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Equity Risk and Insecure Property Rights

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Abstract

How much of the equity risk premium puzzle can be attributed to the insecure property rights of shareholders? This paper develops a version of the CCAPM with insecure property rights. The model implies that the current expected equity premium can be reconciled with a coefficient of risk aversion of 3.76, thus resolving the equity premium puzzle.

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I. **Introduction**

The focus of this research is to model and quantify the economic impact of insecure property rights on the equity risk premium. It should be noted that insecure property rights manifest themselves in many different forms. Here we are interested in insecure property rights that might lead to a collapse in real equity values that would not much affect the real values of bonds, especially government bonds. For example, if the U.S. government were to decide to put extraordinarily heavy taxes on corporate profits, dividends, or capital gains or to impose extraordinarily heavy regulatory burdens on corporations, those policies could redirect a substantial amount of cash flow away from shareholders without affecting bond values. The likelihood of such future tax increases or regulatory burdens narrowly targeted on corporate profits appears to be large enough to reconcile the current expected equity premium with a reasonable coefficient of risk aversion. This paper contributes to the literature by developing a version of the CCAPM with insecure property rights. Insecure property rights are modelled by introducing a stochastic tax on the wealth of shareholders. I calculate that the current expected equity premium, calculated by Fama and French, using the dividend growth model, can be reconciled with a coefficient of risk aversion of 3.76, thus resolving the equity premium puzzle.

The paper is organized as follows. Section II develops a version of the CCAPM with insecure property rights. Section III provides calculations. Section IV concludes.

II. **Model**

Consider an infinite horizon model with $n-1$ risky assets and the n^{th} risk-free asset. The vector of asset prices is $p_t \in \mathbb{R}^{\ltimes}$ at period t. The vector of dividends is $d_t \in \mathbb{R}_{+}^{\ltimes}$ at period t. An investor possesses portfolio $z_t \in [0,1]^n$ of assets and consumes $c_t \in \mathbb{R}$ at period t. Let the investor's one-period utility function be $u(c_t)$. Suppose now that τ_t is a stochastic tax imposed on the wealth of stock holders.

Thus, consider investor's optimization problem:

$$\max_{z_t} \sum_{t=0}^{\infty} b^t E\left[u(c_t)\right], \quad (1)$$

where $0 < b < 1$ and $u(\cdot)$ is such that $u'(\cdot) > 0$ and $u''(\cdot) < 0$,

subject to

$$c_t = (1-\tau_t)\sum_{k=1}^{n-1}(p_{kt}+d_{kt})z_{kt} + (p_{nt}+d_{nt})z_{nt} - \sum_{k=1}^{n} p_{kt}z_{kt+1}. \quad (2)$$

Taking first-order condition we obtain

$$-u'(c_t)p_{kt} + bE\left[u'(c_{t+1})\left(1-\tau_{t+1}\right)\left(p_{kt+1}+d_{kt+1}\right)\right] = 0 \quad \text{for } k = 1,...,n-1,$$

(3)

$$-u'(c_t)p_{nt} + bE\left[u'(c_{t+1})\left(p_{nt+1}+d_{nt+1}\right)\right] = 0. \quad (4)$$

Hence,

$$E\left[\tfrac{bu'(c_{t+1})}{u'(c_t)}\left(\ \left(1-\tau_{t+1}\right)R_{kt+1}\right)\right] = 1, \quad \text{for } k = 1,...,n-1, \quad (5)$$

$$E\left[\tfrac{bu'(c_{t+1})}{u'(c_t)}\ \right]R_f = 1. \quad (6)$$

THEOREM *Consider an infinite horizon economy described by* (1) *and* (2). *We further assume that*

a) Investors have one-period utility function $u(c) = \frac{c^{1-\alpha}}{1-\alpha}$.

b) $\ln((1-\tau_{t+1})\,R_{kt+1})$ *and* $\ln\left(b\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right)$ *are bivariate normally distributed with means*

$$\left(E\left[\ln((1-\tau_{t+1})\,R_{kt+1})\right],\ E\left[\ln\left(b\left(\tfrac{C_{t+1}}{C_t}\right)^{-\alpha}\right)\right]\right) = \left(\mu_k, \mu_c\right),$$

and the variance-covariance matrix

$$V = \begin{pmatrix} \sigma_k^2 & \sigma_{kc} \\ \sigma_{kc} & \sigma_c^2 \end{pmatrix} \textit{ for } k = 1, ..., n-1.$$

c) $ln(R_{kt+1})$ is normally distributed for $k = 1, ..., n-1$[1].

d) $ln(1-\tau_{t+1})$ is normally distributed.

