Decentralized Finance

Privacy on the Blockchain

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Can we have private transactions on a public blockchain?

Naïve reasoning:

universal verifiability ⇒ transaction data must be public.

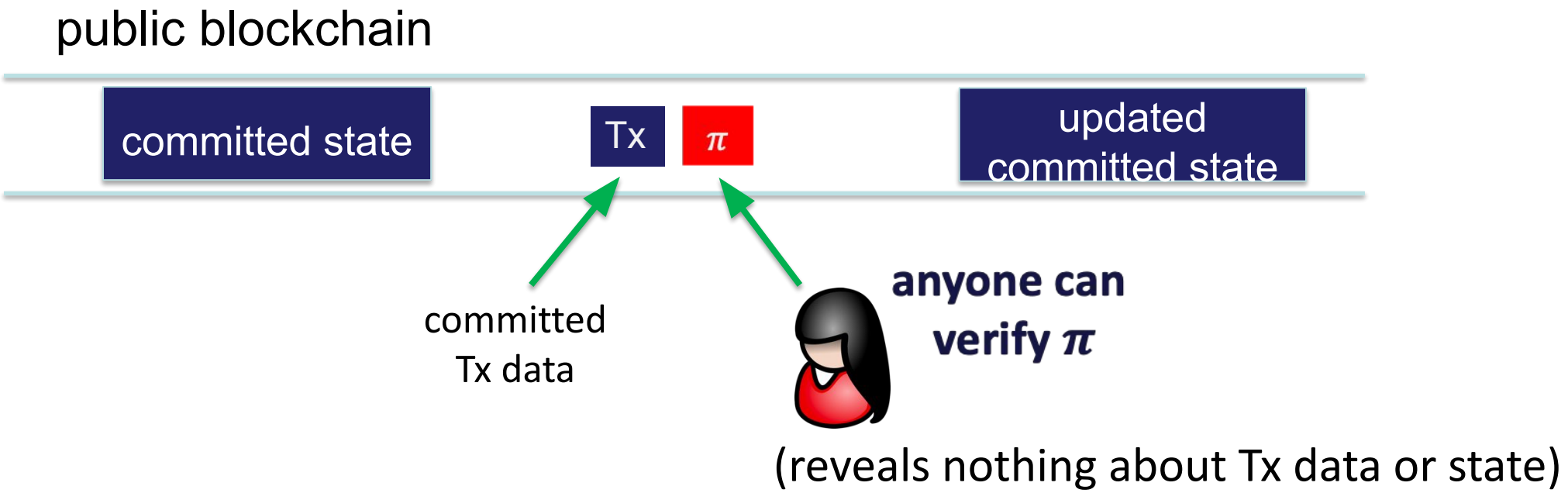
otherwise, how we can verify Tx ??

Goal for this lecture:

crypto magic ⇒ private Tx on a publicly verifiable blockchain

Crypto tools: **commitments** and **zero knowledge proofs**

Private Tx with universal verifiability: how?



Committed data: short (hiding) commitment on chain

Proof π: succinct *zero-knowledge proof* that

(1) committed Tx data is consistent with committed current state, and

(2) committed new state is correct

The need for privacy in the financial system

- Supply chain privacy:

  A car company does not want to reveal
  how much it pays its supplier for tires, wipers, etc.

- Payment privacy:
  - A company that pays its employees in crypto needs to keep list of employees and their salaries private.
  - Privacy for rent, donations, purchases

- Business logic privacy:

  Can the code of a smart contract be private?

Types of Privacy

- **Pseudonymity: (weak privacy)**
  - One consistent pseudonym (e.g. reddit)
    - Pros: Reputation
    - Cons: Linkable posts: one post linked to you $\Longrightarrow$ all posts linked to you

- **Full anonymity:**
  - User's transactions are unlinkable
    - The system cannot tell if two transactions are from the same person
  - Maintaining reputation is possible but more complex

Privacy in Ethereum?

- Accounts:
  - Every account balance is public
  - For Dapps: code and internal state are public
  - All account transactions are linked to account

etherscan.io:

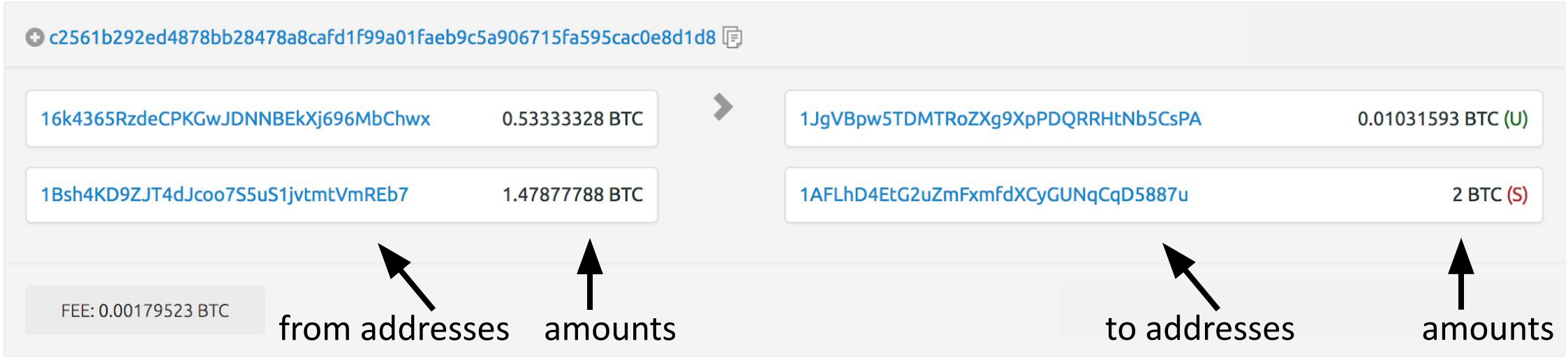
Address 0x1654b0c3f62902d7A86237

…

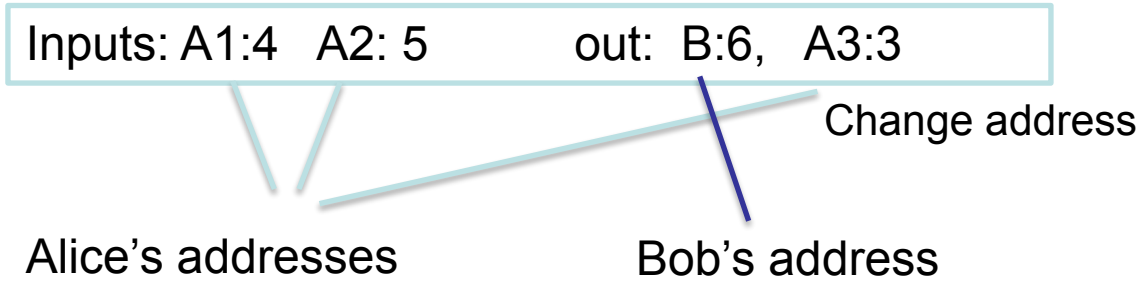
Balance:	1.114479450024297906 Ether
Ether Value:	$4,286.34 (@ $3,846.05/ETH)

	Txn Hash	Method ⓘ	Block
👁	0x0269eff8b4196558c07…	Set Approval For…	13426561
👁	0xa3dacb0e7c579a99cd…	Cancel Order_	13397993
👁	0x73785abcc7ccf030d6a…	Set Approval For…	13387834
👁	0x1463293c495069d61c…	Atomic Match_	13387703

Privacy in Bitcoin?



Alice can have many addresses (creating address is free)



Transaction data can be used
to link addresses to a single owner
and to a physical entity

(chainalysis)

Privacy of Digital Payments

Payments publicly visible and linkable

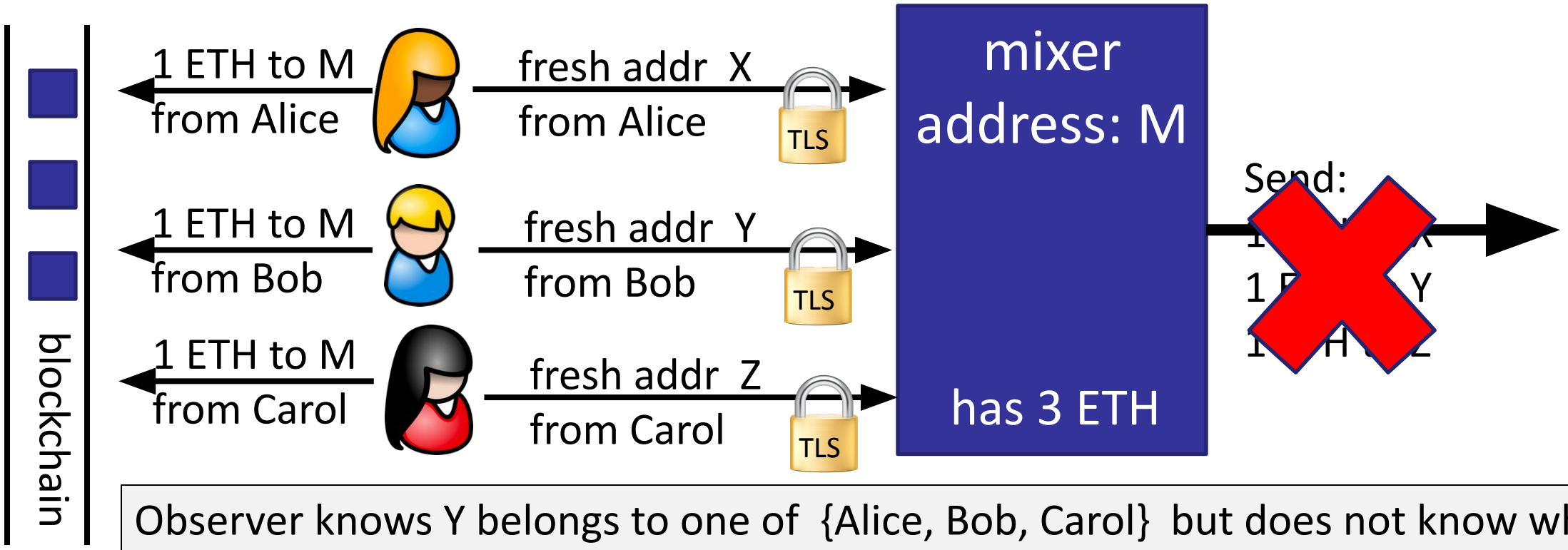
Payments only visible to bank

private payments

Less private

More private

Simple blockchain anonymity via mixing



Observer knows Y belongs to one of {Alice, Bob, Carol} but does not know which one

$\Longrightarrow$ anonymity set of size 3.

$\Longrightarrow$ Bob can mix again with different parties to increase anonymity set.

Problems: (i) mixer knows all, (ii) mixer can abscond with 3 ETH !!

Mixing without a mixer? on Bitcoin: **CoinJoin** (e.g., Wasabi), on Ethereum: **Tornado cash**

Negative aspects of privacy in finance

Criminal activity:

- Tax evasion, ransomware, …

Can we support positive applications of private payments, but prevent the negative ones?

- Can we ensure legal compliance while preserving privacy?
- Yes! With proper use of zero knowledge proofs

Next segment: commitments

An important tool


Cryptographic
Commitments
https://defi-learning.org/

Cryptographic commitments

Cryptographic commitment: emulates an envelope

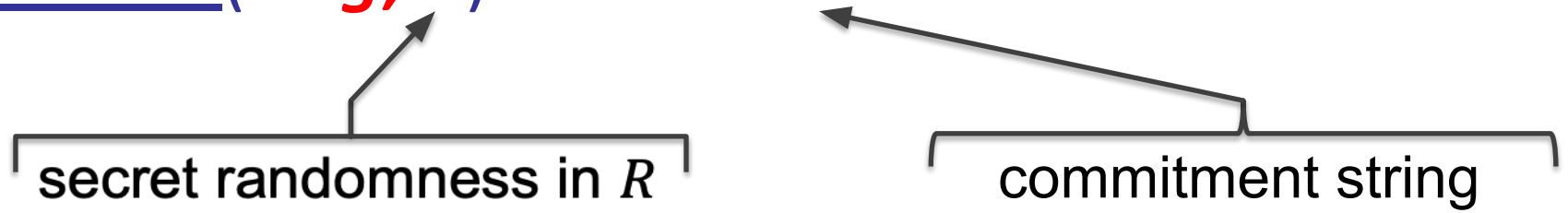
Many applications: e.g., a DAPP for a sealed bid auction

- Every participant **commits** to its bid,
- Once all bids are in, everyone opens their commitment

Cryptographic Commitments

Syntax: a commitment scheme is two algorithms

- **commit**(*msg*, *r*) → *com*



- **verify**(*msg*, *com*, *r*) → accept or reject

anyone can verify that commitment was opened correctly

Commitments: security properties

- **binding**: Bob cannot produce two valid openings for ***com***.

  More precisely: no efficient adversary can produce

  $$\textbf{\textit{com}},\ (m_1, r_1),\ (m_2, r_2)$$

  such that verify(m_1, ***com***, r_1) = verify(m_2, ***com***, r_2) = accept

  and $m_1 \neq m_2$.

