

# STAT151A Quiz 2 (Feb 13th)

Please write your full name and email address:

For this quiz, we'll consider the linear models

$$y_n = \boldsymbol{\beta}^\top \mathbf{x}_n + \varepsilon_n \quad \text{and} \quad y_n = \boldsymbol{\gamma}^\top \mathbf{z}_n + \eta_n$$

with

$$\mathbf{x}_n = (1, x_n)^\top \quad \text{and} \quad \mathbf{z}_n = (1, z_n)^\top \quad \text{where}$$
$$\bar{x} := \frac{1}{N} \sum_{n=1}^N x_n \quad \text{and} \quad z_n := x_n - \bar{x}.$$

Assume that  $x_n$  is not a constant (i.e., for at least one pair  $n$  and  $m$ ,  $x_n \neq x_m$ ).

Let  $\mathbf{X}$  denote the  $N \times 2$  matrix whose  $n$ -th row is  $\mathbf{x}_n^\top$ , and  $\mathbf{Z}$  denote the  $N \times 2$  matrix whose  $n$ -th row is  $\mathbf{z}_n^\top$ .

Recall that the inverse of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

**You have 20 minutes for this quiz.**

**There are three parts, (a), (b), and (c), each weighted equally..**

**(a)**

Find a  $2 \times 2$  matrix  $\mathbf{A}$  such that  $\mathbf{Z} = \mathbf{XA}$ .

**(b)**

Suppose I tell you that the OLS estimate of  $\beta$  is given by  $\hat{\beta} = (2, 3)$ , and that  $\bar{x} = 4$ . What is the value of  $\hat{\gamma}$ , the OLS estimate of  $\gamma$ ?

**(c)**

In general, can you say whether one regression will provide a better fit than the other? That is, can you say which of  $\frac{1}{N} \sum_{n=1}^N (y_n - \mathbf{z}_n^\top \hat{\gamma})^2$  and  $\frac{1}{N} \sum_{n=1}^N (y_n - \mathbf{x}_n^\top \hat{\beta})^2$  is smaller? Argue why or why not.