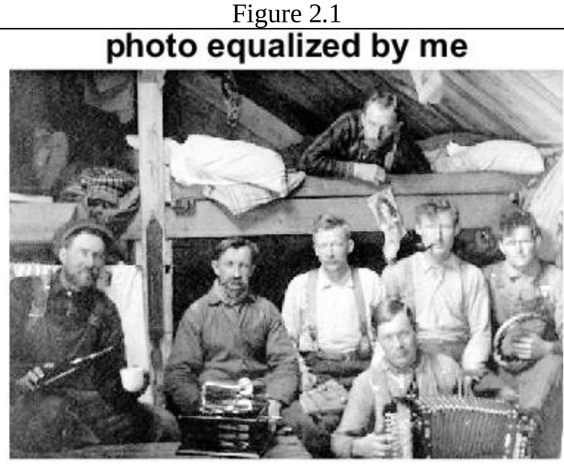


figure 2.2

Figure 2.3

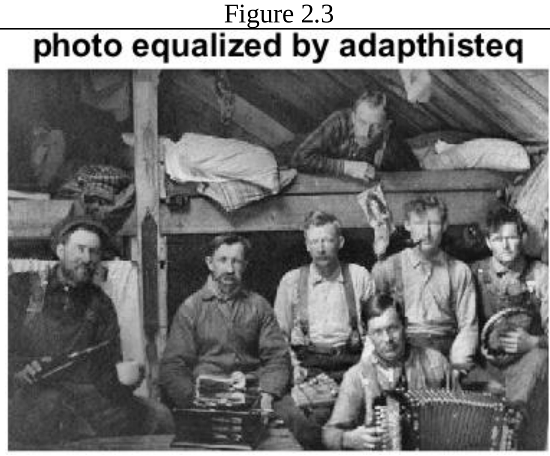


Figure 2.4

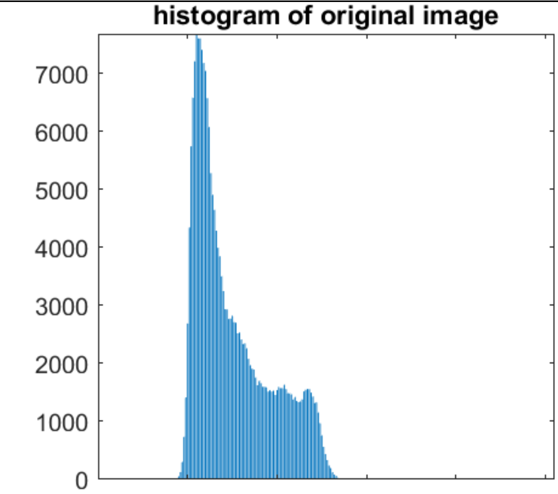


Figure 2.5

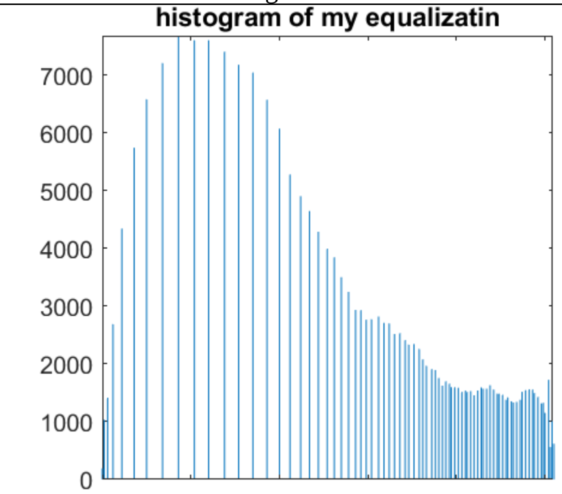


Figure 2.6

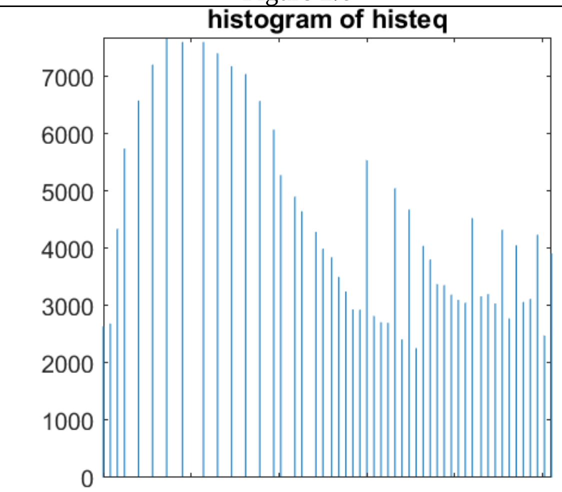


Figure 2.7

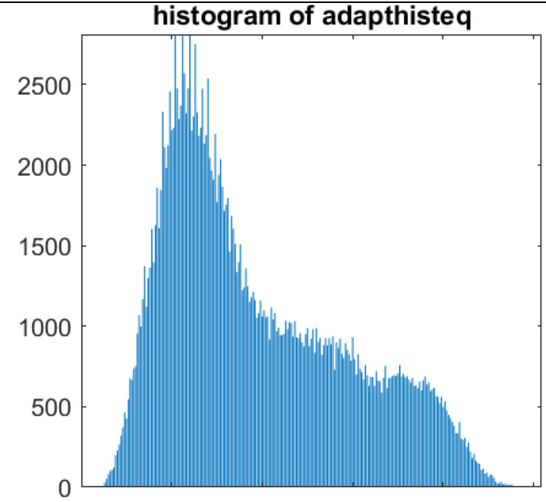


Figure 2.8

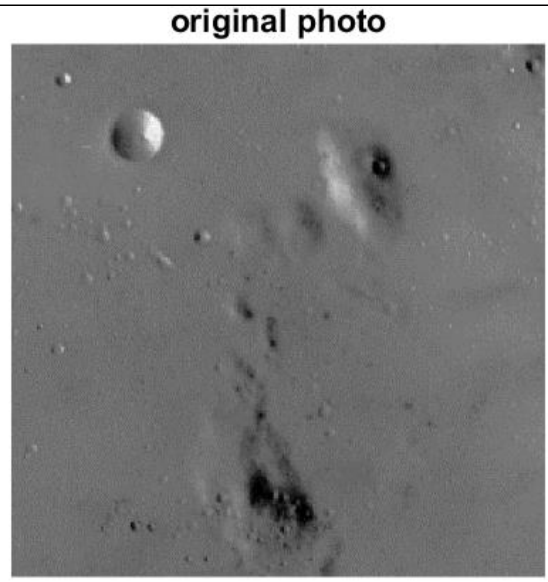


Figure 2.9

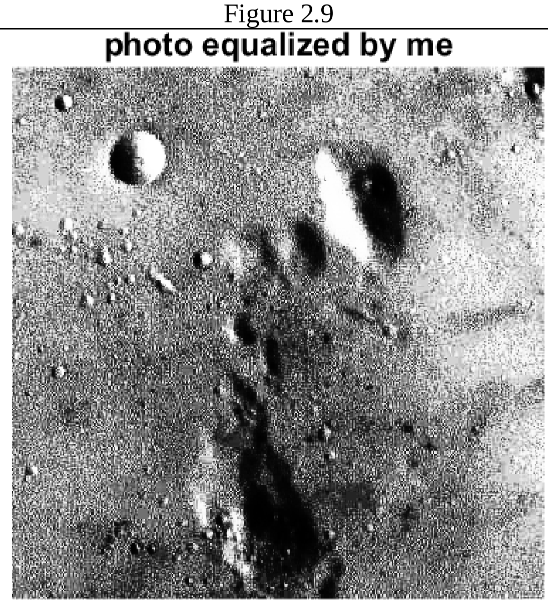


Figure 2.10

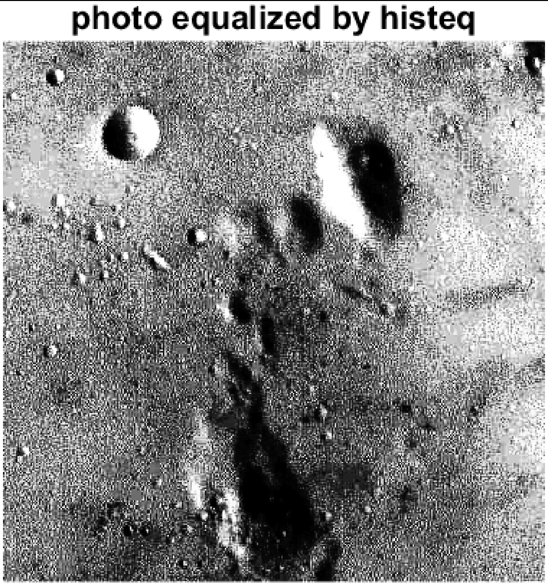


Figure 2.11

Figure 2.12

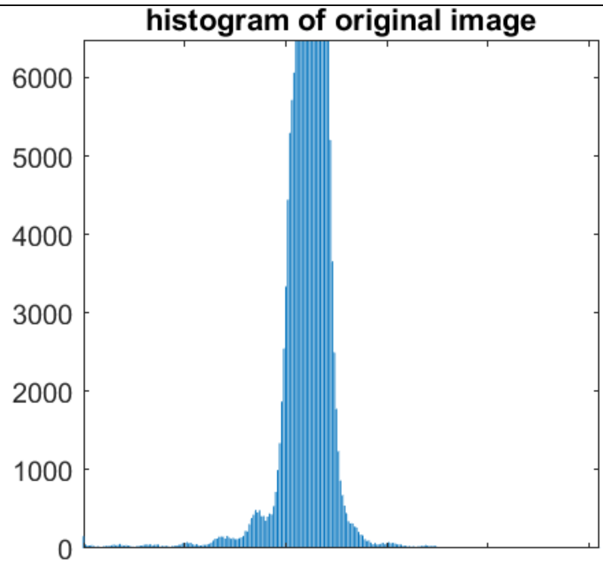


Figure 2.13

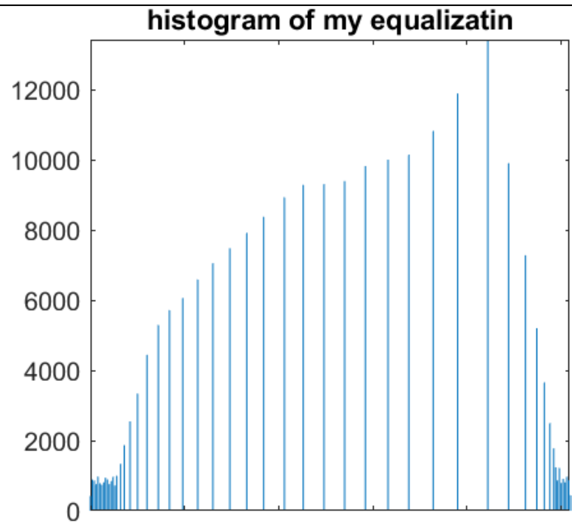


Figure 2.14

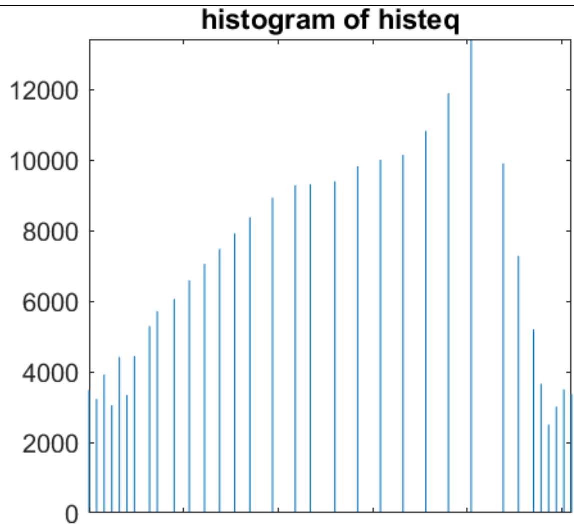


Figure 2.15

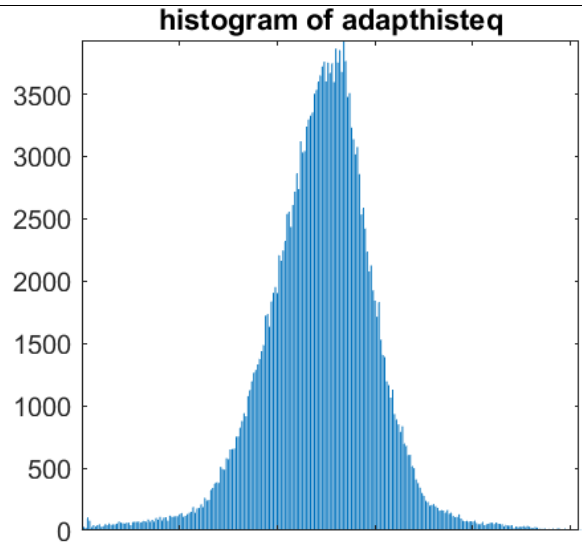


Figure 2.16

The images of my equalization function (figure 2.2 and figure 2.10) and the images of histeq function (figure 2.3 and figure 2.11) are almost