- **hiding**: ***com*** reveals nothing about committed data

  commit(m, r) $\rightarrow$ ***com***, and r is sampled uniformly in R,

  then ***com*** is statistically independent of m

Example 1: hash-based commitment

Fix a hash function $H: M \times R \rightarrow C$ (e.g., SHA256)

where H is collision resistant, and $|R| \gg |C|$

- commit($m \in M$, $r \leftarrow R$): $\boldsymbol{com} = H(m, r)$
- verify(m, $\boldsymbol{com}$, r): accept if $\boldsymbol{com} = H(m, r)$

binding: follows from collision resistance of H

hiding: follows from a mild assumption on H

Example 2: Pedersen commitment

G = finite cyclic group = $\{1, g, g^2, \ldots, g^{q-1}\}$ where $g^i \cdot g^j = g^{(i+j \bmod q)}$

q = $|G|$ is called the **order** of G. Assume q is a prime number.

Fix g, h in G and let $R = \{0, 1, \ldots, q-1\}$. For m, r $\in R$ define

$$H(m, r) = g^m \cdot h^r \in G$$

Fact: for a "cryptographic" group G, this H is collision resistant.

$\Rightarrow$ commitment scheme: **commit** and **verify** as in example 1

$$\textbf{commit}(m \in R,\ r \leftarrow R) = H(m, r) = g^m \cdot h^r$$

An interesting “homomorphic” property

- $\textbf{commit}(m \in R,\ r \leftarrow R) = H(m,r) = g^m \cdot h^r$

Suppose: $\text{commit}(m_1 \in R,\ r_1 \leftarrow R) \rightarrow com_1$

$\text{commit}(m_2 \in R,\ r_2 \leftarrow R) \rightarrow com_2$

Then: $com_1 \times com_2 = g^{m1+m2} \cdot h^{r1+r2} = \text{commit}(\underline{m_1+m_2},\ r_1+r_2)$

$\Rightarrow$ anyone can sum committed value

Next segment: zero knowledge proofs

An important privacy tool


What is a zk-SNARK?
Succinct Non-interactive ARgument of Knowledge
https://defi-learning.org/

zk-SNARK: Blockchain Applications

Scalability:

- SNARK Rollup (zk-SNARK for privacy from public)

Privacy: Private Tx on a public blockchain

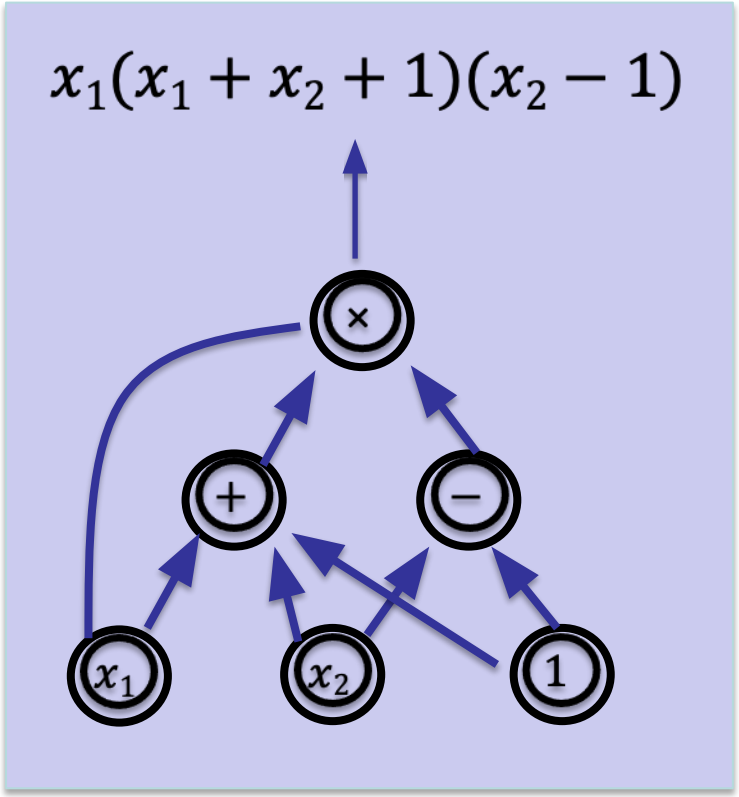
- Confidential transactions
- Tornado cash
- Private Dapps: Aleo

Compliance:

- Proving solvency in zero-knowledge
- Zero-knowledge taxes

(1) arithmetic circuits

- Fix a finite field $\mathbb{F} = \{0, \ldots, p-1\}$ for some prime p>2.
- **Arithmetic circuit**: $C: \mathbb{F}^n \to \mathbb{F}$
  - directed acyclic graph (DAG) where
    - internal nodes are labeled +, −, or ×
    - inputs are labeled $1, x_1, \ldots, x_n$
  - defines an n-variate polynomial with an evaluation recipe
- $|C|$ = # gates in C



Interesting arithmetic circuits

Examples:

- $C_{hash}(h, \mathbf{m})$: outputs 0 if SHA256($\mathbf{m}$) = h , and ≠0 otherwise

  $C_{hash}(h, \mathbf{m}) = \big(h - \text{SHA256}(\mathbf{m})\big)$, $|C_{hash}| \approx$ 20K gates

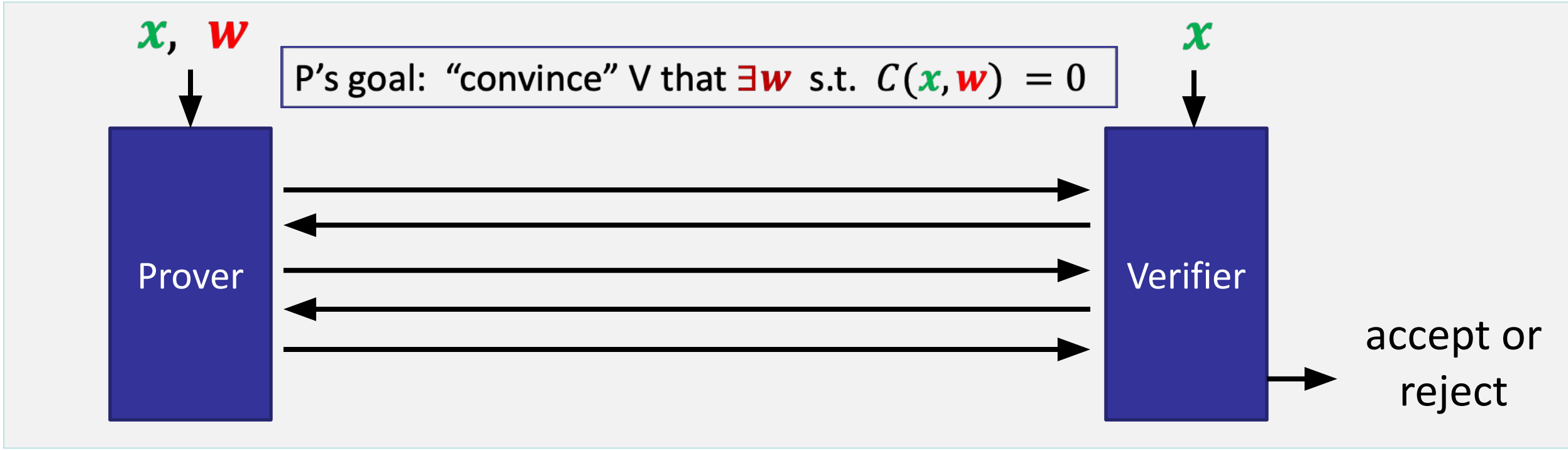
- $C_{sig}(pk, m, \sigma)$: outputs 0 if σ is a valid ECDSA signature on m with respect to pk

(2) Argument systems (for NP)

Public arithmetic circuit: $C(x, w) \rightarrow \mathbb{F}$

public statement in $\mathbb{F}^n$

secret witness in $\mathbb{F}^m$



Two types of argument systems: interactive vs. non-interactive

Interactive: proof takes multiple P↔V rounds of interaction

- Useful when there is a single verifier, e.g. a compliance auditor
- Example: zero-knowledge proof of taxes to tax authority

Non-interactive: prover sends a single message (proof) to verifier

- Used when many verifiers need to verify proof, e.g., Rollup systems
- SNARK: short proof that is fast to verify

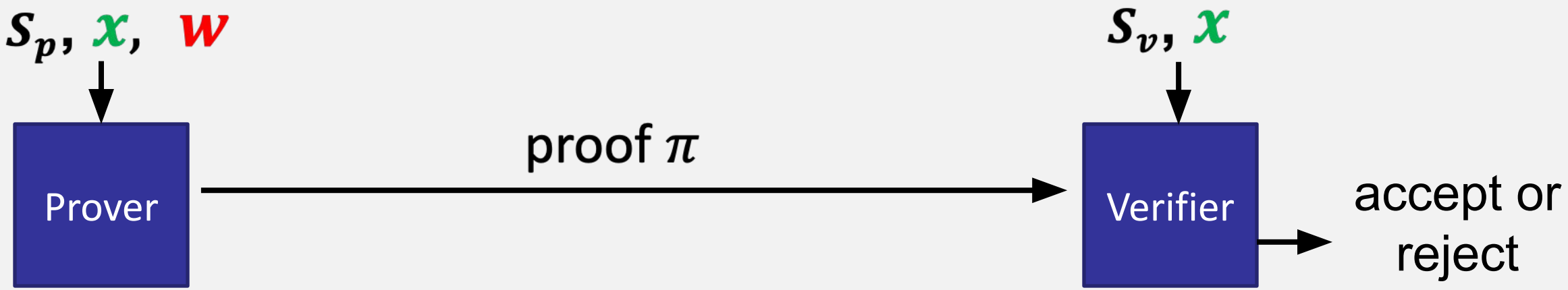
(non-interactive) Preprocessing argument system

Public arithmetic circuit: $C(x, w) \rightarrow \mathbb{F}$

public statement in $\mathbb{F}^n$

secret witness in $\mathbb{F}^m$

Preprocessing (setup): $\mathbf{S}(C) \rightarrow$ public parameters (S_p, S_v)



Preprocessing argument System

A non-interactive argument system is a triple (S, P, V):

- **S**$(C) \rightarrow$ public parameters (S_p, S_v) for prover and verifier
- **P**$(S_p, \boldsymbol{x}, \boldsymbol{w}) \rightarrow$ proof π
- **V**$(S_v, \boldsymbol{x}, \boldsymbol{\pi}) \rightarrow$ accept or reject

Argument system: requirements (informal)

Prover P(S_p, x, w) — Verifier V (S_v, x, π)

proof π →

accept or reject

Complete: $\forall x, w$: $C(x, w) = 0$ ⇒ Pr[V(S_v, x, P(S_p, x, w)) = accept] = 1

Argument of knowledge: V accepts ⇒ P "knows" w s.t. $C(x, w) = 0$

P* does not "know" w ⇒ Pr[V(S_v, x, π) = accept] < negligible

Optional: **Zero knowledge**: (S_v, x, π) "reveals nothing" about w

Preprocessing **SNARK**

A succinct non-interactive argument system is a triple (S, P, V):

- **S**$(C) \rightarrow$ public parameters (S_p, S_v) for prover and verifier
- **P**$(S_p, \boldsymbol{x}, \boldsymbol{w}) \rightarrow$ short proof π ; $|\pi| = O(\log(|\boldsymbol{C}|), \lambda)$
- **V**$(S_v, \boldsymbol{x}, \boldsymbol{\pi}) \rightarrow$ accept or reject ; time(V) = $O(|x|, \log(|\boldsymbol{C}|), \lambda)$

short "summary" of circuit

Why preprocess C??

Preprocessing **SNARK**

A succinct non-interactive argument system is a triple (S, P, V):

- **S**$(C) \rightarrow$ public parameters (S_p, S_v) for prover and verifier
- **P**$(S_p, \boldsymbol{x}, \boldsymbol{w}) \rightarrow$ short proof π ; $|\pi| = O(\mathbf{log}(|\boldsymbol{C}|), \lambda)$
- **V**$(S_v, \boldsymbol{x}, \boldsymbol{\pi}) \rightarrow$ accept or reject ; time(V) = $O(|x|, \mathbf{log}(|\boldsymbol{C}|), \lambda)$

If (S, P, V) is **succinct** and **zero-knowledge** then we say that it is a **zk-SNARK**

The trivial argument system

(a) Prover sends $\boldsymbol{w}$ to verifier,

(b) Verifier checks if $C(\boldsymbol{x}, \boldsymbol{w}) = 0$ and accepts if so.

