Modeling fertility over age and time as striving toward a goal: Individual behavior and aggregate rates

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Theoretical perspectives on reproductive aging

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Age patterns of female reproduction vary widely among iteroparous animal species with determinate growth. Often fertility declines with age, but in other cases, it may be flat or rise across age. Sometimes fertility ceases altogether, leaving a substantial post-reproductive life span. In this article, we discuss theories that may provide some insights into how these diverse patterns might evolve. We present a simple optimal life history model and consider circumstances in which fertility might rise or fall with age. In our model, without assuming that costs per birth rise with age, that foraging efficiency

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PERSPECTIVE

Reproduction and production in a social context: Group size, reproductive skew and increasing returns

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Abstract
Evolutionary success requires both production (acquisition of food, protection and warmth) and reproduction. We suggest that both may increase disproportionately as group size grows, reflecting 'increasing returns' or 'group augmentation benefits', raising fitness in groups that cooperate in production and limit reproduction to one or a few high fertility females supported by non-reproductives, with high reproductive skew. In our optimisation theory both Allee effects (when individual fitness increases with group size or density) and reproductive skew arise when increasing returns determine optimal group size and proportion of reproductive females. Depending on which of food or maternal time is more important for

Questions

- Why is age specific fertility bell shaped? (Not all biology!)
- Did fert in LDCs decline due to family planning programs or decline in demand for births? (Was once a hot debate, e.g. Pritchett)
- Same for US Baby Bust –contraception vs demand?
- How do contraceptive efficacy and abortion access affect fertility over time?
- Do fertility intensions in surveys predict future fertility?
- What aggregate fertility variables to use for analysis of temporal change?

Perspectives on quantum and tempo

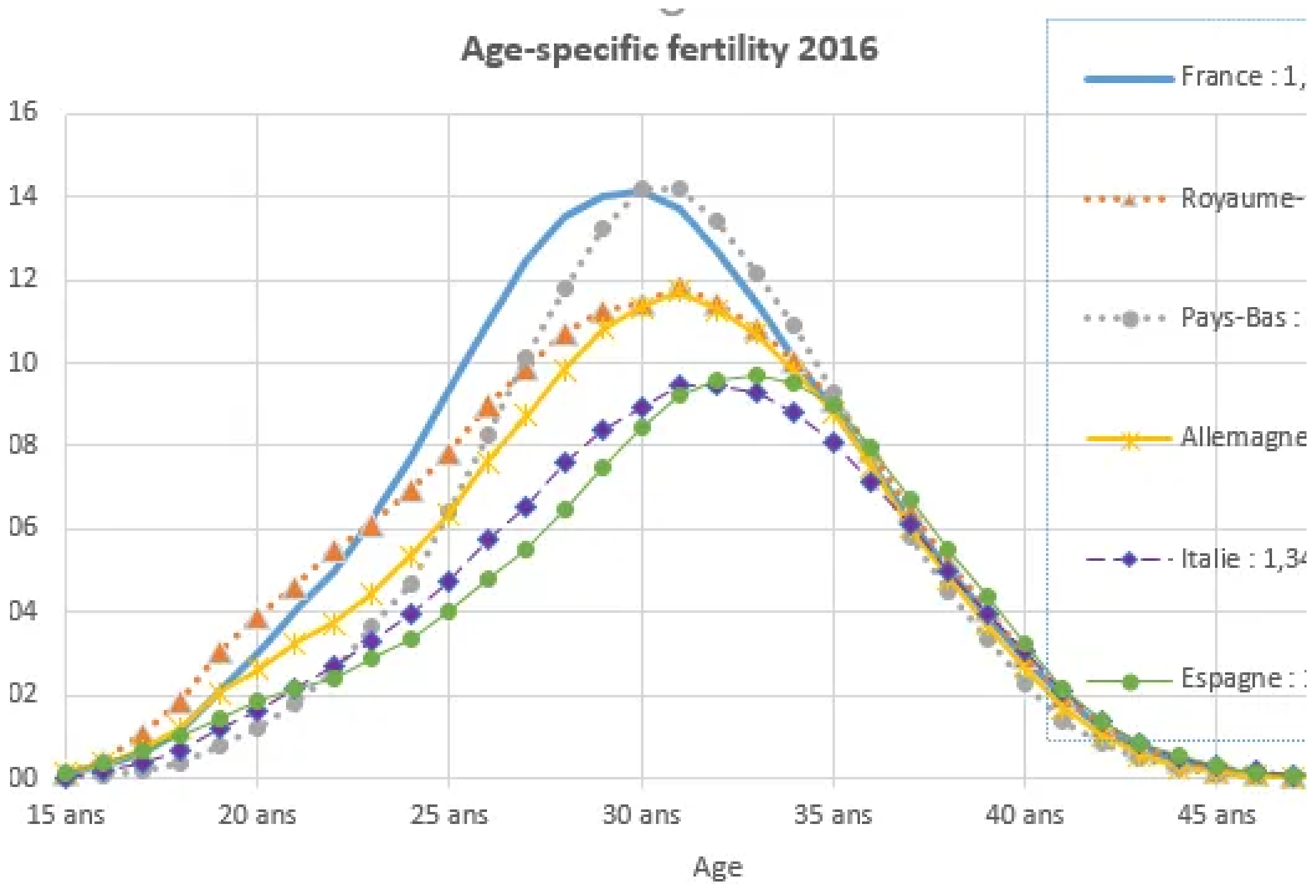
1. A traditional view:
   - Couple formulates at marriage its desired completed fertility (D) "quantum"
   - Constant throughout life.
   - A normal pattern of timing and spacing leads to outcome D. "tempo"
   - Period socioeconomic context may lead to changes in tempo
   - Tempo changes then influence period TFR.
2. Bongaarts-Feeney:
   - Reject causal force of quantum as pole star.
   - Focus on advancing or postponing timing.
3. Moving target or stock adjustment approach
   - Fertility is intentional effort to achieve target, but subject to accident as well.
   - Both target D and spacing vary with period conditions, and changing D can cause changes in timing and spacing that would show up as tempo in other frameworks.

Why is fertility by age bell shaped?

Partly biology, but set that aside.

Demography

1. Age distribution of initiation of childbearing. (this used to be age at marriage, now much more complicated)
2. Distribution by duration of fertility once initiated.



Source : Eurostat, Age-specific fertility in Europe [demo_frate], update: 03/08/2018

Fertility of those who have initiated childbearing (Romania, av of 1985-2005), by union type and duration.

Other than consensual union, exponential decline is approximately right.

For direct marriage without cohab, there is delay in conception following marriage.

For married after cohab, looks like they get married following pregnancy.