Then,

$$\underbrace{ln\left(E\left[R_{kt+1}\right]\right) - ln\left(R_f\right) = a \cdot COV\left[\ln(R_{kt+1}),\ \ln\left(\frac{C_{t+1}}{C_t}\right)\right]}_{Traditional\ Relation} +$$

$$+a \cdot COV\left[\ln(1-\tau_{t+1}),\ \ln\left(\tfrac{C_{t+1}}{C_t}\right)\right] - \ln\left(E\left[1-\tau_{t+1}\right]\right) - COV[ln\left(R_{kt+1}\right), \ln\left(1-\tau_{t+1}\right)],$$

for $k = 1, ..., n-1$.

PROOF: See appendix.

III. Calculations

Using the dividend growth model, Fama and French (2002) estimate the current expected equity premium to be

[1] Since the sum of lognormally distributed random variables is not lognormally distributed, I will later need to assume that not all risky assets satisfy assumptions b) and c) to allow these assumption to be imposed on the market portfolio of risky assets.

$$\ln\left(E\left[R_{mt+1}\right]\right) - \ln\left(R_f\right) = 0.0255.$$[2]

Also,

$$COV\left[\ln(R_{mt+1}),\ \ \ln\left(\tfrac{C_{t+1}}{C_t}\right)\right] = 0.00125,$$

where $R_{mt+1} = 1+r_{mt+1}$ is the gross rate of return on the market portfolio of risky assets,

$R_f = 1 + r_f$ is the gross risk-free rate of return.

I estimate tax τ_{t+1} imposed on the wealth of stockholders as

$$\tau_{t+1} = \tfrac{\tau^d_{t+1}d_{t+1}+\tau^{SCG}_{t+1}SCG_{t+1}+\tau^{LCG}_{t+1}LCG_{t+1}}{p_{t+1}+d_{t+1}} =$$

$$= \underbrace{\frac{\tau^d_{t+1}d_{t+1} + \tau^{SCG}_{t+1}SCG_{t+1} + \tau^{LCG}_{t+1}LCG_{t+1}}{p_t}}_{Tax\ Yield,\ TY_{t+1}} \cdot \underbrace{\frac{p_t}{p_{t+1} + d_{t+1}}}_{1/R_{mt+1}} = \tfrac{TY_{t+1}}{R_{mt+1}},$$

where

τ^d_{t+1} is the dividend tax,

τ^{SCG}_{t+1} is the tax on short-term capital gains,

τ^{LCG}_{t+1} is the tax on long-term capital gains,

SCG_{t+1} are realized short-term capital gains,

LCG_{t+1} are realized long-term capital gains, and

[2]Fama and French (2002) demonstrate that the dividend growth model produces a superior measure of the expected equity premium than using the average stock return.