same. Their histograms are similar. My result can be validated by checking histogram plots. However, I think, adapthisteq function is more successful and satisfying. The most similar image to original image is the image equalized adapthisteq function (figure 2.4 and figure 2.12). Its histogram is different from my equalization function and histeq function structurally. On the histograms at figure 2.6, 2.7, 2.14, 2.15 (from my function and histeq function), there aren't all intensity level. Some levels are zero. It means when the transformation (s = T(r)) is made for the equalization, all gray values (0-255) aren't transformed all gray values. Because these functions work globally. Some pixels which are far from a pixel can affect this pixel's values when the transformation is made. It causes some distortions. However, adapthisteq function works locally and uses all gray levels. Because of this, the peak values at the histograms of my equalization and histeq function (figure 2.6, 2.7, 2.14, 2.15) are higher than the peak values at the adaptive histogram.

The images equalized with adapthisteq function are similar their original images. The most significant difference between them is contrast. The images equalized with adapthisteq have higher contrast than original images.

When these two images and other images (beach, kugu) are equalized with adapthisteq, the results are better than other equalization method. I think we can always obtain better results with adaptive histograms. Because, the images after adaptive equalization don't have distortions and have higher contrast.

Q3. Histogram Flattening and Matching

Consider the four face images.

A) Do histogram equalization of each image, and

B) Do histogram matching wrt. reference Image A.

