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Koç University
COMP341 : Introduction to Artificial Intelligence
Assignment 3

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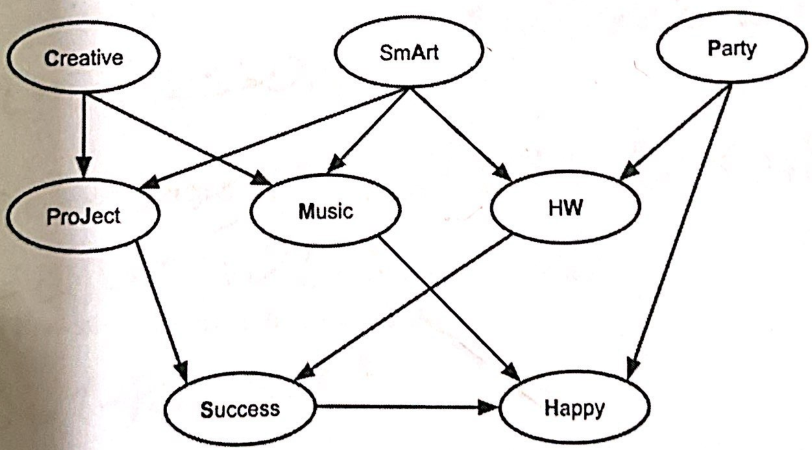
Due Date: January 7 2024, 23:59
Submission Through: Blackboard

Make sure you read and understand every part of this document

- This homework includes Probability, Bayesian Networks, Decision Networks and Value of Information related problems.
- By submitting this homework, you agree to fully comply with Koç University Student Code of Conduct, and accept any punishment in case of failure to comply.
- You are expected to provide clear and concise answers. Gibberish will not receive any credit. Do not overly crowd your answers. Conciseness is a virtue. Write only what is relevant.
- Your answers need to be readable by a human, illegible writing is not gradable hence there is a strong chance that such answers will not get any credit.
- **Submit a single pdf with your solutions, your name and your ID.**

Q1 (10 points) Use the given BN below to answer the conditional independence questions with True or False.



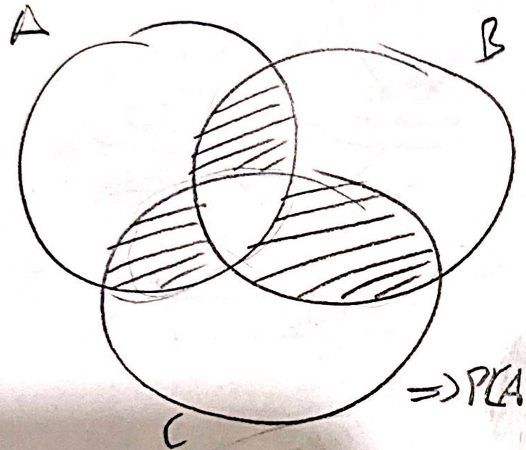
1. False Happy is independent of Smart given Music.
2. True Success is independent of Smart given Project and HW.
3. True Success is independent of Creative given Project.
4. False Smart is independent of Party given Success.
5. True Creative is independent of Party given Music.

Q2 (15 Points) Let A, B and C be three events with given probabilities where $A \cup B$ represents events occurring together or by themselves (OR) and $A \cap B$ represents the events occurring together (AND):

- $P(A) = 0.4$
- $P(B) = 0.7$
- $P(C) = 0.3$
- $P(A \cup B) = 0.8$
- $P(B \cap C) = 0.2$
- $P(C \cap (A \cup B)) = 0.2$
- $P(B \cap (A \cup C)) = 0.4$

Find the probabilities that:

1. (10 points) Exactly two of the events among A, B and C occurs.



$P(A\cap B) = P(A)+P(B)-P(A\cup B)$
$= 0.4+0.7-0.8 = 0.3$

$P(A\cap C) = P(A)+P(C)-P(A\cup B)$
$= \underline{P(C\cap(A\cup B))} - P(B\cap C) + P(A\cap B\cap C)$

$\Rightarrow P(A\cap B\cap C) = P(A\cap B)+P(B\cap C)-P(B\cap(A\cup C))$
$= 0.3+0.2-0.4 = 0.1$

$\Rightarrow P(A\cap C) = 0.2-0.2+0.1 = 0.1$

$\Rightarrow P(\text{exactly two event among } A,B,C) = P(A\cap B)+P(A\cap C)+P(B\cap C) - 3P(A\cap B\cap C)$
$= 0.3+0.1+0.2-3\cdot 0.1 = 0.3$ ✓

2. (5 points) None of the events A, B and C occurs.

