Case Summary

Case ID: ignition_mask_bp_convergence
Objective: Justify the Monte Carlo (MC) method used by ELMFIRE for ignition sampling by deriving the expected time-of-arrival (ToA) statistics at a target pixel when ignition is drawn uniformly from a specified mask (square or circle). We compare the empirical ToA distribution from ensembles to the mask-implied expectation, and quantify how many ensembles are needed for convergence.

0.1 Assumptions

- Flat terrain: slope = 0, aspect arbitrary but unused for $SLP = 0$.
- Homogeneous fuel: `IFBFM = 102`, fixed moisture: $M_1 = 0.03$, $M_{10} = 0.04$, $M_{100} = 0.05$, $M_{\text{lh}} = 0.30$, $M_{\text{lw}} = 0.60$.
- Steady wind: direction WD20 = 180° (blowing to the north), speed WS20 = 15 mph; canopy terms set to zero ($CC = CH = 0$).
- No spotting/suppression; acceleration factor = 1; length/width cap $\text{LOW}_{\max} = 8$.
- Coordinates are meters; the ToA along a ray from source to target uses the directional ROS returned by the ellipse kinematics.
- Ignition location X is uniformly distributed on the mask region Ω (square or circle).

0.2 Simulation Setup

0.2.1 Parameter Table

Fuel model	IFBFM	102
Dead fuel moisture	M_1, M_{10}, M_{100}	0.03, 0.04, 0.05
Live fuel moisture	$M_{\text{lh}}, M_{\text{lw}}$	0.30, 0.60
Wind (20-ft)	$(\text{WD20}, \text{WS20})$	$(180°, 15 \text{ mph})$
Slope / Aspect	(SLP, ASP)	$(0°, 0°)$
Adj. factors	(ADJ, ACCEL)	$(1, 1)$
Ellipse cap	$\text{LOW}_{\max}$	8
Target (m)	$\mathbf{y}$	$(50, 200)$
Square mask (m)	$\Omega_{\square}$	$[0, 100] \times [0, 100]$
Circle mask (m)	$\Omega_{\circ}$	center $(50, 50)$, radius $R = 20$

0.2.2 Method (MATLAB Reference)

For a source $\mathbf{x} \in \Omega$, we compute a direction angle $\theta = \text{atan2}(y_y - x_y, y_x - x_x)$, set the fireline normal to $(\cos\theta, \sin\theta)$, update ellipse kinematics via

UX_AND_UY_ELLIPTICAL, and obtain the directional rate of spread (ROS) magnitude $v(\mathbf{x})$ (ft/min). The straight-line source-to-target distance is $d(\mathbf{x}) = \|\mathbf{y} - \mathbf{x}\|$. Time of arrival is

$$T(\mathbf{x}) = \begin{cases} \dfrac{d(\mathbf{x})}{v(\mathbf{x}) \cdot \frac{0.3048}{60}}, & \text{if coordinates in meters} \\ \dfrac{d(\mathbf{x})}{v(\mathbf{x})}, & \text{if coordinates in feet.} \end{cases} \tag{1}$$

The MATLAB driver (provided) evaluates $T(\mathbf{x})$ for all grid points inside the mask (uniform discrete sampling) and produces histograms and sample moments (mean/min/max).

0.2.3 Input Data

Describe input rasters, constants, initial conditions.

0.2.4 Numerical Controls

Mesh resolution, Time step(CFL), level-set solver options, etc.

