STAT151A Quiz 2.5 (Feb 22nd)

Please write your full name and email address:

For this quiz, we'll consider the linear models

$$y_n = \boldsymbol{\beta}^\intercal \boldsymbol{x}_n + \varepsilon_n \quad \text{and} \quad y_n = \boldsymbol{\gamma}^\intercal \boldsymbol{z}_n + \eta_n$$

with

$$\begin{aligned} &\boldsymbol{x}_n = (x_{n1}, x_{n2})^\intercal \quad \text{and} \quad \boldsymbol{z}_n = (z_{n1}, z_{n2})^\intercal \text{ where} \\ &z_{n1} := x_{n1} - x_{n2} \quad \text{and} \quad z_{n2} := x_{n1} + x_{n2} \end{aligned}$$

Let $\boldsymbol{X}$ denote the $N \times 2$ matrix whose n–th row is $\boldsymbol{x}_n^\intercal$, and $\boldsymbol{Z}$ denote the $N \times 2$ matrix whose n–th row is $\boldsymbol{z}_n^\intercal$.

Assume that the matrix $\boldsymbol{X}$ is full rank.

Recall that the inverse of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

You have 20 minutes for this quiz.

There are three parts, (a), (b), and (c), each weighted equally..

(a)

Using the definitions given on the first page, find a 2×2 matrix $\boldsymbol{A}$ such that $\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{A}$.

(b)

Use the definitions given on the first page of the quiz. Suppose I tell you that the OLS estimate of β is given by $\hat{\beta} = (2, 4)$. What is the value of $\hat{\gamma}$, the OLS estimate of γ?

(c)

Use the definitions given on the first page of the quiz. Consider the residual variance estimators for the two regressions:

$$\hat{\sigma}_x^2 := \frac{1}{N}\sum_{n=1}^{N}\hat{\varepsilon}_n^2 = \frac{1}{N}\sum_{n=1}^{N}(y_n - \boldsymbol{x}_n^\intercal\hat{\beta})^2 \quad \text{and} \quad \hat{\sigma}_z^2 := \frac{1}{N}\sum_{n=1}^{N}\hat{\eta}_n^2 = \frac{1}{N}\sum_{n=1}^{N}(y_n - \boldsymbol{z}_n^\intercal\hat{\gamma})^2.$$

Please select which of (a), (b), (c), or (d) is correct:

a) It is always the case that $\hat{\sigma}_x^2 > \hat{\sigma}_z^2$
b) It is always the case that $\hat{\sigma}_x^2 < \hat{\sigma}_z^2$
c) It is always the case that $\hat{\sigma}_x^2 = \hat{\sigma}_z^2$
d) In general, we cannot determine the relationship between $\hat{\sigma}_x^2$ and $\hat{\sigma}_z^2$ using the information provided.

Briefly justify your answer.