The Formal Demography of Peak Population

Berkeley Formal Demography Workshop

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June 2, 2023

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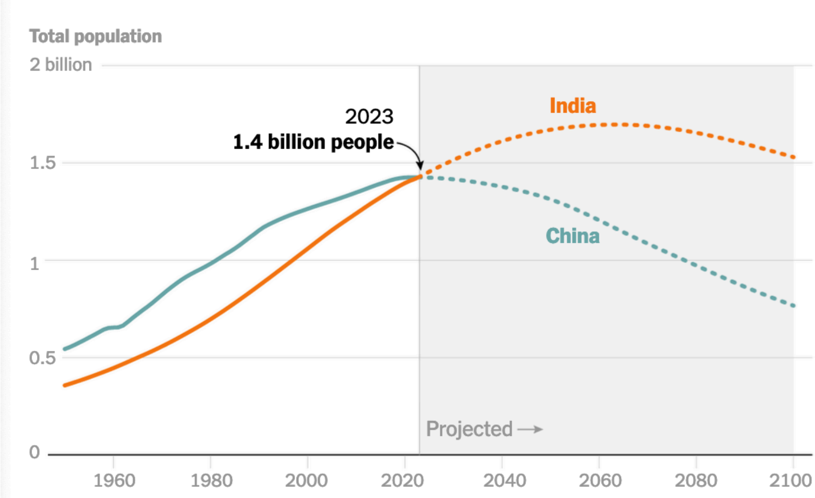
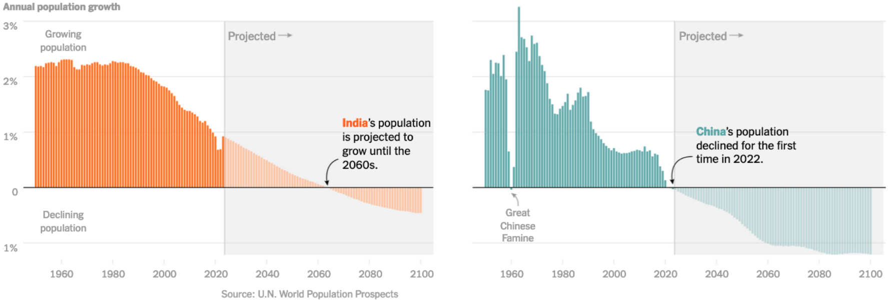
India and China



Source: U.N. World Population Prospects, estimated populations at midyear.

India and China growth rates



UN World Population Projections

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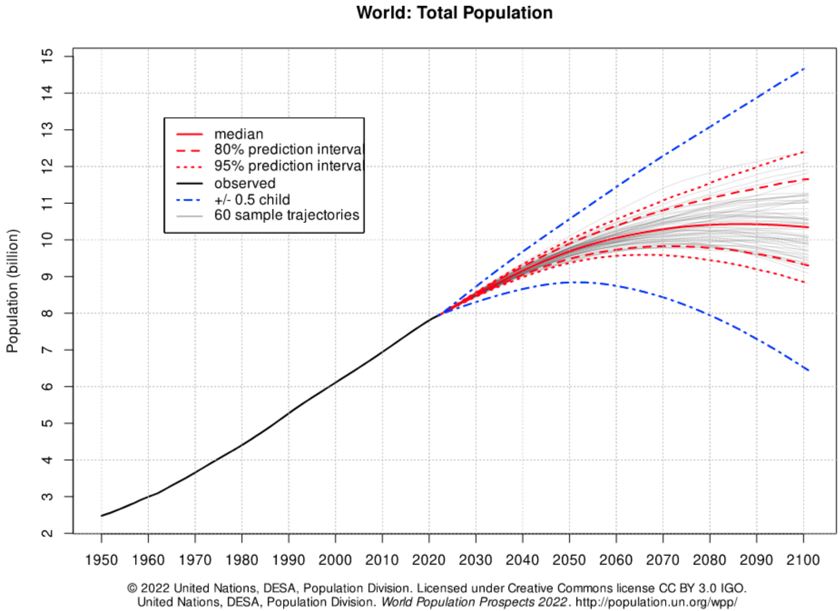
These projections come from historical experience combined with complex projected trajectories of vital rates.


World: Total Population
Population (billion)
median
80% prediction interval
95% prediction interval
observed
+/- 0.5 child
60 sample trajectories
1950 1960 1970 1980 1990 2000 2010 2020 2030 2040 2050 2060 2070 2080 2090 2100
2 3 4 5 6 7 8 9 10 11 12 13 14 15
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United Nations, DESA, Population Division. *World Population Prospects 2022*. http://population.un.org/wpp/

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- How much do changes in life expectancy impact the timing of peak population?
- How much do changes in immigration impact the timing of peak population?
- Can we tell a simpler story about the dynamics of peak population?

