ELMFIRE Verification Suite — Master Report

Verification Team

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Contents

How to Read This Report

Each section includes a full PDF of a verification case report. Cases are ordered by directory name in `cases/`. To add or remove a case from the master report, run the automation script `tools/build_all.sh`.

1 ignition_mask_bp_convergence

Case Summary

Case ID: ignition_mask_bp_convergence
Objective: Justify the Monte Carlo (MC) method used by ELMFIRE for ignition sampling by deriving the expected time-of-arrival (ToA) statistics at a target pixel when ignition is drawn uniformly from a specified mask (square or circle). We compare the empirical ToA distribution from ensembles to the mask-implied expectation, and quantify how many ensembles are needed for convergence.

1.1 Assumptions

- Flat terrain: slope = 0, aspect arbitrary but unused for $SLP = 0$.
- Homogeneous fuel: `IFBFM = 102`, fixed moisture: $M_1 = 0.03$, $M_{10} = 0.04$, $M_{100} = 0.05$, $M_{\mathrm{lh}} = 0.30$, $M_{\mathrm{lw}} = 0.60$.
- Steady wind: direction WD20 = 180° (blowing to the north), speed WS20 = 15 mph; canopy terms set to zero ($CC = CH = 0$).
- No spotting/suppression; acceleration factor = 1; length/width cap $\mathrm{LOW}_{\mathrm{max}} = 8$.
- Coordinates are meters; the ToA along a ray from source to target uses the directional ROS returned by the ellipse kinematics.
- Ignition location X is uniformly distributed on the mask region Ω (square or circle).

1.2 Simulation Setup

1.2.1 Parameter Table

Fuel model	IFBFM	102
Dead fuel moisture	M_1, M_{10}, M_{100}	0.03, 0.04, 0.05
Live fuel moisture	$M_{\mathrm{lh}}, M_{\mathrm{lw}}$	0.30, 0.60
Wind (20-ft)	(WD20, WS20)	$(180°, 15\text{ mph})$
Slope / Aspect	(SLP, ASP)	$(0°, 0°)$
Adj. factors	(ADJ, ACCEL)	$(1, 1)$
Ellipse cap	$\mathrm{LOW}_{\mathrm{max}}$	8
Target (m)	$\mathbf{y}$	$(50, 200)$
Square mask (m)	$\Omega_{\square}$	$[0, 100] \times [0, 100]$
Circle mask (m)	$\Omega_{\circ}$	center $(50, 50)$, radius $R = 20$

1.2.2 Method (MATLAB Reference)

For a source $\mathbf{x} \in \Omega$, we compute a direction angle $\theta = \mathrm{atan2}(y_y - x_y, y_x - x_x)$, set the fireline normal to $(\cos\theta, \sin\theta)$, update ellipse kinematics via `UX_AND_UY_ELLIPTICAL`, and obtain the directional rate of spread (ROS) magnitude $v(\mathbf{x})$ (ft/min). The straight-line source-to-target distance is $d(\mathbf{x}) = \|\mathbf{y} - \mathbf{x}\|$. Time of arrival is

$$T(\mathbf{x}) = \begin{cases} \dfrac{d(\mathbf{x})}{v(\mathbf{x}) \cdot \frac{0.3048}{60}}, & \text{if coordinates in meters} \\ \dfrac{d(\mathbf{x})}{v(\mathbf{x})}, & \text{if coordinates in feet.} \end{cases} \tag{1}$$

The MATLAB driver (provided) evaluates $T(\mathbf{x})$ for all grid points inside the mask (uniform discrete sampling) and produces histograms and sample moments (mean/min/max).

1.2.3 Input Data

Describe input rasters, constants, initial conditions.

1.2.4 Numerical Controls

Mesh resolution, Time step(CFL), level-set solver options, etc.