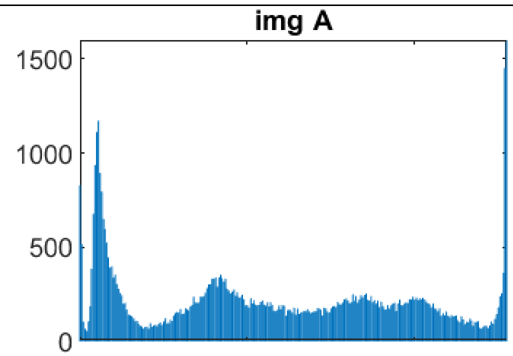


Figure 3.1

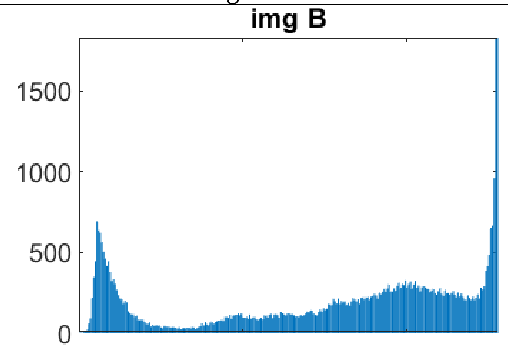


Figure 3.2

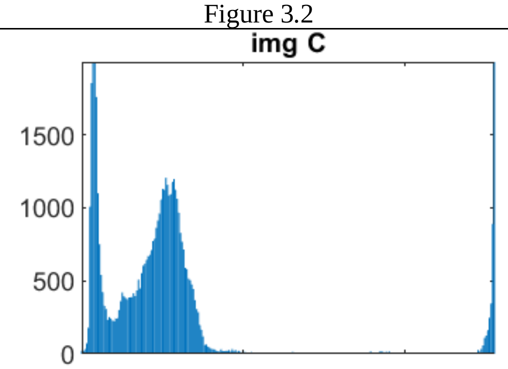


Figure 3.3

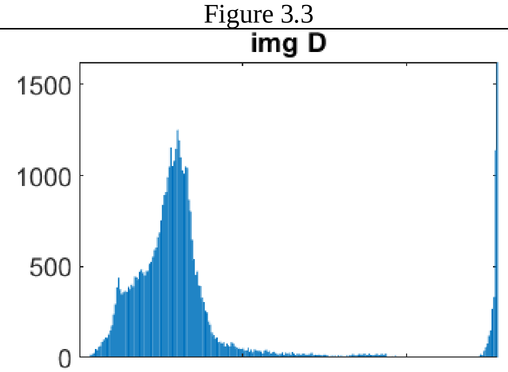


Figure 3.4

Figure: Histograms of original images

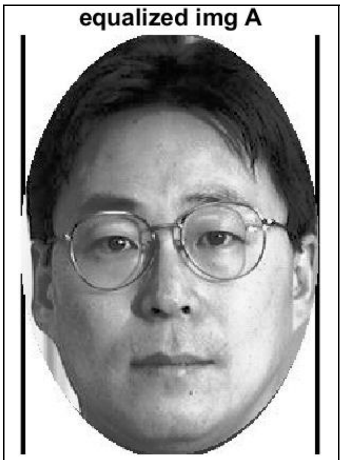


Figure 3.5

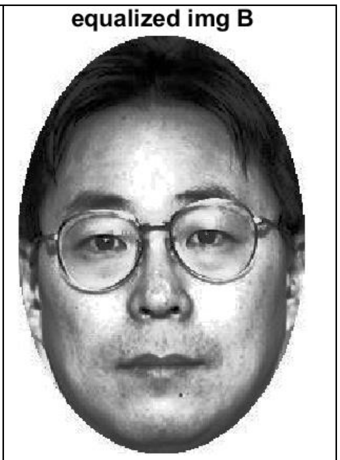


