# Angular Momentum

**Lesson Type**: Module
**Target Grade:** Middle School
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**Semester**: Fall 2016

## Introduction

Students will learn the basics about angular momentum without going into the nitty-gritty details of the math involved. We will be exploring how objects rotate, and be taking a look at the science behind gyroscopes, tops, and frisbees!

## Teaching Goals

- Linear momentum keeps something in a line: “an object in motion stays in motion”
- Angular momentum is its rotational analog and is a dynamical property that makes an object want to spin in the same plane of motion (direction)
- Angular momentum depends on the rate of spinning and the weight distribution of the object
- Friction will cause an object to slow its spinning, therefore decreasing its angular momentum. When it loses enough angular momentum, the object will fall
- Gyroscopes work due to precession. If a spinning object is unable to stay in its same plane of motion (in this case, due to gravity) it will move about a secondary axis (precession) to stay close to the original direction of spinning
- Frisbees, tops, and bikes use conservation of angular momentum to stay stable

## Careers and Applications

Many different types of engineers will face angular momentum at some point in their lives. It is applicable to any rotating object, like wheels; in fact, this is how bikes work. Without the conservation of angular momentum, going faster on a bike wouldn’t stabilize you, and it would take a lot more balance to ride a two-wheeled bike! Conservation of angular momentum can also be seen in things like football (spinning the football when you throw it makes it more stable and aerodynamic) and throwing frisbees. Gyroscopic precession is also an important stabilizer that is present in planes and helicopters-- a typical plane usually has about a dozen gyroscopes on board to stabilize its turning!

## Agenda

- **Module 0- Introduction to Angular Momentum (5 min)**
- **Module 1- Gyroscopes (15 min)**
- **Module 2- Flying Gyros (15 min)**
- **Module 3- Tops (20 min)**
- **Conclusion (5 min)**

## Background on Momentum and Forces:

In this lesson, we are going to be exploring the rotational analog of momentum, so in case you have never seen momentum before, I'm going to give a brief background on it. **Momentum** is a property that all moving objects have. The momentum of an object is given by the formula $\vec{p} = m\vec{v}$, where $m$ is the mass of the object and $\vec{v}$ is the velocity (speed with a direction) of the object. So, a heavier object has more momentum than a lighter object, and an object has more momentum when it is moving faster. The **conservation of angular momentum** is a law that states that in the absence of external forces, momentum is conserved. This means that if no forces act on an object, it will keep moving with the same velocity. **Newton's Laws** govern the *dynamics* of moving bodies; they describe how their motion changes under external influence. The three laws are as follows:

1. Inertial frames exist. In an inertial frame, **an object in motion stays in motion**. This is a central point of this lesson, as we will be making the connection to spinning objects.
2. $\vec{F} = \Delta\vec{p}/\Delta t$. $\vec{F}$ is the force on an object, and $\vec{p}$ is its momentum. $\Delta\vec{p}/\Delta t$ is the time derivative of momentum, and it means "change in momentum per change in time". **This law says that if a force acts on an object, the momentum will change**. Intuitively, this should make sense because if I push a moving object, I will change its speed or direction it is moving. We will be exploring precession later in this lesson, and precession is essentially the rotational equivalent of this.
3. For every action, there is an equal and opposite reaction. This was just included in here for completeness, we won't really use it at all during this lesson. Basically, if you push on something with a force $F$, it will push back on you with a force $F$.

I will go more in depth with background in the respective modules, but for a general overview, from both force and momentum you can find a rotational version that acts in the same way. The **angular momentum** of an object is like momentum for spinning objects. All spinning objects have angular momentum and as they spin faster or increase their mass they gain angular momentum. The rotational equivalent of force is **torque**. Torque is just the tendency of a force to rotate an object, so a force spinning a wheel has torque while a force pushing a wall does not (because the wall doesn't want to rotate). Just like linear momentum, angular momentum is conserved in the absence of external torques, which is why it is important. This means that **an object spinning stays spinning** in the absence of external torques, which is one of the central ideas of this lesson. To change the angular momentum, you must have a torque; this will create a phenomenon known as **precession**, which will make the object rotate about a second axis.

