Case Summary

Case ID: wue_transient_heatflux
Objective: Verify the WU–E (urban structure) heat-flux prototype that couples a wind-aligned anisotropic fire ellipse (Hamada-style regression) with a piecewise transient design-fire curve for HRRPUA. We check that (i) geometric transformations, (ii) ellipse reach/coverage, (iii) per-cell HRR normalization, and (iv) heat-flux partitions (direct flame contact vs. radiation) reproduce expected behavior and conserve energy consistently.

0.1 Methods

0.1.1 Theory

Coordinate Transformations

Geometric angle from source to target cell:

$$\theta = \arctan\big(j,\ i\big) \quad \text{(rad)}. \tag{1}$$

Wind-aligned major-axis angle (radians) from the 20-ft met direction (blowing *from*):

$$\theta_{\text{wind}} = \frac{\pi}{180}\big(270 - \text{WD}_{20ft}\big). \tag{2}$$

Relative angle for ellipse formulas:

$$\theta_f = \theta - \theta_{\text{wind}}, \qquad R = \sqrt{(i\,\Delta)^2 + (j\,\Delta)^2}. \tag{3}$$

Wind-Aligned Ellipse (Hamada-Style)

Convert wind speed to m/s:

$$V = 0.447\,\text{WS}_{20ft}. \tag{4}$$

Downwind ($D_\downarrow$), upwind ($D_\uparrow$), and sidewind ($D_\perp$) distances (m) are piecewise functions of V with coefficients depending on (A, D) and scaled by W_p:

$$\text{Low wind } (V < 10): \quad D_\downarrow = W_p\,(D_1 V + D_2),\ \ D_\perp = W_p\,(S_1 V + S_2),\ \ D_\uparrow = W_p\,(U_1 V + U_2), \tag{5}$$

$$\text{High wind } (V > 17.3): \quad D_\downarrow = W_p\,(D_1 V + D_2),\ \ D_\perp = W_p\,(S_1 V + S_2),\ \ D_\uparrow = W_p\,(U_1 V + U_2), \tag{6}$$

$$\begin{aligned} \text{Moderate } (10 \le V \le 17.3): \quad & D_\downarrow = W_p\,(D_1 V^2 + D_2 V + D_3), \\ & D_\perp = W_p\,(S_1 V^2 + S_2 V + S_3), \\ & D_\uparrow = W_p\,(U_1 V^2 + U_2 V + U_3). \end{aligned}$$

Ellipse parameters:

$$a = \frac{D_\downarrow + D_\uparrow}{2}, \qquad \varepsilon = \min\Big(\tfrac{a}{2},\ a - D_\uparrow\Big), \qquad E_{b2} = 1 - \Big(\tfrac{\varepsilon}{a}\Big)^2, \tag{7}$$

$$b = \begin{cases} \dfrac{D_\perp}{\sqrt{E_{b2}}}, & E_{b2} > 0, \\ 0, & E_{b2} \le 0. \end{cases} \tag{8}$$

State vector:

$$\mathbf{E} = \big[a,\ b,\ \varepsilon,\ D_\downarrow\big]^\top.$$

Ellipse Reach and Coverage

Maximum reach (scalar):

$$R_{\max} = 0.3\, D_{\downarrow}\, \frac{a-\varepsilon}{b^2}. \tag{9}$$

Directional reach along θ_f:

$$R_{\text{ell}}(\theta_f) = \frac{R_{\max}\, b^2}{a-\varepsilon\, \cos\theta_f}. \tag{10}$$

DFC coverage fraction (cell-centered, clipped to $[0,1]$):

$$C_{\text{DFC}} = \max\left\{ \min\left(\frac{R_{\text{ell}}(\theta_f) + \frac{1}{2}\Delta - R}{\Delta}, 1 \right), 0 \right\}. \tag{11}$$

Radiation annulus outside DFC, bounded by cutoff:

$$R_{\text{rad limit}}(\theta_f) = R_{\text{ell}}(\theta_f) + R_{\text{rad}}, \tag{12}$$

$$\Delta_{\text{rad}} = \max\left\{ \min\left(\frac{R_{\text{rad limit}}(\theta_f) + \frac{1}{2}\Delta - R}{\Delta}, 0 \right), 1 \right\}, \tag{13}$$

$$F_{\text{rad}} = \Delta_{\text{rad}}\big(1 - C_{\text{DFC}}\big). \tag{14}$$

Per-Cell HRR Normalization

To conserve total HRRPUA over the ellipse footprint, use the adjuster

$$C_{\text{HRR}} = \frac{\Delta^2}{\pi\,(b/a)\,a\,b} = \frac{\Delta^2}{\pi b^2}. \tag{15}$$