Figure 3.6

Figure 3.7

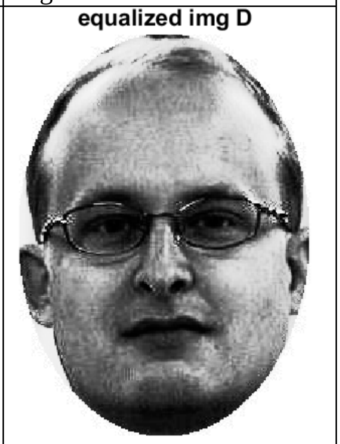


Figure 3.8

Figure 3.1: Equalized Images

Figure 3.10

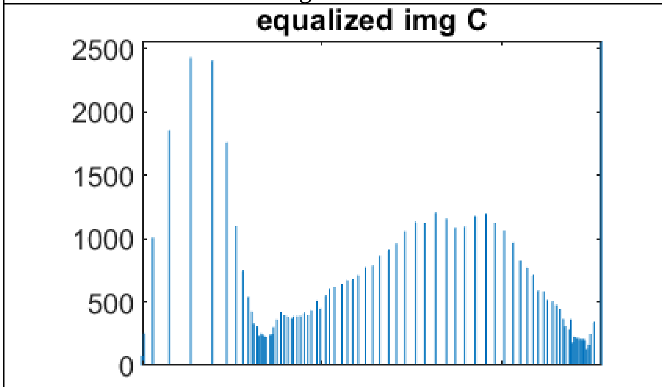


Figure 3.11

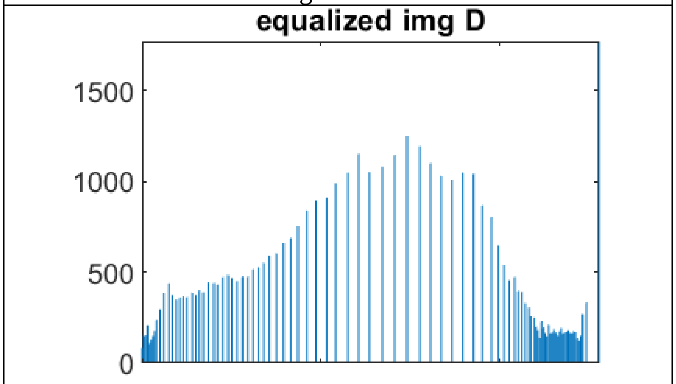


Figure 3.12

Figure: Histograms of Equalized Images

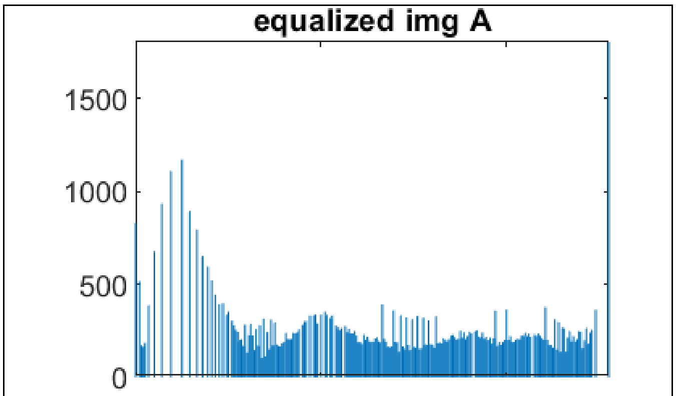
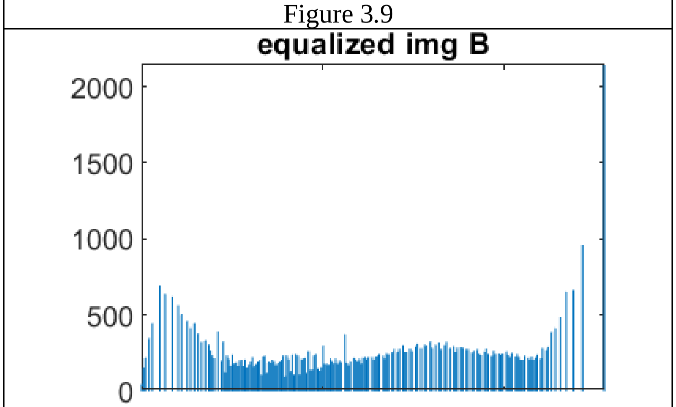