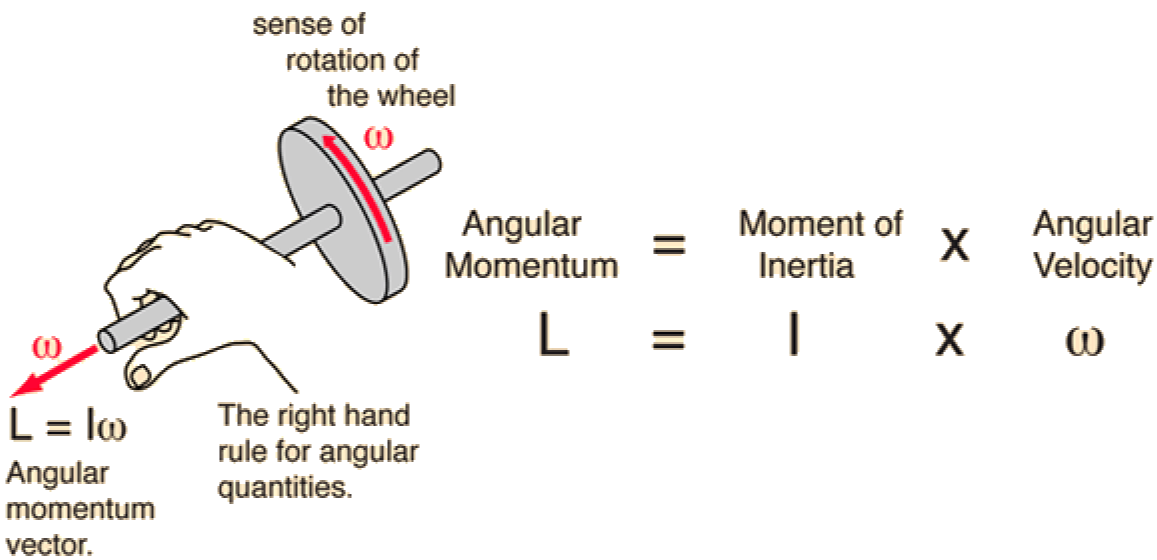
# Module 0: Introduction to Angular Momentum

## Introduction

- Students will be introduced to the idea of angular momentum
- We will examine how frisbees fly

### Background for Mentors

- Angular momentum ($\vec{L}$) is given by the formula: $\vec{L} = I\vec{\omega}$

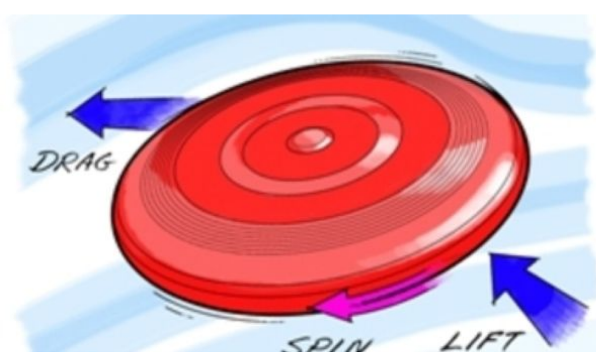



- Where $I$ is the **moment of inertia** and $\omega$ is the rotational speed
    - Moment of inertia is the rotational equivalent of mass-- it depends on the mass of an object and the distance from the axis of rotation, which we will get into later
    - This has a clear analog to linear momentum, where $\vec{p} = m\vec{v}$
- $\vec{L}$ is a *vector* (which I will denote with arrows), a quantity with a magnitude and a direction; this direction is in the same direction as $\omega$
    - To find this direction, we use the right hand rule. Curl your fingers in the direction of the spin like the picture, and your thumb will point in the direction of $\omega$ and $\vec{L}$
- In the absence of external torques ("rotational force" $\tau = r \times F$ ), angular momentum is conserved
    - This means that an object spinning wants to stay spinning-- if an object has angular momentum and is rotating at rate $\omega$, then the product $I\omega$ must stay constant
    - Generally $I$ doesn't change, so the rotational speed is constant, so **"an object spinning stays spinning"**
    - Of course, in the real world we have friction; friction produces a force (and therefore a torque) on spinning objects, causing them to slow down
        - The action of friction on frisbees is pretty much negligible, so they won't stop spinning in midair
- The conservation of angular momentum allows frisbees to stabilize themselves in the air,

