```
% THIS PROGRAM WILL ANALYZE THE CANTILEVER BEAM SUBJECTED TO DISTRIBUTED
% LOAD
%% METHODS: POLYNOMIAL CHAOS, MONTE CARLO SIMULATION & GAUSSIAN PROCESS MODEL (HOME↙
ASSIGNMENT 2)
% SUBMITTED BY: Rudraprasad Bhattacharyya

% w and EI are the random fields
clc;
clear all;close all;
%-------------------------------------------------------------------------------
%% Polynomial Chaos Expansion Model
%-------------------------------------------------------------------------------
%collocation points
P=[0 0;1.732 0;0 1.732;-1.732 0;0 -1.732;1.732 -1.732;
    -1.732 1.732;1.732 1.732;-1.732 -1.732];
L=1;                % Length of beam
x=0.25:0.25:1;      % Dividing the beam in 4 regions
% -------------------------------------------------------------------------------
%Auto-correlation matrix for EI
for i=1:1:4
    for j=1:1:4
        tau=(j-i)*0.25;
        Cor_ei(i,j)=exp(-abs(tau));
    end
end
Cor_ei = corrcov(Cor_ei);
% KL expansion for EI
[f1,lambda1]=eigs(Cor_ei,2);
for j=1:9
    EI(:,j)=1+0.2*(sqrt(lambda1(1,1)).*f1(:,1)*P(j,1)+sqrt(lambda1(2,2)).*f1(:,2)*P(j,2));
end

%Auto-correlation matrix for w
for i=1:1:4
    for j=1:1:4
        tau=(j-i)*0.25;
        Cor_w(i,j)=exp(-abs(tau));

    end
end
% KL expansion for w
[f2,lambda2]=eigs(Cor_w,2);
for j=1:9
    w(:,j)=0.5+0.1*(sqrt(lambda2(1,1)).*f2(:,1)*P(j,1)+sqrt(lambda2(2,2)).*f2(:,2)*P(j,↙
2));
end


% Analytical expression for tip displacement
for m=1:9
    for n=1:9
        del_tip(m,n)= w(1,n)*L^4/2048*EI(1,m)+...
            (19*w(1,n)/3072+7*w(2,n)/6144)*L^4/EI(2,m)+...
            (61*w(1,n)/3072+31*w(2,n)/3072+11*w(3,n)/6144)*L^4/EI(3,m)+...
            (127*w(1,n)/3072+85*w(2,n)/3072+43*w(3,n)/3072+5*w(4,n)/2048)*L^4/EI(4,m);
    end
```

```
end
Y=[del_tip(1,:) del_tip(2,:) del_tip(3,:) del_tip(4,:) del_tip(5,:) del_tip(6,:) del_tip
(7,:) del_tip(8,:) del_tip(9,:)]';

% 81 training points
collo1=-1.732;
collo2=0;
collo3=1.732;
xi=TrPoints(collo1,collo2,collo3);
%-------------------------------------------------------------------------
% PCE Surrogate model
for i=1:81
    X(i,:)=[1 xi(i,1) xi(i,2) xi(i,3) xi(i,4) (xi(i,1))^2 - 1 (xi(i,2))^2 - 1 (xi(i,3))^2
- 1 (xi(i,4))^2 - 1  xi(i,1)*xi(i,2) xi(i,1)*xi(i,3) xi(i,1)*xi(i,4) xi(i,2)*xi(i,3) xi(i,
2)*xi(i,4) xi(i,3)*xi(i,4)];
end
b=inv([X'*X])*[X'*Y]; % PCE Surrogate model parameters

% Generate pdf and cdf for PCE model
N_simulations=10000;
for sim=1:N_simulations
    si=randn(1,4);
    tip(:,sim)=b(1)+b(2)*si(1)+b(3)*si(2)+b(4)*si(3)+b(5)*si(4)+...