Ansley Coale's Analysis

In *Growth and Structure of Human Populations* (1972), Coale investigated the the timing of peak births and of peak population in a closed society with constant mortality undergoing a steady decline in fertility rates.

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- Total population: $N(t) = \int B(t-a)\ell(a)\,da$ for survival $\ell(a)$

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$\phi(a,t) = e^{kt}\phi_0(a)$ where $k < 0$ and $\int \phi_0(a)da = 1$,

i.e. net maternity declines exponentially at each age, and
the Net Reproductive Ratio (NRR) is one at time $t = 0$.

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Conclude: $B(t) \approx B(t-\mu)e^{kt}$

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A denotes the mean age of the stationary age distribution. The approximate year of peak population is

$$A - \mu/2$$

i.e. $A \approx 40$ years after births peak, or 25 years after fertility reaches replacement.

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3 demographic factors: increases in longevity, migration, and a slow-down in the fertility decline

Coale with Longevity Increases

We consider longevity improvements only at older ages.
We allow survival $\ell(a, c)$ to be a function of age and cohort, and express population as

$$N(t) = \int B(t-a)\ell(a, t-a)\, da \approx B(t-A) \int \ell(a, t-a)\, da$$

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Assuming that log CAL increases at rate ρ, population peaks around time

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With $k = -.01$ and $\rho = .002$ we find that population peaks 6 years later than in the Coale scenario.

Coale with Migration

Following Alho (2008), we assume that immigrants have the same fertility and mortality as natives and arrive as a constant proportion γ of native births. We also assume that immigrants arrive prior to the onset of childbearing.

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With $k = -.02$ (e.g. Japan) and $\gamma = .25$ (e.g. New Zealand), population decline is delayed by about 12.5 years.

Coale with slowing fertility decline

Recall Coale's assumption $\phi(a, t) = e^{kt}\phi_0(a)$ producing a log quadratic birthstream and population peaking around year

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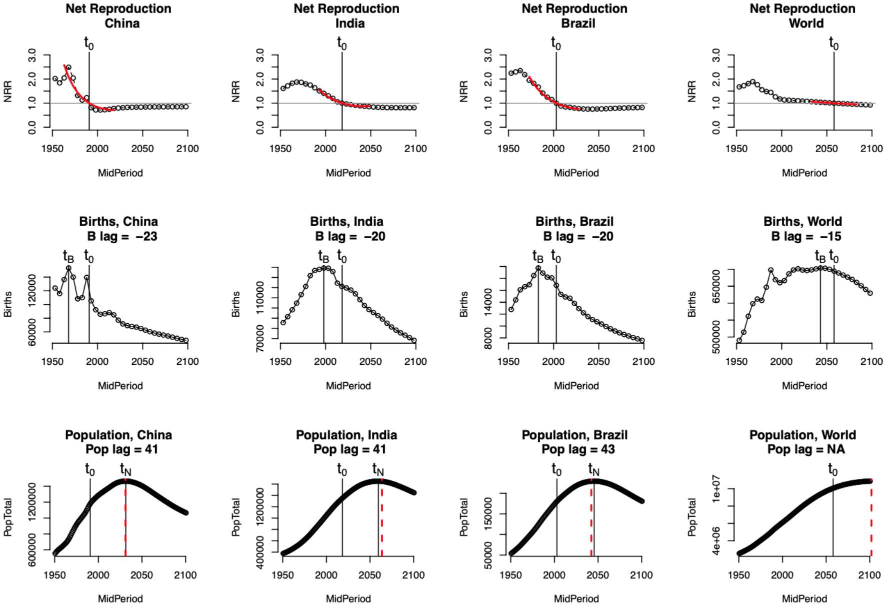
$$A - \mu/2 + \frac{k_2}{-k_1}\sigma^2$$

where σ^2 is the variance of age in the stationary population.


Net Reproduction
China
t0
NRR
0.0
1.0
2.0
3.0
1950
2000
2050
2100
MidPeriod
Net Reproduction
India
Net Reproduction
Brazil
Net Reproduction
World
Births, China
B lag = −23
tB
Births
60000
120000
Births, India
B lag = −20
70000
110000
Births, Brazil
B lag = −20
8000
14000
Births, World
B lag = −15
500000
650000
Population, China
Pop lag = 41
tN
PopTotal
600000
1200000
Population, India
Pop lag = 41
400000
1200000
Population, Brazil
Pop lag = 43
50000
150000
Population, World
Pop lag = NA
4e+06
1e+07

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- Substantial immigration that does not influence the fertility rate could postpone population decline by about a decade.
- Simple extensions of the Coale scenario can match the population trajectories projected by the UN.
- The relative timing of peak births, replacement level fertility, and peak population can be studied in a simple framework revealing the impact of changed hypotheses.