because they want to stay spinning in the same direction
- Frisbees actually fly because of a lift force. However, the stabilization from spinning helps the frisbees glide over the air, and if the spinning wasn't present they would be unstable and not fly well
    - This lesson won't be going into lift from the air, but if a curious student asks why frisbees can move up in the air, the answer is Bernoulli's principle:
        - http://web.wellington.org/miller/How%20the%20Frisbee%20Flies.htm

## Materials
- 1 Frisbee

## Procedure
- Ask the students if they know what momentum is. Explain that momentum is something that all moving objects have, and heavier objects have more momentum. Forces (pushes or pulls) can increase or decrease the momentum. Without forces, momentum is conserved and *objects in motion stay in motion*
- Explain that there is a type of momentum that has to do with spinning-- it is called *angular momentum*
    - All spinning objects have angular momentum
    - Heavier objects have more angular momentum
    - Objects can increase their angular momentum by *spinning* faster
    - Just like the other type of momentum, *angular momentum is conserved*
        - Stress to the students that: **"an object spinning stays spinning"** (in the same direction)
- Here are some good questions to ask the students:
    - Who here knows how to ride a bike? Is it harder to ride a bike when you're moving fast or when you're moving really slow?
        - Explain that it is easier to ride a bike when moving fast-- once the wheels start spinning, they want to stay spinning. This helps to stabilize you and prevent you from tipping over
    - Does anyone know how to throw a football? What do you do when you throw it?
        - Explain that when you throw a football, you spin it to stabilize it in the air. Once it begins spinning, it wants to stay spinning and so it becomes more stable and aerodynamic
    - Why do we spin frisbees?




- To answer this question, find a place that you will have enough room to throw a frisbee (probably outside)
- Have one of the mentors (who knows how to throw a frisbee) throw it as far as they can
    - Hopefully, it goes pretty far! Have them remember roughly

how far it went

- Then, do it again, with a catch: **the frisbee isn't allowed to spin!**
    - This time, it should go a fraction of the original distance
- Explain that the more you spin a frisbee, the more angular momentum it has. Once it has enough angular momentum, it stays in its same *plane of motion* and doesn't wobble-- this makes it glide across the air much better! That's why we always spin frisbees when we throw them!

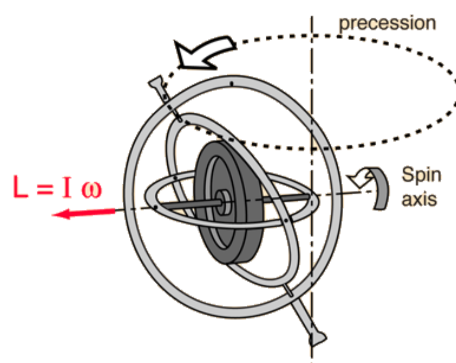
### Additional Notes for Mentors

- This is a pretty long intro, so if you want to skip some of the questions feel free. The point that should be emphasized during this module is that **an object spinning wants to stay spinning**, and this is the reason all the things in this module work like they do.

