

STAT151A Quiz 2.5 (Feb 22nd)

Please write your full name and email address:

For this quiz, we'll consider the linear models

$$y_n = \boldsymbol{\beta}^\top \mathbf{x}_n + \varepsilon_n \quad \text{and} \quad y_n = \boldsymbol{\gamma}^\top \mathbf{z}_n + \eta_n$$

with

$$\begin{aligned} \mathbf{x}_n &= (x_{n1}, x_{n2})^\top \quad \text{and} \quad \mathbf{z}_n = (z_{n1}, z_{n2})^\top \text{ where} \\ z_{n1} &:= x_{n1} - x_{n2} \quad \text{and} \quad z_{n2} := x_{n1} + x_{n2} \end{aligned}$$

Let \mathbf{X} denote the $N \times 2$ matrix whose n -th row is \mathbf{x}_n^\top , and \mathbf{Z} denote the $N \times 2$ matrix whose n -th row is \mathbf{z}_n^\top .

Assume that the matrix \mathbf{X} is full rank.

Recall that the inverse of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

You have 20 minutes for this quiz.

There are three parts, (a), (b), and (c), each weighted equally..

(a)

Using the definitions given on the first page, find a 2×2 matrix \mathbf{A} such that $\mathbf{Z} = \mathbf{XA}$.

(b)

Use the definitions given on the first page of the quiz. Suppose I tell you that the OLS estimate of β is given by $\hat{\beta} = (2, 4)$. What is the value of $\hat{\gamma}$, the OLS estimate of γ ?

(c)

Use the definitions given on the first page of the quiz. Consider the residual variance estimators for the two regressions:

$$\hat{\sigma}_x^2 := \frac{1}{N} \sum_{n=1}^N \hat{\varepsilon}_n^2 = \frac{1}{N} \sum_{n=1}^N (y_n - \mathbf{x}_n^\top \hat{\beta})^2 \quad \text{and} \quad \hat{\sigma}_z^2 := \frac{1}{N} \sum_{n=1}^N \hat{\eta}_n^2 = \frac{1}{N} \sum_{n=1}^N (y_n - \mathbf{z}_n^\top \hat{\gamma})^2.$$

Please select which of (a), (b), (c), or (d) is correct:

- a) It is always the case that $\hat{\sigma}_x^2 > \hat{\sigma}_z^2$
- b) It is always the case that $\hat{\sigma}_x^2 < \hat{\sigma}_z^2$
- c) It is always the case that $\hat{\sigma}_x^2 = \hat{\sigma}_z^2$
- d) In general, we cannot determine the relationship between $\hat{\sigma}_x^2$ and $\hat{\sigma}_z^2$ using the information provided.

Briefly justify your answer.