

STAT151A Quiz 3 (Feb 27th)

Please write your full name and email address:

For this quiz, assume that you have access to a function

$$\Phi(z) = \mathbb{P}(\tilde{z} \leq z),$$

as well as its inverse,

$$\Phi^{-1}(p) = z \text{ such that } \mathbb{P}(\tilde{z} \leq z) = p, \text{ for } p \in [0, 1],$$

where $\tilde{z} \sim \mathcal{N}(0, 1)$ denote a scalar-valued standard normal random variable.

You have 20 minutes for this quiz.

There are three parts, (a), (b), and (c), each weighted equally..

(a)

Suppose that $\tilde{y} \sim \mathcal{N}(\mu, \sigma^2)$, where μ and σ are known. Using only the functions $\Phi(\cdot)$, $\Phi^{-1}(\cdot)$, and the known quantities μ , and σ , find a quantity a such that

$$\mathbb{P}(\tilde{y} \leq a) = 0.90.$$

Note that $\Phi(\cdot)$ is only for a standard normal random variable. Please **do not assume** that you have direct access to the distribution and quantile functions of generic normal random variables.

Please justify your answer carefully.

(b)

In the same setting as (a), please find b_ℓ and b_u such that

$$\mathbb{P}(b_\ell \leq \tilde{y} \leq b_u) = 0.80.$$

Note the change from 0.90 in (a) to 0.80 on the right-hand side in the current problem.

Please justify your answer carefully. You may use your result from part (a).

(c)

Now suppose that $x_{\text{new}} \in \mathbb{R}^P$, $\beta \in \mathbb{R}^P$, and $\sigma \in \mathbb{R}$ all denote **known, non-random** quantities.

Suppose that you also know that $y_{\text{new}} = \beta^\top x_{\text{new}} + \varepsilon_n$, where $\varepsilon_n \sim \mathcal{N}(0, \sigma^2)$.

Using only these known quantities and the functions $\Phi(\cdot)$, $\Phi^{-1}(\cdot)$, find c_ℓ and c_u such that

$$\mathbb{P}(c_\ell \leq y_{\text{new}} \leq c_u) = 0.80.$$

Please justify your answer carefully. You may use your results from parts (a) and (b).