Stat 238, Fall 2025
Project Minis

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Due: by **5:00 pm** Friday, May 2nd, 2025

Mini Project 8: Advanced MCMC Methods

Policies

- This is a group project. You may work with up to two partners.
- All submissions must be properly type-set and sourced. This means providing formal citations and a complete bibliography when sources are used.
- Project minis should be uploaded to Gradescope under the appropriate mini-assignment.
- Please export any notebooks as a single pdf and merge all components into one file. Only one group member should submit to gradescope. They must tag the other members when they submit.

Prompts

This is a mixed coding/reading project. It builds directly on Lab 8. You will be asked to complete the mixing time experiments outlined at the end of the lab, use these to identify a model setting where standard Metropolis-Hastings (MH) algorithms mix slowly, then implement two modifications to the MH algorithm which often speed mixing. These are *simulated tempering* (see BDA 12.3) and *ensemble MCMC* methods. Ensemble methods evolve a population of random walkers together in parallel rather than a single walker.

(a) (8 points) *Mixing Times for MH in the Ising Model.*

Complete the mixing time experiment marked optional at the end of Lab 8 using the Metropolis-Hastings algorithm with Proposal I (independent spin flips). Identify a setting (grid size and temperature) where the process mixes slowly (either, too slowly for MH, or, slowly enough the MH is barely feasible).

Report the results of your experiment. In particular, is mixing slowest near criticality, or at an extreme (very high or very low) temperature?

(b) (8 points) *Simulated Tempering*

1. Read BDA Chapter 12.3 on simulated tempering.

2. Show that, if we apply simulated tempering with p_0 uniform, then the artificial temperature parameter defined in simulated tempering can be absorbed into the choice of temperature in the Ising model. Argue that, simulated tempering for the Ising model can be viewed as a MCMC technique that generates coupled random walks for a series of parallel Ising processes each at a different temperature.
3. Adapt your MH algorithm (with independent spin flips) to incorporate simulated tempering. Choose p_0 to be the uniform distribution over spin arrangements. Use your exploratory analysis in part (a) to choose a temperature ladder.
4. Is your algorithm the same as an importance sampling, rejection sampling, or importance resampling procedure that uses a high temperature Ising model as a proposal for a lower temperature target? Discuss the conceptual similarities and differences between this approach and the simulated tempering approach.
5. Apply your simulated tempering algorithm to the Ising model example you identified as challenging in part (a). Is your simulated tempering method more or less computationally efficient (does it take more or less computational effort to achieve the same effective sample size)? Why or why not?

(c) (6 points) *Ensemble Methods*

An ensemble method (or particle filtering method) is an MCMC method that evolves many chains in parallel rather than a single chain. The chains interact, with interaction rules chosen to speed convergence. At any time, an ensemble method consists of a population of particles, each assigned a state that updates according to an MCMC procedure.

For example, we could track the importance weights assigned to each particle had we used the collection of particles as a proposal. Particles with large weights have discovered regions where the target is under-represented by the ensemble, while particles with small weights occupy over-represented regions. This information can be used to move particles from over-represented regions to under-represented regions. In this sense, particles that discover an underexplored region of the target can rapidly recruit other particles to join their neighborhood. In general, these methods do not help the MCMC method discover new modes, but can drastically reduce the mixing time needed to balance mass between, and to fully explore, separate modes.

1. Read Lindsey, Weare and Zhang. *Ensemble Markov Chain Monte Carlo with Teleporting Walkers.* (2022). Briefly summarize the method proposed here, how it shares information between particles/walkers to speed convergence, and some example target distributions where it might be an effective tool.[1]
2. Propose, in pseudocode, an ensemble particle method for sampling from the Ising model. Are there parameter settings for the Ising model where you think an ensemble method might be useful?[2]

[1] For further reading on related methods see Lu, Lu, and Nolen. *Accelerating Langevin Sampling with Birth-death.* (2019)

[2] Think about the symmetry of the model and the number of modes in the target distribution. Are there situations where the target is multimodal and standard MH fails to discover, or balance between, all the modes? Do you need an ensemble method to improve MH in this setting or are there simpler ways to exploit symmetry?