Problems with this:

(1) $\boldsymbol{w}$ might be secret: prover does not want to reveal $\boldsymbol{w}$ to verifier

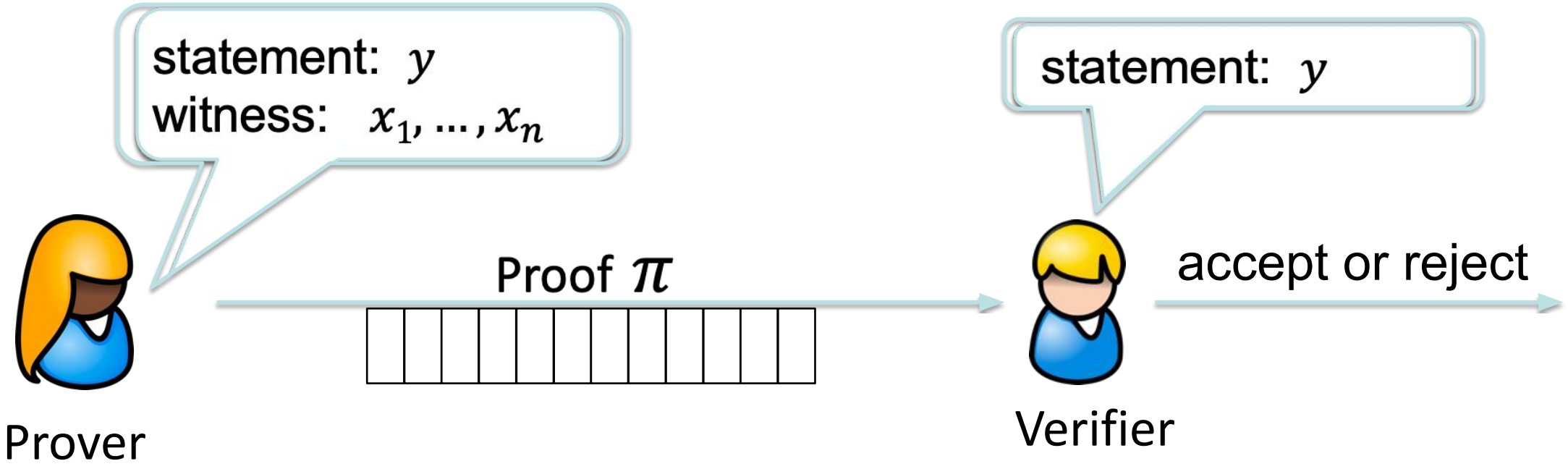
(2) $\boldsymbol{w}$ might be long: we want a "short" proof

(3) computing $C(\boldsymbol{x}, \boldsymbol{w})$ may be hard: we want a "fast" verifier

An example

Prover: I know $(x_1, \ldots, x_n) \in X$ such that $H(x_1, \ldots, x_n) = y$

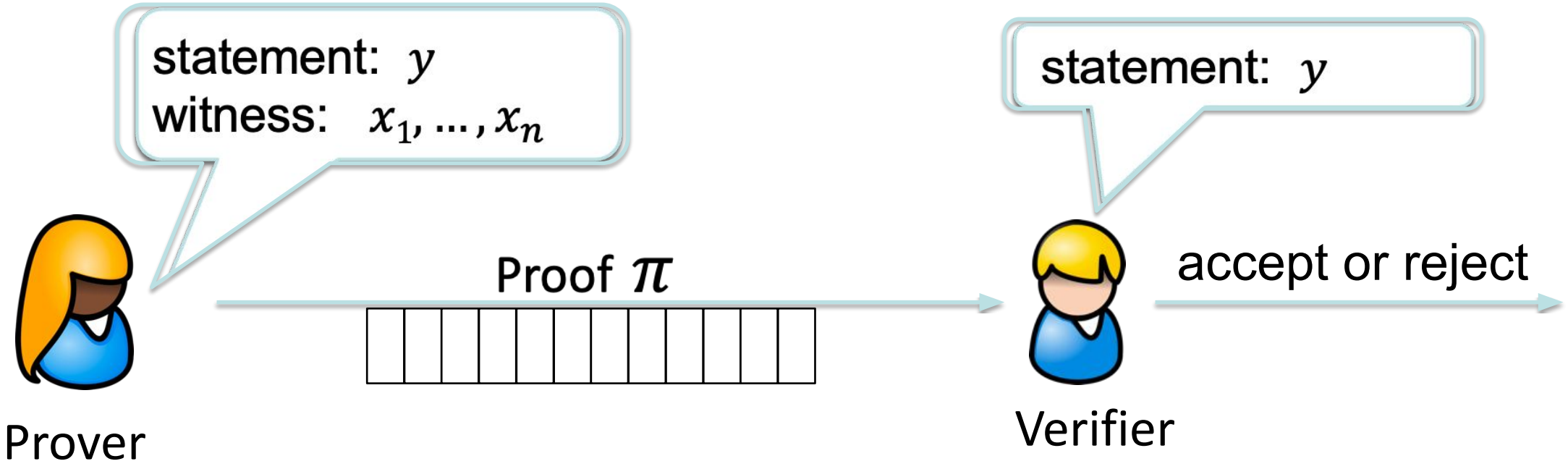
SNARK: $\text{size}(\pi)$ and $\text{VerifyTime}(\pi)$ is $O(\log n)$!!



An example

How is this possible ???

SNARK: $\text{size}(\pi)$ and $\text{VerifyTime}(\pi)$ is $O(\log n)$!!



Types of preprocessing Setup

Recall setup for circuit C: $\mathbf{S}(C) \rightarrow$ public parameters (S_p, S_v)

Types of setup:

better

trusted setup per circuit: $\mathbf{S}(C)$ uses data that must be kept secret

compromised trusted setup $\Rightarrow$ can prove false statements

trusted but universal (updatable) setup: secrets in $\mathbf{S}(C)$ are independent of C

$\boldsymbol{S} = (S_{init}, S_{pre})$: $\underbrace{S_{init}(\lambda) \rightarrow U,}_{\text{one-time}}$ $\underbrace{S_{pre}(U, C) \rightarrow (S_p, S_v)}_{\text{no secret data}}$

transparent setup: $\mathbf{S}(C)$ does not use secret data (no trusted setup)

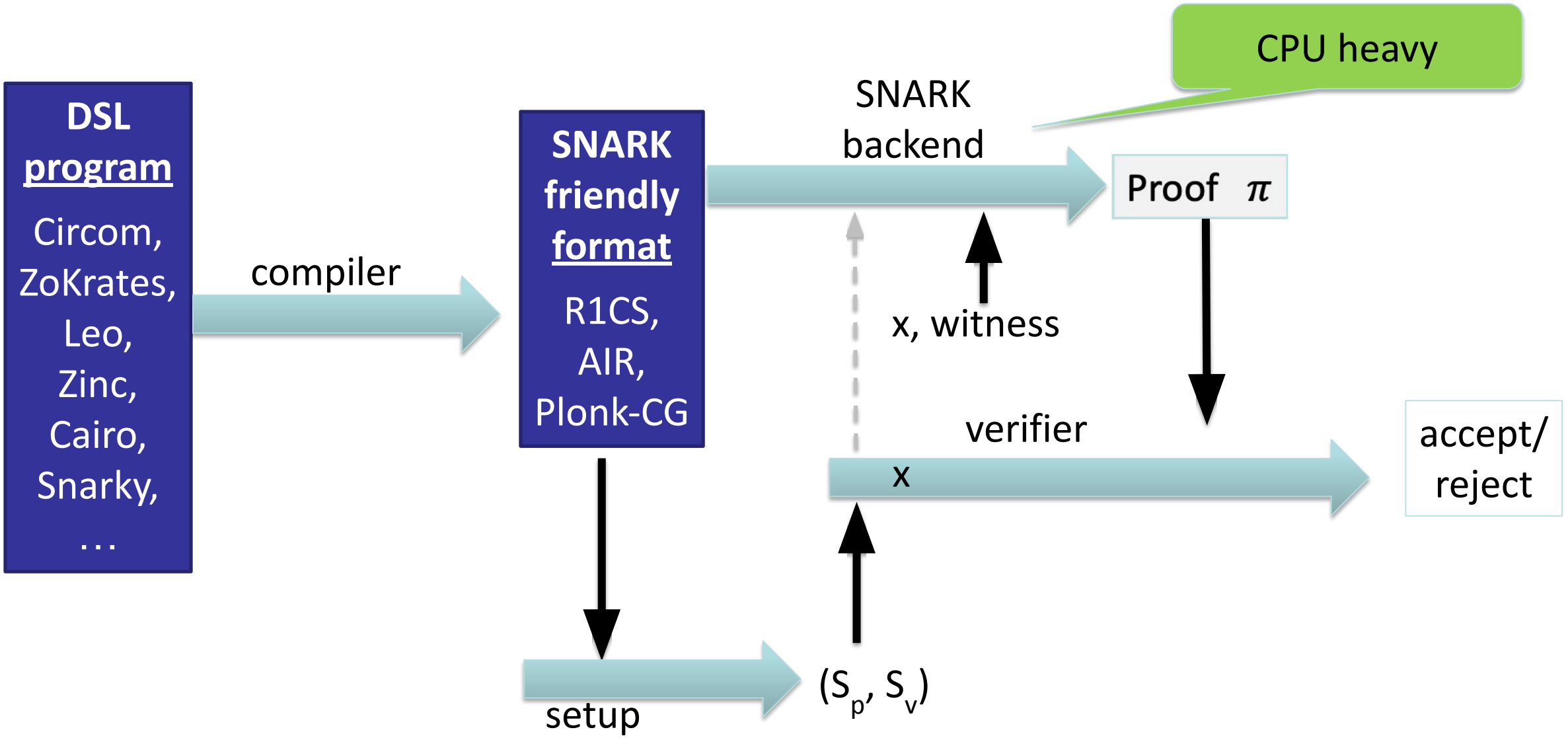
Significant progress in recent years

- **Kilian'92, Micali'94**: succinct transparent arguments from PCP
  - impractical prover time
- **GGPR'13, Groth'16, …**: linear prover time, **constant size proof** $(O_\lambda(1))$
  - **trusted setup per circuit** (setup alg. uses secret randomness)
  - compromised setup ⇒ proofs of false statements
- **Sonic'19, Marlin'19, Plonk'19**, … : universal trusted setup
- **DARK'19, Halo'19, STARK**, … : no trusted setup (transparent)

Types of SNARKs (partial list)

		size of S_p	verifier time	trusted setup?
Groth'16	O(1)		O(1)	yes/per circuit
Plonk/Marlin	O(1)		O(1)	yes/universal
Bulletproofs		O(1)		no
STARK		O(1)		no
DARK		O(1)		no
⋮	⋮			⋮

A SNARK software system



ZoKrates Example

Goal: prove knowledge of a hash (SHA256) preimage for a given $x \in \{0,1\}^{256}$

- For a public x, prover knows $w \in \mathbb{F}_p$ such that SHA256(w) = x
- $\mathbb{F}_p$ is a 254-bit prime field

Compiled into an arithmetic circuits (R1CS) over $\mathbb{F}_p$

```
def main(field x[2],   private field w) -> (field):
   h = sha256packed( w )
   h[0] == x[0]      //  check top 128 bits
   h[1] == x[1]      //  check bottom 128 bits
   return 1
```

How to define “argument of knowledge” and “zero knowledge”?

Definitions: (1) argument of knowledge

Goal: if V accepts then P "knows" $\boldsymbol{w}$ s.t. $C(\boldsymbol{x}, \boldsymbol{w}) = 0$

What does it mean to "know" $\boldsymbol{w}$??

informal def: P knows $\boldsymbol{w}$, if $\boldsymbol{w}$ can be "extracted" from P

P

Definitions: (1) argument of knowledge

Formally: (S, P, V) is an argument **of knowledge** for a circuit C if for every poly. time adversary $A = (A_0, A_1)$ such that

$S(C) \rightarrow (S_p, S_v)$, $(x, st) \leftarrow A_0(S_p)$, $\pi \leftarrow A_1(S_p, x, st)$:

$\Pr[V(S_v, x, \pi) = \text{accept}] > 1/10^6$ (non-negligible)

there is an efficient **extractor** E (that uses A_1 as a black box) s.t.

$S(C) \rightarrow (S_p, S_v)$, $(x, st) \leftarrow A_0(S_p)$, $w \leftarrow E^{A_1(S_p, x, st)}(S_p, x)$:

$\Pr[C(x, w) = 0] > 1/10^6$ (non-negligible)

If holds for **all** A, then (S, P, V) is a **proof of knowledge**.

Definitions: (2) Zero knowledge (against an honest verifier)

- (S, P, V) is **zero knowledge** if for every $x \in \mathbb{F}^n$

  proof π "reveals nothing" about $\boldsymbol{w}$, other than its existence

What does it mean to "reveal nothing" ??

Informal def: π "reveals nothing" about $\boldsymbol{w}$ if the verifier can generate π **by itself** $\Longrightarrow$ it learned nothing new from π

- (S, P, V) is **zero knowledge** if there is an efficient alg. **Sim** s.t. $(S_p, S_v, \pi) \leftarrow \boldsymbol{Sim}(C, x)$ "look like" the real S_p, S_v and π.