Figure 3.9



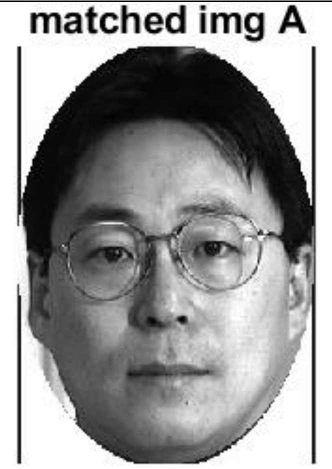


Figure 3.13

Figure 3.14

Figure 3.15

Figure 3.16

Figure: Matched images wrt image A

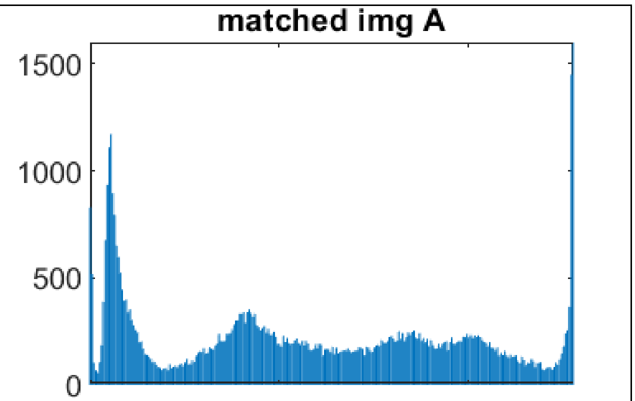


Figure 3.17

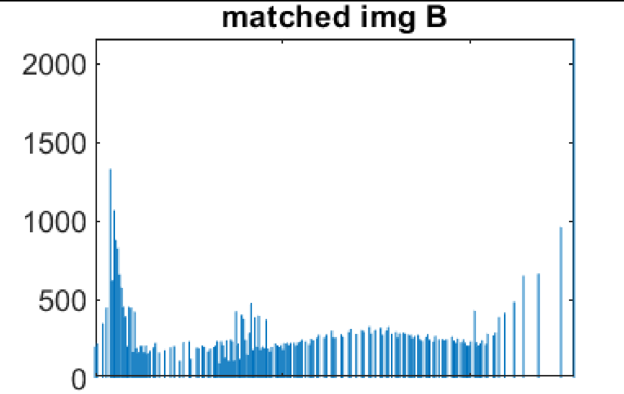


Figure 3.18

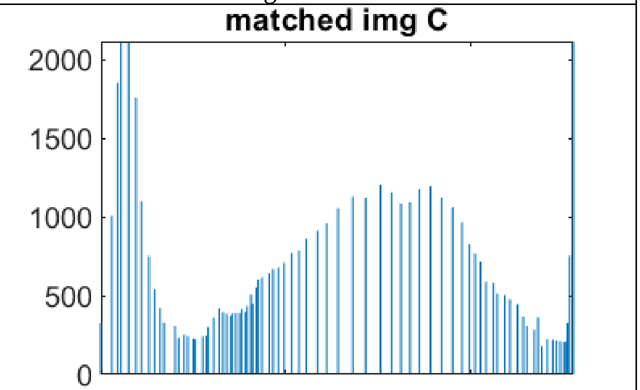


Figure 3.19

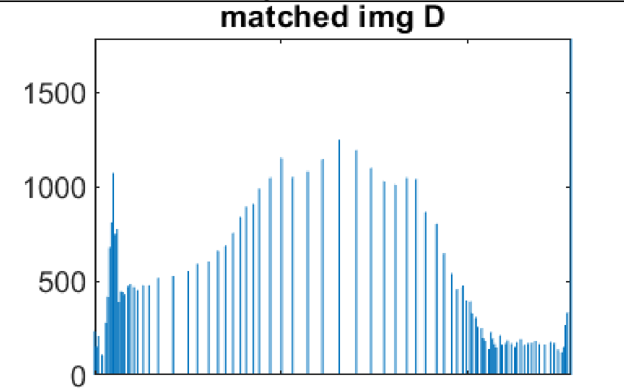


Figure 3.20

Figure: Histograms of Matched Images wrt Image A

The equalized images are not completely equalized because these values (histogram function) are not continuous. However, they are (figure 3.9, 3.10, 3.11, 3.12) more equalized when they are compared to original images (figure 3.1, 3.2, 3.3, 3.4). They spread through all gray level but still they don't use all gray levels. Also, after making matching, histogram of image A (figure 3.17), matchedB (figure 3.18), matchedC (figure 3.19) and matchedD (figure 3.20) are more similar than image B (figure 3.2), image C (figure 3.3), image D (figure 3.4).

