

*Summary + some interesting findings for shiny App creation! (At the end of notes)

Correspondence Analysis I and II

Libraries

- tidyverse

- tidytext

- janeaustenr

- FactoMineR

• this section uses the `janeaustenr` library, which has a function `austen_books()`, which has 6 novels:

- Emma

- Mansfield Park

- Northanger Abbey

- Persuasion

- Pride and Prejudice

- Sense and Sensibility

all texts of
each book can
be accessed by
`austen_books()`

The `janeaustenr` library (Review)

Intro to Correspondence Analysis (CA)

• Correspondence Analysis (CA) allows us to detect associations

Ex: Suppose we want to analyze the use of punctuation in `austen_books`:

punctuations → commas, semicolons, colons, quotations, apostrophes, question marks, exclamation marks, dashes

* Recall from the Regex lectures, we can get a count of a SPECIFIC character by using `str_count()`

To get a data frame for the symbols listed above:

austen_books (columns) → book
 austen_books (columns) → text
 get the books text
 and

crossstable = austen_books() >
 mutate(

commas = str_count(text, ",") ,

colons = str_count(text, ":") ,

Semicolons = str_count(text, ";") ,

(a) quotes = str_count(text, "'") ,

apostrophes = str_count(text, "'") ,

questions = str_count(text, "?") ,

exclamations = str_count(text, "!") ,

dashes = str_count(text, "-") ,

) >

(b) group_by(book) > we want to count for
 summarise(each book

commas = sum(commas) ,

colons = sum(colons) ,

(c) semis = sum(semicolons) ,

quotes = sum(quotes) ,

aposts = sum(apostrophes) ,

quests = sum(questions) ,

bangs = sum(exclamations) ,

dashes = sum(dashes)

what is the above code block doing?

RECALL the austen_books() function:

austen_books() →

book	text
:	:

crosstable

(c) new table
 with each unique
 book + the total counts.

(b) group by
 each unique book

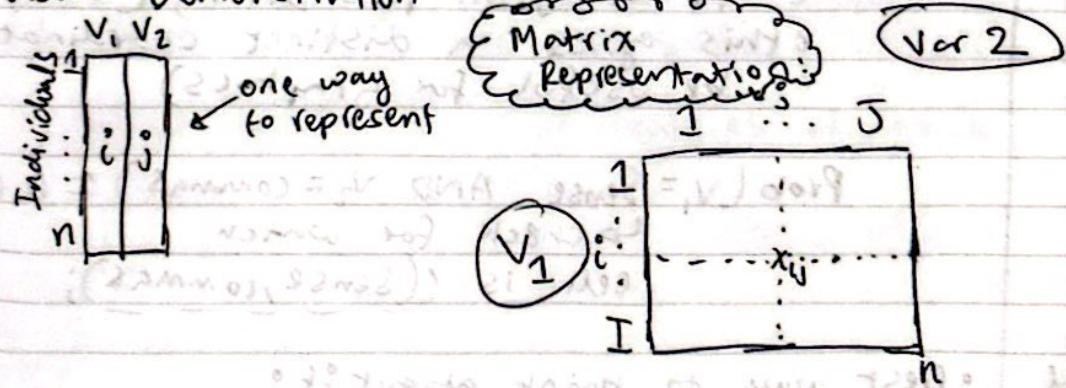
(a) new columns,
counts respective
 character in
 each row of
 text

crosstable	Var 1		Var 2				
	book	commas	colons	...	dashes	(9 columns)	
1 Sense & ...	9900	66	...	11	78		
2 Pride & ...	9132	132	...	395			
3 Mansfield...	12439	339		413			
4 Emma	12020	174	...	3100			
5 Northanger... in	6085	83	...	419			
6 Persuasion	7025	130	...	142			

What is a crosstable?

