

# STAT151A Quiz 1 (Jan 30th)

Please write your full name and email address:

For this quiz, we'll consider the linear model  $y_n = \beta_1 z_n + \beta_2 w_n + \varepsilon_n$ .

**Note that there is no intercept, and instead are two scalar regressors,  $z_n$  and  $w_n$ .**

Recall that the inverse of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

**You have 20 minutes for this quiz.**

**There are three parts, (a), (b), and (c), each weighted equally..**

**(a)**

Write the set of equations

$$y_n = \beta_1 z_n + \beta_2 w_n + \varepsilon_n$$

for  $n \in \{1, \dots, N\}$  in matrix form. That is, let  $\mathbf{X}$  denote an  $N \times 2$  matrix,  $\mathbf{Y}$  and  $\boldsymbol{\varepsilon}$  length- $N$  column vectors, and  $\boldsymbol{\beta} = (\beta_0, \beta_1)^\top$  a length-2 column vector. Then express the matrices  $\mathbf{Y}$ ,  $\mathbf{X}$ , and  $\boldsymbol{\varepsilon}$  in terms of the scalars  $y_n$ ,  $z_n$ ,  $w_n$ , and  $\varepsilon_n$  so that  $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$  is equivalent to the set of regression equations.

**(b)**

Define the following quantities:

$$\bar{z} := \frac{1}{N} \sum_{n=1}^N z_n \quad \bar{w} := \frac{1}{N} \sum_{n=1}^N w_n \quad \bar{y} := \frac{1}{N} \sum_{n=1}^N y_n$$

$$\overline{ww} := \frac{1}{N} \sum_{n=1}^N w_n^2 \quad \overline{zw} := \frac{1}{N} \sum_{n=1}^N z_n w_n \quad \overline{zz} := \frac{1}{N} \sum_{n=1}^N z_n^2 \quad \overline{wy} := \frac{1}{N} \sum_{n=1}^N w_n y_n \quad \overline{zy} := \frac{1}{N} \sum_{n=1}^N z_n y_n.$$

In terms of these quantities and  $N$  alone, write expressions for  $\mathbf{X}^\top \mathbf{X}$ ,  $\mathbf{X}^\top \mathbf{Y}$ , and  $(\mathbf{X}^\top \mathbf{X})^{-1}$ .

**(c)**

Now, for only this part of the quiz, assume that  $\overline{wz} = 0$ . Under this assumption, write an expression for the least squares solution  $\hat{\beta}$  which minimizes

$$\hat{\beta} := \operatorname{argmin}_{\beta} \sum_{n=1}^N (y_n - \beta_1 z_n - \beta_2 w_n)^2.$$