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Kullback-Leibler divergence -based Improved Particle Filter

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***Abstract*—In this paper, we develop an improved particle filtering algorithm for nonlinear states estimation. In case of standard particle filter, the latest observation is not considered for the evaluation of the weights of the particles as the importance function is taken to be equal to the prior density function. This choice of importance sampling function simplifies the computation but can cause filtering divergence. In cases where the likelihood function is too narrow as compared to the prior function, very few particles will have significant weights. Hence a better proposal distribution that takes the latest observation into account is desired. The proposed algorithm consists of a particle filter based on minimizing the Kullback-Leibler divergence distance to generate the optimal importance proposal distribution. The proposed algorithm allows the particle filter to incorporate the latest observations into a prior updating scheme using the estimator of the posterior distribution that matches the true posterior more closely. In the comparative study, the state variables are estimated from noisy measurements of these variables, and the various estimation techniques are compared by computing the estimation root mean square error with respect to the noise-free data. The simulation results show that the proposed algorithm, outperforms the standard particle filter, the unscented Kalman filter, and the extended Kalman filter algorithms.**

***Keywords*—Particle filter, Kullback-Leibler divergence.**

I. Introduction

The states estimation problem that is addressed here can be viewed as an optimal filtering problem [1], in which the posterior distribution of the unobserved state, given the sequence of observed data and the state evolution model, is recursively updated. Several state estimation techniques have been developed and used in practice. These techniques include the extended Kalman filter (EKF), the unscented Kalman filter (UKF), and more recently the particle filter (PF). The classical Kalman Filter (KF) was developed in the 1960s [2], and has been widely used in various engineering and science applications, including communications, control, machine learning, neuroscience, and many others. In the case where the model describing the system is assumed to be linear and Gaussian, the KF provides an optimal solution [3]. It is known that the KF is computationally efficient; however, it is limited by the non-universal linear and Gaussian modeling assumptions. To relax these assumptions, the extended Kalman filter (EKF) [4] and the unscented Kalman filter (UKF) [5] have been developed. In extended Kalman filtering, the model describing the system is linearized at every time sample (in order to estimate the mean and covariance matrix of the state vector), and thus the model is assumed to be differentiable. Unfortunately, for highly nonlinear or complex models, the EKF does not usually provide a satisfactory performance. On the other hand, instead of linearizing the model to approximate the mean and covariance matrix of the state vector, the UKF uses the unscented transformation to improve the approximation of these moments. In the unscented transformation, a set of samples (called sigma points) are selected and propagated through the nonlinear model, which provides more accurate approximations of the mean and covariance matrix of the state vector, and thus more accurate state estimation. However, for UKF, the number of sigma points could be small and may not represent adequately complicated distributions [6].

For most nonlinear systems and non-Gaussian noise observations, closed-form analytic expression of the posterior distribution of the state is untractable [7]. To overcome this drawback, a non-parametric Monte Carlo sampling based method called Sequential Monte Carlo method (SMC) (also known as particle filtering (PF)) [8] has recently gained popularity. PF methods approximate the posterior probability distribution by a set of weighted samples, called particles. Since real world problems usually involve high dimensional random variables with complex uncertainty, the nonparametric and sample-based estimation of uncertainty (provided by the PF) has thus become quite popular to capture and represent the complex distribution $P(z|y)$ in nonlinear and non-Gaussian models [9]. SMC methods offer a number of significant advantages over other conventional methods. However, since they use the prior distribution as the importance distribution [8], the latest data observation is not considered and not taken into account when evaluating the weights of the particles. Even this choice of the importance sampling distribution has computational advantages, it can cause filtering divergence. In cases where the likelihood distribution is too small compared to the prior distribution, very few particles will have significant weights. Hence, a better proposal distribution that takes the latest observation data into account is needed. Newer adaptive methods need to be further developed that incorporate better feedback and smoothing in the selection or deletion of particles and their weights.

Each of the above estimation methods has its advantages and disadvantages. The particle filter can be applied to large parameter spaces, has better convergence properties and easier to implement than the UKF, and both of them can provide improved accuracy over the EKF. In addition, the proposed

improved Particle filtering (IPF) algorithm provides a significant improvement over the PF because, unlike the PF which depends on the choice of sampling distribution used to estimate the posterior distribution, the proposed improved Particle filtering algorithm yields an optimum choice of the sampling distribution, which also accounts for the observed data. The proposal sampling distribution is obtained by minimizing the Kullback-Leibler divergence (KLD) distance.