        b(6)*(1-si(1)^2)+b(7)*(1-si(2)^2)+b(8)*(1-si(3)^2)+b(9)*(1-si(4)^2)+...
        b(10)*si(1)*si(2)+b(11)*si(1)*si(3)+b(12)*si(1)*si(4)+b(13)*si(2)*si(3)+b(14)*si
(2)*si(4)+b(15)*si(3)*si(4);
end
%
-------------------------------------------------------------------------------------
--------------------
% End of PCE

%% MCS on the original model
%
-------------------------------------------------------------------------------------
--------------------
for m=1:200
    for n=1:200
        w=0.5+0.1*(sqrt(lambda2(1,1))*f2(:,1)*randn+sqrt(lambda2(2,2))*f2(:,2)*randn);
        EI=1+0.2*(sqrt(lambda1(1,1))*f1(:,1)*randn+sqrt(lambda1(2,2))*f1(:,2)*randn);
        mcs_tip(m,n)= w(1)*L^4/2048*EI(1)+...
            (19*w(1)/3072+7*w(2)/6144)*L^4/EI(2)+...
            (61*w(1)/3072+31*w(2)/3072+11*w(3)/6144)*L^4/EI(3)+...
            (127*w(1)/3072+85*w(2)/3072+43*w(3)/3072+5*w(4)/2048)*L^4/EI(4);

    end
end
Z=[mcs_tip(:)]';
%
-------------------------------------------------------------------------------------
-----------
% End of MCS

%% Gaussian Process
%
-------------------------------------------------------------------------------------
```

```
-----------
x_initial=[1.2 1.4 1.1 1.5 0.3];

[xOpt, fval] = fminunc(@gpobjective,x_initial);

%Regression Analysis to find TREND
X_trend=[ones(81,1) xi];
b_trend=(X_trend'*X_trend)\(X_trend'*Y);

%Residaul
residual=Y-X_trend*b_trend;
x=xOpt;
for i=1:81
    for   j=1:81
        k_tt(i,j)= x(5)^2*exp(-0.5*((xi(i,1)-xi(j,1))^2/x(1)^2+(xi(i,2)-xi(j,2))^2/x(2)↙
^2+...
            (xi(i,3)-xi(j,3))^2/x(3)^2+(xi(i,4)-xi(j,4))^2/x(4)^2));

    end
end

for simulation=1:N_simulations
    pred_pt=randn(1,4);
    for   j=1:81
        k_tp(1,j)= x(5)^2*exp(-0.5*((pred_pt(1)-xi(j,1))^2/x(1)^2+(pred_pt(2)-xi(j,2))^2/x↙
(2)^2+...
            (pred_pt(3)-xi(j,3))^2/x(3)^2+(pred_pt(4)-xi(j,4))^2/x(4)^2));
    end
    k_pp=1;
    pred_pt=[1 pred_pt];
    y_star(simulation,1)=k_tp*(inv(k_tt)*residual)+pred_pt*b_trend;

end
%-----------------------------------