Main point: $\boldsymbol{Sim}(C, x)$ simulates π without knowledge of $\boldsymbol{w}$

Definitions: (2) Zero knowledge (against an honest verifier)

Formally: (S, P, V) is (honest verifier) **zero knowledge** for a circuit C

if there is an efficient simulator ***Sim*** such that

for all $x \in \mathbb{F}^n$ s.t. $\exists w: C(x,w) = 0$ the distribution:

(S_p, S_v, x, π): where $(S_p, S_v) \leftarrow S(C)$, $\pi \leftarrow P(S_p, x, \boldsymbol{w})$

is indistinguishable from the distribution:

(S_p, S_v, x, π): where $(S_p, S_v, \pi) \leftarrow \boldsymbol{Sim}(C, x)$

How to build a zk-SNARK?

Recall: A zero knowledge preprocessing argument system.

Prover generates a **short** proof that is **fast** to verify

How to build a zk-SNARK ??

Not in this course …

(see, e.g., cs251)

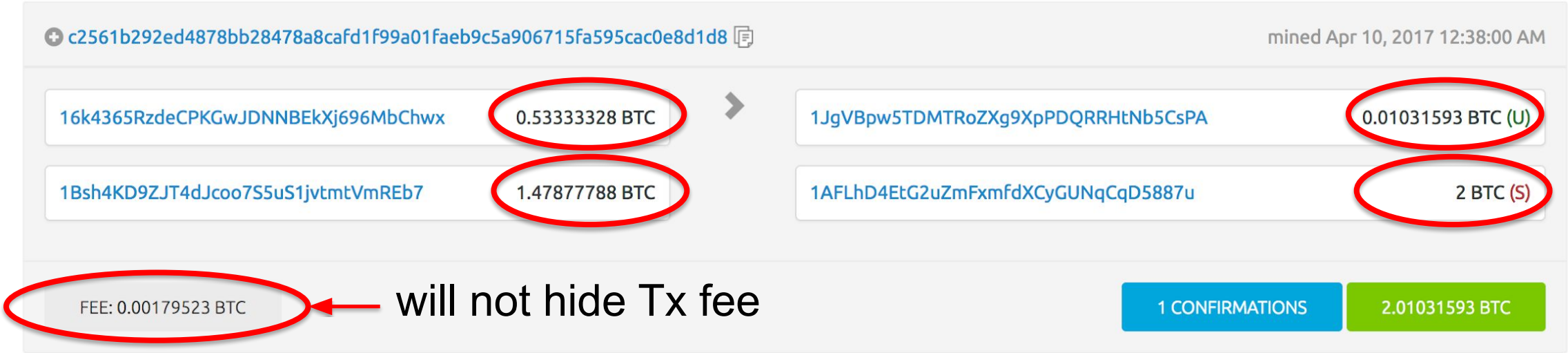
Next segment: confidential transactions


Private Tx Warmup:
Confidential
Transactions
https://defi-learning.org/

Confidential Transactions (CT)

Current Bitcoin Tx expose full payment details:

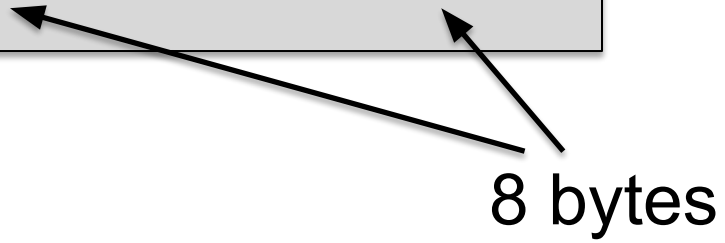
$\Longrightarrow$ Businesses cannot use Bitcoin for supply chain payments, salaries, etc.



Goal: hide amounts in Bitcoin transactions.

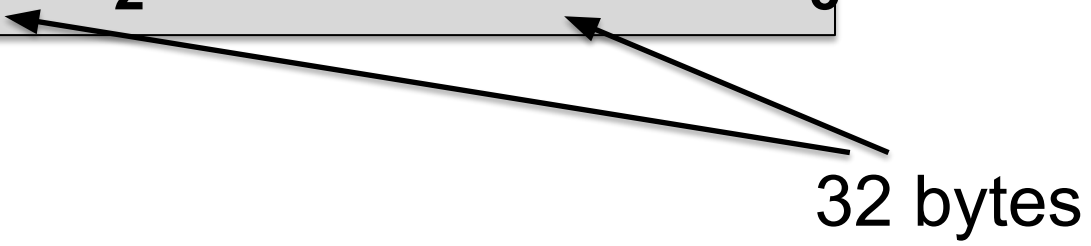
Confidential Tx: how?

Bitcoin Tx today: AcmeCo: **30** → Alice: **1**, AcmeCO: **29**



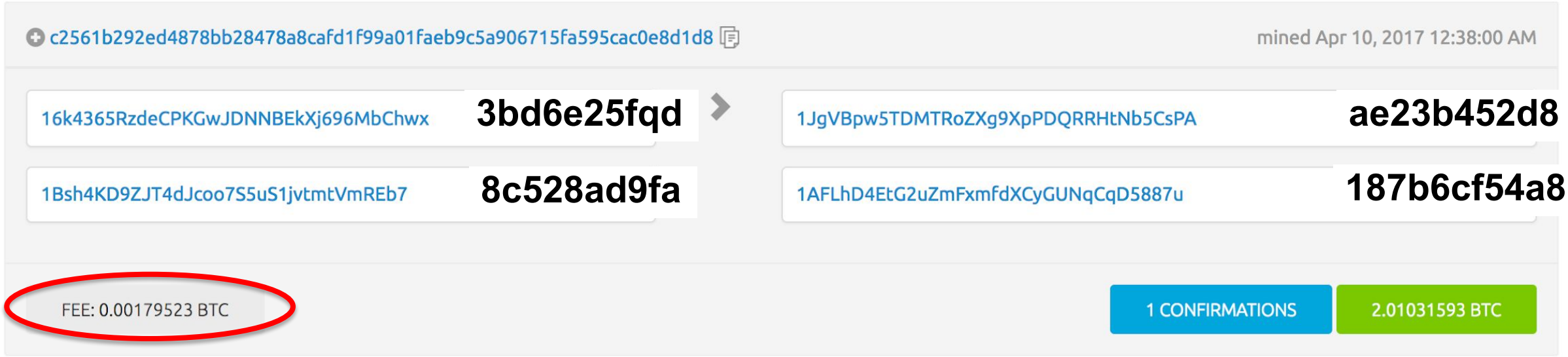
The plan: replace amounts by commitments to amounts

AcmeCo: $\mathbf{com_1}$ → Alice: $\mathbf{com_2}$, AcmeCo: $\mathbf{com_3}$



where $\mathbf{com_1} = \text{commit}(30, r_1)$, $\mathbf{com_2} = \text{commit}(1, r_2)$, $\mathbf{com_3} = \text{commit}(29, r_3)$

Now blockchain hides amounts



How much was transferred ???

The problem: how can miners verify Tx?

AcmeCo: $\mathbf{com_1}$ → Alice: $\mathbf{com_2}$, AcmeCo: $\mathbf{com_3}$

$\mathbf{com_1}$ = commit(m_1=30, r_1), $\mathbf{com_2}$ = commit(m_2=1, r_2), $\mathbf{com_3}$ = commit(m_3=29, r_3)

Solution: zk-SNARK (special purpose, optimized for this problem)

- AcmeCo: (1) privately send r_2 to Alice
  (2) construct a proof π for

statement = x = ($\mathbf{com_1}$, $\mathbf{com_2}$, $\mathbf{com_3}$, **Fees**)
witness = w = (m_1, r_1, m_2, r_2, m_3, r_3)

where circuit $C(x,w)$ outputs 0 iff:

CT arithmetic circuit
- (i) $\mathbf{com_i}$ = commit(m_i, r_i) for i=1,2,3,
- (ii) $m_1 = m_2 + m_3 + \text{Fees}$,
- (iii) $m_2 \geq 0$ and $m_3 \geq 0$

The problem: how can miners verify Tx?

AcmeCo: (1) privately send r_2 to Alice

(2) construct a ZK proof π that Tx is valid

(3) embed π in Tx (need short proof! ⇒ zk-SNARK)

Tx: proof π , AcmeCo: **com$_1$** → Alice: **com$_2$**, AcmeCo: **com$_3$**

Miners: accept Tx if proof π is valid (need fast verification)

⇒ learn Tx is valid, but amounts are hidden

Optimized proof?

circuit $C(x,w)$ outputs 0 if:

(i) $com_i = commit(m_i, r_i)$,

~~(ii) $m_1 = m_2 + m_3 + Fees$,~~

(iii) $m_2 \geq 0$ and $m_3 \geq 0$

Easy to check with Pedersen commitment:

set $com \leftarrow com_1/com_2 \cdot com_3 \cdot g^{Fees}$

(a commitment to $m_1 - m_2 - m_3 - Fees$)

prove that $com = commit(0, r)$

remaining proof is ≈400 bytes

(CT is the beginning of MimbleWimble implemented in the Grin blockchain)