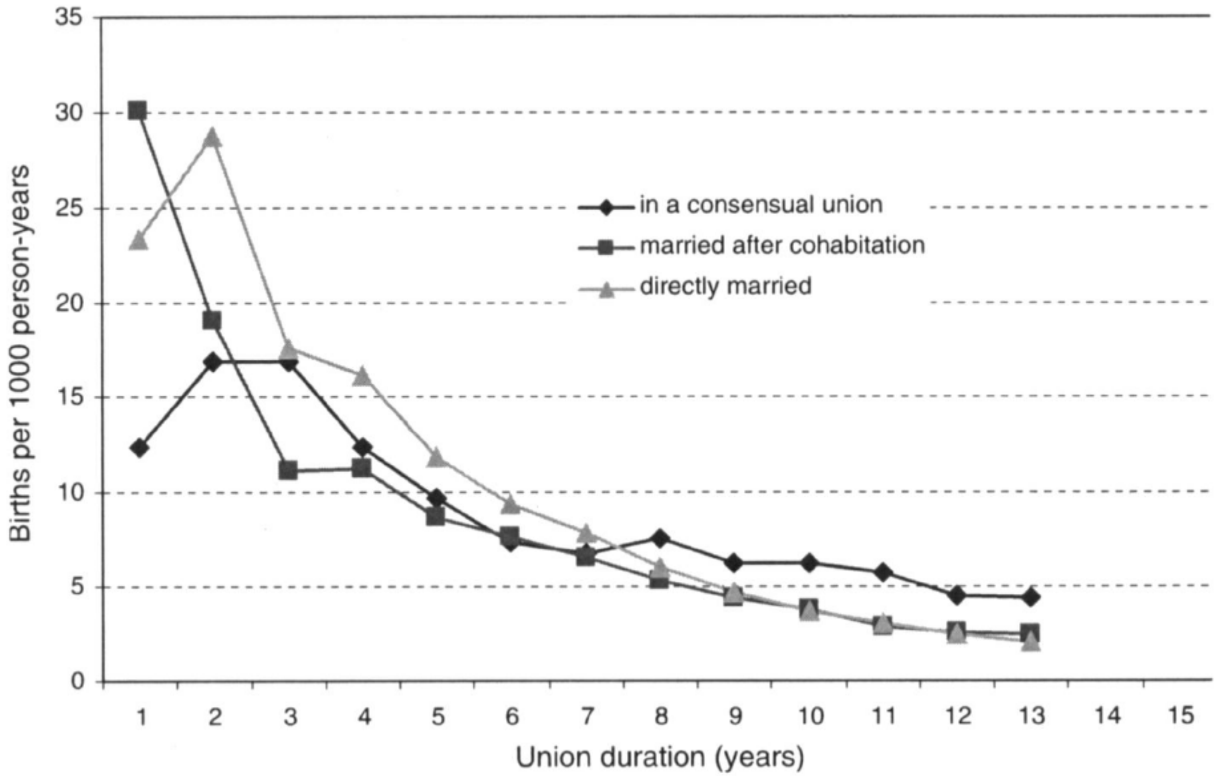


Fig. 4 Fertility rates for partnered Romanian women, 1985–2005, by union duration and type of union. *Notes*: Rates not standardized. Smoothed by a five-term moving average starting in the fourth year of union duration. Computed by the authors from the Romanian GGS of 2005. For married women, the *x*-axis gives the duration of the *marriage*. For women married after cohabitation it indicates the duration of the marriage and does not count in the preceding period of premarital cohabitation

Source: Copied from Hoem and Mureşan (2011)

I will focus on the second, fertility by duration x of “initiators”.

A slide from Josh and Tom.
Apologies for using diff notation.

An equation relating flow of births to stock of children

$$\begin{aligned} \text{fertility} &= \text{rate} \times \text{(unachieved family size target)} \\ f_x &= \alpha \quad \times (D - F_x) \\ &= 0.3 \times (2.0 - 1.0) \end{aligned}$$

- f_x birth rate x years after onset of childbearing
- α rate at which unachieved desires are achieved, constant by duration
- D desired family size target (Ron lets T vary by period).
- F_x children already born

Innovation: to model shocks, we let “fulfillment rate” α vary by period.

$$g(x) = \lambda A(x) = \lambda\left[D - C(x)\right]$$

Basic equations of model

$D =$ desired completed fertility target

$C(x) =$ Children already born

$A(x) = D - C(x) =$ Additional children wanted

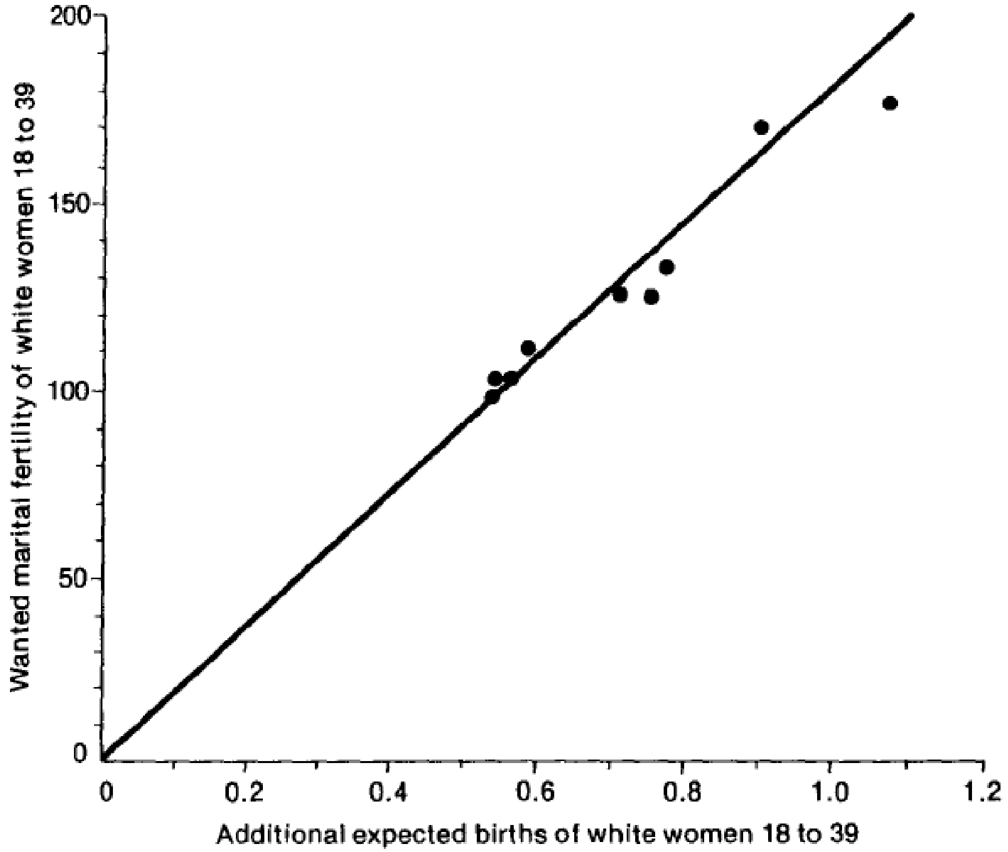
$g(x) = \lambda A(x) = \lambda\left[D - C(x)\right] =$ Fertility of initiators at duration x

Upper limit on x will depend on age at initiation of fertility, but for simplicity I will assume it is 20 years.

Fits pretty well:

Marital fertility 18-39 by Additional Expected Births 18-39, 1955-75 (US married women)

82 Predicting Fertility



Note: Legitimate live births to married women aged 15 to 44 were taken from U.S. Public Health Service, *Natality*, Vital Statistics of the United States, 1975, vol. 1 (Washington, D.C.: U.S. Government Printing Office, 1979). These were multiplied by 1.36 = (45 − 15)/(40 − 18) to get a rate more nearly representative of married women aged 18 to 39. For each preceding

Dynamics – longitudinal equations of motion

$$C'(x) = g(x) = \lambda A(x)$$

Rate of change of Children Ever Born at age x is fertility at age x.

$$A'(x) = \left[D'(x) - C'(x)\right]$$

This is the basic differential equation. Note D can change.

$$A'(x) = \left[D'(x) - \lambda A(x)\right]$$

$$A(x) = e^{-\lambda x}\left[\int_0^x e^{\lambda u} D'(u)\,du + k\right]$$

where k is chosen so $A(0) = D(0)$
at time of initiation, A=D.

If target D is constant, then D'=0 and this becomes:

$$A(x) = e^{-\lambda x} D$$

After initiation of childbearing, fertility declines exponentially. Very small at x=20; for simplicity assume 0.