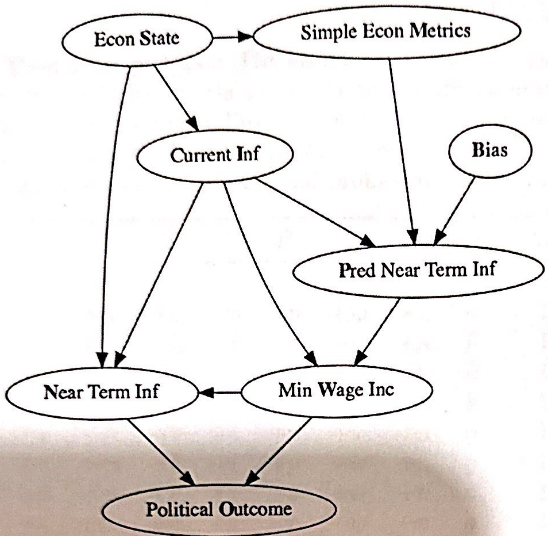
$P(A'\cap B'\cap C') = 1-P(A\cup B\cup C)$

$\Rightarrow P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C) + P(A\cap B\cap C)$

$= 0.4+0.7+0.3-0.3-0.1-0.2+0.1 = 0.9$

$P(A'\cap B'\cap C') = 1-0.9 = 0.1$ ✓

Q3 (23 points) Your economist friend built a bayesian network that relates several economic factors to each other and near term political outcome. The topology of this bayesian network along with the definition and the domain of its random variables, and the conditional probability tables are given below.



- Econ State: The current state of the economy. $S \in \{-s, +s\}$
- Simple Econ Metrics: Simple economic metrics that may not reflect the true state of the economy. $M \in \{-m, +m\}$
- Bias: Bias in predicting the near term inflation. $B \in \{-b, +b\}$
- Current Inf: The current inflation. $I \in \{-i, +i\}$
- Pred Near Term Inf: The official predicted near term inflation. $P \in \{-p, +p\}$
- Near Term Inf: The real near term inflation. $N \in \{-n, +n\}$
- Min Wage Inc: The amount of minimum wage increase. $W \in \{-w, +w\}$
- Political Outcome: The political outcome. $O \in \{b, n, g\}$

P	I	W	$P(W \mid P, I)$
$+p$	$+i$	$+w$	0.80
$+p$	$+i$	$-w$	0.20
$+p$	$-i$	$+w$	0.50
$+p$	$-i$	$-w$	0.50
$-p$	$+i$	$+w$	0.40
$-p$	$+i$	$-w$	0.60
$-p$	$-i$	$+w$	0.00
$-p$	$-i$	$-w$	1.00

S	$P(S)$
$-s$	0.75
$+s$	0.25

N	W	O	$P(O \mid N, W)$
$+n$	$+w$	b	0.30
$+n$	$+w$	n	0.30
$+n$	$+w$	g	0.40
$+n$	$-w$	b	0.90
$+n$	$-w$	n	0.10
$+n$	$-w$	g	0.00
$-n$	$+w$	b	0.00
$-n$	$+w$	n	0.10
$-n$	$+w$	g	0.90
$-n$	$-w$	b	0.10
$-n$	$-w$	n	0.30
$-n$	$-w$	g	0.60