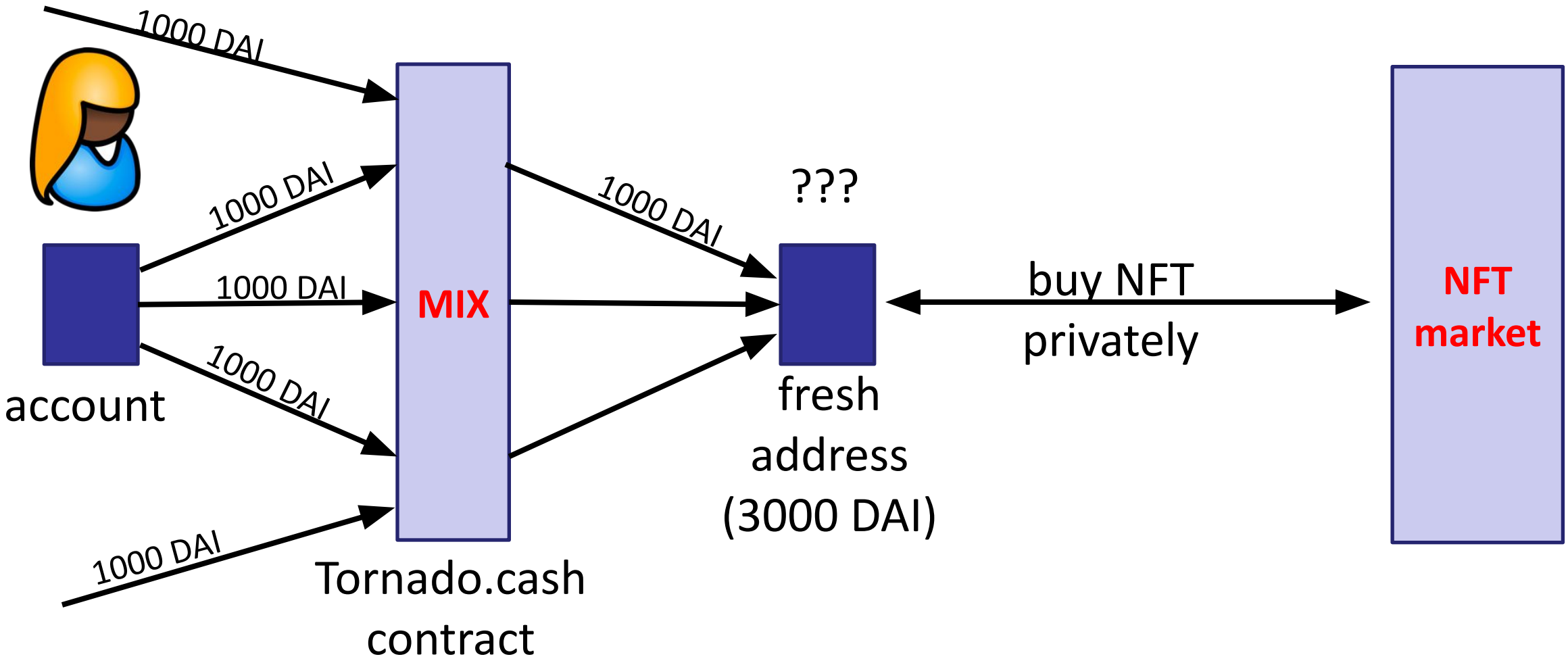
Next segment: anonymous payments


Anonymous Payments:
Tornado Cash and Zcash / IronFish
https://defi-learning.org/

TORNADO CASH: A ZK-BASED MIXER

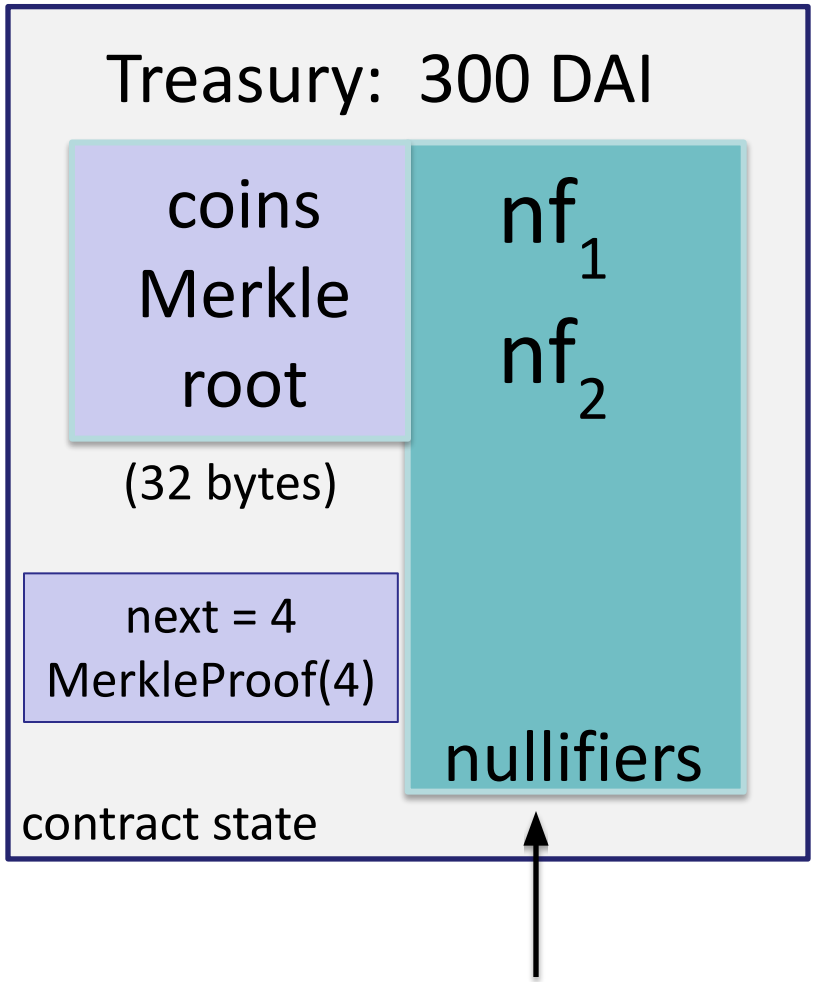
Launched on the Ethereum blockchain on May 2020 (v2)

Tornado Cash: a ZK-mixer

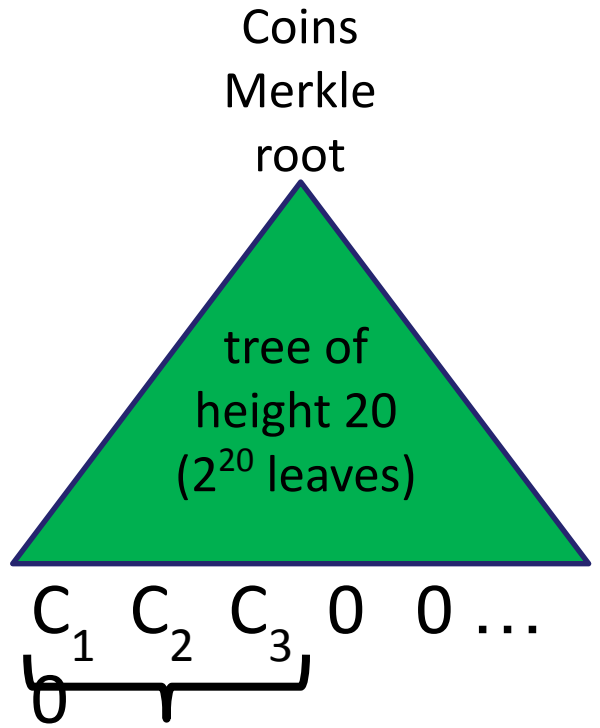


The tornado cash contract (simplified)

<u>100 DAI pool</u>:
each coin = 100 DAI



$H_1, H_2: R \to \{0,1\}^{256}$



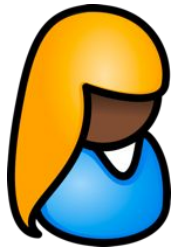
Currently:
- three coins in pool
- contract has 300 DAI
- two nullifiers stored

explicit list:
one entry per **spent coin**

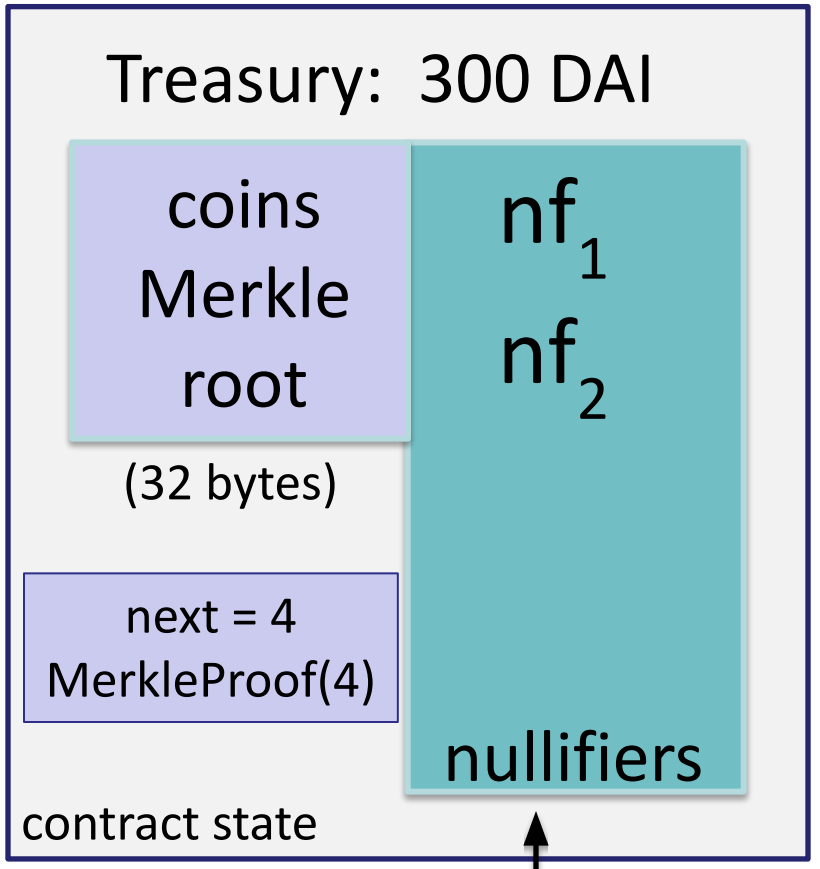
Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:

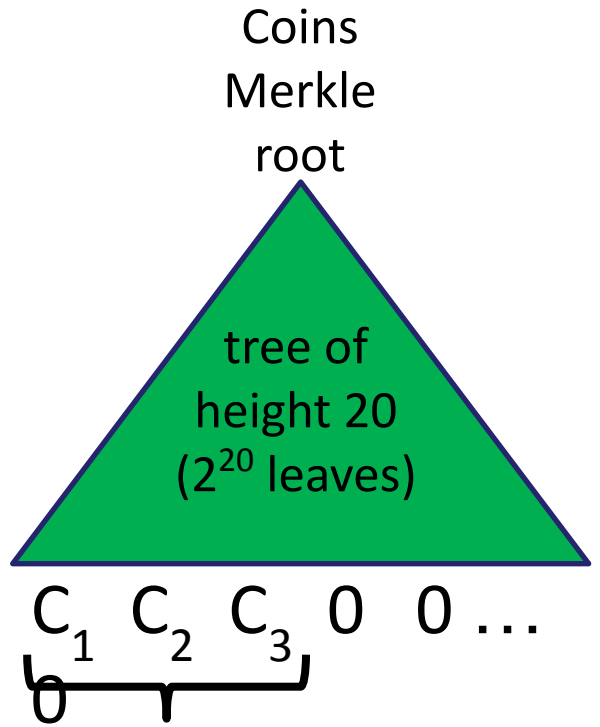


choose random k, r in R
set $C_4 = H_1(k, r)$
write C_4 in leaf #4 in tree
π = MerkleProof(5)



explicit list:
one entry per **spent coin**

$H_1, H_2: R \to \{0,1\}^{256}$

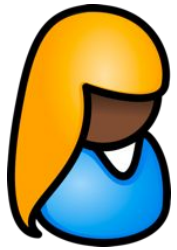


public list of coins

Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:



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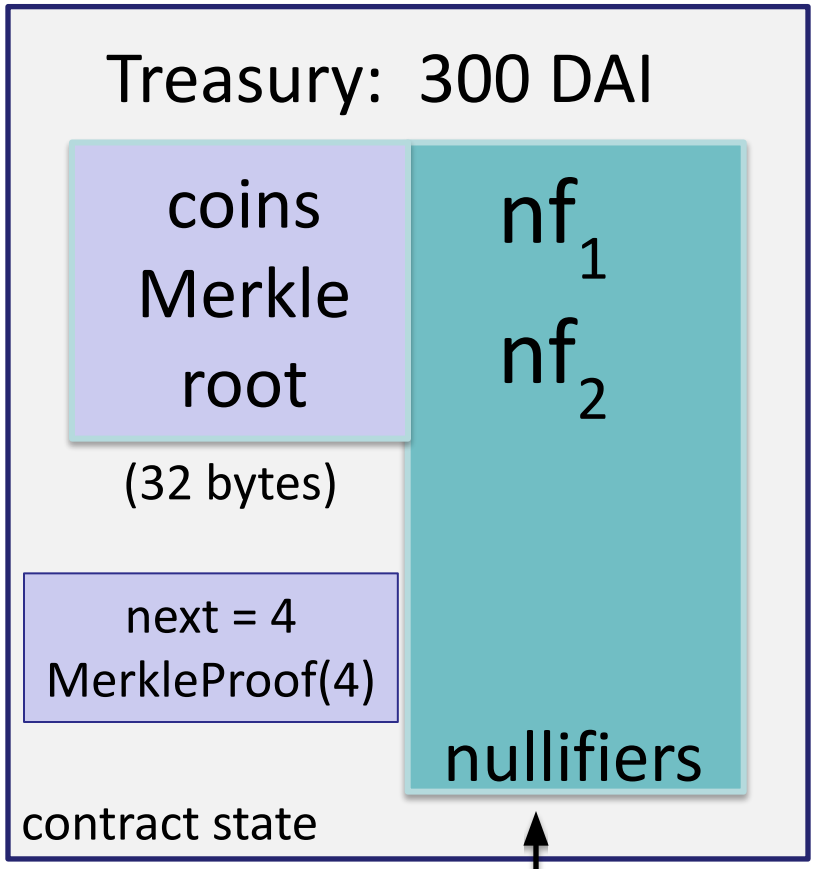
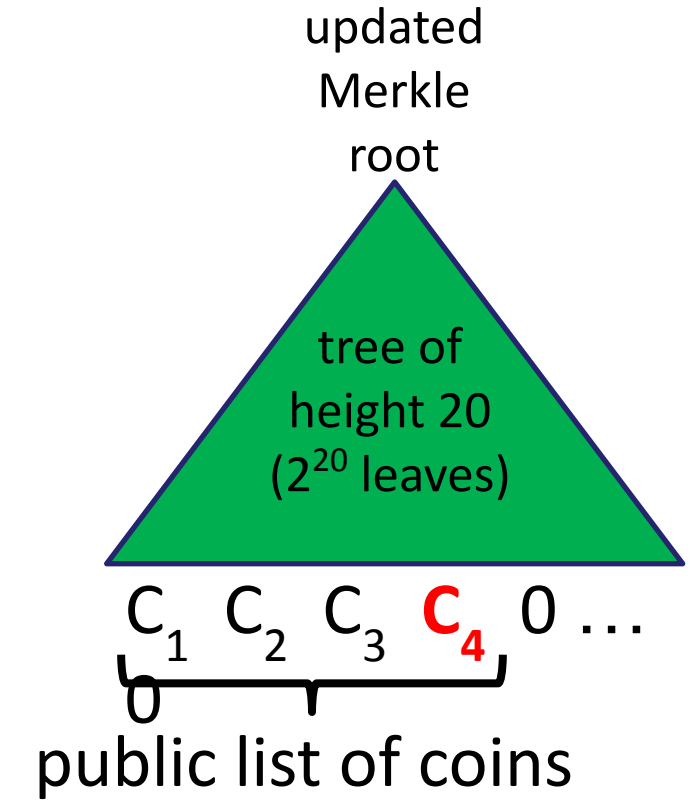


explicit list:
one entry per **spent coin**

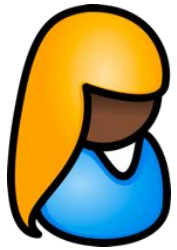
$H_1, H_2: R \to \{0,1\}^{256}$



Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:



100 DAI

C_4 , MerkleProof(5)

choose random k, r in R
set $C_4 = H_1(k, r)$
write C_4 in leaf #4 in tree
π = MerkleProof(5)

Treasury: 300 DAI

coins Merkle root

(32 bytes)

next = 4
MerkleProof(4)

nf_1
nf_2

nullifiers

contract state

explicit list:
one entry per **spent coin**

H_1, H_2: $R \rightarrow \{0,1\}^{256}$

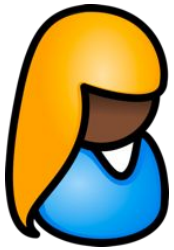
updated Merkle root

tree of height 20 (2^{20} leaves)