1.3 Expected Results and Reasoning

Let $X \sim \mathrm{Unif}(\Omega)$ be the ignition location and $T = T(X)$ the induced ToA random variable. The *mask-implied* expected value and CDF are

$$\mathbb{E}[T] = \frac{1}{|\Omega|}\int_\Omega \frac{d(\mathbf{x})}{\tilde{v}(\mathbf{x})}\,d\mathbf{x}, \quad \tilde{v}(\mathbf{x}) = v(\mathbf{x}) \cdot \frac{0.3048}{60} \ \ (\mathrm{m/min}), \tag{2}$$

$$F_T(t) = \mathbb{P}\left(\frac{d(\mathbf{x})}{\tilde{v}(\mathbf{x})} \le t\right) = \frac{1}{|\Omega|}\int_{\{\mathbf{x}\in\Omega: d(\mathbf{x}) \le t\,\tilde{v}(\mathbf{x})\}} d\mathbf{x}. \tag{3}$$

In special cases:

- If $v(\mathbf{x}) \equiv v_0$ (isotropic ROS), then $T(\mathbf{x}) = d(\mathbf{x})/v_0$ and the ToA distribution is the distance distribution over Ω scaled by $1/v_0$.
- With an anisotropic ellipse (wind-aligned), $v(\mathbf{x})$ varies only through the direction from $\mathbf{x}$ to $\mathbf{y}$; the expectation remains an area integral, efficiently approximated by uniform quadrature on the same grid the script uses.

Thus, the MATLAB "all-points-in-mask" evaluation *is* a numerical quadrature for $\mathbb{E}[T]$ and the full F_T, which we treat as the *expected* mask-implied reference.

1.4 Verification Plan with ELMFIRE Ensembles

ELMFIRE ensembles will use identical physics and environmental settings, but ignition locations will be drawn randomly from Ω with equal probability. For an ensemble size N, define the empirical CDF $\hat{F}_T^{(N)}$ and moments (mean $\hat{\mu}_N$, variance $\hat{\sigma}_N^2$). Convergence is assessed against the mask-implied reference (F_T, μ, σ^2) via:

1. **Mean Relative Error (MRE):** $\mathrm{MRE} = |\hat{\mu}_N - \mu|/\mu$.
2. **RMSE of histogram/bin means** over a fixed partition of t.
3. **Kolmogorov–Smirnov (KS) distance:** $D_N = \sup_t |\hat{F}_T^{(N)}(t) - F_T(t)|$.

A practical sample-size target for the mean uses the CLT:

$$N \gtrsim \left(\frac{z_{1-\alpha/2}\,\sigma}{\varepsilon\,\mu}\right)^2, \tag{4}$$

where ε is the desired relative error tolerance and $z_{1-\alpha/2}$ the normal quantile. We estimate σ from either the mask quadrature or pilot ensembles.

1.5 Acceptance Criteria

- **Mean:** $\mathrm{MRE} \le 5\,\%$.
- **Distribution:** KS distance $D_N \le 0.05$ (typical); and visual agreement of PDF/CDF.
- **Stability:** Increasing N by a factor of two changes $\hat{\mu}_N$ by $\le 2\,\%$ and D_N by ≤ 0.01.

1.6 Results (Mask-Implied Reference)

This section presents the direct evaluation from the MATLAB loop over all source points inside the mask:

- Histogram/PDF of ToA for the *square* mask (uniform over $[0, 100]^2$).
- Histogram/PDF of ToA for the *circle* mask (center $(50, 50)$, $R = 20$).
- Empirical CDFs and summary moments (mean/min/max) for each mask.

1.7 Results (ELMFIRE Ensembles)

After running N-member ELMFIRE ensembles with random ignition draws on the same mask(s):

1. Compare ensemble histograms/CDFs vs. mask-implied reference.
2. Report $\hat{\mu}_N$, $\hat{\sigma}_N$, MRE, RMSE (bins), and KS D_N.
3. Increase N until criteria in § 5 are satisfied; record the smallest N meeting tolerance.

1.8 Discussion

- Interpret any persistent bias (e.g., due to anisotropic ellipse interacting with mask geometry).
- Note sensitivity to wind speed, fuel model, and target placement.
- Document computational cost vs. accuracy trade-offs for ensemble sizing.

Reproducibility

- MATLAB script commit: `<hash>` (code in repo appendix or link).
- ELMFIRE build: `<compiler/flags>`, binary: `<path>`.
- Command(s): `./run_case.sh`; environment: `<modules/conda env>`.
- Logs available under `cases/ignition_mask_bp_convergence/logs/`.

