

**This homework is due on Tuesday, July 14, 2020, at 11:59PM.  
Self-grades are due on Tuesday, July 21, 2020, at 11:59PM.**

## 1 Analyzing Mic Board Circuit

In this problem, we will work up to analyzing a simplified version of the mic board circuit. In lab, we will address the minor differences between the final circuit in this problem and the actual mic board circuit.

The microphone can be modeled as a frequency-dependent current source,  $I_{MIC} = k \sin(\omega t) + I_{DC}$ , where  $I_{MIC}$  is the current generated by the mic (which flows from VDD to VSS),  $I_{DC}$  is some constant current,  $k$  is the force<sup>1</sup> to current conversion ratio, and  $\omega$  is the signal's frequency (in  $\frac{\text{rad}}{\text{s}}$ ). VDD and VSS are 5 V and -5 V, respectively.

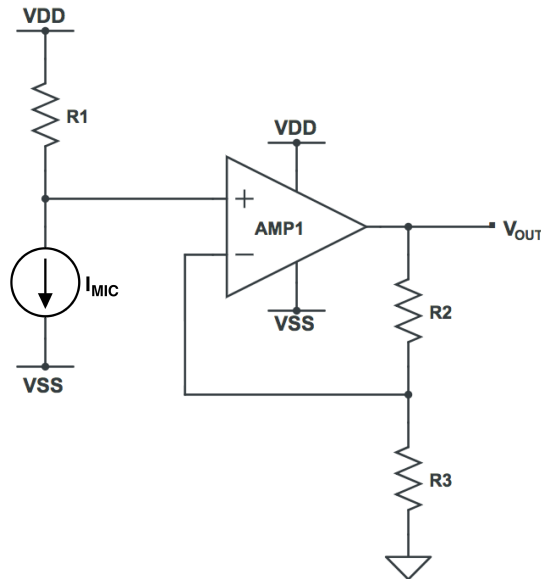


Figure 1: Step 1. The microphone is modeled as a DC current source.

- DC Analysis** Assume for now that  $k = 0$  (so that we can examine just the "DC" response of the circuit), find  $V_{OUT}$  in terms of  $I_{DC}$ ,  $R_1$ ,  $R_2$ , and  $R_3$  (Hint: You do not need to worry about  $V_{SS}$  in your calculations).
- Now, let's include the sinusoidal part of  $I_{MIC}$  as well. We can model this situation as shown below, with  $I_{MIC}$  split into two current sources so that we can analyze the whole circuit using superposition. Let  $I_{AC} = k \sin(\omega t)$ . Find and plot the function  $V_{OUT}(t)$ .

<sup>1</sup>The force is exerted by the soundwaves on the mic's diaphragm.

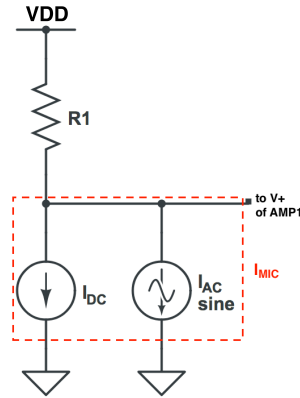


Figure 2: Step 2. The microphone is modeled as the superposition of a DC and a sinusoidal ("AC") current source.

- c) Given that  $V_{DD} = 5\text{ V}$ ,  $V_{SS} = -5\text{ V}$ ,  $R_1 = 10\text{ k}\Omega$ , and  $I_{DC} = 10\text{ }\mu\text{A}$ , find the maximum value of the gain  $G$  of the noninverting amplifier circuit for which the op-amp would not need to produce voltages greater than  $V_{DD}$  or less than  $V_{SS}$  (i.e, find the maximum gain  $G$  we can use without causing the op-amp to clip).
- d) We have modified the circuit as shown below to include a high-pass filter so that the term related to  $I_{DC}$  is removed before we apply gain to the signal. Provide a symbolic expression for  $V_{OUT}$  given that that  $V_{DD0} = 5\text{ V}$ ,  $V_{SS0} = -5\text{ V}$ ,  $V_{DD1} = 3.3\text{ V}$ ,  $V_{SS1} = 0\text{ V}$ . Show your work.