C_1 C_2 C_3 C_4 0 …

public list of coins

Tornado cash: deposit (simplified)



100 DAI

C_4 , π = MerkleProof(5)

coins Merkle root

(32 bytes)

next = 4
MerkleProof(4)

$H_1, H_2: R \rightarrow \{0,1\}^{256}$

updated Merkle root

tree of height 20 (2^{20} leaves)

C_1 C_2 C_3 C_4 0 …

0

public list of coins

Contract does:

(1) use C_4 and MerkleProof(4) to compute updated Merkle root

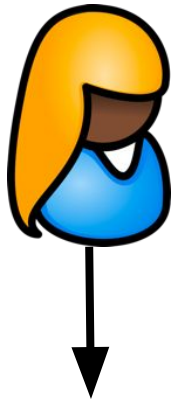
(2) verify π = MerkleProof(5)

(3) if valid: update state

Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:



100 DAI

C_4 , MerkleProof(5)

note: (k, r)
Alice keeps secret
(one note per coin)

Treasury: **400** DAI

updated Merkle root

(32 bytes)

next = **5**
MerkleProof(**5**)

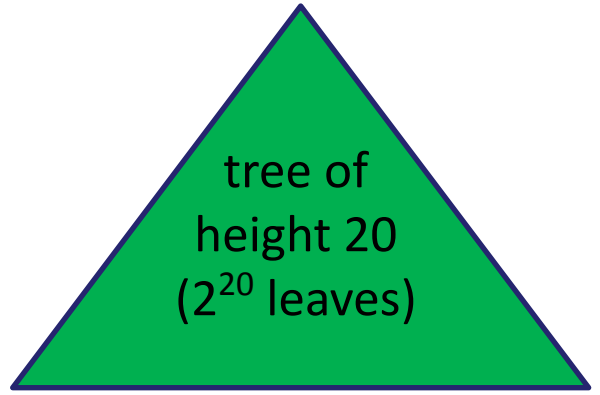
nf_1
nf_2

nullifiers

contract state

Every deposit: new Coin added sequentially to tree

updated Merkle root



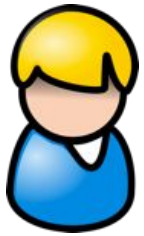
public list of coins

an observer sees who owns which coins

Tornado cash: withdrawal (simplified)

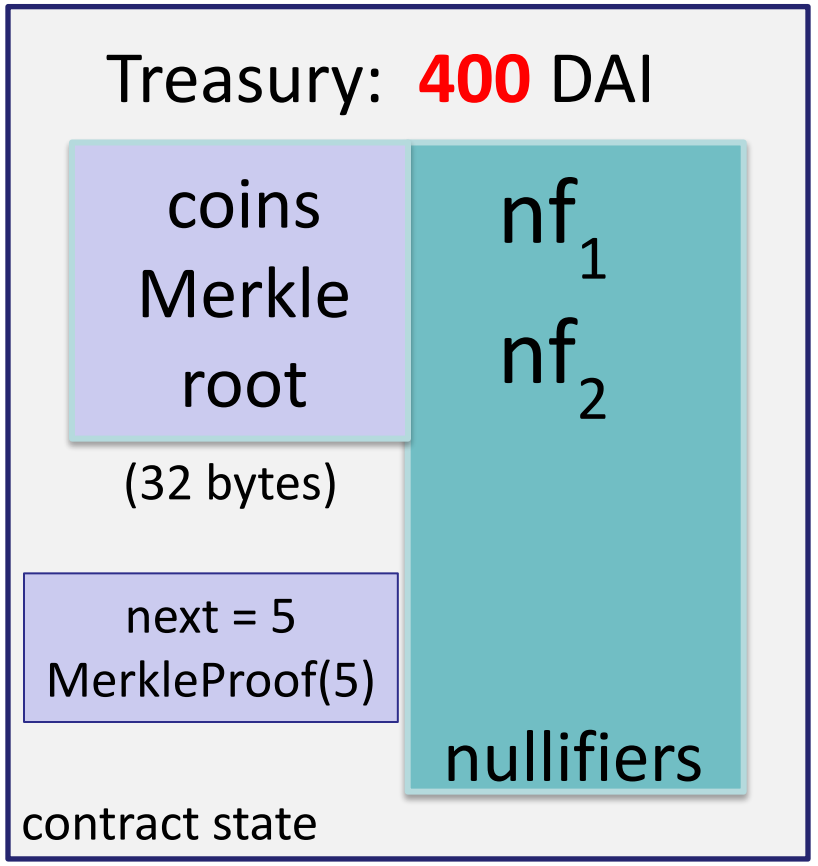
100 DAI pool:
each coin = 100 DAI

Withdraw coin #3 to addr A:

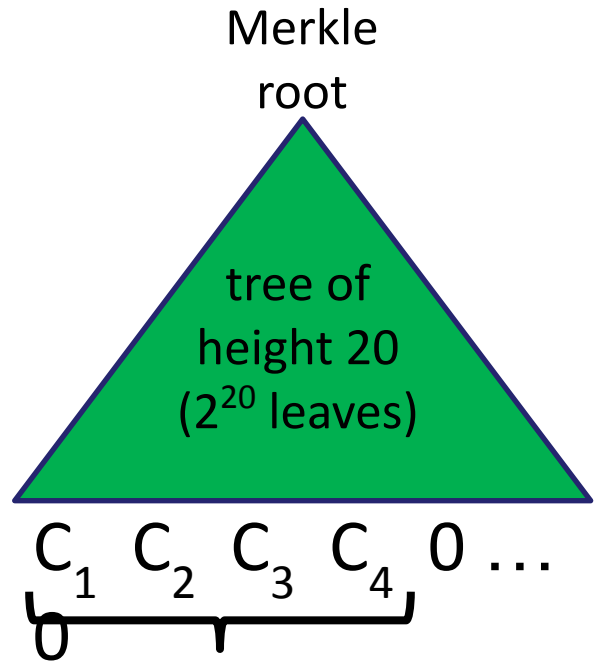


has note= (k', r')

set **nf** = $H_2(k')$



$H_1, H_2: R \rightarrow \{0,1\}^{256}$



Bob proves "I have a note for some leaf in the coins tree, and its nullifier is **nf**"

Tornado cash: withdrawal (simplified)

Withdraw coin #3 to addr A:

has note= (k', r') set **nf** = $H_2(k')$

Bob builds zk-SNARK proof π for

public statement x = (**root**, **nf**, **A**)

secret witness w = (k', r', C_3, MerkleProof(C_3))

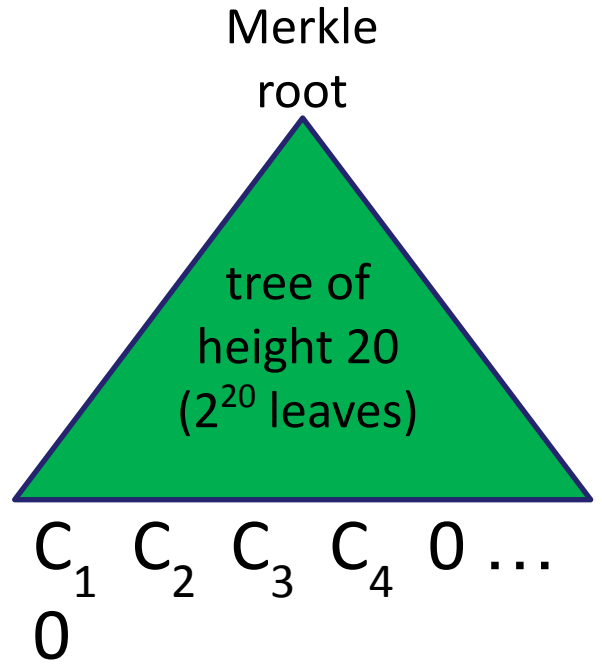
where C(x,w)=0 iff:

(i) C_3 = (leaf #3 of **root**), i.e. MerkleProof(C_3) is valid,

(ii) $C_3 = H_1(k', r')$,

(iii) **nf** = $H_2(k')$.

$H_1, H_2: R \to \{0,1\}^{256}$



(C does not use address A)

Tornado cash: withdrawal (simplified)

$H_1, H_2: R \rightarrow \{0,1\}^{256}$

Withdr

The address A is part of the statement to ensure that a miner cannot change A to its own address and steal funds

Assumes SNARK is non-malleable:

adversary cannot use proof π for x to build a proof π' for some x' ≠ x.

C_1 C_2 C_3 C_4 0 …
0

Bob builds zk-SNARK proof π for

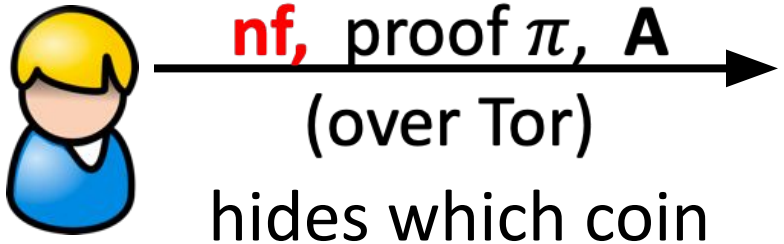
public statement x = (**root**, **nf**, **A**)

secret witness w = (k', r', C_3 , MerkleProof(C_3))

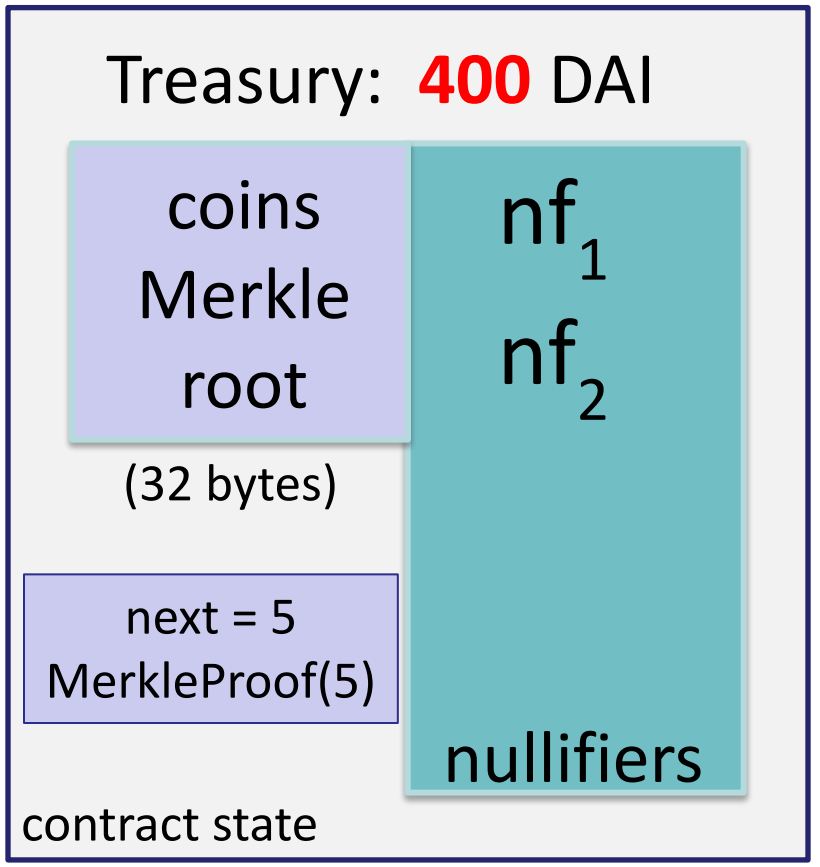
Tornado cash: withdrawal (simplified)

100 DAI pool:
each coin = 100 DAI

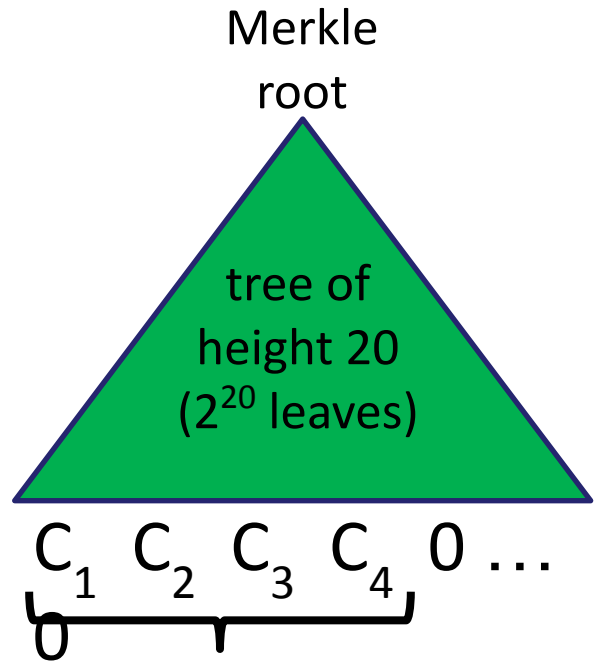
Withdraw coin #3 to addr A:



hides which coin is being spent



$H_1, H_2: R \to \{0,1\}^{256}$

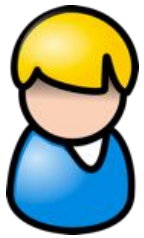


Contract checks (i) proof π is valid, and
(ii) **nf** is not in the list of nullifiers

Tornado cash: withdrawal (simplified)

100 DAI pool:
each coin = 100 DAI

Withdraw coin #3 to addr A:



nf, proof π, **A**
(over Tor)

100 DAI
to address **A**

Treasury: **300** DAI

coins Merkle root
(32 bytes)

next = 5
MerkleProof(5)

nf_1
nf_2
nf

nullifiers

contract state

$H_1, H_2: R \rightarrow \{0,1\}^{256}$

Merkle root

tree of height 20 (2^{20} leaves)

C_1 C_2 C_3 C_4 0 …

0

public list of coins
… but observer does not know which are spent

nf and π reveal nothing about which coin was spent.