- Basically a 2-way table or contingency table (one variable on the left side [book] and the second variable on the right side [commas, colons, ..., dashes])

Visual Demonstration:



Great for finding associations or correlations

Only gets
the first
word

Reformatting
into a
crosstable

Step 1: Represent data frame as a matrix

$X = \text{as.matrix}(\text{crosstable[, -1]})$

$\text{rownames}(X) = \text{str_extract}(\text{crosstable\$book}, "^\wedge \wedge \wedge \wedge \wedge")$

this removes the 'book' name for the column following us to visualize each respective variable.

Side Note: we can calculate probabilities and relative frequencies! (Helps with correlation)

Joint Probability
(Relative Frequency)

In math terms:

$$f_{ij} = \frac{x_{ij}}{n}$$

or current value
new = $\frac{\text{value}}{\text{sum of all counts}}$

$$Xprobs = X / \text{Sum}(X) \leftarrow \begin{array}{l} \text{total count of symbols} \\ \text{round}(Xprobs, 4) \end{array}$$

~~n cases~~ Xprobs^o:

commas colons ... dashes

Sense 0.0967 0.0006 ... 0.0115

Pride 0.0892 0.0013 ... 0.0039

Mansfield 0.1216 0.0033 ... 0.0040

Emma 0.1175 0.0017 ... 0.0303

Northanger 0.0595 0.0008 ... 0.0041

Persuasion 0.0687 0.0013 ... 0.0014

Best way to think about each cell value is a joint probability (V_1, V_2):
this gives a distinct coordinate (will be useful for graphics)

Prob($V_1 = \text{Sense}$ AND $V_2 = \text{commas}$) ≈ 0.0967
↳ check for which cell is ((Sense, commas)),

Marginal Probability

• Best way to think about it:

what would the NEXT probability look like? V_2

V_1, i	j	J
I	$f_{i,j}$	$f_{i,\cdot}$

↳ given the rows and columns

$$f_{i,\cdot} = \sum_{j=1}^J f_{i,j}$$

$$f_{\cdot,j} = \sum_{i=1}^I f_{i,j}$$

Row Margin: Sum of entries of each row, by row

$$f_{i \cdot} = \sum_{j=1}^J f_{ij}$$

R Coding: we can use rowSums(Matrix)

row_margin = rowSums(x_probs)
round(row_margin, 4)

Sense Pride ... Persuasion
0.1732 0.1606 0.1104

These proportions are marginal probabilities

The same for column margins:

Column Margin: Sum of entries of each column, by column

$$f_{\cdot j} = \sum_{i=1}^I f_{ij}$$

R Coding: we can use colSums(Matrix)

col_margin = colSums(x_probs)
round(col_margin, 4)

commas colons ... dashes
0.5531 0.009 0.0552

Independence Model

Probability Review:

$$P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

So now we recall the different probabilities we just covered:

$$P(\text{Joint}) : f_{ij} \quad \text{which means:}$$

$$P(\text{Marginal}_{\text{Row}}) : f_{i\cdot} \quad P(\text{Joint}) = P(\text{M}_{\text{Row}}) \cdot P(\text{M}_{\text{Col}})$$

$$P(\text{Marginal}_{\text{Col}}) : f_{\cdot j} \quad f_{ij} = f_{i\cdot} \cdot f_{\cdot j}$$

Assume V_1 is independent to V_2 :

Then the table of joint probabilities should be approximately the product of the marginal probabilities

R Coding:

$$\begin{aligned} X_{\text{indep}} &= \text{row_margin \% \% \% col_margin} \\ &\text{round}(X_{\text{indep}}, 4) \end{aligned}$$

Note*: when you do something like $1:3 \% \% \% 1:3$ you get:

[1]	[2]	[3]	
[1]	1	2	3
[2]	2	4	6
[3]	3	6	9

∴ This should give us the joint probabilities (an approximate)*

The %%% operators gives the "outer" product of arrays

* because of rounding

+ mosaicplot(x, main, sub, xlab, ylab, sort, off, dir, color, shade, margin, cex.axis, las, border, type)

With our array, we can make many plots (the Correspondence Analysis Notes uses a mosaicplot+)

To make a mosaic plot:

↳ You need some sort of **CONTINGENCY** table

Arguments:

x → Contingency table

main → character string (title)

sub → like a subtitle; character string

xlab, ylab → just like labs() in ggplot, x label and y label

las → numeric (style of axis labels)

border → color of borders of cells

Row Analysis

→ Recall our table of probabilities: Xprobs

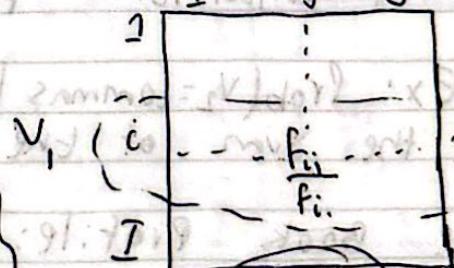
In order to conduct Row Analysis,

we divide each entry (f_{ij}) by the

ROW MARGIN ($f_{i\cdot}$) (aka our marginal probability)

$$\frac{f_{ij}}{f_{i\cdot}} = f_{\cdot j}$$

CA takes these row profiles and compares against the mean profile



This is called a "Row Profile" ↗ a conditional distribution ↗ Mean row profile (the distribution profile for ALL individuals)

How do we get "Row Profiles" in R?

We can use the `sweep()` function

`sweep()`:

$X \rightarrow$ an array (matrix x allowed)

`MARGIN` \rightarrow numeric; correspondence of x to STAT
or char vector of dimension names

`STATS` \rightarrow Summary statistic

`FUN` \rightarrow "[function you want to use]"

Ex: $" - "$ \rightarrow subtract

$" + "$ \rightarrow add

$" / "$ \rightarrow divide

~~Our table rows~~

row_profiles = `sweep(Xprobs, MARGIN=1, STATS=`

~~row-margining, FUN = "/")~~

`round(row_profiles, 4)` \rightarrow rounds to 4 decimal places

Our new table is basically a table of conditional probabilities

Ex: $\text{Prob}(V_2=\text{commas} | V_1=\text{Sense}) = 0.5585$

* All the sum of the rows are ≈ 1

Average Book Profile: ~~Instead of~~

we can use `col_margin` to take into account book profiles

`Rows = rbind(row_profiles, average = col_margin)
round(Rows, 4)`

`mosaicplot(t(Rows), main = "Row Profiles",
las = 1,
border = NA,
col = rainbow(ncol(Rows)),
main = "Row Profiles")`

Based on the mosaicplot, we can make several observations about our Row Analysis

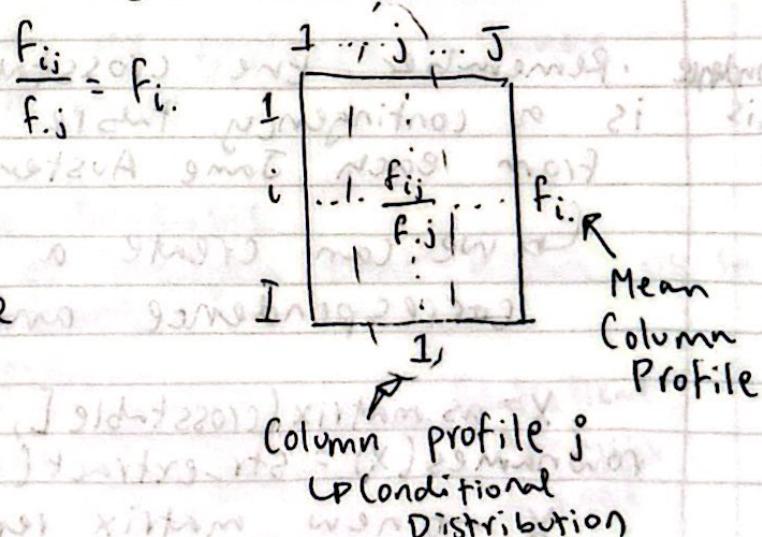
Ex: - commas are the most used punctuation symbol (evenly distributed across all books)