Transient HRRPUA

For burning time τ and parameters $(t_{\text{early}}, t_{\text{dev}}, t_{\text{decay}}, \text{HRRPUA}_{\text{peak}})$:

$$\text{HRRPUA}(\tau) = \begin{cases} \dfrac{\text{HRRPUA}_{\text{peak}}}{t_{\text{early}}}\,\tau, & 0 \le \tau \le t_{\text{early}}, \\ \text{HRRPUA}_{\text{peak}}, & t_{\text{early}} < \tau \le t_{\text{dev}}, \\ \dfrac{\text{HRRPUA}_{\text{peak}}}{t_{\text{dev}} - t_{\text{decay}}}\,(\tau - t_{\text{decay}}), & t_{\text{decay}} < \tau, \\ 0, & \text{otherwise}, \end{cases} \qquad \text{HRRPUA}(\tau) \leftarrow \max\{0, \text{HRRPUA}(\tau)\}. \tag{16}$$

Per-Cell Heat Fluxes

Let $C_{\text{burn}} = 1 - \text{NONBURNABLE\_FRAC}$. Then

$$q''_{\text{DFC}} = C_{\text{burn}}\, C_{\text{DFC}}\, \text{HRRPUA}(\tau)\, C_{\text{HRR}}, \tag{17}$$

$$q''_{\text{rad}} = \frac{0.3\, C_{\text{burn}}\, \alpha\, F_{\text{rad}}\, C_{\text{HRR}}\, \text{HRRPUA}(\tau)\, \Delta^2}{4\pi\, R_{\text{eff}}^2}, \qquad R_{\text{eff}} = \begin{cases} \Delta\,(1 - C_{\text{DFC}}), & 0 < C_{\text{DFC}} < 1, \\ R - R_{\text{ell}}(\theta_f), & \text{otherwise}. \end{cases} \tag{18}$$

Algorithm (Per Time Step)

1. Set $\tau = t - t_0$ and compute HRRPUA(τ).
2. From $(\mathrm{WS}_{20ft}, A, D, W_p)$ compute $\mathbf{E} = [a, b, \varepsilon, D_{\downarrow}]$.
3. For each cell (i, j) with center $(i\,\Delta, j\,\Delta)$:
   3.1. Compute R, θ, θ_f and $R_{\mathrm{ell}}(\theta_f)$.
   3.2. Compute C_{DFC}, F_{rad}, R_{eff}.
   3.3. Evaluate q''_{DFC} and q''_{rad}.

0.1.2 Assumptions

- Urban array represented on a uniform analysis grid of square cells of size Δ (m).
- Structures have a non-burnable fraction NONBURNABLE_FRAC; the remainder contributes to heat release/flux.
- HRRPUA follows a piecewise transient curve with early growth, plateau, and decay; negative segments are clipped to zero.
- Wind-aligned ellipse is derived from 20-ft wind inputs $(\mathrm{WD}_{20ft}, \mathrm{WS}_{20ft})$ and geometric parameters (A, D) with a proportionality W_p.
- Radiation is applied outside the ellipse up to a cutoff radius R_{rad}; convective/design-fire contact (DFC) acts within the ellipse footprint.
- Units: distances in meters; heat flux in kW/m^2.

0.1.3 Simulation Setup

Parameter Table (defaults)

Properties	Symbols	Values
Absorptivity	α	0.89
Radiation cutoff (m)	R_{rad}	100
Analysis cell size (m)	Δ	20
Wind direction (deg, from)	WD_{20ft}	0
Wind speed (mph)	WS_{20ft}	40
Footprint dim. (m)	A	10
Separation (m)	D	10
Wind proportionality	W_p	1
Non-burnable fraction	NONBURNABLE_FRAC	0
Early, dev., decay times (s)	$(t_{\mathrm{early}}, t_{\mathrm{dev}}, t_{\mathrm{decay}})$	(300, 3900, 4200)
Peak HRRPUA (kW/m^2)	$\mathrm{HRRPUA}_{\mathrm{peak}}$	400

Grid and Indices

Cells are indexed by $i, j \in \{-5, \ldots, 5\}$ with centers $(x, y) = (i\,\Delta,\ j\,\Delta)$ relative to the burning structure at $(0, 0)$.