% End of GP model
%% RESULTS-----------------------
%-------------------
% Ploting PDF
%-------------------
figure(1);
% PCE pdf
%hist(tip)
[fp,xp]=ksdensity(tip);
plot(xp,fp,'--r')
hold on;
% MCS pdf
%hist(MCS_tip)
[fm,xm] = ksdensity(Z);
plot(xm,fm,'--b')
hold on;
% GP pdf
[fg,xg]=ksdensity(y_star);
plot(xg,fg,'-*k');
hold on;
xlabel('Tip displacement');
ylabel('PDF');
```

```
legend('PCE','MCS','GP');
%-------------------
% Ploting CDF
%-------------------
figure(2);
cdfplot(tip);%PCE
hold on;
cdfplot(Z);%MCS
hold on;
cdfplot(y_star);%GP
hold on;
xlabel('Tip displacement');
ylabel('CDF');
legend('PCE','MCS','GP');
```

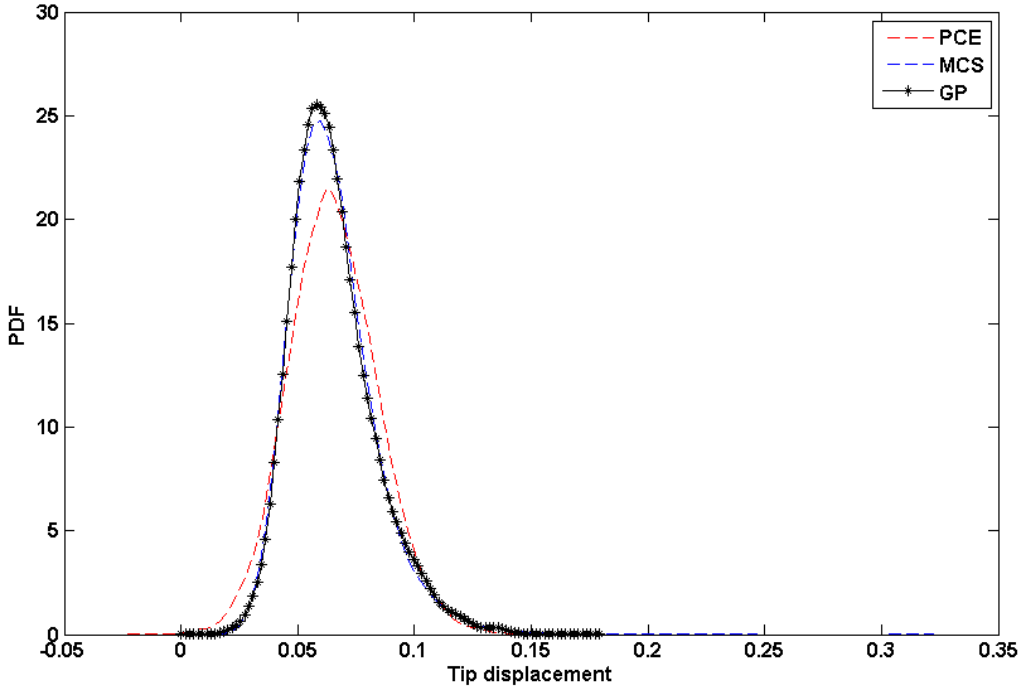
```
function xi=TrPoints(collo1,collo2,collo3)
xi=zeros(81,4);
index1=1;
index2=2;
stop=3;
int_counter=1;
counter=1;
repeat=0;
col=1;
while repeat <= 3
    if repeat~=0
        index1=index1*3;
        index2=index2*3;
        stop=stop*3;
        int_counter=1;
        counter=1;
        col=1+repeat;
    end
    while counter<=81

        if int_counter<=index1
          xi(counter,col)=collo1;
          int_counter=int_counter+1;
          counter=counter+1;
        elseif index1<=int_counter && int_counter<=index2
          xi(counter,col)=collo2;
          int_counter=int_counter+1;
          counter=counter+1;
        elseif index2<int_counter && int_counter<=stop
          xi(counter,col)=collo3;
          int_counter=int_counter+1;
          counter=counter+1;
        else
            int_counter=1;
        end

    end
    repeat=repeat+1;
end
end
```

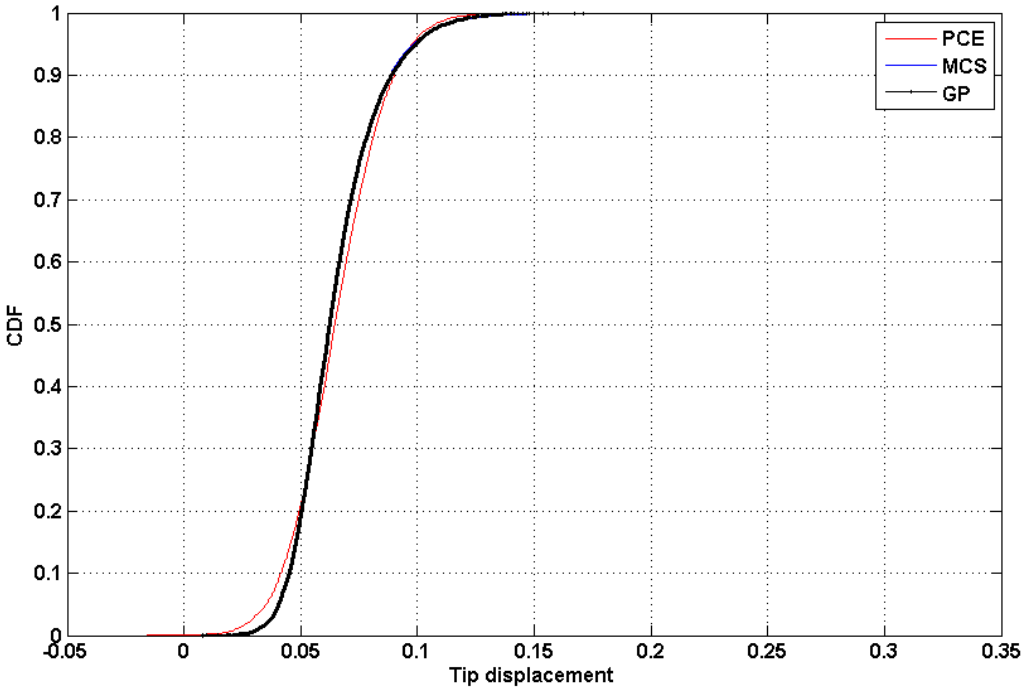
```
function f = gpobjective(x)

xi=[0   0   0   0   ;
    0   0   1.732   0   ;
    0   0   0   1.732   ;
    0   0   -1.732  0   ;
    0   0   0   -1.732  ;
    0   0   1.732   -1.732  ;
    0   0   -1.732  1.732   ;
    0   0   1.732   1.732   ;
    0   0   -1.732  -1.732  ;
    1.732   0   0   0   ;
    1.732   0   1.732   0   ;
    1.732   0   0   1.732   ;
    1.732   0   -1.732  0   ;
    1.732   0   0   -1.732  ;
