STAT151A Quiz 3 (Feb 27th)

Please write your full name and email address:

For this quiz, assume that you have access to a function

$$\Phi(z) = \mathbb{P}\left(\tilde{z} \leq z\right),$$

as well as its inverse,

$$\Phi^{-1}(p) = z \text{ such that } \mathbb{P}\left(\tilde{z} \leq z\right) = p, \text{ for } p \in [0,1],$$

where $\tilde{z} \sim \mathcal{N}(0,1)$ denote a scalar-valued standard normal random variable.

You have 20 minutes for this quiz.

There are three parts, (a), (b), and (c), each weighted equally..

(a)

Suppose that $\tilde{y} \sim \mathcal{N}\left(\mu, \sigma^2\right)$, where μ and σ are known. Using only the functions $\Phi(\cdot)$, $\Phi^{-1}(\cdot)$, and the known quantities μ, and σ, find find a quantity a such that

$$\mathbb{P}\left(\tilde{y} \leq a\right) = 0.90.$$

Note that $\Phi(\cdot)$ is only for a standard normal random variable. Please **do not assume** that you have direct access to the distribution and quantile functions of generic normal random variables.

Please justify your answer carefully.

(b)

In the same setting as (a), please find b_ℓ and b_u such that

$$\mathbb{P}\left(b_\ell \leq \tilde{y} \leq b_u\right) = 0.80.$$

Note the change from 0.90 in (a) to 0.80 on the right-hand side in the current problem. Please justify your answer carefully. You may use your result from part (a).

(c)

Now suppose that $\boldsymbol{x}_{\text{new}} \in \mathbb{R}^P$, $\boldsymbol{\beta} \in \mathbb{R}^P$, and $\sigma \in \mathbb{R}$ all denote **known**, **non-random** quantites.

Suppose that you also know that $y_{\text{new}} = \boldsymbol{\beta}^\intercal \boldsymbol{x}_{\text{new}} + \varepsilon_n$, where $\varepsilon_n \sim \mathcal{N}\left(0, \sigma^2\right)$.

Using only these known quantities and the functions $\Phi(\cdot)$, $\Phi^{-1}(\cdot)$, find c_ℓ and c_u such that

$$\mathbb{P}\left(c_\ell \leq y_{\text{new}} \leq c_u\right) = 0.80.$$

Please justify your answer carefully. You may use your results from parts (a) and (b).