1.9 Discussion

2 wue_transient_heatflux

Case Summary

Case ID: wue_transient_heatflux
Objective: Verify the WU–E (urban structure) heat-flux prototype that couples a wind-aligned anisotropic fire ellipse (Hamada-style regression) with a piecewise transient design-fire curve for HRRPUA. We check that (i) geometric transformations, (ii) ellipse reach/coverage, (iii) per-cell HRR normalization, and (iv) heat-flux partitions (direct flame contact vs. radiation) reproduce expected behavior and conserve energy consistently.

2.1 Methods

2.1.1 Theory

Coordinate Transformations

Geometric angle from source to target cell:

$$\theta = \arctan\big(j,\ i\big) \quad \text{(rad)}. \tag{5}$$

Wind-aligned major-axis angle (radians) from the 20-ft met direction (blowing *from*):

$$\theta_{\text{wind}} = \frac{\pi}{180}\big(270 - \mathrm{WD}_{20ft}\big). \tag{6}$$

Relative angle for ellipse formulas:

$$\theta_f = \theta - \theta_{\text{wind}}, \qquad R = \sqrt{(i\,\Delta)^2 + (j\,\Delta)^2}. \tag{7}$$

Wind-Aligned Ellipse (Hamada-Style)

Convert wind speed to m/s:

$$V = 0.447\,\mathrm{WS}_{20ft}. \tag{8}$$

Downwind ($D_\downarrow$), upwind ($D_\uparrow$), and sidewind ($D_\perp$) distances (m) are piecewise functions of V with coefficients depending on (A, D) and scaled by W_p:

$$\text{Low wind } (V < 10): \quad D_\downarrow = W_p\,(D_1 V + D_2),\ \ D_\perp = W_p\,(S_1 V + S_2),\ \ D_\uparrow = W_p\,(U_1 V + U_2), \tag{9}$$

$$\text{High wind } (V > 17.3): \quad D_\downarrow = W_p\,(D_1 V + D_2),\ \ D_\perp = W_p\,(S_1 V + S_2),\ \ D_\uparrow = W_p\,(U_1 V + U_2), \tag{10}$$

$$\begin{aligned} \text{Moderate } (10 \le V \le 17.3): \quad & D_\downarrow = W_p\,(D_1 V^2 + D_2 V + D_3), \\ & D_\perp = W_p\,(S_1 V^2 + S_2 V + S_3), \\ & D_\uparrow = W_p\,(U_1 V^2 + U_2 V + U_3). \end{aligned}$$

Ellipse parameters:

$$a = \frac{D_\downarrow + D_\uparrow}{2}, \qquad \varepsilon = \min\Big(\tfrac{a}{2},\ a - D_\uparrow\Big), \qquad E_{b2} = 1 - \Big(\tfrac{\varepsilon}{a}\Big)^2, \tag{11}$$

$$b = \begin{cases} \dfrac{D_\perp}{\sqrt{E_{b2}}}, & E_{b2} > 0, \\ 0, & E_{b2} \le 0. \end{cases} \tag{12}$$

State vector:

$$\mathbf{E} = \big[a,\, b,\, \varepsilon,\, D_{\downarrow}\big]^{\mathsf{T}}.$$

Ellipse Reach and Coverage

Maximum reach (scalar):

$$R_{\max} = 0.3\, D_{\downarrow}\, \frac{a-\varepsilon}{b^2}. \tag{13}$$

Directional reach along θ_f:

$$R_{\mathrm{ell}}(\theta_f) = \frac{R_{\max}\, b^2}{a - \varepsilon\, \cos\theta_f}. \tag{14}$$

DFC coverage fraction (cell-centered, clipped to $[0,1]$):

$$C_{\mathrm{DFC}} = \max\left\{ \min\left(\frac{R_{\mathrm{ell}}(\theta_f) + \frac{1}{2}\Delta - R}{\Delta},\, 1 \right),\, 0 \right\}. \tag{15}$$

Radiation annulus outside DFC, bounded by cutoff:

$$R_{\mathrm{rad\,limit}}(\theta_f) = R_{\mathrm{ell}}(\theta_f) + R_{\mathrm{rad}}, \tag{16}$$

$$\Delta_{\mathrm{rad}} = \max\left\{ \min\left(\frac{R_{\mathrm{rad\,limit}}(\theta_f) + \frac{1}{2}\Delta - R}{\Delta},\, 0 \right),\, 1 \right\}, \tag{17}$$

$$F_{\mathrm{rad}} = \Delta_{\mathrm{rad}}\big(1 - C_{\mathrm{DFC}}\big). \tag{18}$$