    1.732   0   1.732   -1.732  ;
    1.732   0   -1.732  1.732   ;
    1.732   0   1.732   1.732   ;
    1.732   0   -1.732  -1.732  ;
    0   1.732   0   0   ;
    0   1.732   1.732   0   ;
    0   1.732   0   1.732   ;
    0   1.732   -1.732  0   ;
    0   1.732   0   -1.732  ;
    0   1.732   1.732   -1.732  ;
    0   1.732   -1.732  1.732   ;
    0   1.732   1.732   1.732   ;
    0   1.732   -1.732  -1.732  ;
    -1.732  0   0   0   ;
    -1.732  0   1.732   0   ;
    -1.732  0   0   1.732   ;
    -1.732  0   -1.732  0   ;
    -1.732  0   0   -1.732  ;
    -1.732  0   1.732   -1.732  ;
    -1.732  0   -1.732  1.732   ;
    -1.732  0   1.732   1.732   ;
    -1.732  0   -1.732  -1.732  ;
    0   -1.732  0   0   ;
    0   -1.732  1.732   0   ;
    0   -1.732  0   1.732   ;
    0   -1.732  -1.732  0   ;
    0   -1.732  0   -1.732  ;
    0   -1.732  1.732   -1.732  ;
    0   -1.732  -1.732  1.732   ;
    0   -1.732  1.732   1.732   ;
    0   -1.732  -1.732  -1.732  ;
    1.732   -1.732  0   0   ;
    1.732   -1.732  1.732   0   ;
    1.732   -1.732  0   1.732   ;
    1.732   -1.732  -1.732  0   ;
    1.732   -1.732  0   -1.732  ;
    1.732   -1.732  1.732   -1.732  ;
    1.732   -1.732  -1.732  1.732   ;
    1.732   -1.732  1.732   1.732   ;
    1.732   -1.732  -1.732  -1.732  ;
```

```
    -1.732  1.732   0   0   ;
    -1.732  1.732   1.732   0   ;
    -1.732  1.732   0   1.732   ;
    -1.732  1.732   -1.732  0   ;
    -1.732  1.732   0   -1.732  ;
    -1.732  1.732   1.732   -1.732  ;
    -1.732  1.732   -1.732  1.732   ;
    -1.732  1.732   1.732   1.732   ;
    -1.732  1.732   -1.732  -1.732  ;
    1.732   1.732   0   0   ;
    1.732   1.732   1.732   0   ;
    1.732   1.732   0   1.732   ;
    1.732   1.732   -1.732  0   ;
    1.732   1.732   0   -1.732  ;
    1.732   1.732   1.732   -1.732  ;
    1.732   1.732   -1.732  1.732   ;
    1.732   1.732   1.732   1.732   ;
    1.732   1.732   -1.732  -1.732  ;
    -1.732  -1.732  0   0   ;
    -1.732  -1.732  1.732   0   ;