S	M	$P(M \mid S)$
$+s$	$+m$	0.90
$+s$	$-m$	0.10
$-s$	$+m$	0.20
$-s$	$-m$	0.80

S	I	$P(I \mid S)$
$+s$	$+i$	0.50
$+s$	$-i$	0.50
$-s$	$+i$	0.75
$-s$	$-i$	0.25

B	$P(B)$
$+b$	0.60
$-b$	0.40

M	B	I	P	$P(P \mid M, B, I)$
$+m$	$+b$	$+i$	$+p$	0.20
$+m$	$+b$	$+i$	$-p$	0.80
$+m$	$+b$	$-i$	$+p$	0.00
$+m$	$+b$	$-i$	$-p$	1.00
$+m$	$-b$	$+i$	$+p$	0.80
$+m$	$-b$	$+i$	$-p$	0.20
$+m$	$-b$	$-i$	$+p$	0.10
$+m$	$-b$	$-i$	$-p$	0.90
$-m$	$+b$	$+i$	$+p$	0.30
$-m$	$+b$	$+i$	$-p$	0.70
$-m$	$+b$	$-i$	$+p$	0.10
$-m$	$+b$	$-i$	$-p$	0.90
$-m$	$-b$	$+i$	$+p$	1.00
$-m$	$-b$	$+i$	$-p$	0.00
$-m$	$-b$	$-i$	$+p$	0.20
$-m$	$-b$	$-i$	$-p$	0.80

W	I	S	N	$P(N \mid W, I, S)$
$+w$	$+i$	$+s$	$+n$	0.75
$+w$	$+i$	$+s$	$-n$	0.25
$+w$	$+i$	$-s$	$+n$	1.00
$+w$	$+i$	$-s$	$-n$	0.00
$+w$	$-i$	$+s$	$+n$	0.40
$+w$	$-i$	$+s$	$-n$	0.60
$+w$	$-i$	$-s$	$+n$	0.60
$+w$	$-i$	$-s$	$-n$	0.40
$-w$	$+i$	$+s$	$+n$	0.50
$-w$	$+i$	$+s$	$-n$	0.50
$-w$	$+i$	$-s$	$+n$	0.70
$-w$	$+i$	$-s$	$-n$	0.30
$-w$	$-i$	$+s$	$+n$	0.10
$-w$	$-i$	$+s$	$-n$	0.90
$-w$	$-i$	$-s$	$+n$	0.40
$-w$	$-i$	$-s$	$-n$	0.60

Part a (2 points): What is the joint probability distribution of this BN, $P(S, M, B, I, P, W, N, O)$?

$P(S,M,B,I,P,W,N,O) = P(S) \cdot P(B) \cdot P(M|S) \cdot P(I|S) \cdot P(P|M,B,I) \cdot P(N|W,I,S) P(W|P,I) \cdot P(O|N,W)$

Part b (9 points): The economic metrics are good ($M = +m$) but the inflation is high ($I = +i$). You want to calculate the political outcome distribution ($P(O|M = +m, I = +i)$) in these circumstances. You ran rejection sampling for 100 total samples and got the samples given below (note that samples inconsistent with the evidence are removed). Calculate $P(P|M = +m, I = +i)$ based on these samples and fill in the empty conditional probability table. Show your work in the available space.

S	M	B	I	P	W	N	O	Count
$+s$	$+m$	$+b$	$+i$	$-p$	$-w$	$+n$	b	2
$-s$	$+m$	$+b$	$+i$	$-p$	$+w$	$+n$	b	1
$-s$	$+m$	$+b$	$+i$	$-p$	$-w$	$+n$	b	1
$-s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	b	1
$-s$	$+m$	$-b$	$+i$	$+p$	$-w$	$+n$	b	1
$-s$	$+m$	$+b$	$+i$	$+p$	$+w$	$+n$	g	1
$+s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	g	1
$+s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	n	1
$-s$	$+m$	$+b$	$+i$	$-p$	$-w$	$-n$	g	1
$+s$	$+m$	$-b$	$+i$	$+p$	$-w$	$-n$	g	1
$+s$	$+m$	$-b$	$+i$	$-p$	$-w$	$-n$	g	1
$+s$	$+m$	$+b$	$+i$	$-p$	$-w$	$-n$	n	1
$+s$	$+m$	$-b$	$+i$	$+p$	$+w$	$-n$	n	1
$+s$	$+m$	$-b$	$+i$	$-p$	$-w$	$-n$	n	1

M	I	O	$P(O\|M,I)$
+m	+i	b	6/15 = 0.4 = 40%
+m	+i	n	4/15 = 0.267 = 26.7%
+m	+i	g	5/15 = 0.33 = 33.3%

M	I	P	$P(P\|M=+M, I=+i)$
+M	+i	+P	7/15 = 0.467 = 46.7%
+M	+i	-P	8/15 = 0.533 = 53.3%

$P(P=+p|M=+M, I=+i) = 0.467$

$P(P=-p|M=+M, I=+i) = 0.533$

Part c (6 points): You realized a lot of the samples have been rejected and as a result you wanted to apply likelihood weighting. You got the samples given below. Calculate the weights of the samples (do not multiply by their counts!). **Hint:** Samples may share the same weight.