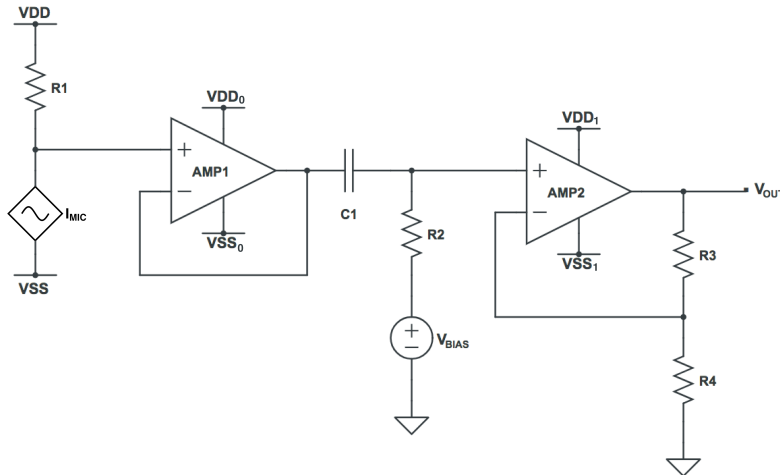
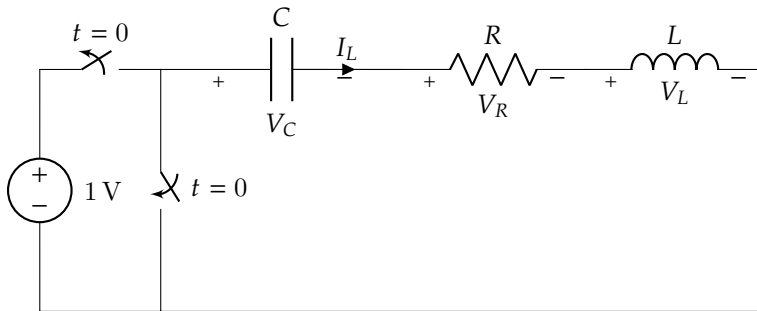


Figure 3: Step 3. Approaching the real mic board circuit. The microphone is still modeled as the superposition of a DC and a sinusoidal ("AC") current source.

- e) We would now like to choose  $V_{BIAS}$  so that we can get as much gain  $G$  out of the non-inverting amplifier circuit (AMP2) as possible without causing AMP2 to clip (i.e, the output of AMP2 must stay between  $0\text{ V}$  and  $3.3\text{ V}$ ). What value of  $V_{BIAS}$  will achieve this goal? If  $k = 10^{-5}$  and  $R_1 = 10\text{ k}\Omega$ , what is the maximum value of  $G$  you can use without having AMP2 clip?

## 2 RLC Responses: Initial Part

Consider the following circuit like you saw in lecture:



Assume the circuit above has reached steady state for  $t < 0$ . At time  $t = 0$ , the switch changes state and disconnects the voltage source, replacing it with a short.

- Write a vector differential equations  $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$  using state variables  $x_1(t) = V_C(t)$  and  $x_2(t) = I_L(t)$  that describes this circuit for  $t \geq 0$ . Remember to find the initial conditions  $x_1(0)$  and  $x_2(0)$ . Also, leave the system symbolic in terms of  $L$ ,  $R$ , and  $C$ .
- Find the eigenvalues of the  $A$  matrix symbolically.  
(Hint: the quadratic formula will be involved.)
- Under what condition on the circuit parameters  $R, L, C$  are the eigenvalues of  $A$ 
  - distinct, and purely real?
  - distinct, and purely imaginary?
  - distinct, and have nonzero real and imaginary parts?
- Now let  $R = 1 \text{ k}\Omega$ ,  $L = 25 \text{ }\mu\text{H}$ ,  $C = 10 \text{ nF}$  and solve for state variables  $x_1(t)$  and  $x_2(t)$ . Recall from discussion that if  $\lambda_1$  and  $\lambda_2$  are the eigenvalues of  $A$ , then we can write out our solution as