0.1.4 Input Data

Describe input rasters, constants, initial conditions.

0.1.5 Numerical Controls

Mesh resolution, Time step(CFL), level-set solver options, etc.

0.2 Expected Results and Reasoning

- **Geometric consistency:** As V increases, $D_{\downarrow}$ grows faster than $D_{\perp}$ and $D_{\uparrow}$, increasing a and eccentricity (smaller b/a); $R_{\text{ell}}(\theta_f)$ elongates downwind.
- **Coverage partition:** Cells inside the ellipse ($C_{\text{DFC}} > 0$) receive q''_{DFC} proportional to HRRPUA and C_{HRR}; outer annulus receives q''_{rad} diminishing with R_{eff}^{-2}.
- **Conservation:** With C_{HRR}, summing q''_{DFC} over the footprint tracks the design HRRPUA(τ) (up to discretization error).
- **Limits:** For $V \to 0$, the ellipse tends toward isotropic ($D_{\downarrow} \approx D_{\perp} \approx D_{\uparrow}$); for very large V, footprint becomes highly elongated downwind; q''_{rad} shifts outward.

0.3 Acceptance Criteria

- **Energy consistency**: $\sum_{\text{cells}} |Q_{simulation} - Q_{analytical}|/Q_{analytical} \leq 0.5\,\%$, Q will be HRRPUA, transient DFC and radiative heat fluxes.
- **Partition sanity**: $q''_{\text{rad}} \to 0$ as $R_{\text{eff}} \to \infty$ and vanishes inside pure DFC cells when $F_{\text{rad}} = 0$.
- **Directional response**: Downwind flux peak $>$ side $>$ upwind for V in moderate/high ranges.

0.4 Results

Metric	Value
HRR mean relative error	0.000314
DFC mean relative error	1.8e-05
RAD mean relative error	0.000288

Table 1: Comparison errors (analytic vs simulation).

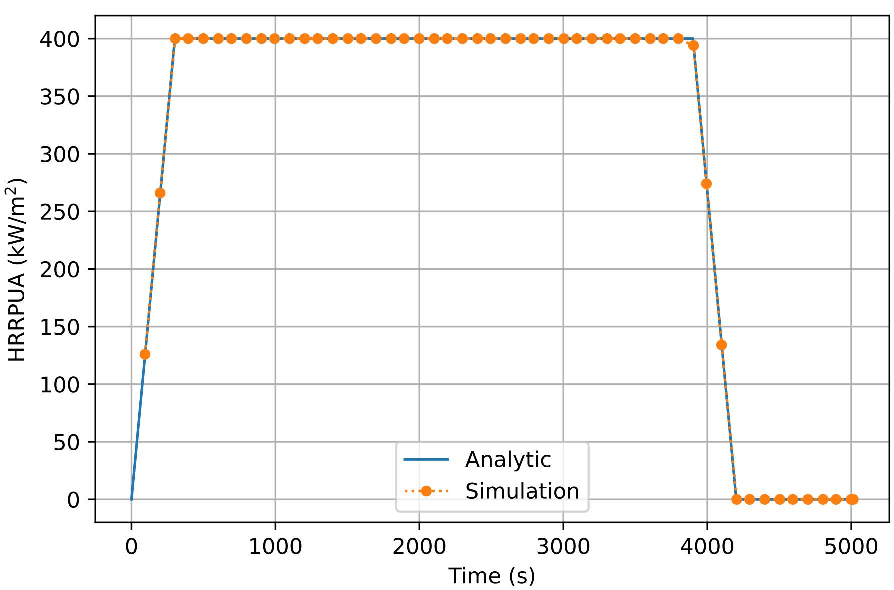

Figure 1: Transient HRR (analytic vs. simulation).

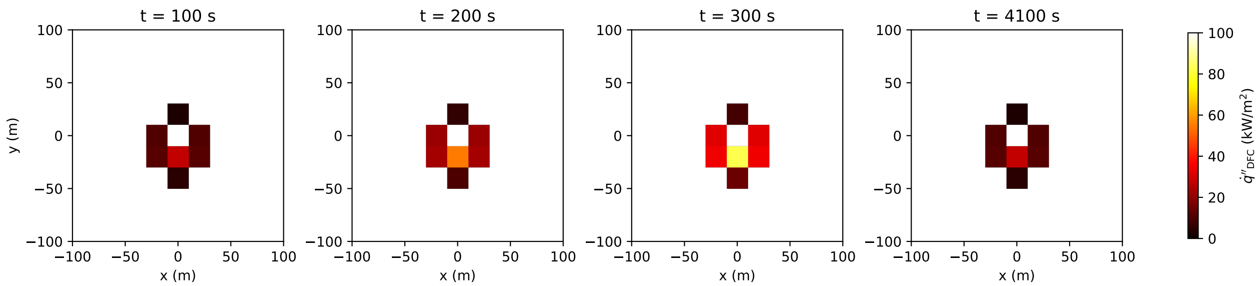

Figure 2: Analytic DFC heat flux at selected times.