The contribution of this work is to propose new improved particle filtering (IPF) algorithm for states estimation with better proposal distribution based on minimizing the Kullback-Leibler distance.

The rest of the paper is organized as follows. In Section II, a statement of the problem addressed in this paper is presented, followed by description of new improved particle filtering (IPF) technique in Section II-B. Then, in Section III, the performances of the various state estimation techniques are compared through their application to estimate the state variables. Finally, some concluding remarks are presented in Section IV.

II. Problem Formulation

In this section, the states estimation problem is formulated, and then a comparative performance analysis of states estimation using improved particle filter, particle filter, unscented Kalman filter, and extended Kalman filter will be conducted for states estimation.

A. Problem Statement

Here, the estimation problem of interest is formulated. Let a nonlinear state space model be described as follows:

$$\begin{aligned} \dot{z} &= g(z, u, \theta, w), \\ \dot{y} &= l(z, u, \theta, v), \end{aligned} \tag{1}$$

where $z \in \mathbb{R}^n$ is a vector of the state variables, $u \in \mathbb{R}^p$ is a vector of the input variables, $\theta \in \mathbb{R}^q$ is a known parameter vector, $y \in \mathbb{R}^m$ is a vector of the measured variables, and g and l are nonlinear differentiable functions. Discretizing the state space model (1), the discrete model can be written as follows:

$$\begin{aligned} z_k &= f(z_{k-1}, u_{k-1}, \theta_{k-1}, w_{k-1}), \\ y_k &= h(z_k, u_k, \theta_k, v_k), \end{aligned} \tag{2}$$

which describes the state variables at some time step (k) in terms of their values at a previous time step $(k-1)$. Let the process and measurement noise vectors have the following properties: $\mathbf{E}[w_k] = 0$, $\mathbf{E}[w_k w_k^T] = \mathbf{Q}_k$, $\mathbf{E}[v_k] = 0$ and $\mathbf{E}[v_k v_k^T] = \mathbf{R}_k$. The objective is to estimate the state vector z_k, given the measurements vector y_k.

B. Description of States Estimation Techniques

Here, the formulations as well as the algorithms of interest (PF and IPF) are described.

1) Particle Filter: A particle filter is an implementation of a recursive Bayesian estimator [10]. Bayesian estimation relies on computing the posterior $p(z_k|y_{1:k})$, which is the density function of the unobserved state vector, z_k, given the sequence of the observed data $y_{1:k} \equiv \{y_1,\ y_2,\ \cdots,\ y_k\}$. However, instead of describing the required posterior distribution in a functional form, in this particle filter scheme, it is represented approximately as a set of random samples of the posterior distribution. These random samples, which are called the particles of the filter, are propagated and updated according to the dynamics and measurement models [10]. The advantage of the PF is that it is not restricted by the linear and Gaussian assumptions, which makes it applicable in a wide range of applications. The basic form of the PF is simple, but may be computationally expensive. Thus, the advent of cheap, powerful computers over the last ten years has been a key to the introduction and utilization of particle filters in various applications.

For a given dynamical system describing the evolution of the states that we wish to estimate, the estimation problem can be viewed as an optimal filtering problem [10], in which the posterior distribution, $p(z_k|y_{1:k})$, is recursively updated. Here, the dynamical system is characterized by a Markov state evolution model, $p(z_k|z_{1:k-1}) = p(z_k|z_{k-1})$, and an observation model, $p(y_k|z_k)$. In a Bayesian context, the task of state estimation can be formulated as recursively calculating the predictive distribution $p(z_k|y_{1:k-1})$ and the filtering distribution $p(z_k|y_{1:k})$ as follows,

$$p(z_k|y_{1:k-1}) = \int_{\mathbb{R}^n} p(z_k|z_{k-1}) p(z_{k-1}|y_{1:k-1}) dz_{k-1}, \tag{3}$$

$$\text{and} \quad p(z_k|y_{1:k}) = \frac{p(y_k|z_k) p(z_k|y_{1:k-1})}{p(y_k|y_{1:k-1})},$$

$$\text{where} \quad p(y_k|y_{1:k-1}) = \int_{\mathbb{R}^x} p(y_k|z_k) p(z_k|y_{1:k-1}) dz_k.$$