Per-Cell HRR Normalization

To conserve total HRRPUA over the ellipse footprint, use the adjuster

$$C_{\mathrm{HRR}} = \frac{\Delta^2}{\pi\,(b/a)\,a\,b} = \frac{\Delta^2}{\pi b^2}. \tag{19}$$

Transient HRRPUA

For burning time τ and parameters $(t_{\mathrm{early}}, t_{\mathrm{dev}}, t_{\mathrm{decay}}, \mathrm{HRRPUA}_{\mathrm{peak}})$:

$$\mathrm{HRRPUA}(\tau) = \begin{cases} \dfrac{\mathrm{HRRPUA}_{\mathrm{peak}}}{t_{\mathrm{early}}}\,\tau, & 0 \le \tau \le t_{\mathrm{early}}, \\ \mathrm{HRRPUA}_{\mathrm{peak}}, & t_{\mathrm{early}} < \tau \le t_{\mathrm{dev}}, \\ \dfrac{\mathrm{HRRPUA}_{\mathrm{peak}}}{t_{\mathrm{dev}} - t_{\mathrm{decay}}}\,(\tau - t_{\mathrm{decay}}), & t_{\mathrm{decay}} < \tau, \\ 0, & \text{otherwise,} \end{cases} \qquad \mathrm{HRRPUA}(\tau) \leftarrow \max\{0, \mathrm{HRRPUA}(\tau)\}. \tag{20}$$

Per-Cell Heat Fluxes

Let $C_{\mathrm{burn}} = 1 - \mathrm{NONBURNABLE\_FRAC}$. Then

$$q''_{\mathrm{DFC}} = C_{\mathrm{burn}}\, C_{\mathrm{DFC}}\, \mathrm{HRRPUA}(\tau)\, C_{\mathrm{HRR}}, \tag{21}$$

$$q''_{\mathrm{rad}} = \frac{0.3\, C_{\mathrm{burn}}\, \alpha\, F_{\mathrm{rad}}\, C_{\mathrm{HRR}}\, \mathrm{HRRPUA}(\tau)\, \Delta^2}{4\pi\, R_{\mathrm{eff}}^2}, \qquad R_{\mathrm{eff}} = \begin{cases} \Delta\,(1 - C_{\mathrm{DFC}}), & 0 < C_{\mathrm{DFC}} < 1, \\ R - R_{\mathrm{ell}}(\theta_f), & \text{otherwise.} \end{cases} \tag{22}$$

Algorithm (Per Time Step)

1. Set $\tau = t - t_0$ and compute HRRPUA(τ).
2. From $(\mathrm{WS}_{20ft}, A, D, W_p)$ compute $\mathbf{E} = [a, b, \varepsilon, D_{\downarrow}]$.
3. For each cell (i, j) with center $(i\,\Delta, j\,\Delta)$:
   3.1. Compute R, θ, θ_f and $R_{\mathrm{ell}}(\theta_f)$.
   3.2. Compute C_{DFC}, F_{rad}, R_{eff}.
   3.3. Evaluate q''_{DFC} and q''_{rad}.

2.1.2 Assumptions

- Urban array represented on a uniform analysis grid of square cells of size Δ (m).
- Structures have a non-burnable fraction NONBURNABLE_FRAC; the remainder contributes to heat release/flux.
- HRRPUA follows a piecewise transient curve with early growth, plateau, and decay; negative segments are clipped to zero.
- Wind-aligned ellipse is derived from 20-ft wind inputs $(\mathrm{WD}_{20ft}, \mathrm{WS}_{20ft})$ and geometric parameters (A, D) with a proportionality W_p.
- Radiation is applied outside the ellipse up to a cutoff radius R_{rad}; convective/design-fire contact (DFC) acts within the ellipse footprint.
- Units: distances in meters; heat flux in kW/m^2.