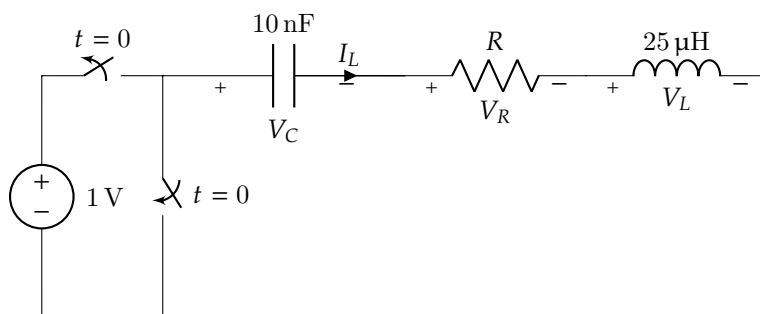
$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 e^{\lambda_1 t} + \alpha_2 e^{\lambda_2 t} \\ \beta_1 e^{\lambda_1 t} + \beta_2 e^{\lambda_2 t} \end{bmatrix} \quad (1)$$

Feel free to round your answers to two significant figures.

- Now let  $R = 1 \text{ }\Omega$ ,  $L = 10 \text{ }\mu\text{H}$ ,  $C = 400 \text{ nF}$  and solve for state variables  $x_1(t)$  and  $x_2(t)$ .
- Our answer for the previous part was in terms of complex exponentials. Why did the final voltage and current waveforms end up being purely real?

### 3 RLC Responses: Critically Damped Case

Building on the previous problem, consider the following circuit with specified component values: (Notice  $R$  is not specified yet. You'll have to figure out what that is.)



Assume the circuit above has reached steady state for  $t < 0$ . At time  $t = 0$ , the switch changes state and disconnects the voltage source, replacing it with a short.

For this problem, we use the same notations as in Problem 1.

- For what value of  $R$  is there going to be a single eigenvalue of  $A$ ?
- Find the eigenvalues and eigenspaces of  $A$ . What are the dimensions of the corresponding eigenspaces and its implications? (i.e. how many linearly independent eigenvectors can you find associated with this eigenvalue?)
- In the provided Jupyter notebook, move the sliders to the resistance value you found in the first part and  $C = 10\text{nF}$ . Sketch  $V_c(t)$  and comment on its appearance. Additionally, sketch the location of the eigenvalues on the complex plane. What happens to the voltage and eigenvalues as you slightly increase or decrease  $R$ ?

**Note:** You will not be required to turn in the Jupyter Notebook, but you should still run the cells to sketch and comment on the behavior of the circuit.

## 4 Basic Orthonormality Proofs

In this problem, we ask you to establish several important properties of orthonormal bases. Let  $U = [\vec{u}_0 \ \vec{u}_1 \ \cdots \ \vec{u}_{n-1}]$  be an  $n$  by  $n$  matrix, where its columns  $\vec{u}_0, \vec{u}_1, \dots, \vec{u}_{n-1}$  form an orthonormal basis of  $\mathbb{C}^n$ .

- a) **Show that a set of orthonormal vectors  $\{\vec{u}_0, \dots, \vec{u}_{n-1}\}$  must be linearly independent.**

(Hint: Suppose  $\vec{w} = \sum_{i=0}^{n-1} \alpha_i \vec{u}_i$ , then first show that  $\alpha_i = \langle \vec{w}, \vec{u}_i \rangle$ . From here ask yourself whether a nonzero linear combination of the  $\{\vec{u}_i\}$  could ever be identically zero.)

This basic fact shows how orthogonality is a very nice special case of linear independence.