The state vector z_k is assumed to follow a Gaussian model, $z_k \sim \mathcal{N}(\mu_k, \lambda_k)$, where at any time instant k, the expectation μ_k and the covariance matrix λ_k are both constants. Thus, the marginal state distribution is obtained by integrating over the mean and covariance matrix as follows,

$$p(z_k|z_{k-1}) = \int \mathcal{N}(z_k|\mu_k, \lambda_k) p(\mu_k, \lambda_k|z_{k-1}) d\mu_k d\lambda_k, \tag{4}$$

The nonlinear nature of the system model leads to intractable integrals when evaluating the marginal state distribution, $p(z_k|z_{k-1})$. Therefore, Monte Carlo approximation is utilized, where the joint posterior distribution, $p(z_{0:k}|y_{1:k})$, is approximated by the point-mass distribution of a set of weighted samples (particles) $\{z_{0:k}^{(i)}, \ell_k^{(i)}\}_{i=1}^N$, i.e.,:

$$\hat{p}_N(z_{0:k}|y_{1:k}) = \sum_{i=1}^{N} \ell_k^{(i)} \delta_{z_{0:k}^{(i)}}(d\, z_{0:k}) / \sum_{i=1}^{N} \ell_k^{(i)}, \tag{5}$$

where $\delta_{z_{0:k}^{(i)}}(d\, z_{0:k})$ denotes the Dirac function, and N is the total number of particles. Based on the same set of particles,

the marginal posterior probability of interest, $p(z_k|y_{1:k})$, can also be approximated as follows:

$$\hat{p}_N(z_k|y_{1:k}) = \sum_{i=1}^{N} \ell_k^{(i)} \delta_{z_k^{(i)}}(d\, z_k) / \sum_{i=1}^{N} \ell_k^{(i)}. \quad (6)$$

In this Bayesian importance sampling (IS) approach, the particles $\{z_{0:k}^{(i)}\}_{i=1}^{N}$ are sampled from the following distribution,

$$\pi(z_{0:k}|y_{1:k}) = \int \mathcal{N}(z_k|\mu_k, \lambda_k) p(\mu_k, \lambda_k|z_{k-1}) d\mu_k d\lambda_k, \quad (7)$$

Then, the estimate of the augmented state $\widehat{z}_k$ can be approximated by a Monte Carlo scheme as follows:

$$\widehat{z}_k = \sum_{i=1}^{N} \ell_k^{(i)} z_k^{(i)}, \quad (8)$$

where $\ell_k^{(i)}$ are the corresponding importance weights:

$$\ell_k^{(i)} \propto \frac{p(y_{1:k}|z_{0:k}^{(i)}) p(z_{0:k}^{(i)})}{\pi(z_{0:k}^{(i)}|y_{1:k})}. \quad (9)$$

The PF algorithm for state/parameter estimation is summarized in Algorithm 1.

Algorithm 1: Particle Filtering algorithm

Input: y_k, μ_0, λ_0
Output: $\widehat{z}_k$
for $i = 1, 2, \ldots$ **do**
  Importance sampling step:
  Sample $\widetilde{z}_k^{(i)} \sim \pi(z_k^{(i)}|z_{0:k-1}^{(i)}, y_{1:k})$, according the equation (4), and set $\widetilde{z}_{0:k}^{(i)} = (z_{0:k-1}^{(i)}, z_k^{(i)})$;
  Compute the approximated joint distribution, $\hat{p}_N(z_{0:k}|y_{1:k})$, using equation (5);
  Evaluate importance weights using equation (9);
  Normalize importance weights:

$$\widetilde{\ell}_k^{(i)} = \frac{\ell_k^{(i)}}{\sum_{j=1}^{N}(\ell_k^{(j)})}$$

  Selection step:
  If $N_{eff} = \frac{1}{\sum_{i=1}^{N}(\ell_k^{(i)})^2} < N_{threshold}$ Resample with replacement N particles $\{z_{0:k}^{(i)}\}_{i=1}^{N}$ from the set $\{\widetilde{z}_{0:k}^{(i)}\}_{i=1}^{N}$ according to the normalised importance weights, $\ell_k^{(i)} = \widetilde{\ell}_k^{(i)}$;
  Compute the estimated state using equation (8);
end
Return the augmented state estimation $\widehat{z}_k$.