## Module 1: Gyroscopes

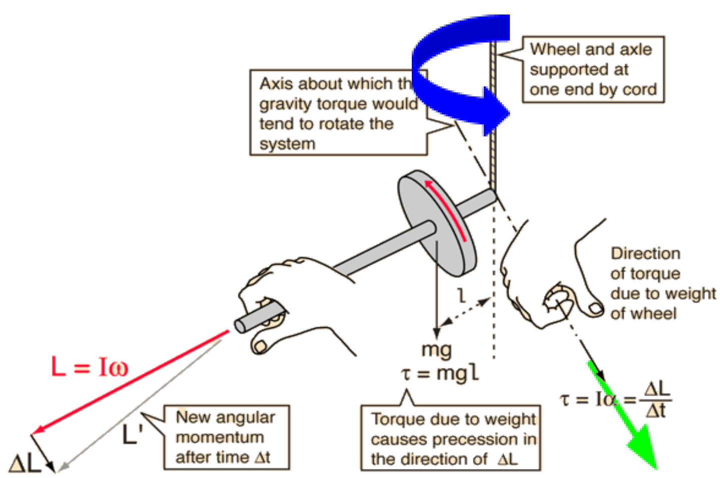
### Introduction

In this module, the students will be learning the physics of gyroscopes and how it relates to the angular momentum. They will be learning about gyroscopic motion and **precession**




### Background for Mentors

- If you have never seen a gyroscope before, I suggest watching this video:
    - https://www.youtube.com/watch?v=cquvA_IpEsA
- Gyroscopes are consequences of the rotational version of Newton's second law and the strangeness of the vector cross product
- Torque is the rotational analog of force, and given by $\tau = \vec{r} \times \vec{F}$ , where $\vec{r}$ is distance and $\vec{F}$ is applied force. The cross product means that torque is at a right angle to $\vec{r}$ and $\vec{F}$ , a direction given again by the right hand rule



- In the picture above, the disk is spinning with angular momentum in the direction of the red arrow (remember that $L$ points in the plane of spinning according to the right hand rule)
    - The force on the wheel is downward due to gravity, and the position is radially outward
    - By the right hand rule, this gives a torque in the direction of the green arrow
    - Newton's second law says that:

$$\vec{\tau} = \Delta\vec{L}/\Delta t$$

    - This means torque creates a change of angular momentum in the same direction as the torque
        - The red arrow, the direction of angular momentum, must therefore change and rotate in the direction of the green arrow, the direction of torque
- This rotation about the central axis is called **precession**; motion is said to be **gyroscopic** if it processes about a different axis
- *Now, there's a lot of math in here so be sure not to get caught up in it. The main idea is this: Spinning objects want to stay spinning. When they can't because of a torque (rotational version of force) they spin around another axis, causing precession. This is all due to the weird properties of vector multiplication and the right hand rule, which you shouldn't get into with the students*

## Materials

- 4 Gyroscopes/class
- String

## Procedure

- Split the students up into 4 groups, and have a mentor (or two) at each group. In each group, the mentor(s) will have a gyroscope
- Begin the module by setting the gyroscope on the stand and asking the students what they think will happen if you let go. They will probably say it will fall. Release the gyroscope and let it fall to the table
- Now, do the same thing again but this time pull the string on the gyroscope and spin it as fast as you can. Ask them if they think anything different will happen. Let go of the gyroscope and watch it precess
- Explain that gyroscopes work due to conservation of angular momentum, or lack thereof. The key topic in this module is precession
    - **Big idea to explain to students**: Just like how pushing on an object in motion will cause it to change how it is moving, pushing on a rotating object will also change how it is moving. In the gyroscope's case, it precesses.
    - This movement is called precession, and any spinning object will experience it when acted on by a force
- Allow each student in the group to have a turn with the gyroscope. Each time the

gyroscope is used, you will have to rewrap the string
- You can give the students ideas for cool tricks to do with the gyroscope (most of these require the gyroscope to be spinning fast, so make sure the students to pull the string as fast as they can):
  - Tightrope: Balance the gyroscope on the string when it is being held taut
  - Balance on a pen: If you balance the gyroscope on the tip of a pen, it will precess on the tip
  - Balance on finger: Same idea with pen, but easier to do
  - Loop in string: If you make a loop in the string, you can put the bottom of the gyroscope in it and it will rotate around (sort of hard to explain, but pretty clear in the video below)
  - https://www.youtube.com/watch?v=p9zhP9Bnx-k
- After each mentee has taken his/her turn with the gyroscope, reconvene the class and begin the next module