- b) **Show that  $U^{-1} = U^*$ , where  $U^*$  is the conjugate transpose of  $U$ .**
- c) **Show that  $U$  preserves inner products, i.e. if  $\vec{v}, \vec{w}$  are vectors of length  $n$ , then**

$$\langle \vec{v}, \vec{w} \rangle = \langle U\vec{v}, U\vec{w} \rangle.$$

Recall that the inner-product is defined to be  $\langle \vec{v}, \vec{w} \rangle = \vec{w}^* \vec{v}$ .

- d) Let  $M$  be a matrix which can be diagonalized by  $U$ , i.e.  $M = U\Lambda U^*$ , where  $\Lambda$  is a diagonal matrix with the eigenvalues  $\lambda_0, \dots, \lambda_{n-1}$  along the diagonal. **Show that  $M^*$  has the same set of eigenvectors  $U$ , while the eigenvalues of  $M^*$  are  $\overline{\lambda_0}, \dots, \overline{\lambda_{n-1}}$ .**
- e) Let  $V$  be another  $n$  by  $n$  matrix, where the columns also form an orthonormal basis. **Show that the columns of the product,  $UV$ , also form an orthonormal basis.**

## 5 Change of Basis Potpourri

- a) Consider a linear transformation  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  represented by an  $n \times n$  matrix  $A$  that maps  $\vec{x} \in \mathbb{R}^n$  to  $\vec{y} \in \mathbb{R}^n$ :

$$\vec{y} = A\vec{x}$$

Suppose we want to change to another basis for  $\mathbb{R}^n$ , given by the columns of the matrix  $P$ :

$$P = \begin{bmatrix} | & & | \\ \vec{p}_1 & \dots & \vec{p}_n \\ | & & | \end{bmatrix}$$

Then we would like to find the representation  $\tilde{A}$  of  $A$  in this new basis such that

$$\tilde{A}\tilde{\vec{x}} = \tilde{\vec{y}}$$

where  $\tilde{\vec{x}}$  and  $\tilde{\vec{y}}$  are the vectors  $\vec{x}$  and  $\vec{y}$  in the new basis. **Show that  $\tilde{A} = P^{-1}AP$ .**

- b) **Suppose we want to transform to an orthonormal basis, so it can be represented by the columns of a unitary matrix  $U$ . Show that if  $A$  is Hermitian, then  $\tilde{A} = U^{-1}AU$  is also Hermitian, i.e. Hermiticity is preserved.**
- c) **Suppose we want to transform to an orthonormal basis, so it can be represented by the columns of a unitary matrix  $U$ . Show that if  $A$  is unitary, then  $\tilde{A} = U^{-1}AU$  is also unitary, i.e. unitarity is preserved.**
- d) **Prove that the determinant of a matrix  $A$  is invariant under change of basis. Further show that for a diagonalizable matrix, the determinant is equal to  $\prod_{k=1}^n \lambda_k$ , where the  $\lambda_k$  are the eigenvalues of  $A$ .**
- e) **Prove that the trace of a matrix  $A$  is invariant under change of basis. Further show that for a diagonalizable matrix, the trace is equal to  $\sum_{k=1}^n \lambda_k$ , where the  $\lambda_k$  are the eigenvalues of  $A$ .**

As a refresher, the trace of a matrix is defined as

$$\text{Tr}(A) = \sum_i^n A_{ii}$$

i.e., the trace is the sum of the diagonal entries of the matrix.

A helpful property of the trace is that it is *cyclic*:

$$\text{Tr}(ABCD) = \text{Tr}(DABC) = \text{Tr}(CDAB) = \text{Tr}(BCDA)$$

## 6 SVD of the derivative operator

The continuous functions from the Real interval  $[0, 2\pi)$  to  $\mathbb{C}$  are a vector space. In this problem, fix a natural number  $N$  and a vector subspace

$$V = \text{Span}\{f_n\}_{n=-N, \dots, N}, \text{ (} 2N + 1 \text{ in total)}$$

where  $f_n$  is the function defined by

$$f_n(\theta) = e^{jn\theta}.$$

Define an inner product by

$$\langle u, v \rangle = \frac{1}{2\pi} \int_0^{2\pi} u(\theta) \overline{v(\theta)} d\theta.$$