2.1.3 Simulation Setup

Parameter Table (defaults)

Properties	Symbols	Values
Absorptivity	α	0.89
Radiation cutoff (m)	R_{rad}	100
Analysis cell size (m)	Δ	20
Wind direction (deg, from)	WD_{20ft}	0
Wind speed (mph)	WS_{20ft}	40
Footprint dim. (m)	A	10
Separation (m)	D	10
Wind proportionality	W_p	1
Non-burnable fraction	NONBURNABLE_FRAC	0
Early, dev., decay times (s)	$(t_{\mathrm{early}}, t_{\mathrm{dev}}, t_{\mathrm{decay}})$	(300, 3900, 4200)
Peak HRRPUA (kW/m^2)	$\mathrm{HRRPUA}_{\mathrm{peak}}$	400

Grid and Indices

Cells are indexed by $i, j \in \{-5, \ldots, 5\}$ with centers $(x, y) = (i\,\Delta,\ j\,\Delta)$ relative to the burning structure at $(0, 0)$.

2.1.4 Input Data

Describe input rasters, constants, initial conditions.

2.1.5 Numerical Controls

Mesh resolution, Time step(CFL), level-set solver options, etc.

2.2 Expected Results and Reasoning

- **Geometric consistency:** As V increases, $D_{\downarrow}$ grows faster than $D_{\perp}$ and $D_{\uparrow}$, increasing a and eccentricity (smaller b/a); $R_{\text{ell}}(\theta_f)$ elongates downwind.
- **Coverage partition:** Cells inside the ellipse ($C_{\text{DFC}} > 0$) receive q''_{DFC} proportional to HRRPUA and C_{HRR}; outer annulus receives q''_{rad} diminishing with R_{eff}^{-2}.
- **Conservation:** With C_{HRR}, summing q''_{DFC} over the footprint tracks the design HRRPUA(τ) (up to discretization error).
- **Limits:** For $V \to 0$, the ellipse tends toward isotropic ($D_{\downarrow} \approx D_{\perp} \approx D_{\uparrow}$); for very large V, footprint becomes highly elongated downwind; q''_{rad} shifts outward.

2.3 Acceptance Criteria

- **Energy consistency**: $\sum_{\text{cells}} |Q_{simulation} - Q_{analytical}| / Q_{analytical} \leq 0.5\,\%$, Q will be HRRPUA, transient DFC and radiative heat fluxes.
- **Partition sanity**: $q''_{\text{rad}} \to 0$ as $R_{\text{eff}} \to \infty$ and vanishes inside pure DFC cells when $F_{\text{rad}} = 0$.
- **Directional response**: Downwind flux peak $>$side $>$upwind for V in moderate/high ranges.

2.4 Results

Metric	Value
HRR mean relative error	0.000314
DFC mean relative error	1.8e-05
RAD mean relative error	0.000288

Table 1: Comparison errors (analytic vs simulation).

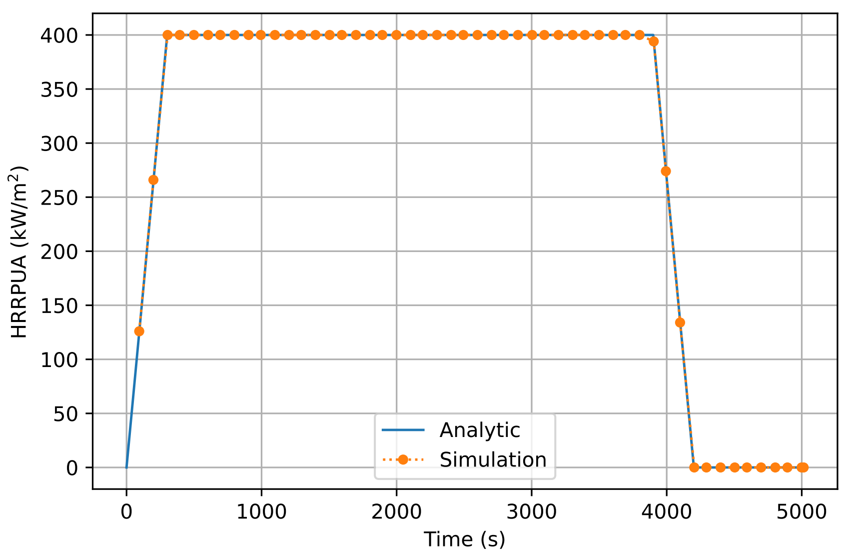

Figure 1: Transient HRR (analytic vs. simulation).