In summary, particle filtering suffers from one major drawback. Its efficient implementation requires the ability to sample from $p(z_k|z_{k-1})$, which does not take into account the current observed data, y_k, and thus many particles can be wasted in low likelihood (sparse) areas. This issue is addressed by the proposed improved particle filter (IPF), which is described in the next sub-section.

2) Improved Particle Filter: The choice of optimal proposal function is one of the most critical design issues in importance sampling schemes. In [10], the optimal proposal distribution $\hat{p}(z_k|z_{0:k-1}, y_{0:k})$ is obtained by minimizing the variance of the importance weights given the states $z_{0:k-1}$ and the observations data $y_{0:k}$. This selection has also been studied by other researchers. However, this optimal choice suffers from one major drawback. The particles are sampled from the prior density $p(z_k|z_{0:k-1})$ and the integral over the new state need to be computed. In the general case, closed form analytic expression of the posterior distribution of the state is untractable [10]. Therefore, the distribution $p(z_k|z_{0:k-1})$ is the most popular choice of proposal distribution. One of its advantages is its simplicity in sampling from the prior functions $p(z_k|z_{0:k-1})$ and the evaluation of weights $\ell_k^{(i)}$ (as presented in the previous section). However, the latest observation is not considered for the computation of the weights of the particles as the importance density is taken to be equal to the prior density. The transition prior $p(z_k|z_{0:k-1})$ does not take into account the current observation data y_k, and many particles can be wasted in low likelihood areas. This choice of importance sampling function simplifies the computational complexity but can cause filtering divergence [11]). In cases where the likelihood density is too narrow as compared to the prior function, very few particles will have considerable weights. Next, we present an overview of KLD-based improved particle filter.

Improved Particle Filter based on KLD minimization: As mentioned above, the distribution of interest for the state takes the form of a marginal posterior distribution $p(z_k|y_{1:k})$. The proposed extended Bayesian sampling algorithm (also named as improved particle filtering, IPF) is proposed for approximating intractable integrals arising in Bayesian statistics. By using a separable approximating distribution $\hat{q}(z_k) = \hat{q}(z_k|z_{0:k-1}, y_{0:k}) = \prod_i q(z_k^i)$ to lower bound the marginal likelihood, an analytical approximation to the posterior probability $p(z_k|y_{1:k})$ is provided by minimizing the Kullback-Leibler divergence (KLD):

$$D_{\mathrm{KL}}(\hat{q}||p) = \int \hat{q}(z_k|z_{0:k-1}, y_{0:k}) \log \frac{\hat{q}(z_k|z_{0:k-1}, y_{0:k})}{p(z_k|z_{0:k-1}, y_{0:k}|y_{1:k})} dz_k, \quad (10)$$

where,

$$\hat{q}(z_k|z_{0:k-1}, y_{0:k}) = \prod_i \hat{q}(x_k^i|x_{0:k-1}, y_{0:k}) = \hat{q}(z_k)\hat{q}(\mu_k)\hat{q}(\lambda_k).$$

Minimizing the KLD subject to the constraint $\int q(z_k)dz_k = \prod_i \int q(z_k^i)dz_k^i = 1$, the Lagrange multiplier scheme is used to yield the following approximate distribution [12], [13],

$$\hat{q}(z_k^i) \propto \exp\left[E\left(\log\, p(y_{1:k}, z_k)\right)_{\prod_{j\neq i}\hat{q}(z_k^j)}\right], \quad (11)$$

where $E(.)_{q(z_k^j)}$ denotes the expectation operator relative to the distribution $q(z_k^j)$. Therefore, these dependent parameters can be jointly and iteratively updated. Taking into account the separable approximate distribution $\hat{q}(z_{k-1})$ at time $k-1$, the posterior distribution $p(z_k|y_{1:k})$ is sequentially approximated according to the following scheme:

$$\begin{aligned} \hat{p}(z_k|y_{1:k}) &\propto p(y_k|z_k)p(z_k, \lambda_k|\mu_k)q_p(\mu_k), \quad (12) \\ \text{where} \quad q_p(\mu_k) &= \int p(\mu_k|\mu_{k-1})\hat{q}(\mu_{k-1})d\mu_{k-1}. \end{aligned}$$