Column Analysis

• Very similar to Row Analysis
↳ we use col_margin

Analysis by Column:

Once again CA compares column profile to the mean profile



```
col_profiles = sweep (Xprobs, MARGIN=2, STATS=col_margin,  
FUN="/")  
round (col_profiles, 4)
```

Average Symbol Profile: Take into account the average symbol profile (row-margin)

```
cols = cbind (col_profiles, average=row_margin)  
round (cols, 4)
```

```
mosaicplot (
```

```
t (cols),  
las=1,  
border=NA,
```

```
col=rainbow (nrow (cols)),  
main="Column Profiles")
```

Example observation based on the plot:

- Mansfield has the largest proportion of colors

Correspondence Analysis Map

- Remember the crosstable which is a contingency table of punctuations from each Jane Austen novel

- ↳ we can create a map for our correspondence analysis

```
X = as.matrix (crosstable [, -1])  
rownames (X) = str_extract (crosstable $ book, " \| w+ ")  
X → new matrix representation  
of crosstablp.
```

Simultaneous
Rep of
Rows and
Columns

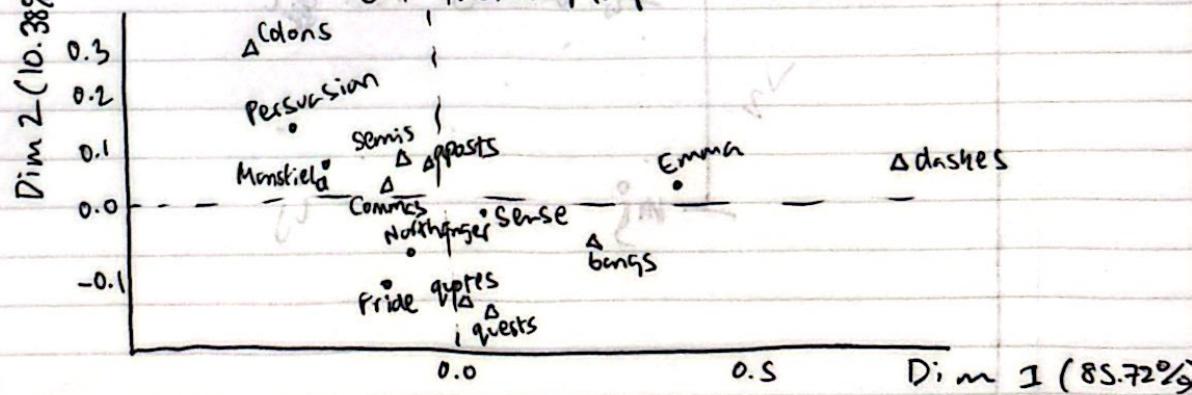
- `CA()` → from "FactoMineR" package
 - allows us to perform that Correspondence Analysis using a matrix

Ex:

`austen_ca1 = CA(x)`

* by default, creates a Scatterplot to visually represent the categories

CA factor Map

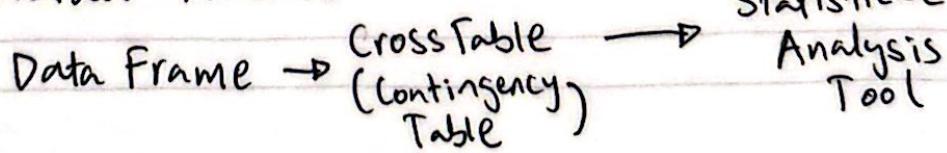


- We can disable the default graph with the argument `graph = FALSE`
 - ↳ we can use `ggplot()` to make custom CA maps

Summary

- Correspondence Analysis
 - ↳ a type of analysis that allows us to detect associations

Overall Process



Some Analyses we can do:

- Row Analysis
- Column Analysis
- Creating a Map

* Contingency Tables are a data frame mapped with 2 variables:

		V ₁				
		1 _{ij}	2	3	..	Σ_i
V ₂		1 _{ij}	
		m_j	Σ_j	