Fertility goals changed systematically before, during, and after Baby Boom

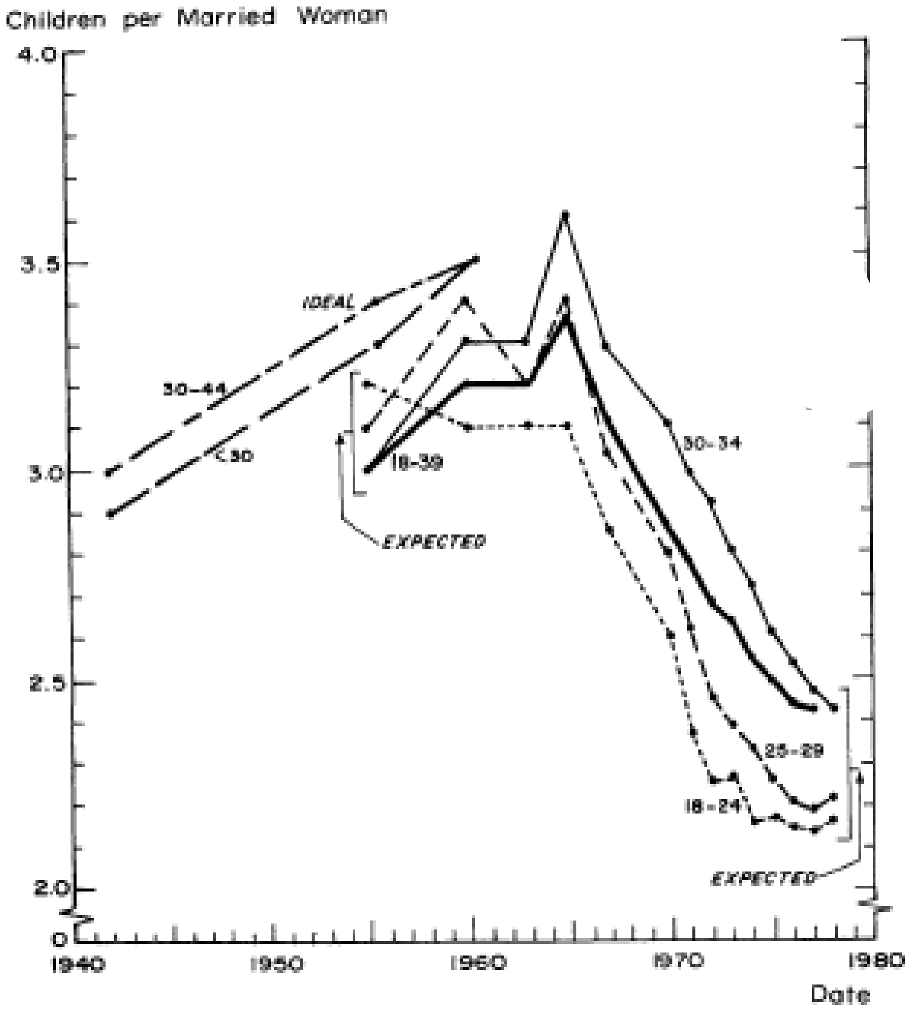
Survey responses on “Ideal” and “Expected” lifetime fertility.

All measures are “period”.

“Expected” are contaminated by contraceptive number failures.

Note that cohort targets appear to change in period manner, that is **all ages (birth cohorts) seem to change in same way at same time**.

Measures of fertility goals for US women, 1940-1980 (from Lee, 1980, “Aiming”)



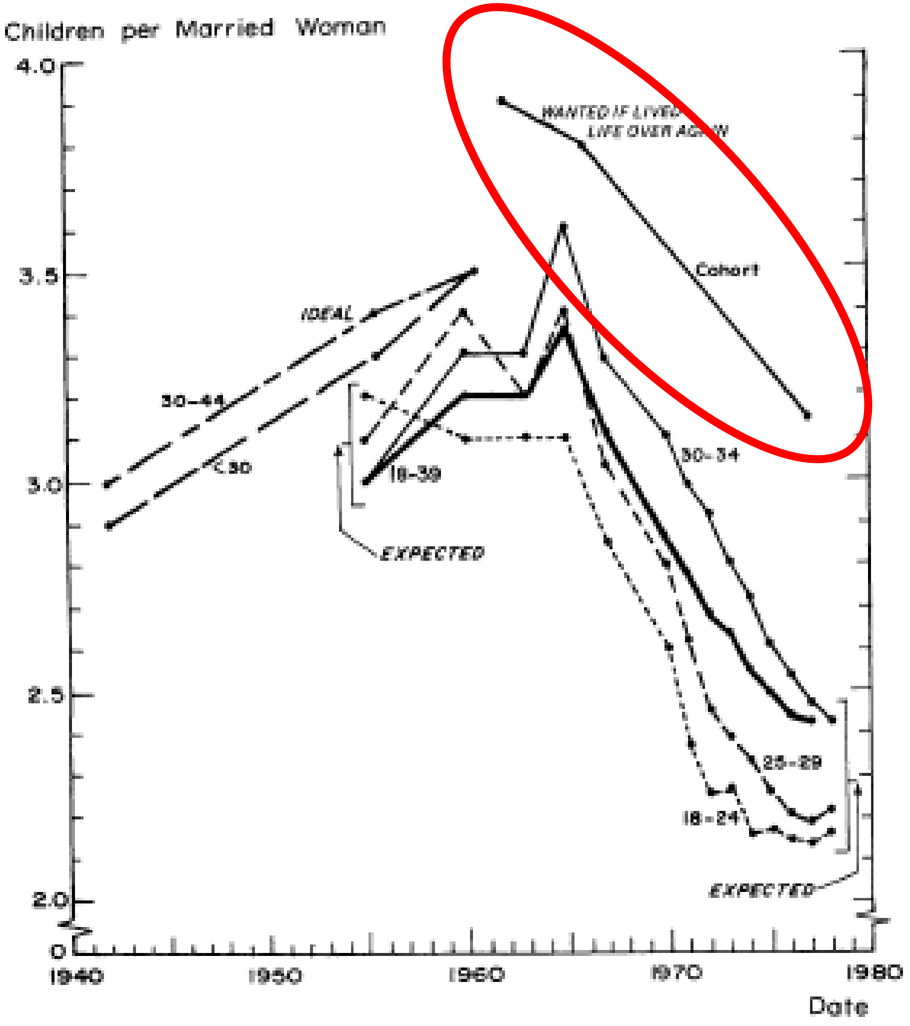
Remarkable longitudinal data from Detroit Area Study (DAS) show target change within a cohort and among the same women.

All are period measures except for longitudinal survey of Detroit Area Women, asked “Number of children wanted if life could be lived over again?”.

1962 3.9
1966 3.8
1977 3.1

Longitudinal declines similarly to period.

Measures of fertility goals for US women, 1940-1980 (from Lee, 1980, “Aiming”)



Dynamics with linear change in D(t)

Assume $D(t) = a + bt$

After bunch of math we find

$$A(x) = \left(\frac{b}{\lambda}\right)\left(1 - e^{-\lambda x}\right) + e^{-\lambda x}\left[a + b(t - x)\right]$$

If b=0 then D=a and

$$A(x) = e^{-\lambda x} D$$

That described fertility of initiation cohorts. Now get period fertility of initiators F(t)

Index each variable on x and on t.

Integrate g(x,t) over x at given t to find F(t).

After more math we find for linear case, assuming exp(lambda*20)=0:

$$F(t) \doteq D(t) + b\left(20 - \tfrac{2}{\lambda}\right)$$

(after x=20, fertility is 0 by assumption)

When b=0 D is constant and F(t)=D.

Evaluate for lambda=.2 and b=.1: **F(t) = D(t) + 1.0.**

For decline at .1 we get: **F(t)=D(t) – 1.0.**

Period total fert when D changes linearly then is constant.