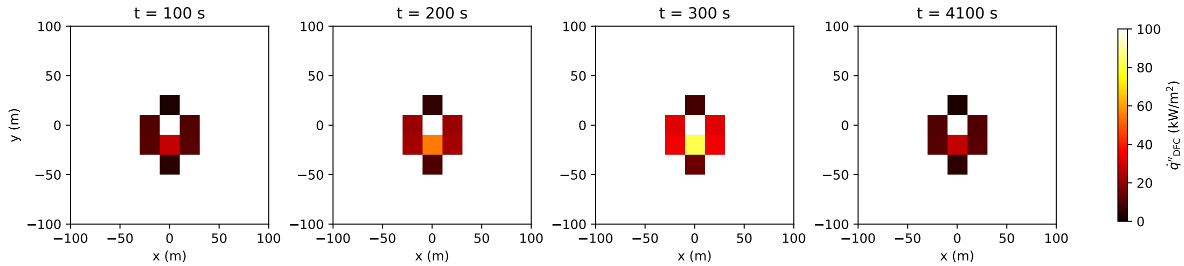

Figure 2: Analytic DFC heat flux at selected times.

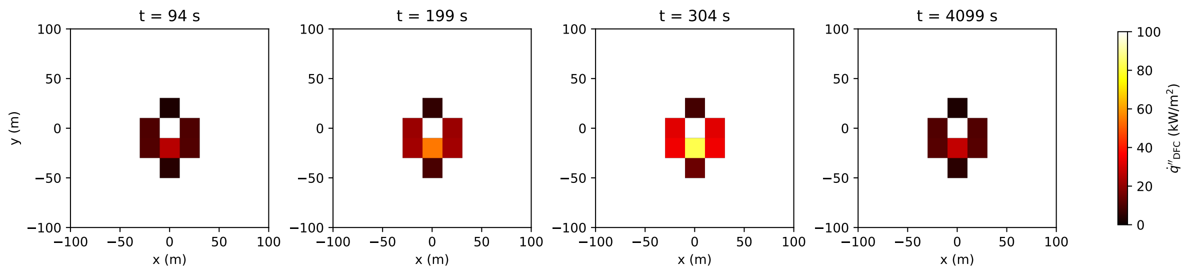

Figure 3: Simulated DFC heat flux at selected times.

2.5 Discussion

- Clarify parameter sensitivities (e.g., A, D, W_p) and wind-direction convention: $\theta_{\text{wind}} = \frac{\pi}{180}(270-\text{WD}_{20ft})$ points the major axis toward $+x$ when $WD_{20ft} = 0°$.
- Document any discretization effects at coarse Δ and how C_{HRR} compensates for footprint changes.
- Note corner cases: $E_{b2} \le 0$ (degenerate b), transition regions in the piecewise wind regression, and clipping of coverage fractions.

Reproducibility

- MATLAB functions: `ellipse_ucb`, `hrr_transient`, `heat_flux_calc`.
- Command(s): `./run_case.sh`; environment: `<modules/conda env>`.
- Logs under `cases/wue_transient_heatflux/logs/`; figures in `cases/wue_transient_heatflux/figures/`.

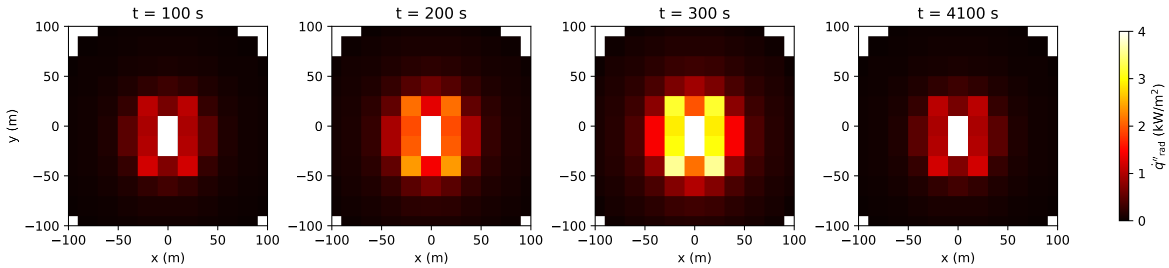

Figure 4: Analytic radiative heat flux at selected times.