### Additional Notes for Mentors
- This lesson is intended mostly just to show the students something that I think is really cool-- expect the students to have a fair amount of trouble conceptualizing what is going on
- The one concept that should be hammered home is precession. When angular momentum **can't** be conserved because of a force acting on a spinning object (in this case, because of gravity) objects will **precess**
- Also, you don't need to balance the gyroscope on a stand-- it will precess just as well if it is standing on the desk, and it usually precesses for longer too

# Module 3: Flying Gyros

## Introduction
In this module, the students will be using the ideas that they have learned about angular momentum to use, creating "flying gyroscopes" out of paper. Try to encourage competition among the students and see who can make one that goes the furthest.

### Background for Mentors
- The flying gyros work in much the same way as frisbees do-- When they spin, angular momentum is created in the direction of motion. This angular momentum is conserved, so the ring keeps spinning and tries to stay in the same plane of motion as it travels, stabilizing the ring in the air
  - There's also an added lift factor from Bernoulli's principle (like the frisbee) that we won't be getting into with the students

## Materials
- Paper (one per student, or more if they have problems folding or want to build more)

### Procedure

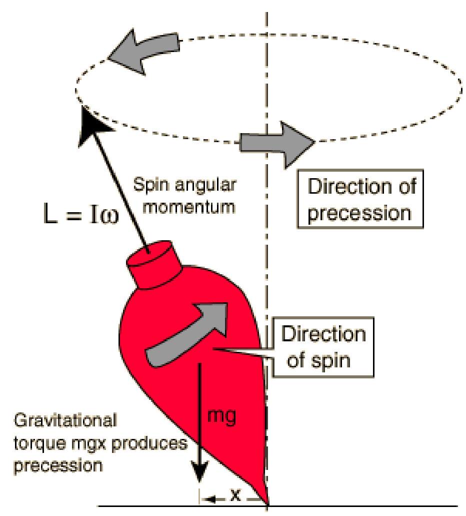
- The entire procedure is at this link with pictures: http://www.instructables.com/id/Flying-gyroscope-out-of-a-single-sheet-of-paper!!/
- Let the students know that this module will be a competition to see who can create the farthest flying gyro
  - As the module progresses, mentors can give tips on different ways to tweak the design to have the students see which will go faster:
    - Shorter folds to make the front of the ring denser
    - Longer body vs. shorter body
    - Larger radius vs. smaller radius
- Hold your piece of paper landscape-style
- Fold up the bottom edge ½” to 1” wide
  - Don’t make the folds too small, or your ring will be too short and too dense and will have trouble flying
- Keep folding over your first fold until you have the desired length of your ring: Should be ~6”
- Fold your paper into a cylinder, with the wrapped edges coming to each other
- Put the wrapped edges inside one another-- this is hard to explain, but when you are done, the paper should hold the ring shape
- Once the ring is completed, the students can throw it
  - Throw with the heavy side forward, and try to put some spin on it
  - Have them compare the distances when they spin it and when they don’t-- why does one go further?

## Module 4: Tops

### Introduction

The goal of this module is to create a top out of household items. Again, this module can be made into a competition between the students to see who can create the longest spinning top.

#### Background for Mentors




- Tops work much like gyroscopes: they have some spin angular momentum, and torque from gravity causes them to precess
- In this module, we are challenging the kids to make the longest spinning top
- Tops will fall down due to friction-- the less angular momentum they have, the quicker the friction will eat away at the angular momentum they have and eventually cause them to fall
- To make a long spinning top, we should be aiming to increase the angular momentum of our tops so that friction takes longer to take away the angular

momentum

- We can do this by changing the **moment of inertia** of the top
    - The moment of inertia ($I$) is the angular equivalent of mass: it is given by the formula:

