DIGITAL CONTROL OF AN ANALOG PLANT

In this project, we design a digital controller $C(z)$ for a given analog plant $P(s)$. First, we start with a continuous-time control system with an analog controller and look at its root locus behavior. Then, we approximate the analog controller by a digital controller and look at the effects resulting from digitization of the controller. Finally, we model digital approximation of the analog controller in continuous time and observe the changes in the root-locus of the end-to-end system between using analog vs. digital controllers.

The structure of an analog feedback control system, where r is the reference input and y is the output, is shown below.

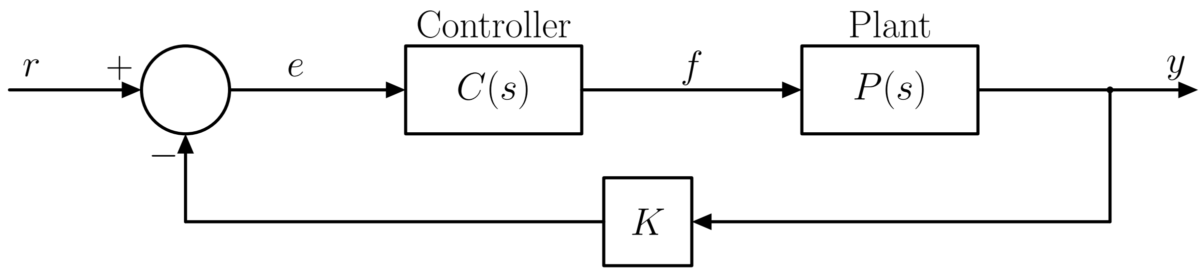

Figure 1: The feedback control system.

1. The transfer function of the plant is given by

$$P(s) = \frac{3}{as + b}$$

where $a = 0.1$ and $b = -1$. Suppose the transfer function of the analog controller is given by

$$C(s) = \frac{0.1s + 3.75}{\frac{16}{600}s^2 + 1.1s + 19.75}.$$

Is the overall system with feedback gain $K = 1$ stable? You should justify your answer by plotting the step response of the closed-loop system.

2. Root locus diagram is a graph of location of the poles of the closed-loop system as the feedback gain K changes. Plot the root locus diagram of the closed-loop system using the MATLAB command `rlocus`. Choose a value for K such that the overall system is stable. Explain how you pick that gain value. Plot the step response of the closed loop system with the gain that you pick.

3. Now, assume that we want to implement the controller on a digital platform, say, a computer. Then, the controller $C(s)$ needs to be approximated as shown in Figure 2. Convert the analog controller into a digital controller using impulse sampling with sampling period $T_s = 1\,\text{ms}$ using the command `c2d` (*don't forget to look at its options*). What is the transfer function of the digital controller $C(z)$? Is there any mathematical relationship between $C(s)$ and $C(z)$? What is that relationship? Show it by comparing the poles of both controllers, i.e., by mapping poles from s-domain to z-domain or vice versa.

4. To convert the digital output of $C(z)$ to an analog output, we need to use a zero-order hold (ZOH) block. Find the transfer function of the analog controller $\tilde{C}(s)$ obtained from the digital controller using the command `d2c`. Is there any difference

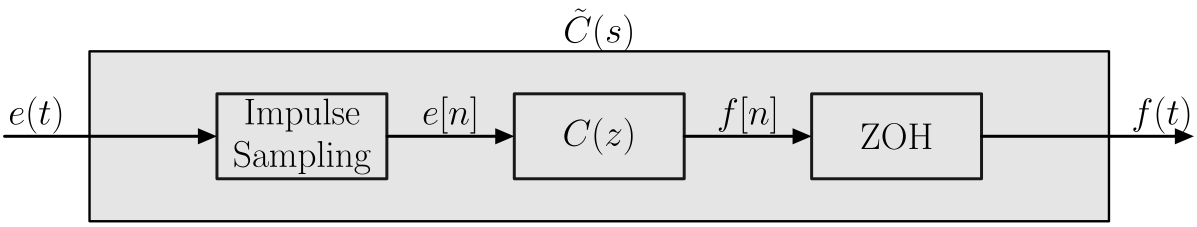

Figure 2: Approximation of an analog controller by a digital controller.

between the original $C(s)$ and the "converted" response $\tilde{C}(s)$? Now, use the gain value that you pick in question 2 and plot the step response of the system. Explain what you see.

Note: You can submit the code for this homework in a single `.m` *file. There is no need for separate files for each question.*