STAT151A Quiz 2 (Feb 13th)

Please write your full name and email address:

For this quiz, we'll consider the linear models

$$y_n = \beta^\top x_n + \varepsilon_n \quad \text{and} \quad y_n = \gamma^\top z_n + \eta_n$$

with

$$x_n = (1, x_n)^\top \quad \text{and} \quad z_n = (1, z_n)^\top \text{ where}$$

$$\bar{x} := \frac{1}{N}\sum_{n=1}^{N} x_n \quad \text{and} \quad z_n := x_n - \bar{x}.$$

Assume that x_n is not a constant (i.e., for at least one pair n and m, $x_n \neq x_m$.).

Let X denote the $N \times 2$ matrix whose n–th row is $x_n^\top$, and Z denote the $N \times 2$ matrix whose n–th row is $z_n^\top$.

Recall that the inverse of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

You have 20 minutes for this quiz.

There are three parts, (a), (b), and (c), each weighted equally..

(a)

Find a 2×2 matrix A such that $Z = XA$.

(b)

Suppose I tell you that the OLS estimate of β is given by $\hat{\beta} = (2, 3)$, and that $\bar{x} = 4$. What is the value of $\hat{\gamma}$, the OLS estimate of γ?

(c)

In general, can you say whether one regression will provide a better fit than the other? That is, can you say which of $\frac{1}{N} \sum_{n=1}^{N} (y_n - z_n^\top \hat{\gamma})^2$ and $\frac{1}{N} \sum_{n=1}^{N} (y_n - x_n^\top \hat{\beta})^2$ is smaller? Argue why or why not.