Find the histogram distances as a measure of how well we have accomplished histogram matching. Among many possible histogram distances, consider the following two (L-1 is the highest gray value):

i) *The chi-square histogram distance between histograms p and q, namely:*

$$\chi_{p,q} = \sqrt{\frac{1}{2}\sum_{i=0}^{L-1}\frac{|p(i)-q(i)|^2}{p(i)+q(i)}}.$$

ii) *The Kullback-Leibler distance*

$$KL_{pq} = \frac{1}{2}\left[\sum_{i=0}^{L-1} p(i)log_2\left(\frac{q(i)}{p(i)}\right) + \sum_{i=0}^{L-1} q(i)log_2\left(\frac{p(i)}{q(i)}\right)\right]$$

Compare the histogram distances before and after histogram equalization and histogram matching by listing them in two 4x4 tables.

0			
78	0		
167	179	0	
158	169	91	0

Figure 3.21: Chi Distances before matching

0			
0.95e4	0		
1.48e4	2.09e4	0	
3.18e4	1.09e4	3.76e4	0

Figure 3.22: KL Distances before matching

0			
103	0		
168	161	0	
152	148	166	0

Figure 3.23: Chi Distances after matching

0			
1.52e4	0		
5.98e4	3.74e4	0	
3.48e4	1.80e4	1.66e4	0

Figure 3.24: KL Distances after matching

When I check the histograms, it can be said that the matching implementations for image B, C and D with respect to image A are sufficient, their histograms are similar than histograms of original images (figure 3.2, 3.3, 3.4). However, when I calculate the distances for matchings using chi-square histogram distance and Kullback-Leibler distance, some results are better and some of them are worse. The worse results are shown as red and better results as green at figure 3.23 and 3.24. I think the reason that there are some worse results is that this function and data aren't continuous. The histogram of original images use all gray level and have a value other than zero. It's like continuous. However, at the histogram of matched images, many gray level values are zero. For example, when I make zoom in at the histogram of original image B (figure 3.25), there are values rather than zero for all gray level (135-136-137-…-143) whereas at the histogram of matched image B (figure 3.26), the value of many gray levels(137-138-140-141) is zero. It's like rounding the first gray levels to specific gray levels. So, some of gray levels aren't used and the values of gray levels to which other gray levels round are higher than the same gray level at the original image. At the figure 3.25, the value of 135 is 114 whereas at the figure 3.26, it's 293. I think this situation affects these distance results. If we used a function which envelopes the histogram function to calculate distances, we could get better results.

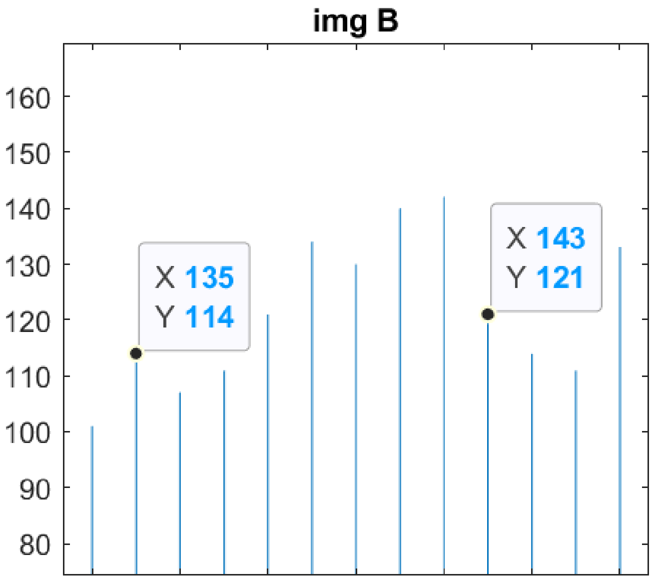


Figure 3.25 (original image B)

