EE550 HW6

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1 Part 1

- dynamic equations:

$$\dot{v_1} = -v_1 + \frac{2}{\pi} \arctan \frac{\lambda \pi v_2}{2} \tag{1}$$

$$\dot{v_2} = -v_2 + \frac{2}{\pi} \arctan \frac{\lambda \pi v_1}{2} \tag{2}$$

The points where the derivatives are zero are equilibrium points(minimum energy). The equilibrium points are $(0,0), (-0.5730, -0.5730)$ and $(0.5730, 0.5730)$. However, the point$(0,0)$ is unstable.

- activation function:

$$g(u) = \frac{2}{\pi} \arctan \frac{\lambda \pi u}{2} \tag{3}$$

$$g^{-1}(\xi) = \frac{2}{\pi\lambda} \tan \frac{\pi \xi}{2} \tag{4}$$

- energy function:

$$E(v_1, v_2) = -T_{12} v_1 v_2 + \int_0^{v_1} g^{-1}(\xi) d\xi + \int_0^{v_2} g^{-1}(\xi) d\xi \tag{5}$$

$$\int g^{-1}(\xi) d\xi = \int \frac{2}{\pi\lambda} \tan \frac{\pi\xi}{2} d\xi \tag{6}$$

$$= \frac{-4}{\pi^2 \lambda} \ln \cos \frac{\pi \xi}{2} + C \tag{7}$$

$$\int_0^v g^{-1}(\xi) d\xi = \frac{-4}{\pi^2\lambda} \ln \cos \frac{\pi v}{2} \tag{8}$$

$$E(v_1, v_2) = -T_{12} v_1 v_2 + \frac{-4}{\pi^2\lambda} \ln \cos \frac{\pi v_1}{2} + \frac{-4}{\pi^2\lambda} \ln \cos \frac{\pi v_2}{2} \tag{9}$$

I generated a input space covering (-1,+1) with precise 0.01 for v_1, v_2 and calculated the energy at each point for $\lambda = 1.4$. I plotted the energy contour maps with 6 different energy levels at 0.449, 0.156, 0.017, 0.003, 0.023 and 0.041.

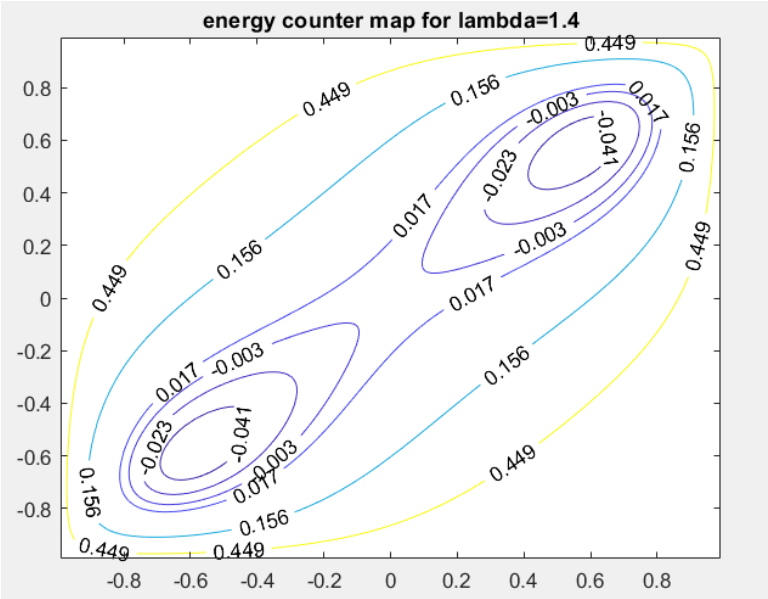

Figure 1: Energy Contour Map

2 Part 2

I selected a point$(0.2, 0.6)$ to simulate the dynamic system. I used iterative algorithm with learning rate 0.01 to update the potentials of the nodes.

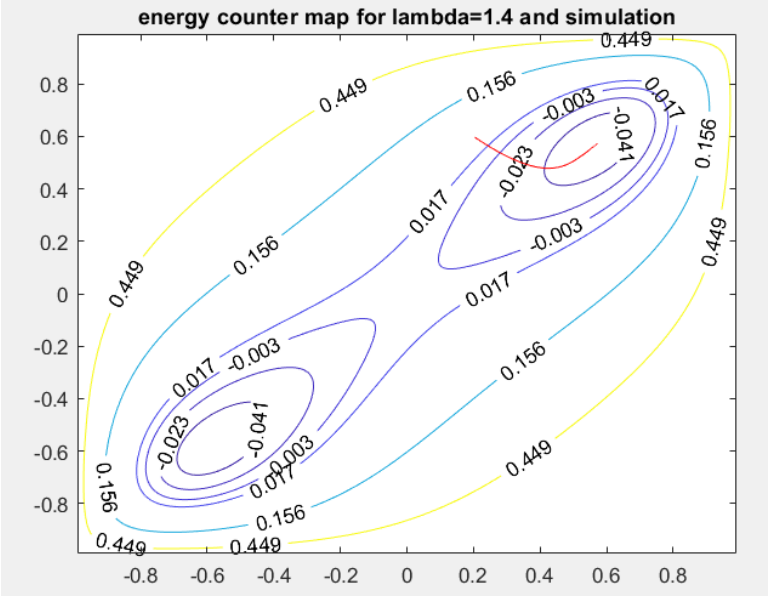

Figure 2: Convergence to the Eq Point

The initial point converged (red path) to the point$(0.572, 0.572)$. (the stopping criteria is that the

distance between current point and any equilibrium points must be less than 0.001).

3 Part 3

I calculated the equilibrium points for λ starting from 1.4 to 20.0 with 0.1 increment.

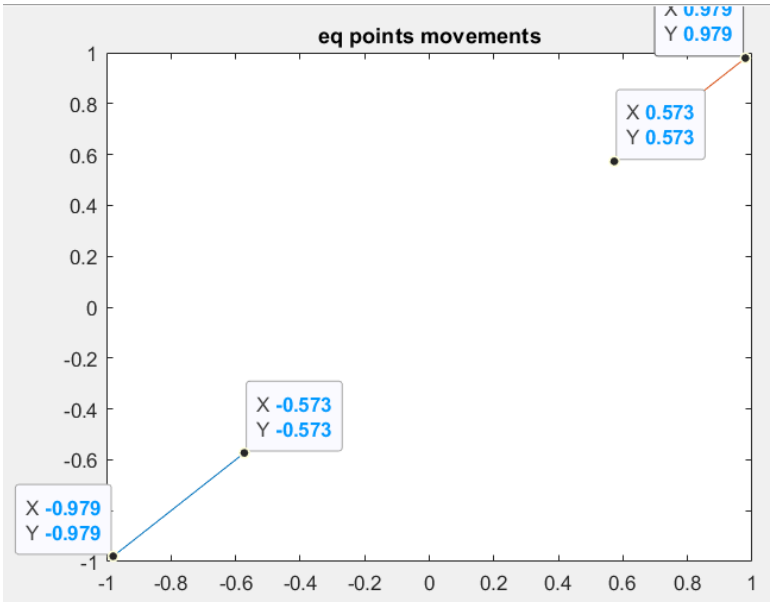

Figure 3: Equilibrium Points

Initial points are $(-0.573, -0.573)$ and $(0.573, 0.573)$ for $\lambda = 1.4$. When λ increases, equilibrium points move through the corners. The equilibrium points for $\lambda = 20.0$ are $(-0.979, -0.979)$ and $(0.979, 0.979)$.