S	M	B	I	P	W	N	O	#	w
$-s$	$+m$	$+b$	$+i$	$-p$	$-w$	$+n$	b	12	$0.2 \times 0.75 = 0.15$
$-s$	$+m$	$+b$	$+i$	$-p$	$+w$	$+n$	g ✓	10	0.15
$-s$	$+m$	$+b$	$+i$	$+p$	$+w$	$+n$	g ✓	10	0.15
$-s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	g ✓	9	0.15
$-s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	$\boxed{n}$	8	0.15
$-s$	$+m$	$+b$	$+i$	$-p$	$+w$	$+n$	$\boxed{n}$	5	0.15
$-s$	$+m$	$+b$	$+i$	$+p$	$+w$	$+n$	b	5	0.15
$-s$	$+m$	$+b$	$+i$	$-p$	$+w$	$+n$	b	5	0.15
$-s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	b	4	0.15
$-s$	$+m$	$-b$	$+i$	$-p$	$-w$	$+n$	b	3	0.15
$-s$	$+m$	$-b$	$+i$	$+p$	$-w$	$-n$	g ✓	3	0.15
$-s$	$+m$	$+b$	$+i$	$+p$	$-w$	$+n$	b	3	0.15
$+s$	$+m$	$+b$	$+i$	$-p$	$-w$	$-n$	g ✓	3	$0.9 \times 0.5 = 0.45$
$-s$	$+m$	$+b$	$+i$	$-p$	$-w$	$+n$	$\boxed{n}$	3	0.15
$+s$	$+m$	$+b$	$+i$	$+p$	$-w$	$+n$	b	2	0.45 −
$+s$	$+m$	$+b$	$+i$	$-p$	$+w$	$+n$	$\boxed{n}$	2	0.45 +
$-s$	$+m$	$-b$	$+i$	$+p$	$-w$	$+n$	b	2	0.15
$+s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	b	2	0.45 −
$+s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	g ✓	1	0.45
$+s$	$+m$	$+b$	$+i$	$-p$	$-w$	$+n$	b	1	0.45 −
$+s$	$+m$	$+b$	$+i$	$-p$	$-w$	$-n$	b	1	0.45 −
$+s$	$+m$	$+b$	$+i$	$-p$	$-w$	$-n$	$\boxed{n}$	1	0.45 +
$+s$	$+m$	$+b$	$+i$	$-p$	$+w$	$-n$	g ✓	1	0.45
$-s$	$+m$	$-b$	$+i$	$+p$	$-w$	$-n$	b	1	0.15
$+s$	$+m$	$+b$	$+i$	$-p$	$+w$	$+n$	b	1	0.45 −
$+s$	$+m$	$-b$	$+i$	$+p$	$+w$	$+n$	$\boxed{n}$	1	0.45 +
$+s$	$+m$	$+b$	$+i$	$+p$	$+w$	$+n$	$\boxed{n}$	1	0.45 +

if $S = -s \Rightarrow 0.15$

if $S = +s \Rightarrow 0.45$

(use $P(M|S)$ and $P(I|S)$ tables)

Part d (6 points): Based on your answer to the previous part, calculate $P(O|M=+\text{m}, I=+\text{i})$ and fill in the empty conditional probability table. Show your work in the available space.