Hence, the particles $\{z_{0:k}^{(i)}\}_{i=1}^{N}$ are sampled according to the following optimal function:

$$\hat{q}(z_{0:k}^{(i)}|y_{1:k}) = \int \mathcal{N}(z_k^{(i)}|\mu_k, \lambda_k) p(\mu_k, \lambda_k|z_{k-1}^{(i)}) p(y_k|z_k^{(i)}) d\mu_k d\lambda_k. \tag{13}$$

The recursive estimate of the importance weights can be derived as follows:

$$\ell_k^{(i)} = \ell_{k-1}^{(i)} \frac{p(y_{1:k}|z_{0:k}^{(i)}) p(z_{0:k}^{(i)})}{\hat{q}(z_{0:k}^{(i)}|y_{1:k})}. \tag{14}$$

Equation (14) provides a mechanism to sequentially update the importance weights, given an appropriate choice of proposal distribution, $\hat{q}(z_{0:k}^{(i)}|y_{1:k})$. Then, the estimate of the augmented state $\widehat{z}_k$ can be approximated by a Monte Carlo scheme as follows:

$$\widehat{z}_k = \sum_{i=1}^{N} \ell_k^{(i)} z_k^{(i)}, \tag{15}$$

The improved particle filter which based on minimizing KLD for proposal distribution generation within a particle filter framework is depicted in Algorithm 2.

Algorithm 2: Improved Particle Filtering algorithm

Input: y_k, μ_0, λ_0
Output: $\widehat{z}_k$
for $i = 1, 2, \ldots$ **do**
  Importance sampling step:
  Sample $\widetilde{z}_k^{(i)} \sim \hat{q}(z_k^{(i)}|z_{0:k-1}^{(i)}, y_{1:k})$, according the equation (13), and set $\widetilde{z}_{0:k}^{(i)} = (z_{0:k-1}^{(i)}, z_k^{(i)})$;
  Compute the approximated joint distribution, $\hat{p}(z_{0:k}|y_{1:k})$, as the equation (12);
  Evaluate importance weights, as the equation (14);
  Normalize importance weights:

$$\widetilde{\ell}_k^{(i)} = \frac{\ell_k^{(i)}}{\sum_{j=1}^{N}(\ell_k^{(j)})}$$

  Selection step:
  If $N_{eff} = \frac{1}{\sum_{i=1}^{N}(\ell_k^{(i)})^2} < N_{threshold}$ Resample with replacement N particles $\{z_{0:k}^{(i)}\}_{i=1}^{N}$ from the set $\{\widetilde{z}_{0:k}^{(i)}\}_{i=1}^{N}$ according to the normalised importance weights, $\ell_k^{(i)} = \widetilde{\ell}_k^{(i)}$;
  Compute the estimated state, as the equation (15);
end
Return the augmented state estimation $\widehat{z}_k$.

III. Simulation Results Analysis

In this section, the state estimation techniques (i.e., EKF, UKF, PF and IPF) are compared through their utilization to estimate the state variables of the Cad System in E. coli (CSEC) [14]. First, a description of the CSEC process model is presented, and then one comparative study is conducted to assess the performances of the proposed algorithm (IPF) compared to EKF, UKF, and PF techniques. In the comparative study, the state estimation techniques are used to estimate the four state variables (x_1, x_2, x_3 and x_4 for a model of the Cad System in E. coli (CSEC) [14]) from noisy measurers of these variables.

Next, the model of CSEC [14], that will be used in our analysis, will be described.

A. Cad System in E. coli (CSEC) Model

Using the metabolic pathways, one can specify the kinetic orders that are zero. For instance, if x_j does not directly affect x_i, the corresponding kinetic orders g_{ij} and h_{ij} are zero. Moreover, based on experience [14], the kinetic orders g_{ii} are set to zero to omit a direct reinforcing effect of a metabolite on its own production. For example, consider the generic branched pathway which has four dependent variables $x_1, ..., x_4$ and one independent variable x_5 [14]. The production of x_1 depends on the independent variable x_5 with an inhibition effect exerted by x_3. Hence, the kinetic orders g_{13}, g_{15} and h_{11} are only non-zero. Similarly, one can determine for each metabolite which kinetic orders are non-zero and which are negative such as g_{13}. These concentrations are the four dependent variables $x_1, ..., x_4$ in the following S-system [14], which also involves one independent variable x_5:

$$\begin{aligned}
\dot{x}_1 &= \alpha_1 x_3^{g_{13}} x_5^{g_{15}} - \beta_1 x_1^{h_{11}} \\
\dot{x}_2 &= \alpha_2 x_1^{g_{21}} - \beta_2 x_2^{h_{22}} \\
\dot{x}_3 &= \alpha_3 x_2^{g_{32}} - \beta_3 x_3^{h_{33}} x_4^{h_{34}} \\
\dot{x}_4 &= \alpha_4 x_1^{g_{41}} - \beta_4 x_4^{h_{44}}
\end{aligned} \tag{16}$$

where $w = [\alpha_1, ..., \alpha_4, \beta_1, ..., \beta_4, g_{13}, g_{15}, g_{21}, g_{32}, g_{41}, h_{11}, h_{22}, h_{33}, h_{34}, h_{44}]$. Based on the fact that the rate coefficients α_i's and β_i's and all the kinetic orders except g_{13} are positive, the parameter pre-specified set is defined as $\mathcal{X}_w = \mathbb{R}_+^8 \times \mathbb{R}_- \times \mathbb{R}_+^9$. Eventually, to perform comparison between the techniques, the estimation root mean square errors (RMSE) criteria will be used and calculated on the states (with respect to the noise free data),

$$\text{RMSE} = \sqrt{E\left((\mathbf{z} - \widehat{\mathbf{z}})^2\right)}, \tag{17}$$

where $\mathbf{z}$ (resp. $\widehat{\mathbf{z}}$) is the true state (resp. the estimated state).

The model (16) is used to simulate the responses of x_1, x_2, x_3 and x_4 for Cad System in E. coli (CSEC) as functions of time as functions. These simulated states, which are assumed to be noise free, are then contaminated with zero mean Gaussian errors, i.e., a measurement noise $v_{k-1} \sim \mathcal{N}(0, \sigma_v^2)$. Considering a value of $\sigma_v^2 = 0.01$ the following data set can be generated. The sampling time used for discretization is 0.01min. Figure 1 shows the changes in the four state variables.

B. Estimation of State Variables from Noisy Measurements using EKF, UKF, PF and IPF

The objective behind this study is compare the performances of various he estimation accuracy of EKF, UKF, PF and IPF algorithms through their application using simulated time-series metabolic data representing the concentrations of four metabolites, i.e., x_1, x_2, x_3 and x_4. Hence, we consider the state vector that we wish to estimate, $z_k = [x_1\ x_2\ x_3\ x_4]^T$, and the model parameters (i.e., α_1, α_2, α_3, α_4, β_1,β_2, β_3, β_4, g_{13}, g_{21}, g_{32}, g_{41}, h_{11}, h_{22}, h_{33}, h_{44}) are assumed to be known. The simulation results of estimating the four states x_1, x_2, x_3, and x_4 using EKF, UKF, PF and IPF are shown in Figures 2(a,b,c,d). Also, the estimation root mean

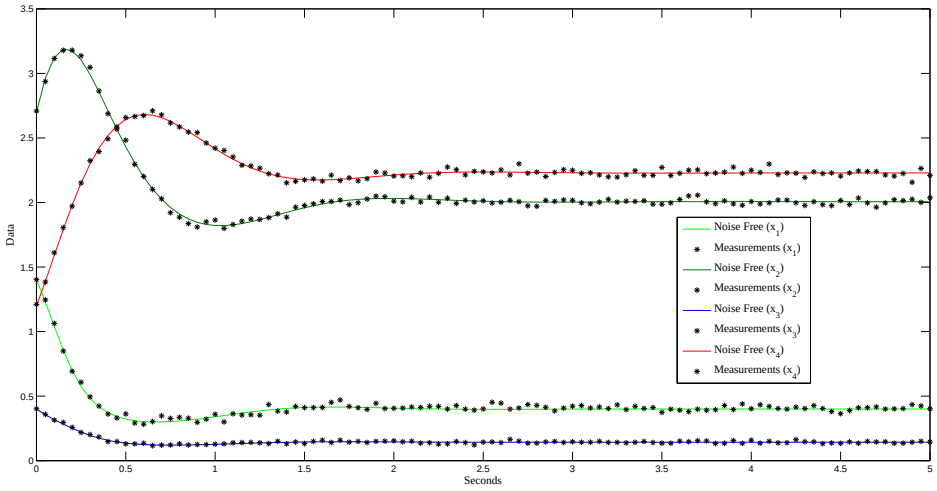

Figure 1. Simulated CSEC data used in estimation: state variables (x_1, x_2, x_3, and x_4).