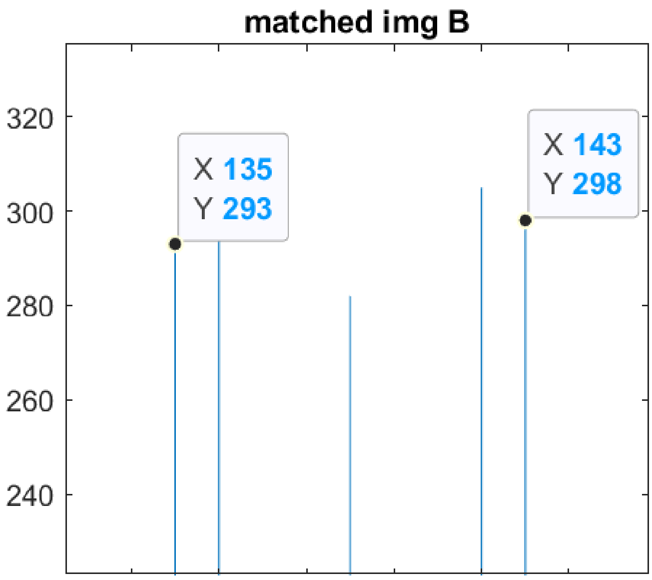


figure 3.26 (matched image B wrt image A)

Q4. Histogram Equalization and Color Images

A) Histogram equalize separately the RBG components of the kugu and the beach images, and then create the color image. Do you get a satisfactory result?
B) Histogram equalize the intensity component only while freezing the hue and saturation components, and then create the color image.
C) Histogram equalize the saturation component only while freezing the hue and intensity components; then create the color image.
D)Comment on the outcomes of the three color image histogram equalization methods.

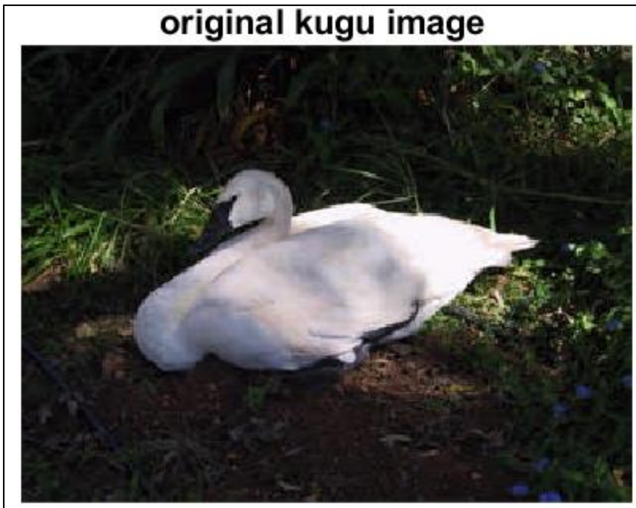


Figure 4.1

Figure 4.2

Figure 4.3

Figure 4.4

Figure 4.5

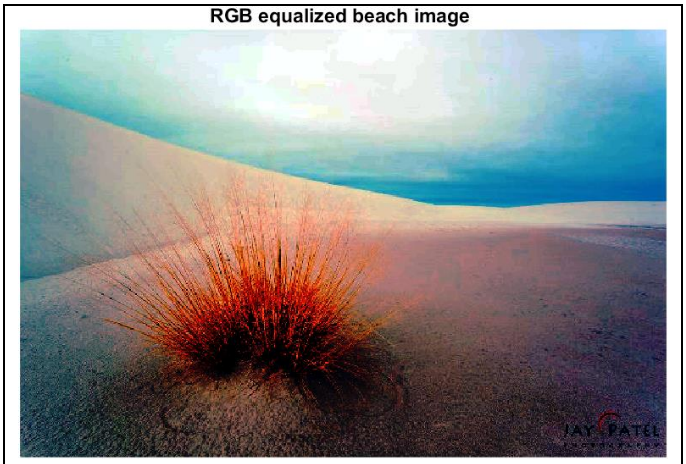


Figure 4.6

Figure 4.7

Figure 4.8

A) When the RGB components are equalized separately, the output images can be seen at figure 4.2 and figure 4.6. The results aren't satisfactory. There are some distortions and some parts are too bright.

B) When the intensity components are equalized, the output images can be seen at figure 4.3 and 4.7. These equalizations increase the contrast of the image without distortions.

C) When the saturation components are equalized, the output images can be seen at figure 4.4 and 4.8. The saturation is about the effect of the color. The higher the saturation is, the more color effect there is and the less grey there is.

D) RGB equalization can create too bright parts or too dark parts and distortions. So, RGB equalization isn't a good idea. The saturation and intensity equalization give better results.