- Verify that the functions  $\{f_n\}_n$  are orthonormal. Conclude that they are a basis for  $V$  and that  $V$  has dimension  $2N + 1$ .
- Define the linear operator  $D : V \rightarrow V$  by

$$Dv(\theta) = \frac{d}{d\theta} v(\theta).$$

Verify that  $D$  is diagonal in the basis  $\{f_n\}_n$ . What are its eigenvalues?

- Does the fact that  $D$  is diagonal in an orthonormal basis imply that it is self-adjoint?
- Define the basis  $\{g_n\}_n$  by

$$g_n = \sigma(n) j f_n = \begin{cases} j f_n, & n \geq 0 \\ -j f_n, & n < 0 \end{cases},$$

where  $\sigma(n) = 1$  if  $n$  is nonnegative and  $-1$  otherwise. Take  $\{f_n\}_n$  as a basis for the domain of  $D$  and  $\{g_n\}_n$  as a basis for the codomain of  $D$ . Verify that this choice of bases is a singular value decomposition of  $D$ . What are the singular values?

## 7 Exam Proctoring Practice

Midterm 1 is on Friday, June 17th, and this will be the last homework you turn in before the exam. We are asking that as a part of this homework, you practice creating a proctoring recording, as you will be required to do for the exam. The requirements and instructions for the recording are the following. Pay careful attention to the order of the steps.

- 1 Use zoom to create your recording.
  - Create a new meeting and make sure you are unmuted and video is on.
  - Your video must capture as much of your face and work area as possible, within reason.
  - You are permitted to use a virtual background if you would like.
- 2 Begin the recording.
- 3 Share screen.
  - Make sure that your front-facing camera is still shown in the corner of your screen while screen sharing.
- 4 Open the exam and begin.
  - For the purposes of this homework, move on to next step.
- 5 Submit your exam to gradescope.
  - Submit your supplementary python file or ipython notebook for the exam to gradescope.
  - For the purposes of this homework, submit this homework assignment and a "hello world" python/ipython notebook file to the HW3 and HW3 Supplementary Code assignments, respectively. (Therefore save this problem until you are ready to turn in your homework.)
- 6 Stop recording and upload.
  - End the zoom meeting (so that video saves).
  - (Optional) Upload the recording mp4 file to your Berkeley google drive.
  - (Optional) Set sharing settings to share with the account `eeecs16b-su20@berkeley.edu`.
  - Copy the link of your recording, and submit the link to the following google form: *Link to form*.
  - This file should be submitted within 3 hours after completing the exam, since it may take some time to process the recording. If your file will not upload within this time limit, take a screenshot and send to an email to `eeecs16b-su20@berkeley.edu` explaining the situation.
  - If 3 hours after the end of the exam is later than 11pm in your timezone, send an email to `eeecs16b-su20@berkeley.edu` explaining the situation, and submit the file by 9am the next morning in your timezone.

Important: we are asking you to do this exercise so you are familiar with the general procedure for audio/video proctoring before the exam happens. We reserve the right to modify this process if necessary before the actual exam.

**If you are unable to perform or have concerns about any part of this procedure, please send an email to `eeecs16b-su20@berkeley.edu` so we can attempt to resolve the issue before the exam.**



## 8 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student! We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

- a) **What sources (if any) did you use as you worked through the homework?**
- b) **If you worked with someone on this homework, who did you work with?** List names and student ID's. (In case of homework party, you can also just describe the group.)
- c) **How did you work on this homework?** (For example, *I first worked by myself for 2 hours, but got stuck on problem 3, so I went to office hours. Then I went to homework party for a few hours, where I finished the homework.*)
- d) **Do you have any feedback on this homework assignment?**
- e) **Roughly how many total hours did you work on this homework?**