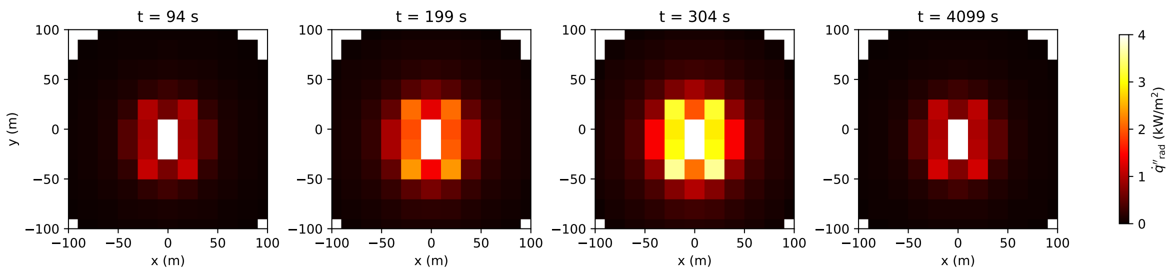

Figure 5: Simulated radiative heat flux at selected times.

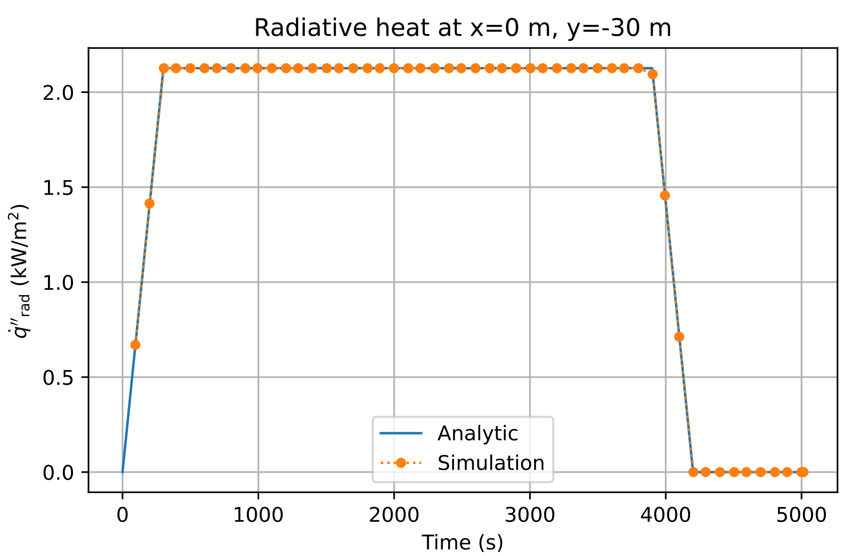

Figure 6: Radiative heat flux time history at $(x, y) = (0, -30)$ m.

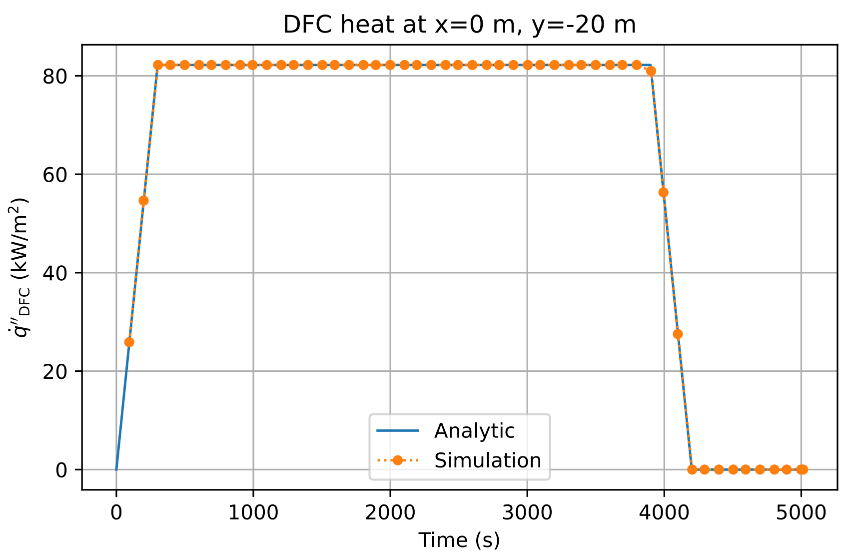

Figure 7: DFC heat flux time history at $(x, y) = (0, -20)$ m.