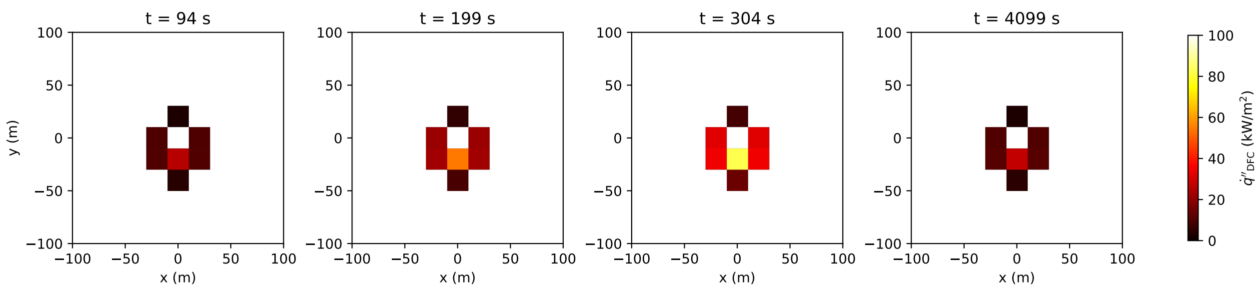

Figure 3: Simulated DFC heat flux at selected times.

0.5 Discussion

- Clarify parameter sensitivities (e.g., A, D, W_p) and wind-direction convention: $\theta_{\text{wind}} = \frac{\pi}{180}(270 - \text{WD}_{20ft})$ points the major axis toward $+x$ when $WD_{20ft} = 0°$.
- Document any discretization effects at coarse Δ and how C_{HRR} compensates for footprint changes.
- Note corner cases: $E_{b2} \le 0$ (degenerate b), transition regions in the piecewise wind regression, and clipping of coverage fractions.

Reproducibility

- MATLAB functions: `ellipse_ucb`, `hrr_transient`, `heat_flux_calc`.
- Command(s): `./run_case.sh`; environment: `<modules/conda env>`.
- Logs under `cases/wue_transient_heatflux/logs/`; figures in `cases/wue_transient_heatflux/figures/`.

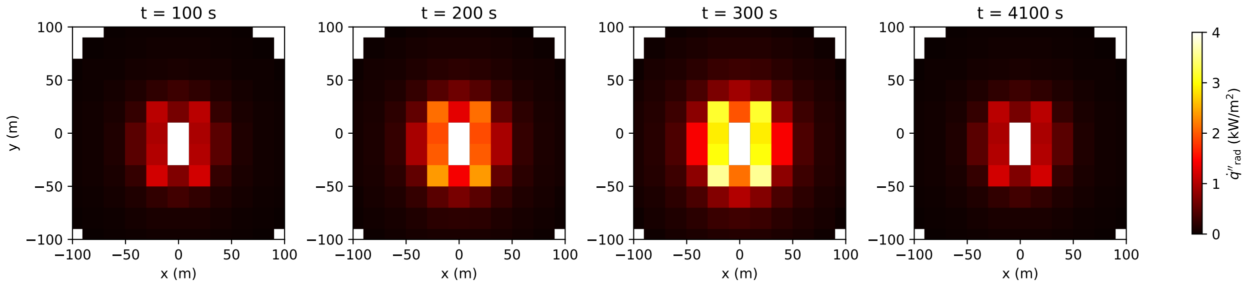

Figure 4: Analytic radiative heat flux at selected times.