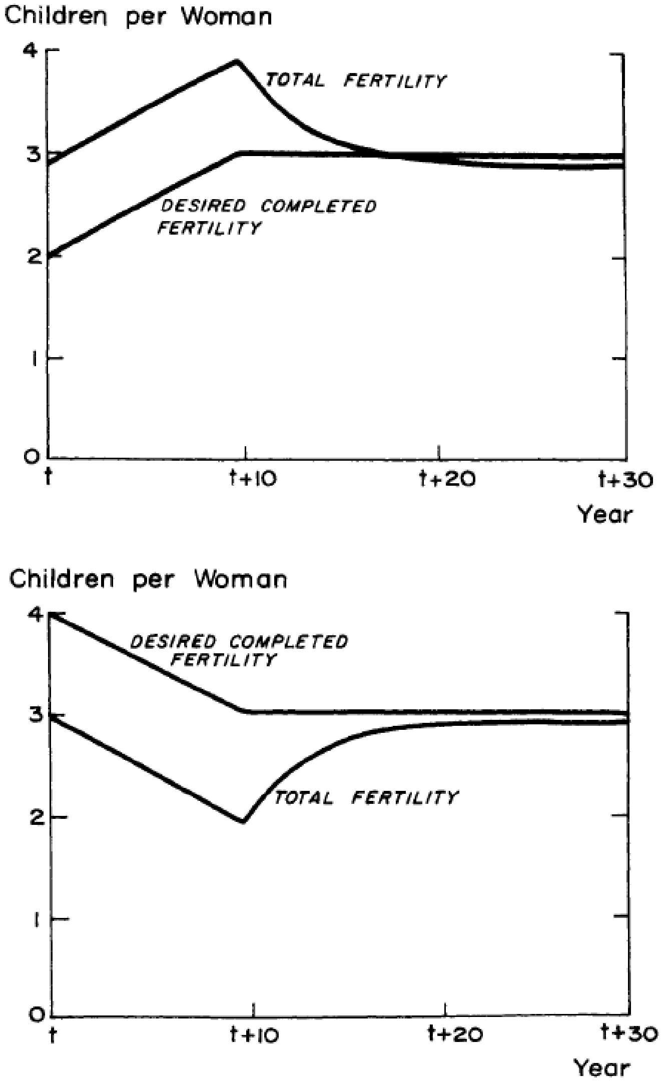


Figure 5. Period total fertility when desired completed fertility changes linearly, then is constant.
Sources: Calculated from Equation 19 with $\lambda = 0.18$, $\beta = 20$, and $b = \pm 0.1$.

Now dynamics with a sinusoidal variation in D, something like Baby Boom and Bust.

$$D(t) = \bar{D} - a\cos\left(\frac{t}{w}\right)$$

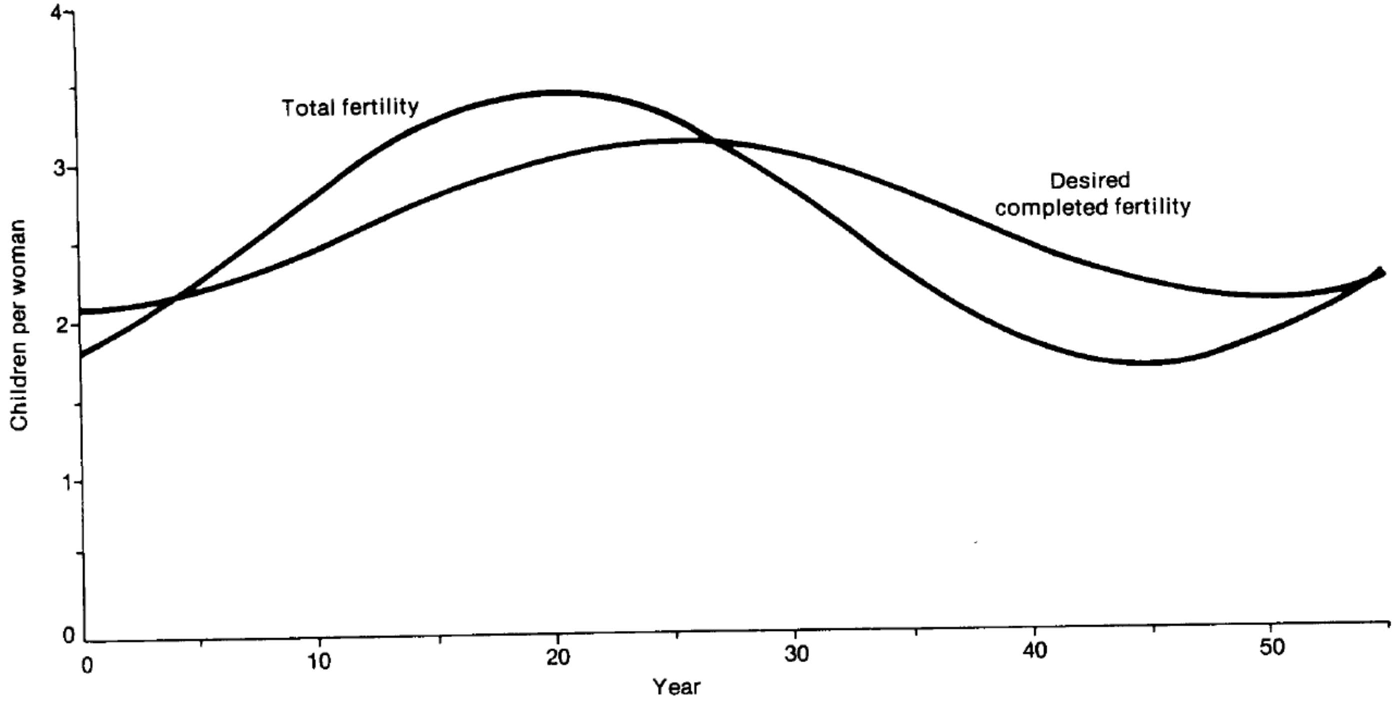
$$D'(t) = \left(a/w\right)\sin\left(\frac{t}{w}\right)$$

After some heroic math/arithmetic we find:

$$t^* = w\tan^{-1}\left\{\frac{\beta y\lambda z - 2y^3 + e^{-\lambda\beta}\left[(y^4 - y^2)\sin\left(\frac{\beta}{w}\right) + 2y\cos\left(\frac{\beta}{w}\right)\right]}{-z\lambda\beta - (y^4 - y^2) + e^{-\lambda\beta}\left[(y^4 - y^2)\cos\left(\frac{\beta}{w}\right) - 2y\sin\left(\frac{\beta}{w}\right)\right]}\right\}$$

Simulated from fitted model in "Aiming" when D(t) varies from 2.1 to 3.1 over 50-year sinusoidal cycle.

*Peak of TFR is 5 years after peak in D
*Amplitude of swing in TFR is twice swing in D.



Note: The length of the cycle (trough to trough) is fifty years, and desired completed fertility varies from 2.1 to 3.1 children per woman. The time path of *TF* was calculated from equation 25 of Ronald Lee, ''Aiming at a Moving Target: Period Fertility and Changing Reproductive Goals,'' *Population Studies* (in press).

Figure 6–3. The Time Path of Period Total Fertility When Desired Completed Fertility Fluctuates Sinusoidally

Why does TFR peak before D?

Recall TFR is related to rate of change of D.

TFR highest when D is rising most rapidly, and declines when it slows, well before its peak.