M	I	O	$P(O\|M,I)$
+m	+i	b	0.4179
+m	+i	n	0.2313
+m	+i	g	0.3507

$$P(O=b|M=+m, I=+i) = \frac{\overbrace{(12+5+5+4+3+3+2+1)}^{35}\times 0.15+\overbrace{(2+2+1+1+1)}^{7}\times 0.45}{(35\times 0.15+7\times 0.45)+[\underbrace{(10+10+9+3)}_{32}\times 0.15+\underbrace{(3+1+1)}_{5}\times 0.45]+[\underbrace{(8+5+3)}_{16}\times 0.15+\underbrace{(2+1+1+1)}_{5}\times 0.45]}$$

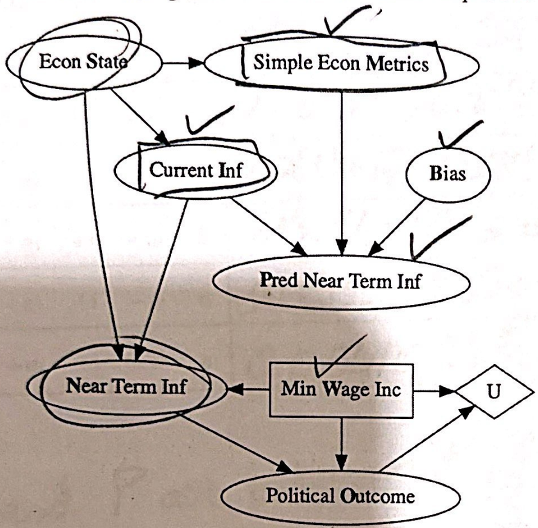
$$= \frac{\overbrace{35\times 0.15+7\times 0.45}^{8.4}}{\underbrace{(35\times 0.15+7\times 0.45)}_{8.4}+\underbrace{(32\times 0.15+5\times 0.45)}_{7.05}+\underbrace{(16\times 0.15+5\times 0.45)}_{4.65}}$$

$$= \frac{8.4}{20.1} = 0.4179 = 41.79\%$$

$$P(O=n|M=+m, I=+i) = \frac{4.65}{20.1} = 0.2313 = 23.13\%$$

$$P(O=g|M=+m, I=+i) = \frac{7.05}{20.1} = 0.3507 = 35.07\%$$

Q4 (52 points) Your friend modified the bayesian network to convert it into a decision network. The Minimum Wage Increase node (W) is changed to a decision variable. A utility node is also added that is affected by the political outcome and minimum wage increase. There is a trade-off. Increasing the minimum wage would improve the political outcome, however, it comes with other costs such as inflation, lay-offs, unregistered workers etc. which are all lumped in to the model. The decision network is given below along with the new conditional probability tables and the utility table.



- Econ State: $S \in \{-s, +s\}$
- Simple Econ Metrics: $M \in \{-m, +m\}$
- Bias: $B \in \{-b, +b\}$
- Current Inf: $I \in \{-i, +i\}$
- Pred Near Term Inf: $P \in \{-p, +p\}$
- Near Term Inf: $N \in \{-n, +n\}$
- Political Outcome: $O \in \{b, n, g\}$
- Min Wage Inc: The amount of minimum wage increase decision. $W \in \{-w, +w\}$, where $+w$ is high increase and $-w$ is low increase.
- Utility: The utility representing the trade-off between political outcome and minimum wage increase cost.

W	O	U
$+w$	b	-120.00
$+w$	n	0.00
$+w$	g	40.00
$-w$	b	-80.00
$-w$	n	20.00
$-w$	g	120.00

S	$P(S)$
$-s$	0.75
$+s$	0.25

N	W	O	$P(O\|N,W)$
$+n$	$+w$	b	0.30
$+n$	$+w$	n	0.30
$+n$	$+w$	g	0.40
$+n$	$-w$	b	0.90
$+n$	$-w$	n	0.10
$+n$	$-w$	g	0.00
$-n$	$+w$	b	0.00
$-n$	$+w$	n	0.10
$-n$	$+w$	g	0.90
$-n$	$-w$	b	0.10
$-n$	$-w$	n	0.30
$-n$	$-w$	g	0.60

