Chapter 14
Confidence in the Prediction of Unmeasured System Output Using Roll-Up Methodology

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Abstract This research is concerned with how to use available experimental data from tests of lower complexity to inform the prediction regarding a complicated system where no test data is available. Typically, simpler test configurations are used to infer the unknown parameters of an engineering system. Then the calibration results are propagated through the system model to predict the uncertainty in the system response. However, it is important to note that parameter estimation results are affected by the quality of the model used to represent the test configuration. Therefore, it is necessary that the model of the test configuration be also subjected to rigorous validation testing. Then the calibration and validation results for the test configurations need to be integrated to produce the distributions of the parameters to be used in the system-level prediction. Such a systematic roll-up methodology that integrates calibration and validation results at multiple levels of test configurations has been previously established (Sankararaman and Mahadevan, Reliab Eng Syst Saf 138:194–209, 2015).

The current work develops an approach to quantify the confidence in the use of lower-level test data (through the roll-up methodology) in predicting system-level response. The propagated roll-up distributions are compared against simulated output distributions from a calibrated system model based on synthetic data; this comparison is done through the model reliability metric (Rebba and Mahadevan, Reliab Eng Syst Saf 93(8):1197–1207, 2008) and results in a quantified roll-up confidence. Then an optimization procedure is formulated to maximize the roll-up confidence by selecting the most valuable calibration and validation tests at the lower levels. The proposed methods for both the forward and inverse UQ problems are applied to the multi-level Sandia dynamics challenge problem (Red-Horse and Paez, Comput Methods Appl Mech Eng 197(29–32):2578–2584, 2008).

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Keywords Multi-level system · Bayesian · Roll-up · Uncertainty quantification · Resource allocation

14.1 Extended Abstract

There are certain critical engineering systems where experiments cannot be directly conducted on the full system, such as aircraft, weapons systems, space rovers, etc. Experimental testing restrictions can be due to cost of testing, international and governmental testing requirements, or physical limitations in the testing equipment and environment. However, the individual components and subsystems of this complex system may be readily conducive to experimentation. The first challenge addressed in this work is how to use available experimental data from levels of lower complexity to inform the prediction regarding a complicated system where no test data is available. The second challenge confronted is that, before any experiments have been conducted, how to best allocate an available budget to lower level tests in order maximize confidence in the roll-up prediction.

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A key assumption in this work is that the levels of lower complexity share one or more model parameters with the complex system whose performance is being predicted. Model calibration focuses on the inference of model parameters so that the prediction by the computational model matches the experimental data [4]. Here Bayesian inference is used for model calibration:

$$f''(\boldsymbol{\theta}) \propto L(\boldsymbol{\theta}) f'(\boldsymbol{\theta}) \tag{14.1}$$

If all the parameters to be calibrated are denoted as $\boldsymbol{\theta}$, Eq. (14.1) of Bayesian inference indicates that the joint posterior distribution $f''(\boldsymbol{\theta})$ is proportional to the product of likelihood function $L(\boldsymbol{\theta})$ and prior distribution $f'(\boldsymbol{\theta})$.

After obtaining the posterior distribution of model parameters in the model calibration, some analysts directly propagate the posterior distributions through the computational model of the system of interest to predict the system output, while others incorporate model validation in the uncertainty quantification of model parameters claiming that it provides information to understand the limitation of calibration [5]. In this work, model validation is performed on the models of each test configuration. Among several available quantitative validation metrics, the model reliability metric is implemented here since it gives an easily interpretable probability value that can be used in the roll-up methodology [1, 2].

The roll-up approach developed by Sankararaman and Mahadevan [6] provides a framework for integrating the results from calibration and validation into the distribution for each model parameter, to be used in system-level prediction:

$$\begin{aligned} f\left(\theta \left| D_1^{C,V}, D_2^{C,V}\right.\right) = {} & P(G_1)\, P(G_2) f\left(\theta \left| D_1^C, D_2^C\right.\right) + P\left(G_1'\right) P(G_2) f\left(\theta \left| D_2^C\right.\right) \\ & + P(G_1)\, P\left(G_2'\right) f\left(\theta \left| D_1^C\right.\right) + P\left(G_1'\right) P\left(G_2'\right) f(\theta) \end{aligned} \tag{14.2}$$

In Eq. (14.2), $P(G_i)$ is the probability of the model at Level i being valid, calculated using the model reliability metric. The term $P\left(G_i'\right)$ is the complementary probabilities of model being invalid, calculated as $1 - P(G_i)$. The integrated (roll-up) distribution from Eq. (14.2) is then propagated through the system model to predict the quantity of interest. This completes the forward problem.