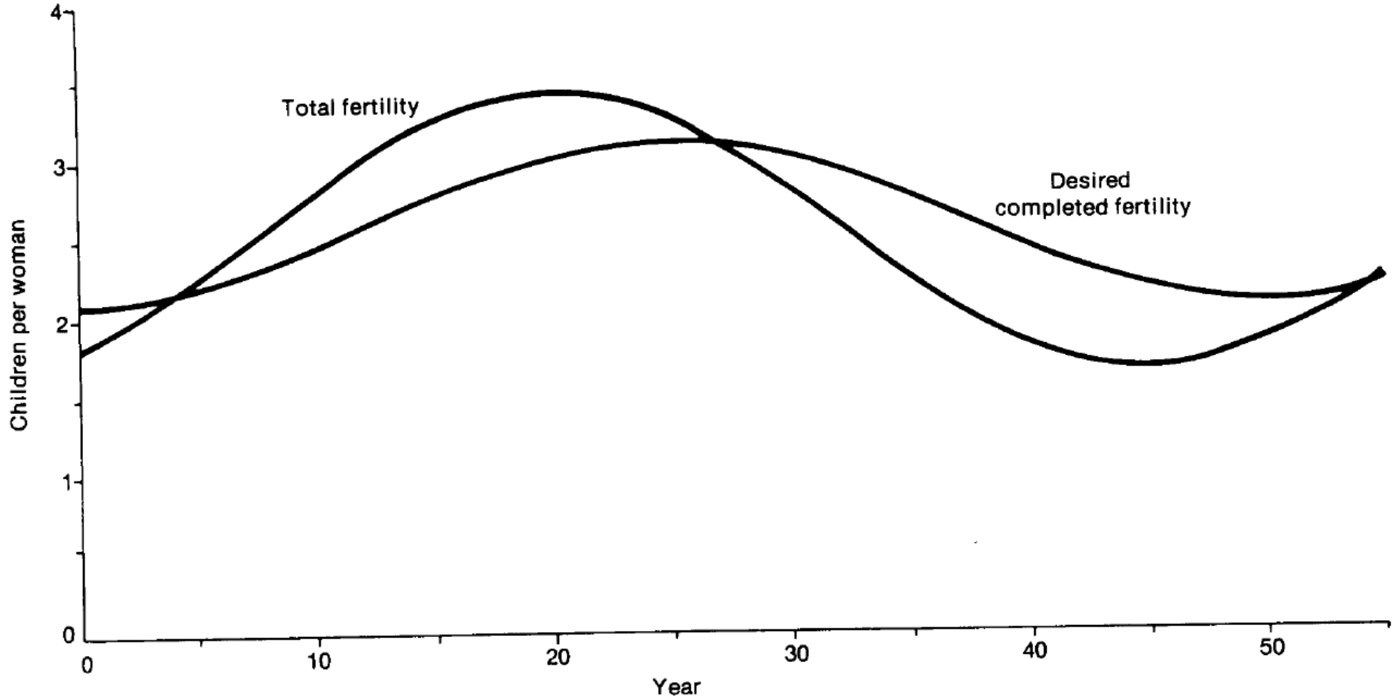


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Figure 6–3. The Time Path of Period Total Fertility When Desired Completed Fertility Fluctuates Sinusoidally

Many questioned the value of survey data on fertility expectations or intentions.

For example, Easterlin observed that period fertility rates declined well before either ideal of expected fertility, and concluded that '' ... changes in behavior precede those in attitudes, rather than vice versa" (Easterlin 1973, p.209). Easterlin's conclusion was cited with approval by Westoff (1978, p. 3).

The idea was that the survey data reflected ex post rationalizations of prior behavior.

Here we see that the earlier downturn in TFR is quite consistent with the expectations type data having real meaning.

Anything troubling about this analysis? Plenty.

- Here are two problems:

“Does this mean that birth intervals are half as long when additional desired fertility is twice as great? That can’t be.”

“Those equations may hold when D is rising, but when D is falling they assume that women can reverse previous births, that is have negative fertility.”

Let’s explore one at a time.

Let’s address first problem by unpacking A

- Let p(x) be the proportion of nonterminators at age x, that is, women who have not yet reached their target.
- Let $\alpha(x)$ be the average additional wanted children at age x for nonterminators. Let m(x) be the birth rate to nonterminators.
- Then: First problem: Let’s break down A, additional wanted.

$$A(x) = \alpha(x)\, p(x)$$

$$g(x) = m(x)\, p(x)$$

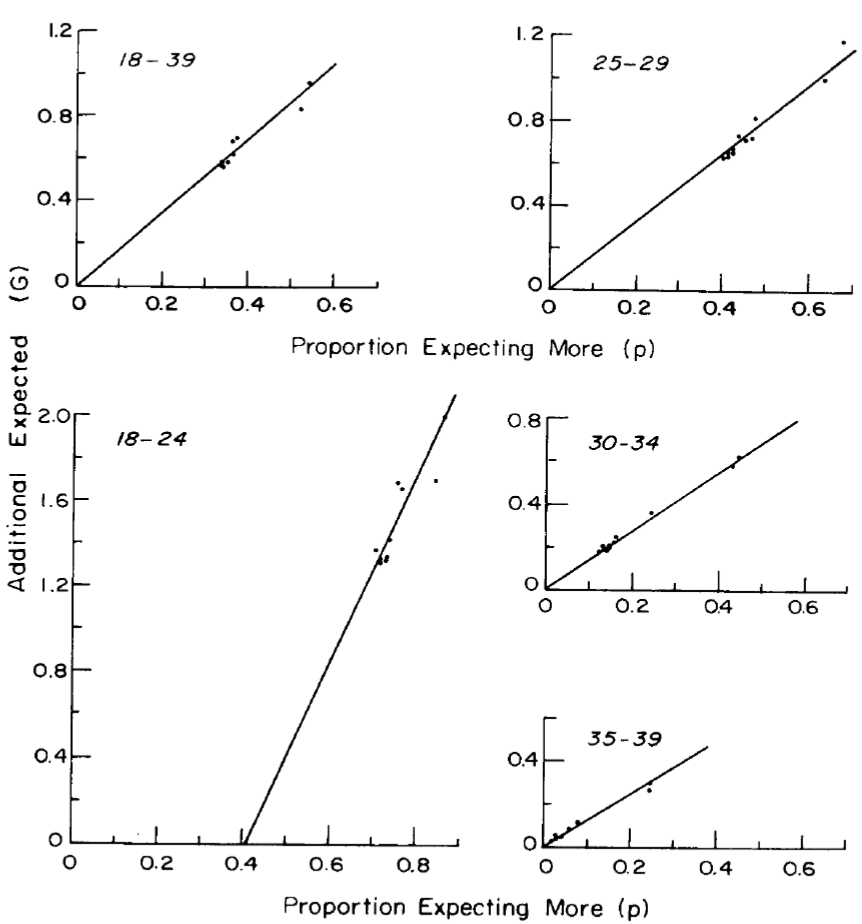
$$g(x) = \frac{m(x)}{\alpha(x)} A(x) = \lambda A(x)$$

Empirically, both m(x) and $\alpha(x)$ rise with age, and their ratio is quite constant at .2.

Chart of A vs p; slope is α

At ages above 25, additional wanted, A(x), is a multiple of proportion wanting more, p(x).

Figure 1. Additional Expected Births by Proportion Expecting More for U.S. Married Women, by Age Group, for Survey Years 1955 to 1978.