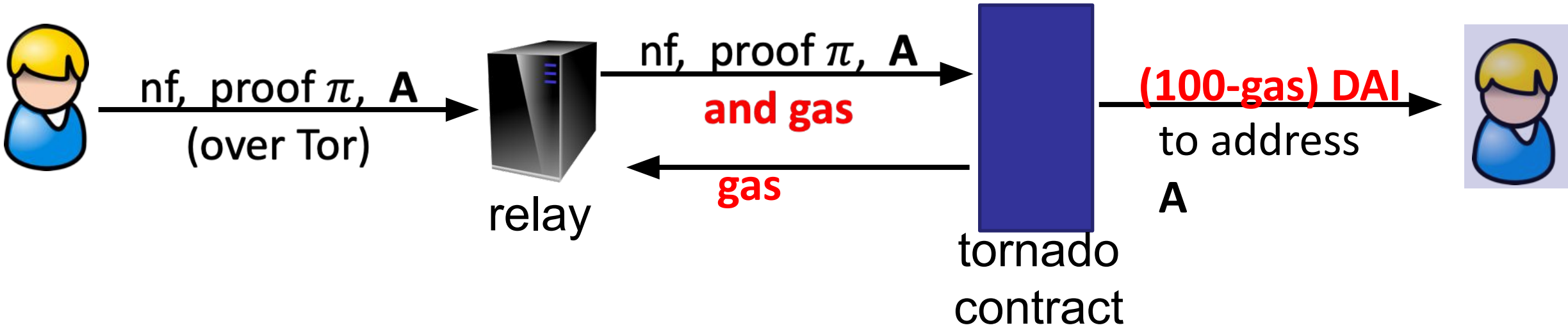
But, coin #3 cannot be spent again, because **$nf = H_2(k')$** is now nullified.

Relayers

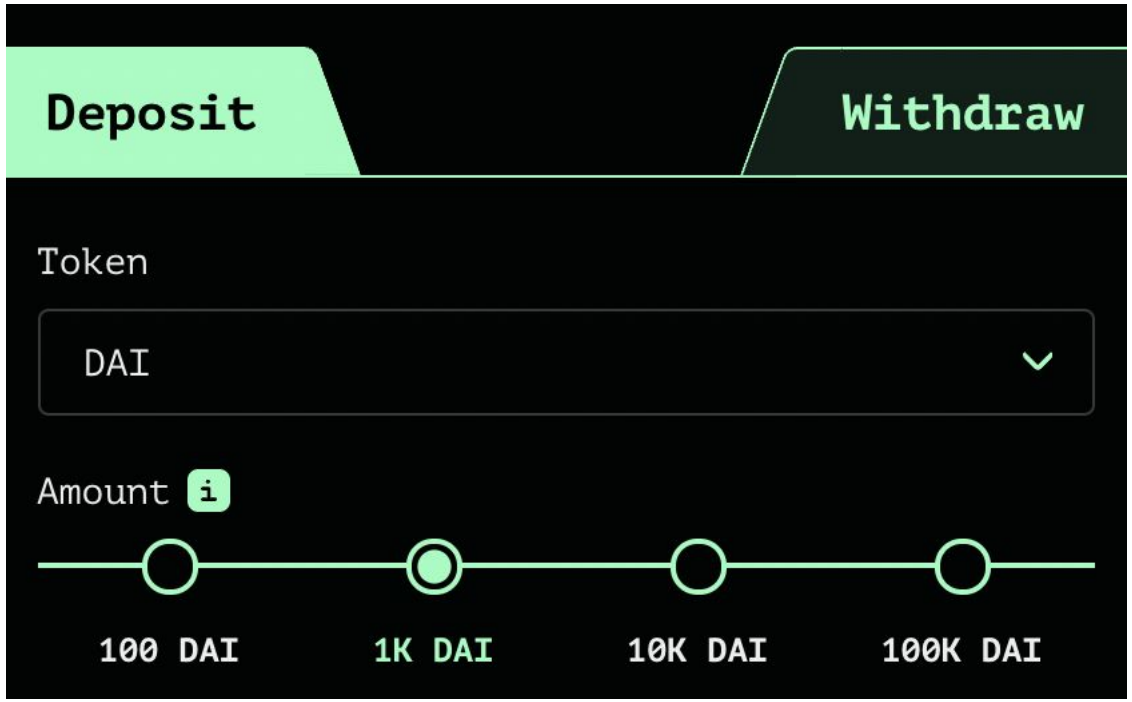
Problem: how does Bob pay for gas for the withdrawal Tx?

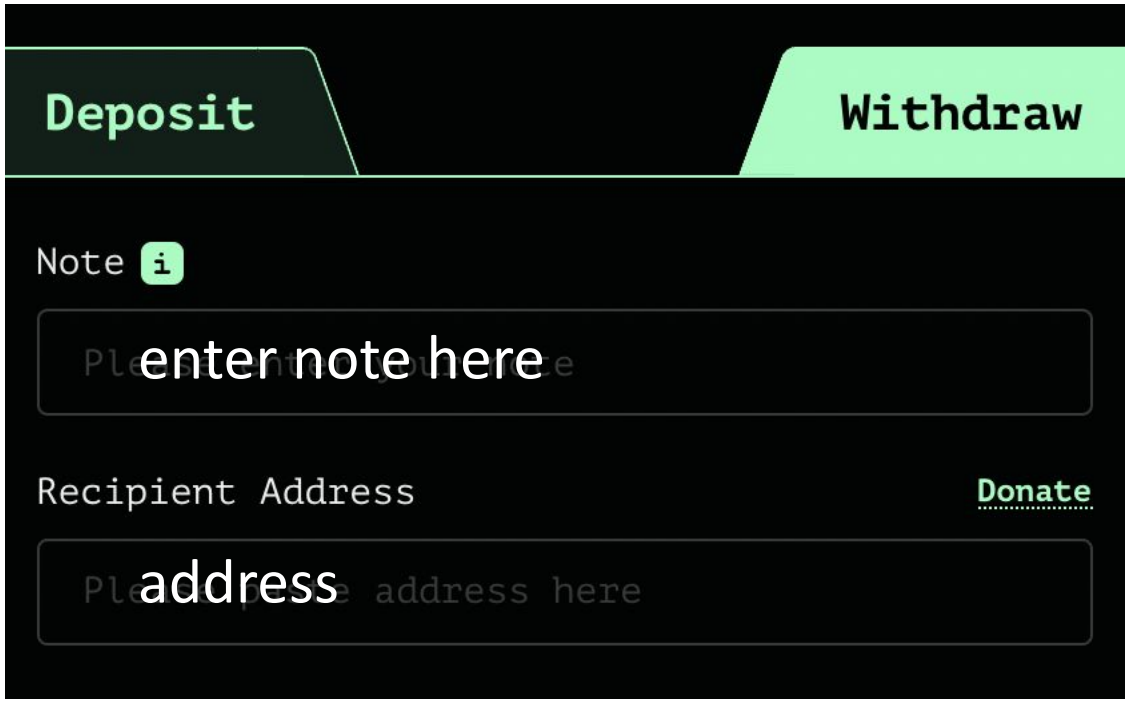
- If paid from Bob's address, then fresh address is linkable to Bob

Tornado's solution: **Bob uses a relay**



Tornado Cash: the UI





After deposit: get a note

Later, use note to withdraw

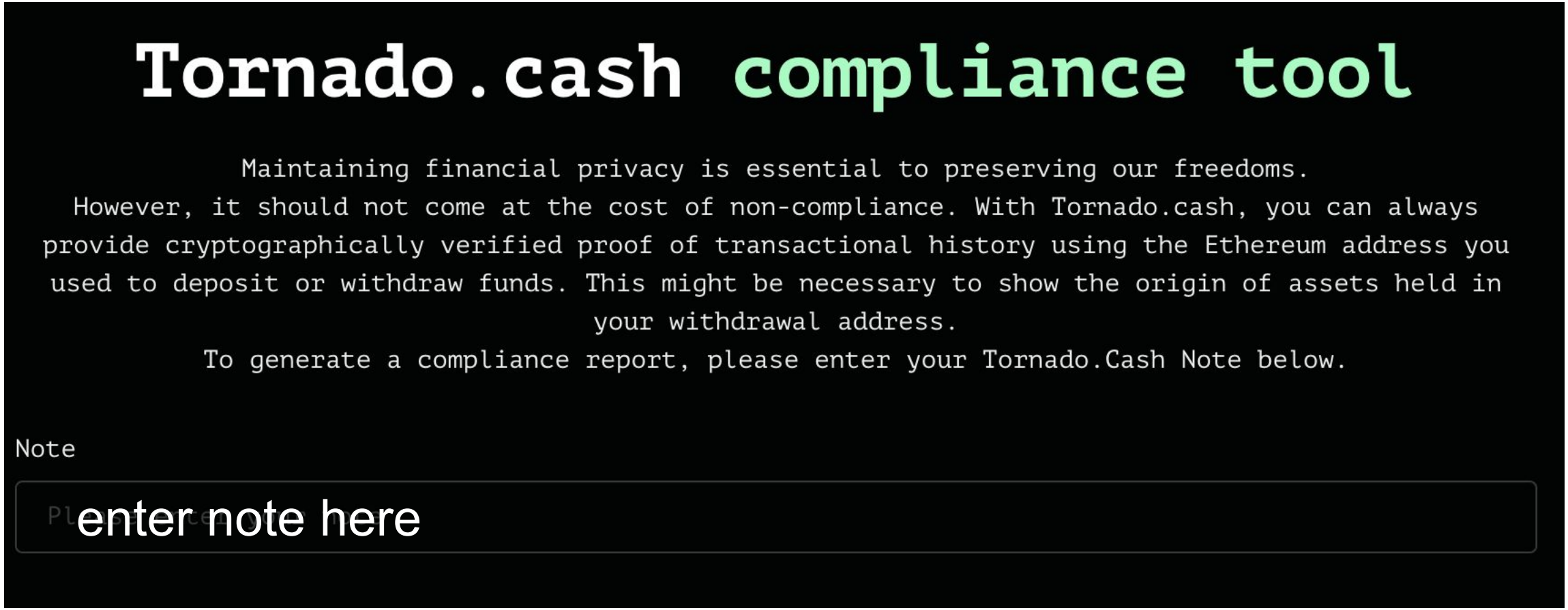
(wait before withdrawing)

Anonymity set

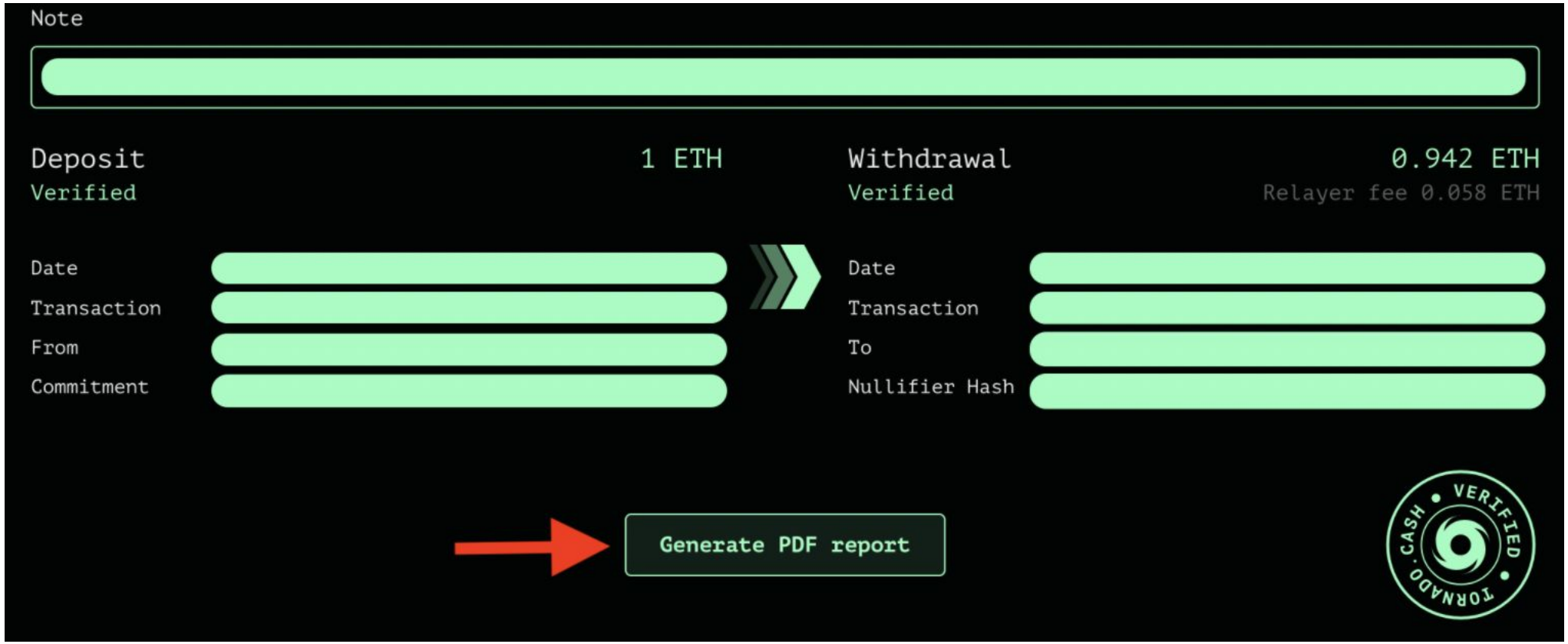
leaves occupied
over all pools

Oct. 2021

Compliance tool



Compliance tool



Reveals source address and destination address of funds

ZCASH / IRONFISH

Two L1 blockchains that extend Bitcoin.