0.3 Expected Results and Reasoning

Let $X \sim \text{Unif}(\Omega)$ be the ignition location and $T = T(X)$ the induced ToA random variable. The *mask-implied* expected value and CDF are

$$\mathbb{E}[T] = \frac{1}{|\Omega|} \int_{\Omega} \frac{d(\mathbf{x})}{\tilde{v}(\mathbf{x})}\, d\mathbf{x}, \quad \tilde{v}(\mathbf{x}) = v(\mathbf{x}) \cdot \frac{0.3048}{60} \ \ (\text{m/min}), \tag{2}$$

$$F_T(t) = \mathbb{P}\left(\frac{d(\mathbf{x})}{\tilde{v}(\mathbf{x})} \le t \right) = \frac{1}{|\Omega|} \int_{\{\mathbf{x} \in \Omega : d(\mathbf{x}) \le t\, \tilde{v}(\mathbf{x})\}} d\mathbf{x}. \tag{3}$$

In special cases:

- If $v(\mathbf{x}) \equiv v_0$ (isotropic ROS), then $T(\mathbf{x}) = d(\mathbf{x})/v_0$ and the ToA distribution is the distance distribution over Ω scaled by $1/v_0$.
- With an anisotropic ellipse (wind-aligned), $v(\mathbf{x})$ varies only through the direction from $\mathbf{x}$ to $\mathbf{y}$; the expectation remains an area integral, efficiently approximated by uniform quadrature on the same grid the script uses.

Thus, the MATLAB "all-points-in-mask" evaluation *is* a numerical quadrature for $\mathbb{E}[T]$ and the full F_T, which we treat as the *expected* mask-implied reference.

0.4 Verification Plan with ELMFIRE Ensembles

ELMFIRE ensembles will use identical physics and environmental settings, but ignition locations will be drawn randomly from Ω with equal probability. For an ensemble size N, define the empirical CDF $\hat{F}_T^{(N)}$ and moments (mean $\hat{\mu}_N$, variance $\hat{\sigma}_N^2$). Convergence is assessed against the mask-implied reference (F_T, μ, σ^2) via:

1. **Mean Relative Error (MRE):** $\mathrm{MRE} = |\hat{\mu}_N - \mu|/\mu$.
2. **RMSE of histogram/bin means** over a fixed partition of t.
3. **Kolmogorov–Smirnov (KS) distance:** $D_N = \sup_t |\hat{F}_T^{(N)}(t) - F_T(t)|$.

A practical sample-size target for the mean uses the CLT:

$$N \gtrsim \left(\frac{z_{1-\alpha/2}\, \sigma}{\varepsilon\, \mu} \right)^2, \tag{4}$$

where ε is the desired relative error tolerance and $z_{1-\alpha/2}$ the normal quantile. We estimate σ from either the mask quadrature or pilot ensembles.

0.5 Acceptance Criteria

- **Mean:** $\mathrm{MRE} \le 5\,\%$.
- **Distribution:** KS distance $D_N \le 0.05$ (typical); and visual agreement of PDF/CDF.
- **Stability:** Increasing N by a factor of two changes $\hat{\mu}_N$ by $\le 2\,\%$ and D_N by ≤ 0.01.

0.6 Results (Mask-Implied Reference)

This section presents the direct evaluation from the MATLAB loop over all source points inside the mask:

- Histogram/PDF of ToA for the *square* mask (uniform over $[0, 100]^2$).
- Histogram/PDF of ToA for the *circle* mask (center $(50, 50)$, $R = 20$).
- Empirical CDFs and summary moments (mean/min/max) for each mask.

0.7 Results (ELMFIRE Ensembles)

After running N-member ELMFIRE ensembles with random ignition draws on the same mask(s):

1. Compare ensemble histograms/CDFs vs. mask-implied reference.
2. Report $\hat{\mu}_N$, $\hat{\sigma}_N$, MRE, RMSE (bins), and KS D_N.

3. Increase N until criteria in § 5 are satisfied; record the smallest N meeting tolerance.

0.8 Discussion

- Interpret any persistent bias (e.g., due to anisotropic ellipse interacting with mask geometry).
- Note sensitivity to wind speed, fuel model, and target placement.
- Document computational cost vs. accuracy trade-offs for ensemble sizing.

Reproducibility

- MATLAB script commit: <hash> (code in repo appendix or link).
- ELMFIRE build: <compiler/flags>, binary: <path>.
- Command(s): ./run_case.sh; environment: <modules/conda env>.
- Logs available under cases/ignition_mask_bp_convergence/logs/.

0.9 Discussion