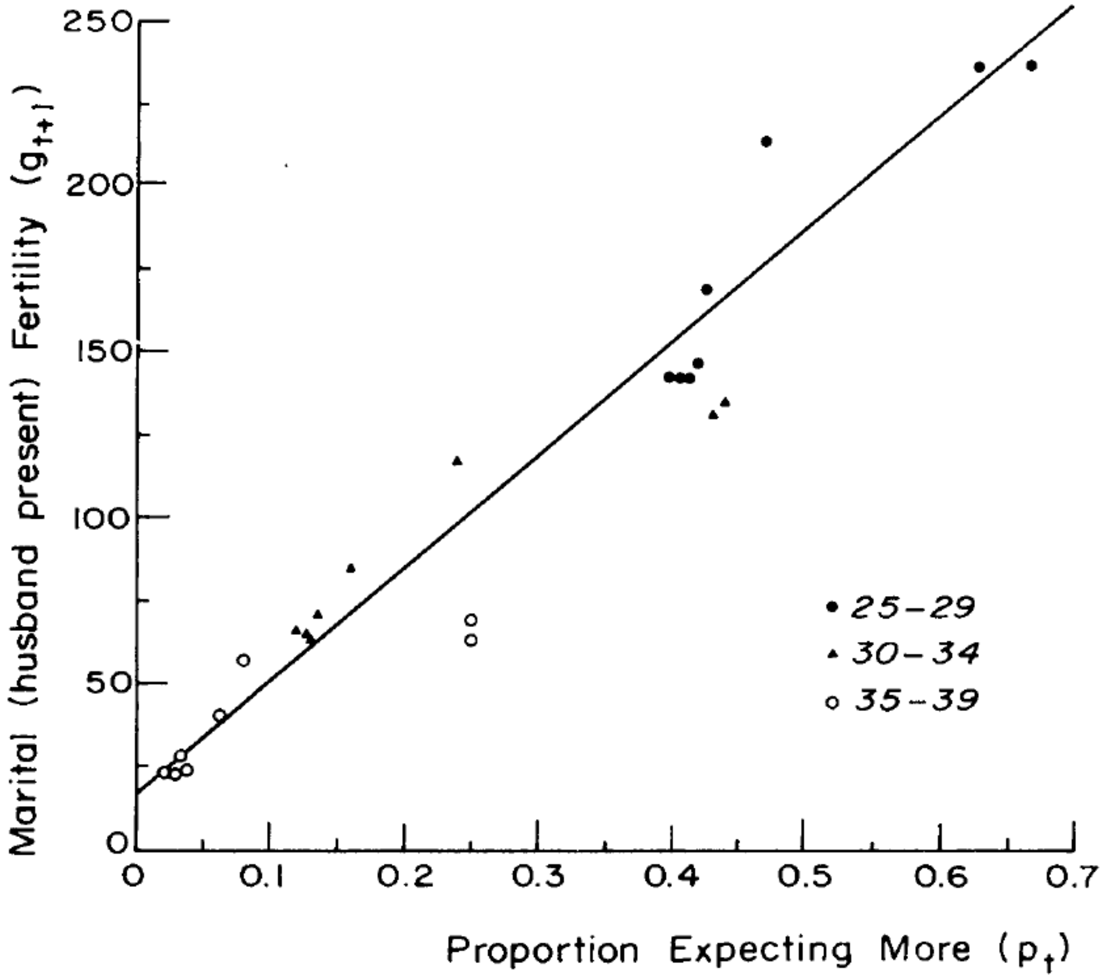


Note

The diagram shows data from 1955, 1960, 1962–64, 1967, 1971–1978, although not all of these years for each group. Sources are: Freedman and Bumpass (1955 to 1962–64); U.S. Bureau of the Census, 1978 (1967–1978). 1955 data refer to white women only.

Fertility is
At ages above 25 marital fertility is closely related to proportion expecting more, p(x,t).
Slope is about .3

Figure 2. Marital Fertility by Proportion Expecting More Births for U.S. Married Women with Husband Present, 1955 to 1974, for Age Groups 25–29, 30–34 and 35–39.



This finesses the first problem:

birth rate of nonterminators is pretty steady at .3 (from last slide).

Additional wanted by nonterminators, alpha, is pretty steady at 1.5.

Their ratio is steady at lambda= .2.

Irreversibility is a big problem. Here is a simulation of a fertility transition with declining D

Ignoring irreversibility we get this nice picture.

As D declines during the transition, the TFR drops even more, but once D stabilizes the TFR returns to its new equilibrium level.

Mean completed fertility will rise in later years.

Lambda = .1 means that completed fertility is well below D.

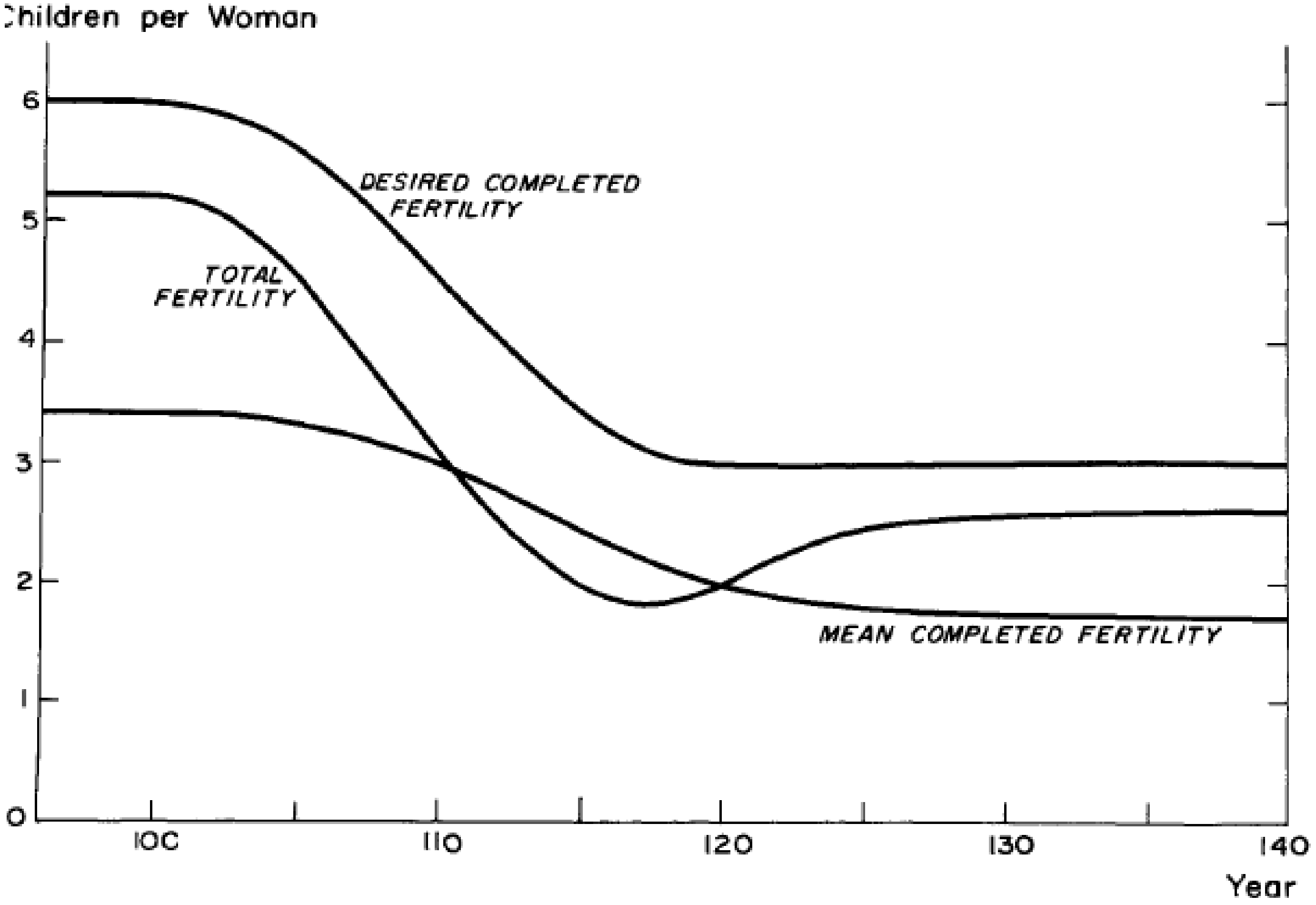


Figure 7. Period Total Fertility During A Demographic Transition

Sources: The transition is from a Desired Completed Fertility of 6.0 to one of 3.0 over a period of 20 years, with the pattern of decline being half a sine wave. Note that $\lambda = 0.1$, not 0.18; $\beta = 20$.