Sapling (Zcash v2) launched in Aug. 2018.

More complicated, but similar use of Nullifiers

Zcash / IronFish (simplified)

Goal: fully private payments … like cash, but across the Internet

Includes mechanisms to let parties abide by financial regulation

Zcash / IronFish supports two types of TXOs:

- **transparent** (as in Bitcoin)
- **shielded** (anonymized)

a Tx can have both types of inputs, both types of outputs

Addresses and coins (notes)

H_1, H_2, H_3: cryptographic hash functions.

sk needed to spend note for address pk

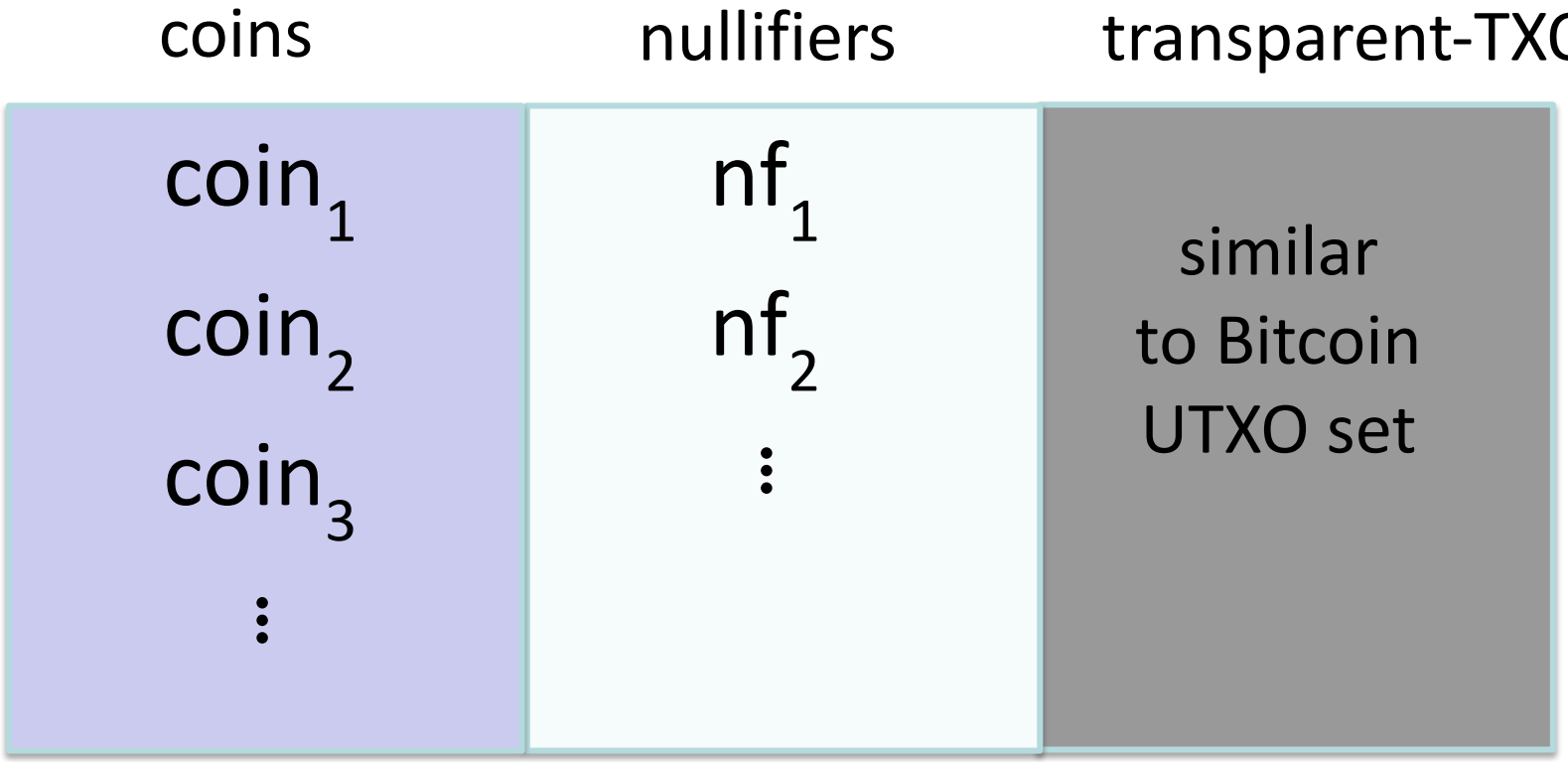
(1) shielded address: random $sk \leftarrow X$, $pk = H_1(sk)$

(2) **shielded coin** owned by address pk:

- coin owner has (from payer): value v and $r \leftarrow R$
- on blockchain: $coin = H_2((pk, v), r)$ (commitment to pk, v)

pk: addr. of owner, v: value of coin, r: random chosen by payer

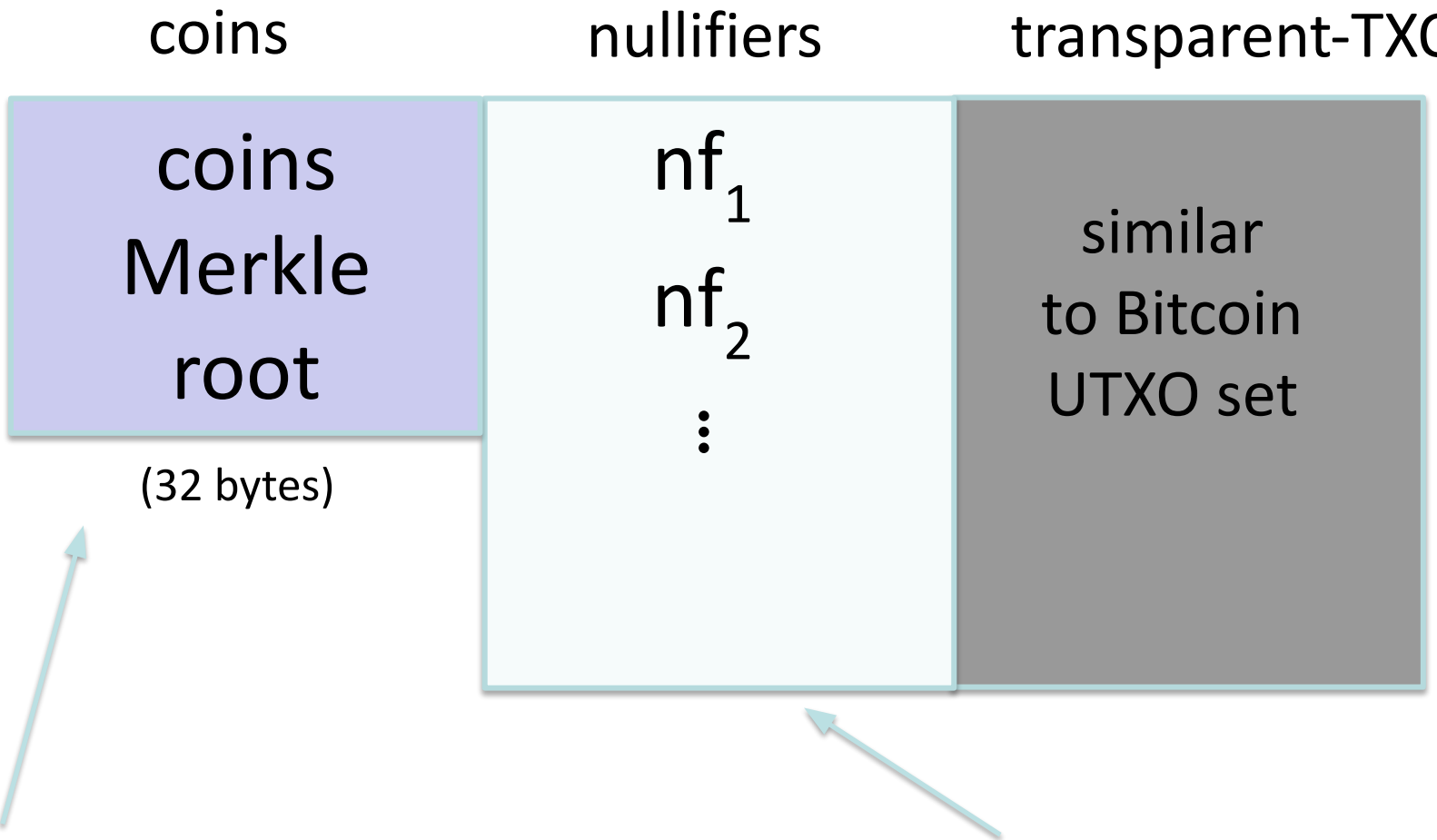
The blockchain



just Merkle root … append only tree
(coins are never removed)

explicit list:
one entry per **spent coin**

The blockchain



Transactions: an example

owner of **coin** = H_2((pk, v) , r)

wants to send **coin** value v to: shielded pk', v'

(v = v' + v'') transp. pk'', v''

step 1: construct new coin: **coin'** = H_2((pk', v') , r')

by choosing random r' ← R (and send (v', r') to owner of pk')

step 2: compute **nullifier** for spent coin **nf** = H_3(sk, index of coin in Merkle tree)

nullifier **nf** is used to "cancel" **coin** (no double spends)

key point: miners learn that some coin was spent, but not which one!

Transactions: an example

step 3: construct a zk-SNARK proof π for

statement = x = (current Merkle root, **coin'**, **nf**, v'')

witness = w = (sk, (v, r), (pk', v', r'), MerkleProof(**coin**))

C(x, w) outputs 0 if: compute **coin** := H_2((pk=H_1(sk), v), r) and check

The Zcash circuit

(1) MerkleProof(**coin**) is valid,

(2) **coin'** = H_2((pk', v') , r')

(3) v = v' + v'' and v' ≥ 0 and v'' ≥ 0

(4) **nf** = H_3(sk, index-of-coin-in-Merkle-tree)

from Merkle proof

What is sent to miners

step 4: send (**coin'**, **nf**, transparent-TXO, proof π) to miners,

send (v' , r') to owner of pk'

step 5: miners verify

(i) proof π and transparent-TXO

(ii) verify that **nf** is not in nullifier list (prevent double spending)

if so, add **coin'** to Merkle tree, add **nf** to nullifier list,

add transparent-TXO to UTXO set.

Summary

- Tx hides which coin was spent

  ⇒ **coin** is never removed from Merkle tree,
  but cannot be double spent thanks to nullifer

note: prior to spending **coin**, only owner knows **nf**:

$$\mathbf{nf} = H_3(sk, \text{index of coin in Merkle tree})$$

- Tx hides address of **coin'** owner

- Miners can verify Tx is valid, but learns nothing about Tx details

End of lecture. Let's do a quick review.

A zk-SNARK for a circuit C:

- Given a public statement x, prover P outputs a proof π that "convinces" verifier V that prover knows w s.t. $C(x, w) = 0$.
- Proof π is short and fast to verify

What is it good for?

- Private payments and private Dapp business logic (Aleo)
- Private compliance and L2 scalability with privacy

More to think about:

- private DAO participation? private governance?

Further topics (see, e.g., cs251)

- How to build a zk-SNARK?
- Recursive SNARKs:
  - Proving knowledge of a SNARK proof

    1-level recursive statement: "I know a proof π that $\exists w: C(x, w) = 0$"
  - Used in systems that keep business logic private
- Privately communicating with the blockchain: Nym
  - And (privately) compensating proxies for relaying traffic


END OF TOPIC
https://defi-learning.org/