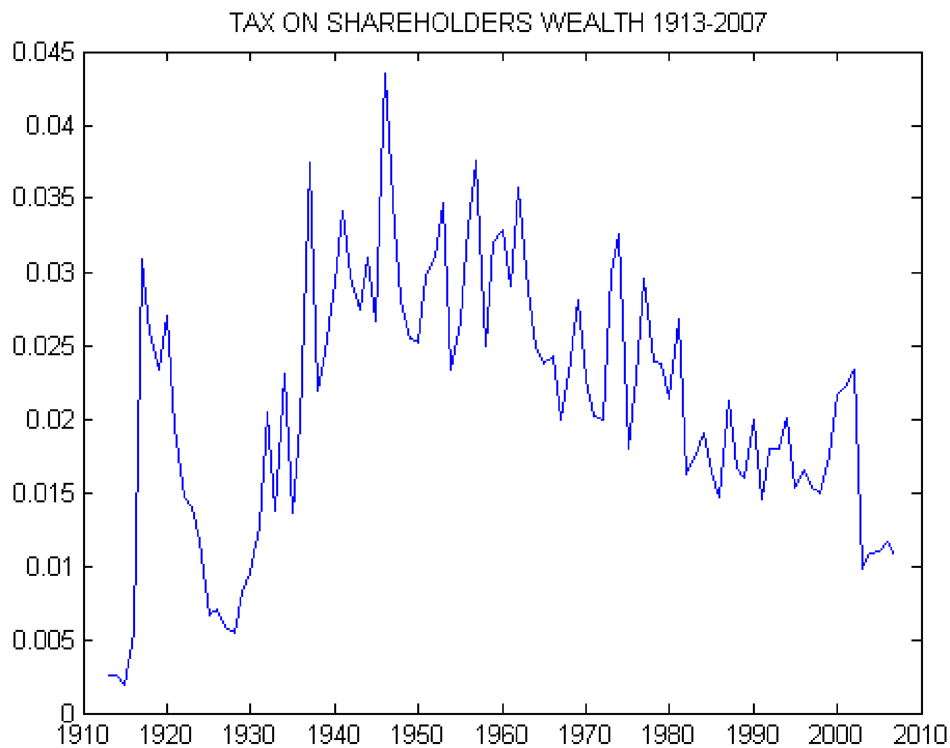

Figure 1:

TY_{t+1} is the tax yield.[3]

[3]Sialm (2008) estimates the tax yield as $TY_{t+1} = \tau^{d}_{t+1} \cdot 0.045 + \tau^{SCG}_{t+1} \cdot 0.001 + \tau^{LCG}_{t+1} \cdot 0.018$.

So, $\frac{d_{t+1}}{p_t} = 0.045$, $\frac{SCG_{t+1}}{p_t} = 0.001$ and $\frac{LCG_{t+1}}{p_t} = 0.018$.

See Figure 1 above. I calculate that for 1913-2007,

$$\ln\left(E\left[1-\tau_{t+1}\right]\right) = -0.0214,$$

$$COV\left[\ln(R_{mt+1}),\;\ln\left(1-\tau_{t+1}\right)\right] = 0.0006,$$

$$COV\left[\ln(1-\tau_{t+1}),\;\ln b\left(\frac{C_{t+1}}{C_t}\right)\right] = 0.0000.$$

The traditional CCAPM without insecure property rights, and with the current expected equity premium of 6%, calculated by Mehra (2003), using simply the average stock return, yields a coefficient of risk aversion of roughly 50:[4]

$$a = \frac{\ln(E[R_{kt+1}])-\ln(R_f)}{COV\left[\ln(R_{kt+1}),\;\ln\left(\frac{C_{t+1}}{C_t}\right)\right]} =$$

$$= \frac{0.07-0.01}{0.00125} = 47.6.$$

After introducing insecure property rights and with the current expected equity premium of 2.55%, calculated by Fama and French (2002), using the dividend growth model, I obtain by the Theorem that for an average investor who realizes short-term and long-term gains in accordance with historical patterns, the coefficient of risk aversion is

$$a = \frac{\ln(E[R_{kt+1}])-\ln(R_f)+\ln(E[1-\tau_{t+1}])+COV[\ln(R_{kt+1}),\;\ln(1-\tau_{t+1})]}{COV\left[\ln(R_{kt+1}),\;\ln\left(\frac{C_{t+1}}{C_t}\right)\right]+COV\left[\ln(1-\tau_{t+1}),\;\ln\left(\frac{C_{t+1}}{C_t}\right)\right]} =$$

$$= \frac{0.0255-0.0214+0.0006}{0.00125+0.0000} = 3.76.$$

[4]Mehra (2003).

Since most of the studies indicate a coefficient of risk aversion between 2 and 4, $a = 3.76$ resolves the puzzle.

IV. Conclusion

This paper develops a version of the CCAPM with insecure property rights. Insecure property rights are modelled by introducing a stochastic tax on the wealth of shareholders. The likelihood of future tax increases or regulatory burdens narrowly targeted on corporate profits appears to be large enough to reconcile the current expected equity premium with a reasonable coefficient of risk aversion. I calculate that the current expected equity premium can be reconciled with a coefficient of risk aversion of 3.76, thus resolving the equity premium puzzle.

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Appendix

PROOF: We have for $k = 1, ..., n-1$,

$$\mu_k = E\left[ln((1-\tau_{t+1})\, R_{kt+1})\right] = E\left[ln(R_{kt+1})\right] + E\left[ln(1-\tau_{t+1})\right].$$

So,

$$\mu_k = E\left[ln(R_{kt+1})\right] + E\left[ln(1-\tau_{t+1})\right].$$

Also, for $k = 1, ..., n-1$,

$$\sigma_k^2 = VAR\left[ln((1-\tau_{t+1})\,R_{kt+1})\right] = VAR\left[ln\left(R_{kt+1}\right) + ln\left(1-\tau_{t+1}\right)\right] =$$

$$= VAR\left[ln\left(R_{kt+1}\right)\right] + VAR\left[ln\left(1-\tau_{t+1}\right)\right] + 2\cdot COV[ln\left(R_{kt+1}\right), ln\left(1-\tau_{t+1}\right)].$$