Introducing contraceptive failure

Terminators are at risk of number failure rates at rate Q_1

Nonterminators are at risk of timing failures at rate Q_2

g(x)* is “wanted fertility”, births wanted sometime.

$$g^* = g - (1 - p)Q_1$$

g(x)** is “planned fertility”, neither number nor timing failures.

$$g^{**} = g - pQ_2 - (1 - p)Q_1$$

More on planned and unplanned fertility

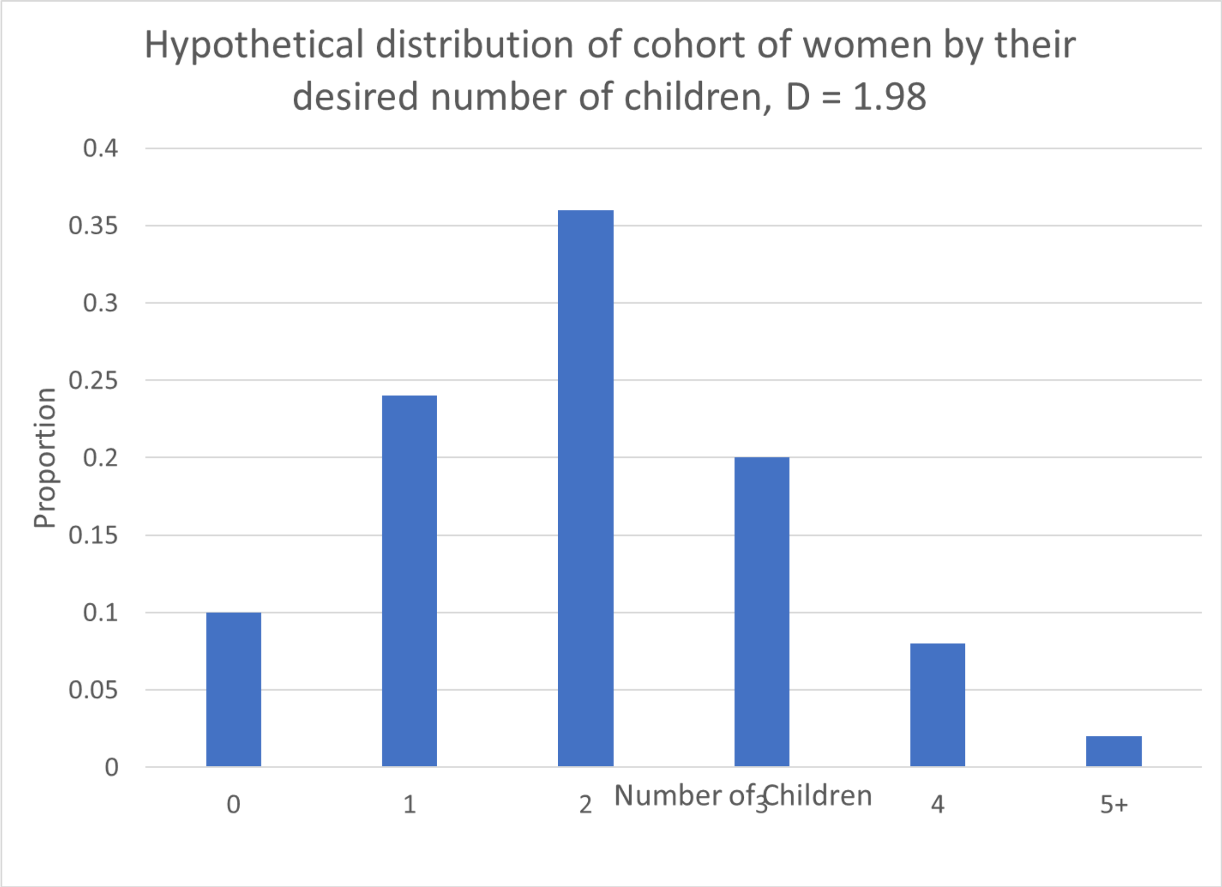
- Introduce m* the *ex ante* planned birth rate of nonterminators and the ex post planned rate of nonterminators g**, since nonterminators experiencing timing failures are not at risk of a planned birth in that yr.

$$g^{**} = m^*\left(1 - Q_2\right) p$$

- This is all pretty complicated. Here is the expression for g:

$$g = p\left[m^*\left(1 - Q_2\right) + \left(Q_2 - Q_1\right)\right] + Q_1$$

Made-up data for f(y)



For an initiation cohort of women with heterogeneous fertility targets (desires, demands)

y = number of births (continuous for simplicity) desired by a woman

f(y) = proportion of women wanting between y and y+dy births.

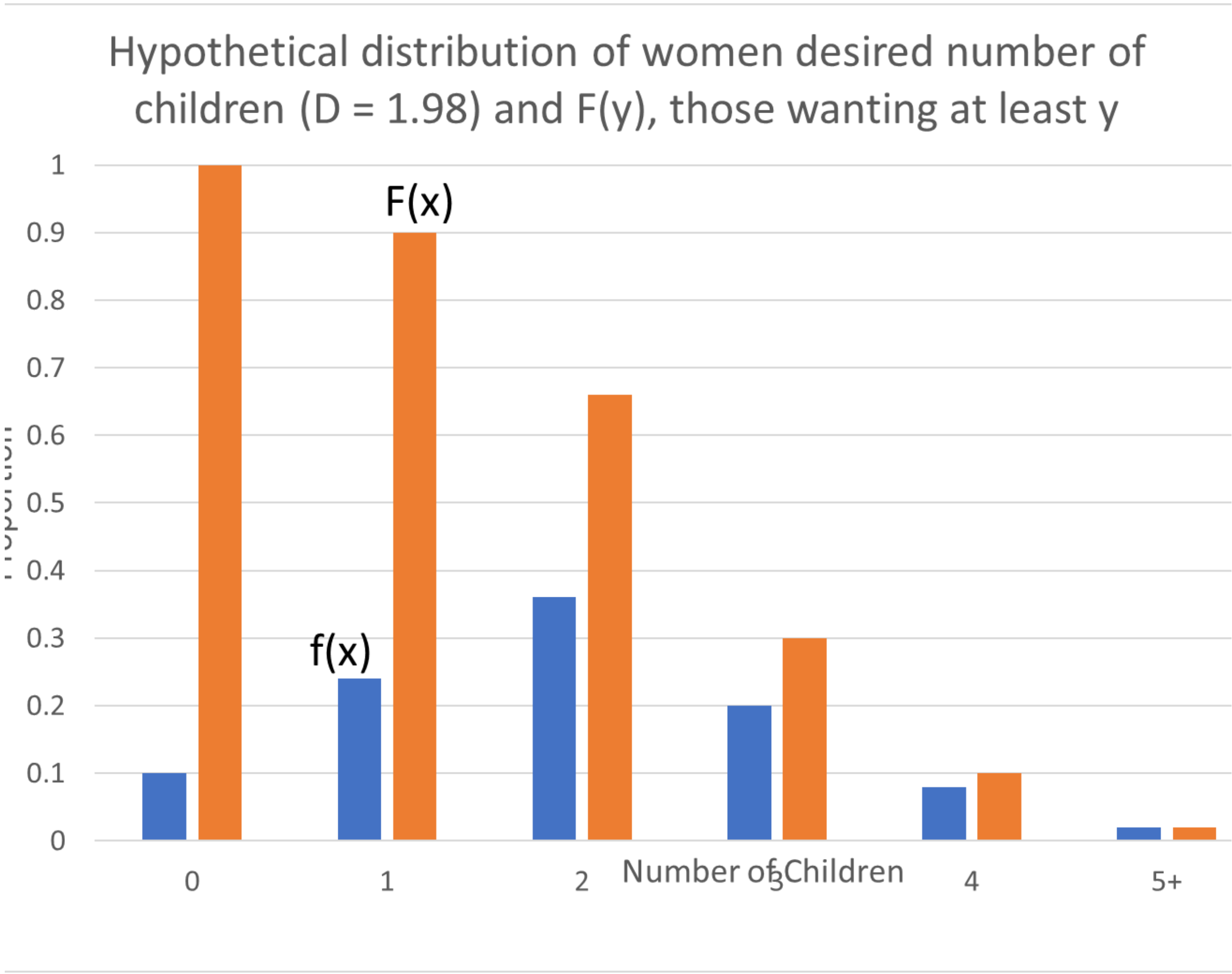
F(y) = proportion of women wanting at least y children

$$F(y)=\int_{y}^{12} f(u)\,du$$

Integral over y of F(y) is mean of f(y), D.

$$D=\int_{0}^{12} F(y)\,dy=\int_{0}^{12} yf(y)\,dy$$

F(y) corresponding to f(y)



Fertility of an initiation cohort at duration x

m = birthrate of nonterminators

x years after initiating fertility a woman has mx births.