S	M	$P(M\|S)$
$+s$	$+m$	0.90
$+s$	$-m$	0.10
$-s$	$+m$	0.20
$-s$	$-m$	0.80

S	I	$P(I\|S)$
$+s$	$+i$	0.50
$+s$	$-i$	0.50
$-s$	$+i$	0.75
$-s$	$-i$	0.25

B	$P(B)$
$+b$	0.60
$-b$	0.40

M	B	I	P	$P(P\|M,B,I)$
$+m$	$+b$	$+i$	$+p$	0.20
$+m$	$+b$	$+i$	$-p$	0.80
$+m$	$+b$	$-i$	$+p$	0.00
$+m$	$+b$	$-i$	$-p$	1.00
$+m$	$-b$	$+i$	$+p$	0.80
$+m$	$-b$	$+i$	$-p$	0.20
$+m$	$-b$	$-i$	$+p$	0.10
$+m$	$-b$	$-i$	$-p$	0.90
$-m$	$+b$	$+i$	$+p$	0.30
$-m$	$+b$	$+i$	$-p$	0.70
$-m$	$+b$	$-i$	$+p$	0.10
$-m$	$+b$	$-i$	$-p$	0.90
$-m$	$-b$	$+i$	$+p$	1.00
$-m$	$-b$	$+i$	$-p$	0.00
$-m$	$-b$	$-i$	$+p$	0.20
$-m$	$-b$	$-i$	$-p$	0.80

W	I	S	N	$P(N\|W,I,S)$
$+w$	$+i$	$+s$	$+n$	0.75
$+w$	$+i$	$+s$	$-n$	0.25
$+w$	$+i$	$-s$	$+n$	1.00
$+w$	$+i$	$-s$	$-n$	0.00
$+w$	$-i$	$+s$	$+n$	0.40
$+w$	$-i$	$+s$	$-n$	0.60
$+w$	$-i$	$-s$	$+n$	0.60
$+w$	$-i$	$-s$	$-n$	0.40
$-w$	$+i$	$+s$	$+n$	0.50
$-w$	$+i$	$+s$	$-n$	0.50
$-w$	$+i$	$-s$	$+n$	0.70
$-w$	$+i$	$-s$	$-n$	0.30
$-w$	$-i$	$+s$	$+n$	0.10
$-w$	$-i$	$+s$	$-n$	0.90
$-w$	$-i$	$-s$	$+n$	0.40
$-w$	$-i$	$-s$	$-n$	0.60

Part a (18 points): You want to calculate the optimal action and the maximum expected utility of this decision network given that good economic metrics ($M = +m$) and high inflation ($I = +i$). You realize you need to calculate the political outcome distrtibution first for both action options, i.e., $P(O|M = +m, I = +i, W = +w)$ and $P(O|M = +m, I = +i, W = -w)$. This time, you are going to use exact inference. I recommend that you use variable elimination. Fill in the conditional probability table below and show your work in the empty spaces. **Hint:** You do not have to include all the nodes in this due to conditional independence relations. Since W is a decision variable, it does not have a cpt. I recommend you keep the results as fractions and use a common denominator to represent them (e.g. 1/2, 1/3, 1/6 as 3/6, 2/6 and 1/6).

M	I	W	O	$P(O\|M = +m, I = +i, W)$
+m	+i	$+w$	b	0.0590625/0.2025 = 0.2917
+m	+i	$+w$	n	0.061875/0.2025 = 0.3056
+m	+i	$+w$	g	0.0815625/0.2025 = 0.4028
+m	+i	$-w$	b	0.1305/0.225 = 0.58
+m	+i	$-w$	n	0.0405/0.225 = 0.18
+m	+i	$-w$	g	0.054/0.225 = 0.24

⇒ B and P are independent of O given M = +m and I = +i

$P(S,M,B,I,P,W,N,O) = P(S).\cancel{P(B)}.P(M|S).P(I|S).\cancel{P(P|M,B,I)}.$
$P(N|W,I,S)\,P(O|N,W)$

Therefore initial factors are: $f_1: P(S)$, $f_2: P(M|S)$, $f_3: P(I|S)$
$f_4: P(N|W,I,S)$, $f_5: P(O|N,W)$

⇒ Hidden variables are S and N, so join $f_1.f_2.f_3.f_4$ to have f_6
(S,M,I,N,W)