$$I = \int r^2 dm$$

    - $r$ is radial distance and $dm$ is a super super small change in mass
    - All the integral sign means is that we have to sum over the whole distribution-- if I have a disk, the $r$ value changes with radius-- we have to consider all the different radii of the disk
    - From this formula, is seems that we can change our $I$ value in 2 ways:
        a. Increase the mass ($dm$)
        b. Increase the radius ($r$) by moving mass further away from the axis
    - So, to maximize the moment of inertia of a disk, we should add mass to the outermost edge where $r$ is greatest
        - For example, a ring and a wheel of the same mass and radius will have different moments of inertia. The ring has all its mass on the outside, while the wheel has some of that mass on the inside-- this means that on average, the mass is further from the axis for the ring than the wheel
    - For this module, if we increase the moment of inertia too much the tops will be hard to spin. This will lead to them having **less** angular momentum than before because they will be spinning slower, so have the students try to find the sweet spot that maximizes rotation time!
- Eventually, the top will fall due to the torque from the pencil. If a smaller pencil is used, there will be less torque and it will rotate longer

## Materials

- Paper plates (1/student)
- Golf pencils (1/student)
- Paper
- Pennies
- Tape

## Procedure

- Start this lesson by reiterating the things we've learned about angular momentum so far
    - Every spinning object has angular momentum
    - Angular momentum is conserved
    - Angular momentum depends on the speed of spinning and the mass of the object
- Explain that we are going to build a top in this lesson, and hand the materials out to each student

    - At this point, you can introduce the challenge to the students of creating the longest spinning top
- Form smaller groups with the students, and assign a mentor to each group. They will each walk through the module with their respective groups
- To create the basic model of the top, poke the golf pencil through the center of the paper plate, and tape it there if necessary
    - Let the students spin the tops. *Make sure to put paper down under the tops so the pencil lead doesn't get all over the desk*
    - Explain that the paper plate increases the angular momentum of the pencil, so when we spin it the system will want to stay spinning
- **For middle schools,** you can explain the moment of inertia to the students; the angular momentum doesn't just depend on the mass, but how the mass is placed. If it's further from the axis, it will increase the angular momentum more
    - How does changing the radial distance of the pennies affect the duration of spinning?
- **For elementary schools**, you should explain that adding weight to the top increases the angular momentum. The more angular momentum, the longer the top will spin
- Have the students try these out with pennies, and see who can use it to maximize the top's spin time

**Additional Notes for Mentors**
- Tops are again trying to reinforce the idea of precession that we explored in module 1
- Some students might have trouble spinning the tops after we build them (unfortunately, they're harder to spin than a store bought top), so be prepared to sit down with them and help them out. As they add more pennies, the tops will also get harder to spin because of the increased angular momentum

## References


- http://hyperphysics.phy-astr.gsu.edu/hbase/mechanics/bicycle.html
- https://www.youtube.com/watch?v=OpQk_apgnMQ
- http://www.explora.us/en/visit/try-this/18-try-this-put-a-spin-on-it
- http://physics.stackexchange.com/questions/9805/how-do-you-explain-spinning-tops-to-a-nine-year-old
- https://www.youtube.com/watch?v=ty9QSiVC2g0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## Summary Materials Table

| Material | Amount per Site | Expected $$ | Vendor (or online link) |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Frisbees | 1 | Inventory? | |
| Gyroscopes | 4 | $10 for 2 gyros. Can be reused across sites | https://www.amazon.com/Originial-TEDCO-Gyroscope-Twin-Pak/dp/B000FGKHZ2/ref=pd_sim_21_5?ie=UTF8&dpID=51UYBOrP%2BeL&dpSrc=sims&preST=_AC_UL160_SR160%2C160_&psc=1&refRID=MNXFXKPVZNSAJQH3PFSP |
| Paper plates | 20 | $5/85 | Staples |
| Paper | 20 | $0 | Staples |
| Golf pencils | 20 | $10/144 | Staples |
| Pennies | 150 | Can be reused | |
| Masking Tape | | | |