$p(x)$ = Proportion of nonterminators after x years.

$$p(x) = F(mx)$$

$g(x)$ = Fertility of the cohort at duration x

$$g(x) = m(x)p(x) = m(x)F(mx)$$

Some implications

- The shape of $g(x)$ is identical to the shape of $F(mx)$. An increase in $f(1)$, or more generally in the proportion wanting few births, makes $g(x)$ fall more rapidly at short durations.
- The greater is m, the more quickly does $g(x)$ decline with duration, and the higher is the level of fertility initially.
- An increase in $f(0)$, the proportion of women wanting no births, affects the level but not the shape of $g(x)$.
- A concentration of women at, say, a desire for 2 children will make $g(x)$ fall more rapidly after about the first 5 years of marriage.

Differentiating we get:

$$g'(x) = -m\, f(mx)$$

Solving for f, and changing variables, we get:

$$f(x) = -g'(x/m)/m$$

Given an empirical age schedule for fertility by duration of marriage (e.g.) we could difference it to find the g'(x) function, and divide by m to get f(x), the distribution of fertility targets.

This actually works quite well, matching the survey data quite closely.

Changing goals – two strong assumptions:

1. At all times, all duration cohorts of women have the same distributions of family size goals
2. When fertility goals for women change, they change for all women by the same amount and in the same direction.

3. How deal with continuous change in discrete distribution?
   - To get D' = .1, let 10% of women at each discrete desired number of children move up by one child more desired. Then D will increase by .1 while the distribution remains discrete.
   - Or – just let each category increase by .1, e.g. 1 becomes 1.1 etc. That works too.
     - ✓ For what follows, I assume this one. The whole distrib just slides left by .1. This requires fractional births.

- write the distribution of y conditional on the value of its mean, thus:

$$f(y \mid D)$$

- Since shape of distribution is assumed invariant, write:

$$f(y \mid D) = h\left(y - D\right)$$

- where h has exactly same shape as f, but is centered on mean 0 rather than D.

- Similarly define

$$F(y \mid D) = H\left(y - D\right)$$

the proportion of women wanting at least y children when the mean desired completed fertility is D.

Now let D change over time: D(t)

- divide women into two groups:
  - Nonterminators
  - Terminators who attained their desired fertility earlier but who may now want more births as a result of revising goal upwards as D rises.
- First group:
  - x yrs after initiating reproduction a proportion P(x,t) of this group wants D(t) and has mx.

$$P(x,t) = F\left(xm \mid D(t)\right) = H\left(xm - D(t)\right)$$

  - These women contribute a flow of births:

$$mF(xm \mid D(t)) = mH(xm - D(t))$$

- Integrate over all durations x for this cohort

$$\int_0^{25} mF\left(xm \mid D\left(t\right)\right)dx \ = \ \int_0^{25} mH\left(xm - D\left(t\right)\right)dx = D\left(t\right)$$

- These women were nonterminators already, so revising target upwards does not change their current behavior.
- Therefore the nonterminators contribute enough births so TFR = D.

How about second group, the terminators?

- Some will switch back to nonterminators status.
- Suppose D'(t)<m, that is demand rises more slowly than the birthrate of nonterminators.
- Then this group will remain constantly on the edge of terminator-nonterminator status, bearing additional children to match D'(t).
- At duration x there are proportion 1-P(x,t) of these women, with birth rate D'(t).
- Integrating over all durations, their contribution, k, is:

$$k = \int_0^{25} D'(t)(1 - P(x,t))dx$$

$$k = D'(t)(\beta - D(t)/m)$$ Where β = 20 is upper duration limit to fecundity.

Combining the two groups of women we get:

$$TFR(t) = D(t) + D'(t)\left[\beta - D(t)/m\right]$$

- With unchanging goals the second term is zero. TFR equals D(t).
- With rising goals D'(t) is positive. Second term adds to TFR.
  - This “Acceleration effect" relates TFR to rate of change of goals, D'(t).
  - The greater is desired family size D, the smaller is acceleration effect, since fewer women will have become terminators.
  - Big enough D gives natural fertility population with no acceleration effect.

Illustration: when D is rising, TFR>D.

- D(t) = 2.1 (replacement)
- Fecund years after initiation β = 20
- m =.3 births/yr (for nonterminators).
- Suppose D is rising .1 per year.
- Acceleration effect k is

$$k = D'(t)(\beta - D(t)/m)$$

$$k = .1(20 - 2.1/.3) = .1*13 = 1.3$$

$$TFR = D + k = 2.1 + 1.3 = 3.4$$

- For Aiming model, the same case yields TFR=3.1. Pretty similar.

The TFR equals desired family size when desired family size is declining.

- If D is declining, no woman switches from terminator to non-terminator status
  - ➢If she wanted no children in the past, she certainly will not want more when she has reduced her overall target.
  - ➢Therefore the only women bearing children will always have been nonterminators
- Already shown that contribution to period fertility of nonterminators is exactly equal to D.
- Therefore, when fertility goals decline TFR=D and there is no overall acceleration effect.
- However, terminators are accumulating kids they would not have chosen to have.

The amplitude of fluctuations in the TFR exceeds the amplitude of fluctuations in desired family size.

- Since TFR exceeds D when D is rising and equals D when D is falling, fluctuations in TFR have greater amplitude.

The peak in TFR precedes the peak in D, just as in Aiming.
Amplitude of fluctuations in TFR exceeds those in D.

- Consider a peak in a smoothly varying time path of desired family size, occurring at t. Then we must have D'(t) = 0 and D"(t)<0.
- Differentiate wrt t: $TFR(t) = D(t) + D'(t)\left[\beta - D(t)/m\right]$
- Get: $TFR'(t) = D''(t)(\beta - D(t)/m)$

- Evaluate the result at t . Since at peak in D, D' =0 and D"<0. So TFR'(t) <0 and it is already past its own peak and falling when D reaches its peak.
- This is one of the main results from Aiming.

Sinusoidal case here is similar to Aiming

- Details in the paper, but here is a comparison of the same example used in Aiming.

	Aiming model	Present Model	Comment
Max TFR – Max D	.38	.37	Very similar on upswing.
Min D – Min TFR	.47	00	Irreversibility gives 0 on downswing.
Lead of Max TFR over Max D in years	5.5	7.7	Very similar.

- Quite remarkable given completely different models.

Adding contraceptive failure is simpler than in the Aiming model

- The whole analysis is oriented around terminators vs nonterminators.
    - Terminators are at risk of number failures.
    - Nonterminators are at risk of timing failures.
- Adding these to this model would be straightforward.
- Increased timing failure rates
    - Would not have a long run effect on TFR
    - But would temporarily raise TFR. Longitudinally, fertility would drop faster with duration.
    - Higher birth rates to nonterminators earlier would be offset by reductions later.
- Increased number failure rates
    - Would raise TFR in short term and long term.
- Reduced access to abortion would be similar to increase in both number and timing failure rates. Abortions leading to timing failures would be offset by later reductions in fertility. Size of offset can be calculated by have not yet done so.

Contraception and the fertility transition

- Declines in D (as with fertility transition) expose women to number failure rates over more years. Changes in age at initiation of childbearing (marriage age?) are also important here.
- One analyst found TFR varied closely with D cross-nationally and concluded family planning programs were unimportant. But at D = 2 exposure to risk of numbers failure is perhaps 15 years whereas with D = 6 it might be only a third or less. If TFR is close to D when D differs from 6 to 2, then number failure rates must be vastly reduced when D is low.

Other applications (see ref list at end)

Forecasting fertility using fertility expectations data. (Probably not a good idea.) See Lee (1981) “Model for forecasting fertility…”

Using these target models to structure econometric analysis of time series of marital fertility in US. See Lee (1981) “Stock Adjustment model…”.

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