Uncertainty Quantification HW #2 Berkcan Kapusuzoglu
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P ~ Type I Extreme Value(mean=2000, cov=0.2)
E ~ Lognormal(mean=30000, cov=0.1)
I ~ Normal(mean=10, cov=0.05)
Create a surrogate model for Y = P/EI using Gaussian Process
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The squared-exponential correlation function is used:

$$R(\mathbf{a},\mathbf{b}) = \exp\left[-\sum_{i=1}^{d}\theta_i(a_i - b_i)^2\right]$$

where the covariance between outputs of the Gaussian process **Z**(.) at points **a** and **b** is defined as:

$$Cov[Z(\mathbf{a}),Z(\mathbf{b})] = \sigma_Z^2 R(\boldsymbol{a},\boldsymbol{b})$$

Generate training points and compute output for each combination. Then, express input in terms of standard random variables and do linear regression analysis to find the coefficients.

1. Choose an initial value for $\boldsymbol{\theta} = [l_1\ l_2\ l_3\ \sigma_Z] = [5\ 200\ .25\ 0.3]$
2. Assuming we have little prior knowledge about $\boldsymbol{\theta}$, we need to maximize $\log p(\mathbf{y}|\mathbf{x},\boldsymbol{\theta})$ for 7 training points for each dimension ($n = 7^3 = 343$).
$$\log p(\mathbf{y}|\mathbf{x},\boldsymbol{\theta}) = -\frac{1}{2}\mathbf{y}^{\boldsymbol{T}}K^{-1}\mathbf{y} - \frac{1}{2}\log|K| - \frac{n}{2}\log 2\pi.$$
3. Compare results with MATLAB built-in function "*fitrgp*".