STAT151A Quiz 2.5 (Feb 22nd)

Please write your full name and email address:

For this quiz, we'll consider the linear models

$$y_n = \beta^\top x_n + \varepsilon_n \quad \text{and} \quad y_n = \gamma^\top z_n + \eta_n$$

with

$$\begin{aligned} x_n &= (x_{n1}, x_{n2})^\top \quad \text{and} \quad z_n = (z_{n1}, z_{n2})^\top \text{ where} \\ z_{n1} &:= x_{n1} - x_{n2} \quad \text{and} \quad z_{n2} := x_{n1} + x_{n2} \end{aligned}$$

Let X denote the $N \times 2$ matrix whose n–th row is $x_n^\top$, and Z denote the $N \times 2$ matrix whose n–th row is $z_n^\top$.

Assume that the matrix X is full rank.

Recall that the inverse of a 2x2 matrix is given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

You have 20 minutes for this quiz.

There are three parts, (a), (b), and (c), each weighted equally..

(a)

Using the definitions given on the first page, find a 2×2 matrix A such that $Z = XA$.

(b)

Use the definitions given on the first page of the quiz. Suppose I tell you that the OLS estimate of β is given by $\hat{\beta} = (2, 4)$. What is the value of $\hat{\gamma}$, the OLS estimate of γ?

(c)

Use the definitions given on the first page of the quiz. Consider the residual variance estimators for the two regressions:

$$\hat{\sigma}_x^2 := \frac{1}{N}\sum_{n=1}^{N}\hat{\varepsilon}_n^2 = \frac{1}{N}\sum_{n=1}^{N}(y_n - x_n^\top\hat{\beta})^2 \quad \text{and} \quad \hat{\sigma}_z^2 := \frac{1}{N}\sum_{n=1}^{N}\hat{\eta}_n^2 = \frac{1}{N}\sum_{n=1}^{N}(y_n - z_n^\top\hat{\gamma})^2.$$

Please select which of (a), (b), (c), or (d) is correct:

a) It is always the case that $\hat{\sigma}_x^2 > \hat{\sigma}_z^2$
b) It is always the case that $\hat{\sigma}_x^2 < \hat{\sigma}_z^2$
c) It is always the case that $\hat{\sigma}_x^2 = \hat{\sigma}_z^2$
d) In general, we cannot determine the relationship between $\hat{\sigma}_x^2$ and $\hat{\sigma}_z^2$ using the information provided.

Briefly justify your answer.