    -1.732  -1.732  0   1.732   ;
    -1.732  -1.732  -1.732  0   ;
    -1.732  -1.732  0   -1.732  ;
    -1.732  -1.732  1.732   -1.732  ;
    -1.732  -1.732  -1.732  1.732   ;
    -1.732  -1.732  1.732   1.732   ;
    -1.732  -1.732  -1.732  -1.732  ;
    ];
Y=[0.0625
    0.043848094
    0.069087642
    0.081151906
    0.055912358
    0.037260453
    0.087739547
    0.050435736
    0.074564264
    0.088439718
    0.062048769
    0.097788907
    0.114830668
    0.07909053
    0.05269958
    0.124179856
    0.071397958
    0.105481479
    0.0552927
    0.038802055
    0.061194361
    0.071783344
    0.049391039
    0.032900394
    0.077685005
    0.044703716
    0.065881684
    0.048419118
```

```
    0.033969843
    0.053519815
    0.062868392
    0.04331842
    0.028869145
    0.06796909
    0.039070541
    0.057767694
    0.07260824
    0.05092575
    0.080157923
    0.09429073
    0.065058557
    0.043376067
    0.101840413
    0.058475433
    0.086741047
    0.110053572
    0.07718238
    0.121461722
    0.142924764
    0.098645423
    0.065774231
    0.154332914
    0.08859053
    0.131516614
    0.043910406
    0.030812868
    0.048581077
    0.057007944
    0.039239736
    0.026142198
    0.061678615
    0.035483539
    0.052337274
    0.075052772
    0.052677111
    0.083129253
    0.097428434
    0.066976292
    0.04460063
    0.105504915
    0.060753591
    0.089351953
    0.054274615
    0.03806998
    0.059933218
    0.070479251
    0.048616013
    0.032411377
    0.076137853
    0.043728582
    0.064820648];

for i=1:81
    for   j=1:81
```

```
        k(i,j)= x(5)^2*exp(-0.5*((xi(i,1)-xi(j,1))^2/x(1)^2+(xi(i,2)-xi(j,2))^2/x(2)^2+...
            (xi(i,3)-xi(j,3))^2/x(3)^2+(xi(i,4)-xi(j,4))^2/x(4)^2));

    end
end

f=-0.5*Y'*inv(k)*Y-0.5*log(det(k))-81/2*log(2*3.14);
f=-f;
```


PCE
MCS
GP
PDF
Tip displacement
30
25
20
15
10
5
0
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35


PCE
MCS
GP
CDF
Tip displacement
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35