In the inverse problem, no experimental tests have yet been conducted. This is where resource allocation is necessary. Tests can be conducted at multiple lower levels and can be used for either calibration or validation of the lower level models. Additionally, there are different associated costs for each type of test and a total budget for testing that cannot be exceeded. The objective is thus to select lower level tests that, when rolled-up, accurately predict the quantity of interest with the least amount of uncertainty, or said differently, minimize bias and variance in the roll-up prediction.

A method is developed in this study to assess how well the roll-up prediction of the QOI agrees with the "perfect" QOI prediction. The "perfect" QOI prediction replicates the ideal case where system level tests are available, by synthetically generating data at the system level to compare with the roll-up prediction. Several different metrics could be considered for such assessment, including KL Divergence, sensitivity comparison, or the model reliability metric. In this work, the model reliability metric is used since it has a clear interpretation and offers a quantitative basis for decision-making. The model reliability metric was previously developed for the purpose of model validation [2], and is extended in this work to assess the roll-up prediction.

Roll-up confidence is used as the objective in test resource allocation. The number of tests at each of the lower levels and whether the tests are used to calibrate model parameters or validate the lower level models is decided based on maximizing the roll-up confidence. Of course, this is subject to a budget constraint.

The methods developed for both the forward problem and the inverse problem are applied to the Sandia dynamics challenge problem [3] (Fig. 14.1).

This example consists of three levels with each of the levels having in common three masses joined by springs and dashpots. The three levels are distinguished by their boundary conditions and input excitations. The stiffnesses and damping coefficients are the model parameters requiring calibration and the quantity of interest is the peak acceleration of the topmost mass at the third (system) level.

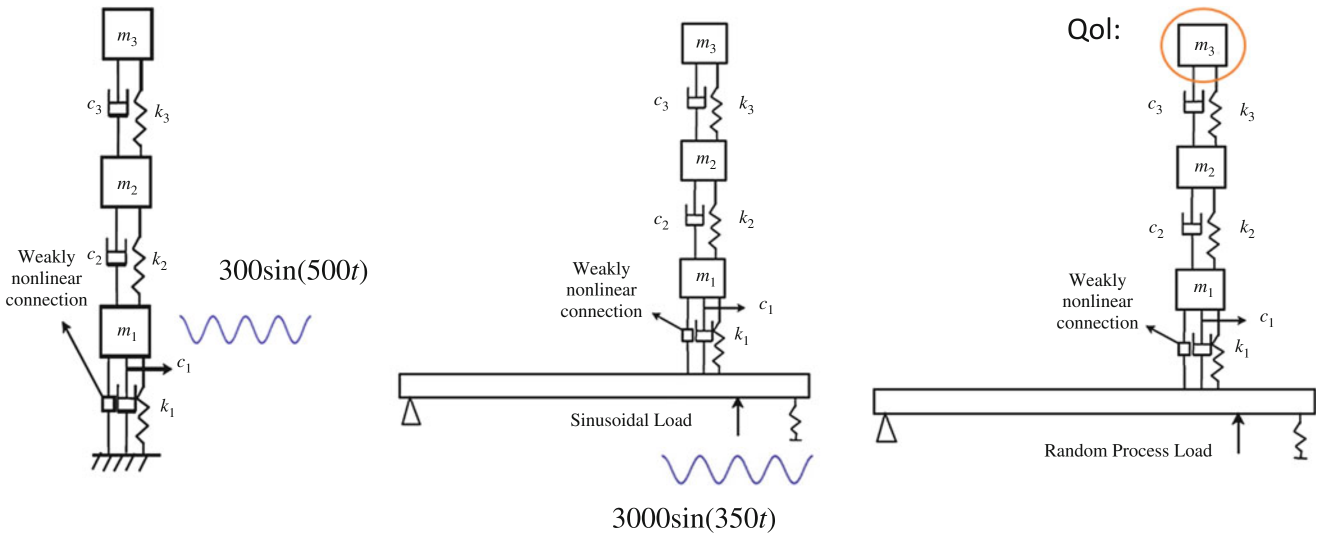

Fig. 14.1 Sandia dynamics challenge problem setup [3]

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