Therefore,

$$\sigma_k^2 = VAR\left[ln\left(R_{kt+1}\right)\right] + VAR\left[ln\left(1-\tau_{t+1}\right)\right] + 2\cdot COV[ln\left(R_{kt+1}\right),\ ln\left(1-\tau_{t+1}\right)].$$

At the same time, for $k = 1, ..., n-1$,

$$\sigma_{kc} = COV\left[ln((1-\tau_{t+1})\,R_{kt+1}),\ \ \ln\left(b\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right)\right] =$$
$$= COV\left[ln(R_{kt+1}) + ln(1-\tau_{t+1}),\ \ \ln\left(b\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right)\right] =$$
$$= -a\cdot COV\left[\ln(R_{kt+1}), \ln\left(\frac{C_{t+1}}{C_t}\right)\right] - a\cdot COV\left[ln(1-\tau_{t+1}),\ \ \ln b\left(\frac{C_{t+1}}{C_t}\right)\right]$$

Thus,

$$\sigma_{kc} = -a\cdot COV\left[\ln(R_{kt+1}), \ln\left(\frac{C_{t+1}}{C_t}\right)\right] - a\cdot COV\left[ln(1-\tau_{t+1}),\ \ \ln b\left(\frac{C_{t+1}}{C_t}\right)\right].$$

Now, using Rubinstein (1976) we obtain

$$\mu_k + \tfrac{1}{2}\sigma_k^2 - ln\left(R_f\right) = -\sigma_{kc} \quad \text{for } k = 1, ..., n-1.$$

So,

$$E\left[ln(1-\tau_{t+1})\right]+E\left[ln(R_{kt+1})\right]+$$

$$\frac{1}{2}\left(VAR\left[ln\left(R_{kt+1}\right)\right]+VAR\left[ln\left(1-\tau_{t+1}\right)\right]+2\cdot COV[ln\left(R_{kt+1}\right),ln\left(1-\tau_{t+1}\right)]\right)-$$

$$-ln\left(R_f\right)=$$

$$=a\cdot COV\left[\ln(R_{kt+1}),\ln\left(\frac{C_{t+1}}{C_t}\right)\right]+a\cdot COV\left[ln(1-\tau_{t+1}),\ \ \ln b\left(\frac{C_{t+1}}{C_t}\right)\right].$$

Therefore,

$$E\left[ln(R_{kt+1})\right]+\frac{VAR[ln(R_{kt+1})]}{2}-ln(R_f)=a\cdot COV\left[\ln(R_{kt+1}),\ln\left(\frac{C_{t+1}}{C_t}\right)\right]+$$

$$a\cdot COV\left[ln(1-\tau_{t+1}),\ \ \ln b\left(\frac{C_{t+1}}{C_t}\right)\right]-$$

$$-E\left[ln(1-\tau_{t+1})\right]-\frac{1}{2}VAR\left[ln\left(1-\tau_{t+1}\right)\right]-COV[ln\left(R_{kt+1}\right),ln\left(1-\tau_{t+1}\right)]$$

But by normality of $ln(R_{kt+1})$ and $ln(1-\tau_{t+1})$ I obtain

$$ln\left(E\left[R_{kt+1}\right]\right)=E\left[ln(R_{kt+1})\right]+\frac{1}{2}VAR\left[ln\left(R_{kt+1}\right)\right]$$

and

$$ln\left(E\left[1-\tau_{t+1}\right]\right)=E\left[ln(1-\tau_{t+1})\right]+\frac{1}{2}VAR\left[ln\left(1-\tau_{t+1}\right)\right].$$

Hence,

$$ln\left(E\left[R_{kt+1}\right]\right)-ln(R_f)=a\cdot COV\left[\ln(R_{kt+1}),\ln\left(\frac{C_{t+1}}{C_t}\right)\right]+$$

$$+a\cdot COV\left[ln(1-\tau_{t+1}),\ \ \ln b\left(\frac{C_{t+1}}{C_t}\right)\right]-ln\left(E\left[1-\tau_{t+1}\right]\right)-$$

$$-COV[ln\left(R_{kt+1}\right),ln\left(1-\tau_{t+1}\right)].\ \blacksquare$$