S	N	W	$f_6 = f_1 \times f_2 \times f_3 \times f_4$	
+s	+n	+w	0.25 × 0.9 × 0.5 × 0.75 = 0.084375	0.196875
+s	−n	+w	0.25 × 0.9 × 0.5 × 0.25 = 0.028125	0.028125
−s	+n	+w	0.75 × 0.2 × 0.75 × 1 = 0.1125	
−s	−n	+w	0.75 × 0.2 × 0.75 × 0 = 0	
+s	+n	−w	0.25 × 0.9 × 0.5 × 0.5 = 0.05625	
+s	−n	−w	0.25 × 0.9 × 0.5 × 0.5 = 0.05625	0.135
−s	+n	−w	0.75 × 0.2 × 0.75 × 0.7 = 0.07875	0.09
−s	−n	−w	0.75 × 0.2 × 0.75 × 0.3 = 0.03375	

⇒ Sum over S in f_6 and obtain $f_7(N, M, I, W)$

$f_7(+n, +m, +i, +w) = 0.196875$ $f_7(+n, +m, +i, -w) = 0.135$

$f_7(-n, +m, +i, +w) = 0.028125$ $f_7(-n, +m, +i, -w) = 0.09$

Step 2: Join f_7 and f_5 to obtain $f_8(N, M, I, W, O)$

N	W	O	$f_8 = f_5 \times f_7$	
+n	+w	b	0.3 × 0.196875	= 0.0590625
+n	+w	n	0.3 × 0.196875	= 0.0590625
+n	+w	g	0.4 × 0.196875	= 0.07875
-n	+w	b	0 × ~~"~~	= 0
-n	+w	n	0.1 × 0.028125	= 0.0028125
-n	+w	g	0.9 × "	= 0.0253125
+n	-w	b	0.9 × 0.135	= 0.1215
+n	-w	n	0.1 × "	= 0.0135
+n	-w	g	0 × "	= 0
-n	-w	b	0.1 × 0.09	= 0.009
-n	-w	n	0.3 × 0.09	= 0.027
-n	-w	g	0.6 × 0.09	= 0.054

⇒ Sum over N to have $f_9(M = +m, I = +i, W, O)$

$f_9(+w, b) = 0.0590625$, $f_9(+w, n) = 0.061875$, $f_9(+w, g) = 0.0815625$

$f_9(-w, b) = 0.1305$, $f_9(-w, n) = 0.0405$, $f_9(-w, g) = 0.054$

⇒ Step 3: Normalize to have C.P.

Part b (6 points): Given your results from the previous part, calculate the Expected Utility (EU) of each action. What is the Maximum Expcted Utility (MEU) and the optimal action?

$EU(+w, +m, +i) = P(b \mid +m, +i) \cdot U(b, +w) + P(n \mid +m, +i) \cdot U(n, +w)$
$+ P(g \mid +m, +i) U(g, +w)$

$\rightarrow$ To find $P(O \mid +m, +i) \Rightarrow$ we sum the table found in part a up over W and normalize

$\Rightarrow P(b \mid +m, +i) = 0.58 + 0.2917 = 0.8717/2 = 0.4358$
$P(n \mid +m, +i) = 0.3056 + 0.18 = 0.4856/2 = 0.2428$
$P(g \mid +m, +i) = 0.4028 + 0.24 = 0.6428/2 = 0.3214$

$EU(+w, +m, +i) = 0.43585 \times (-120) + 0.2428 \times 0 + \underbrace{0.3214 \times 40}_{12.856} =$
$= -39.476$

$EU(-w, +m, -i) = \overbrace{0.43585 \times (-80)}^{-34.864} + \overbrace{0.2428 \times 20}^{4.856} + \overbrace{0.3214 \times 120}$
$= 8.56$ ✓

$\Rightarrow MEU(+m, +i) = 8.56$ with optimal action $+w$

2

Part c (28 points): Your friend does not trust the economic metrics and wonders whether the optimal decision would change or not if we knew it. However, figuring out the real state of the economy is an expensive and time consuming process. What would be the value of this information? In other words, calculate $VPI(S|I = +i, M = +m)$. Some hints:

- You need to do inference for $P(O|M = +m, I = +i, S, W)$. Note that O and M are conditionally independent given S, thus the inference simplifies to $P(O|I = +i, S, W)$.
- $P(S|M, I, W) = P(S|M, I)$ and $P(S = +s|M = +m, I = +i) = 0.5$ (calculated for you). Since both values of S are equally likely, you can ignore this as factor for inference. Note that you need this for VPI calculations!
- For inference, you need to join two factors and marginalize out one hidden variable but the first table, after joining will be large (24 rows)! To keep things simple, you can do the $W = +w, W = -w, S = +s, S = -s$ cases separately.
- See if you need to normalize or not at the end.