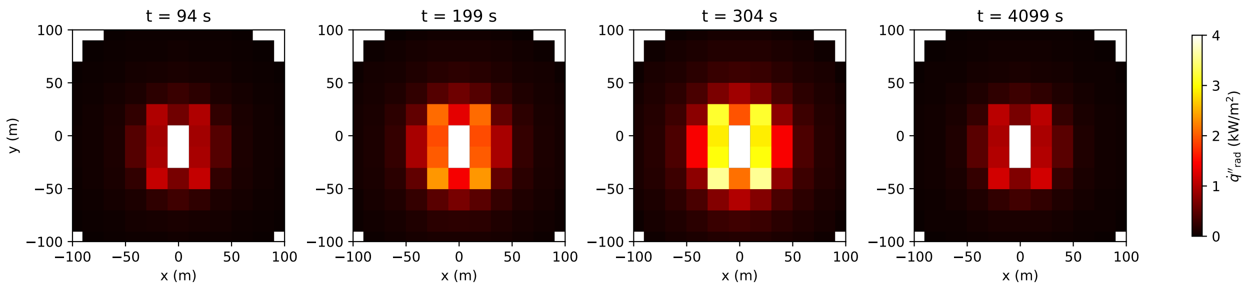

Figure 5: Simulated radiative heat flux at selected times.

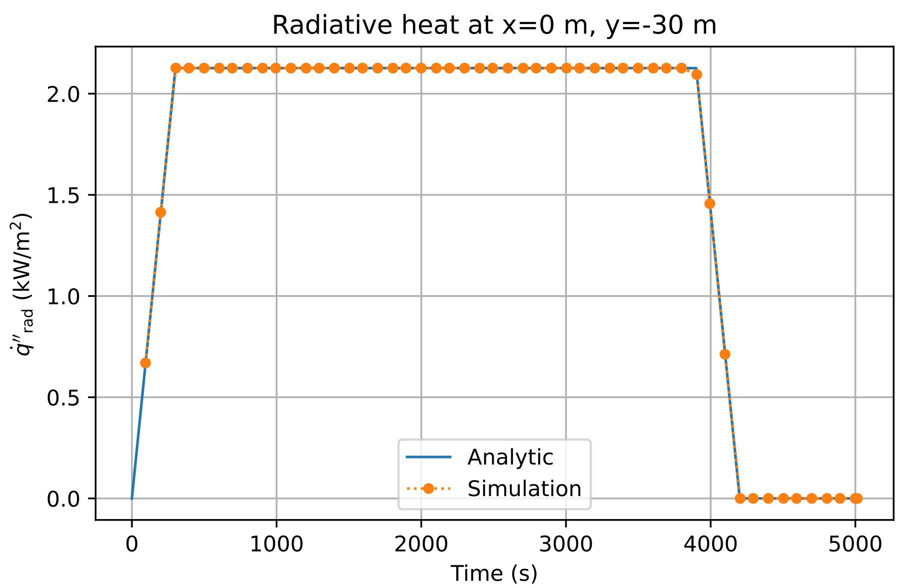

Figure 6: Radiative heat flux time history at $(x, y) = (0, -30)$ m.

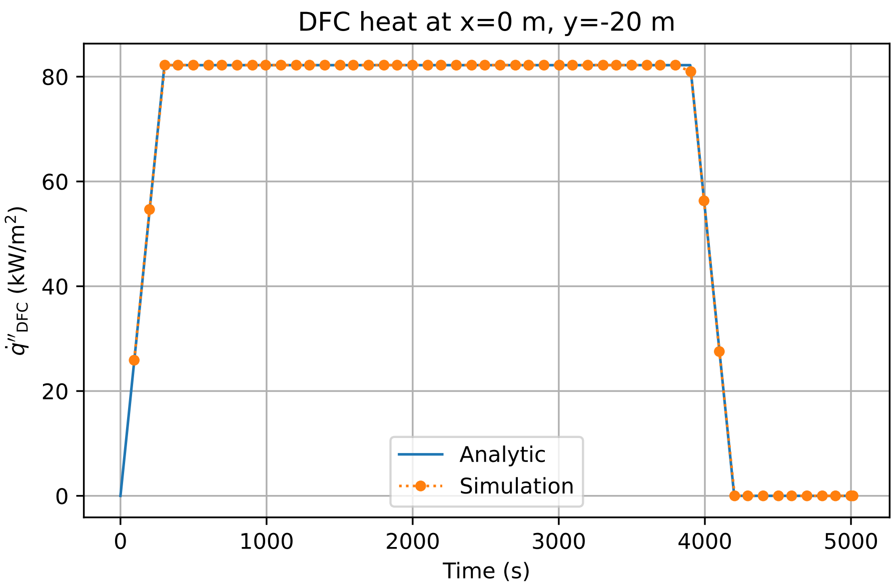

Figure 7: DFC heat flux time history at $(x, y) = (0, -20)$ m.