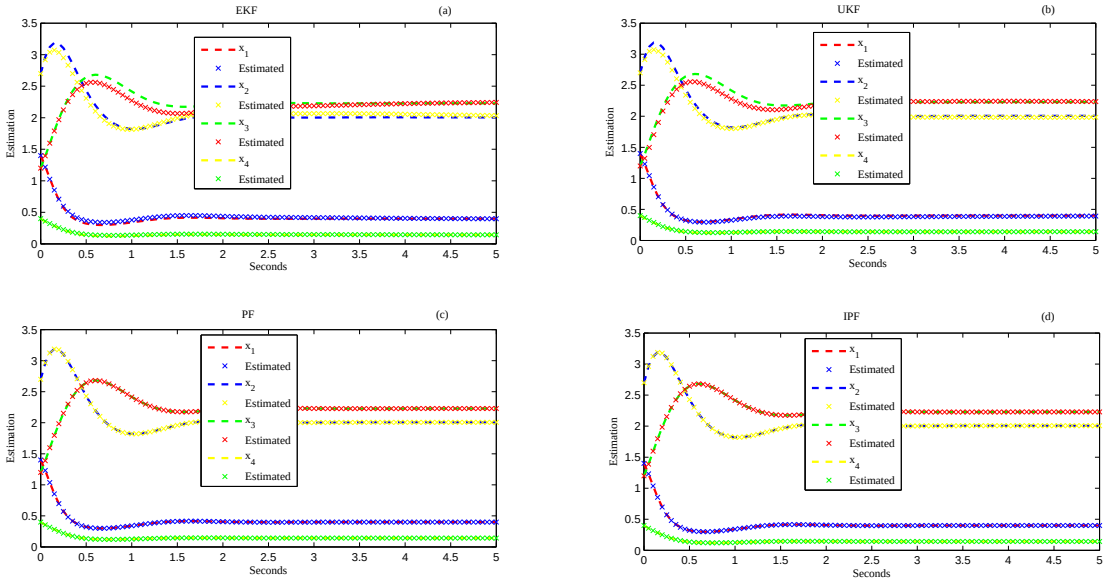

Figure 2. Estimation of state variables using various state estimation techniques

square errors (RMSE) for the estimated states are shown in Table I. It can be observed from Figure 2 and Table I that EKF resulted in the worst performance of all estimation techniques, which is expected due to the limited ability of EKF to accurately estimate the mean and covariance matrix of the estimated states through lineralization of the nonlinear process model. The results also show that the PF provides a significant improvement over the UKF, which is due to the fact that, by using UKF, linearizing the process model does not necessarily provide good estimates of the mean of the state vector and the covariance matrix of the estimation error which are used in state estimation. The results also show that the IPF provides a significant improvement over the PF, which is due to the fact that the IPF yields an optimal choice of the sampling distribution $p(z_k|z_{k-1}, y_k)$ over the estimated states by minimizing a KLD criterion that also utilizes the observed data y_k.

IV. Conclusions

In this paper, we developed an improved particle filtering algorithm for nonlinear and non-Gaussian states estimation. In case of standard particle filter, the latest observation is not considered for the evaluation of the weights of the particles as the importance function is taken to be equal

Table I
ROOT MEAN SQUARE ERRORS (RMSE) OF ESTIMATED STATES FOR EKF, UKF, PF AND IPF

Technique	x_1	x_2	x_3	x_4
EKF	0.0694	0.1160	0.1215	0.0311
UKF	0.0593	0.0937	0.1129	0.0195
PF	0.0009	0.0012	0.0009	0.0012
IPF	0.00078	0.0011	0.00079	0.0009

to the prior density function. This choice of importance sampling function simplifies the computation but can cause filtering divergence. In cases where the likelihood function is too narrow as compared to the prior function, very few particles will have significant weights. Hence a better proposal distribution that takes the latest observation into account is desired. The proposed algorithm consists of a particle filter based on minimizing the Kullback-Leibler divergence distance to generate the optimal importance proposal distribution. The proposed algorithm allows the particle filter to incorporate the latest observations into a prior updating scheme using the estimator of the posterior distribution that matches the true posterior more closely.

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