$P(S,M,B,I,P,W,N,O) = P(S).P(B).P(M|S).P(I|S).P(P|M,B,I).$
$P(N|W,I,S).P(O|N,W)$

$\Rightarrow$ Join $f_1: P(N|W,I,S)$ and $f_2: P(O|N,W)$

N	W	S	O	$f_3 = f_1 \times f_2$	
+n	+w	+s	b	0.75×0.3	$= 0.225$
+n	+w	+s	n	0.75×0.3	$= 0.225$
+n	+w	+s	g	0.75×0.4	$= 0.3$
-n	+w	+s	b	0.25×0	$= 0$
-n	+w	+s	n	0.25×0.1	$= 0.025$
-n	+w	+s	g	0.25×0.9	$= 0.225$
+n	-w	+s	b	0.5×0.9	$= 0.45$
+n	-w	+s	n	0.5×0.1	$= 0.05$
+n	-w	+s	g	0.5×0	$= 0$
-n	-w	+s	b	0.5×0.1	$= 0.05$
-n	-w	+s	n	0.5×0.3	$= 0.15$
-n	-w	+s	g	0.5×0.6	$= 0.3$
+n	+w	-s	b	1×0.3	$= 0.3$
+n	+w	-s	n	1×0.3	$= 0.3$
+n	+w	-s	g	1×0.4	$= 0.4$
+n	-w	-s	b	0.4×0.9	$= 0.36$
+n	-w	-s	n	0.4×0.1	$= 0.04$
+n	-w	-s	g	0.4×0	$= 0$
-n	-w	-s	b	0.6×0.1	$= 0.06$
-n	-w	-s	n	0.6×0.3	$= 0.18$
-n	-w	-s	g	0.6×0.6	$= 0.36$
-n	+w	-s	b	0×0	$= 0$
-n	+w	-s	n	0×0.1	$= 0$
-n	+w	-s	g	0×0.9	$= 0$

Sum over N to get $f_i: P(O \mid I = i, S, W)$

O	S	W	$P(O \mid I=i, S, W)$	$P(O \mid I=i, S, W) \cdot U(W, O)$
b	+s	+w	$0.225 + 0 = 0.225$	$\times -120 = -27$
n	+s	+w	$0.225 + 0.025 = 0.25$	$\times 0 = 0$
g	+s	+w	$0.3 + 0.225 = 0.525$	$\times 40 = 21$
b	+s	-w	$0.45 + 0.05 = 0.5$	$\times -80 = -40$
n	+s	-w	$0.05 + 0.15 = 0.2$	$\times 20 = 4$
g	+s	-w	$0 + 0.3 = 0.3$	$\times 120 = 36$
b	-s	+w	$0.3 + 0 = 0.3$	$\times -120 = -36$
n	-s	+w	$0.3 + 0.1 = 0.3$	$\times 0 = 0$
g	-s	+w	$0.4 + 0 = 0.4$	$\times 40 = 16$
b	-s	-w	$0.36 + 0.06 = 0.42$	$\times -80 = 33.6$
n	-s	-w	$0.04 + 0.18 = 0.22$	$\times 20 = 4.4$
g	-s	-w	$0 + 0.36 = 0.36$	$\times 120 = 43.2$

$EU(+w) = -27 + 0 + 21 - 36 + 0 + 16 = -26$

$EU(-w) = -40 + 4 + 36 + 33.6 + 4.4 + 43.2 = 81.2$

$EU(\phi) = (-26 + 81.2)/2 = 27.6$ (without knowing S)

max = -4

$EU(+s, +w) = -27 + 0 + 21 = -6$, $EU(+s, -w) = -40 + 4 + 36 = -4$

Max = 81.4

$EU(-s, +w) = -36 + 0 + 16 = -20$, $EU(-s, -w) = 33.6 + 4.4 + 43.2 = 81.2$

$$EU(S, W) = \frac{-4 + 81.4}{2} = 38.6$$

$VPI = 38.6 - 27.6 = 11$