

# EECS 16B Summer 2020 Midterm 1 (Form 1)

## Instructions

You have 90 minutes. You may access printed, handwritten, and pre-downloaded materials during this exam. You may not consult the Internet or other human beings during this exam. You may not share the contents of this exam by any medium, verbal or digital, at any time before it is posted on the course website.

Justify your answers. A correct result without justification will not receive full credit.

## Honor Code

COPY, SIGN, and DATE the UC Berkeley Honor Code by hand. (*If you fail to do so, your exam may not be accepted.* Email the instructors if you are unable.)

*As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

.....  
COPY, SIGN, and DATE the following addendum by hand. (*If you fail to do so, your exam may not be accepted.* Email the instructors if you are unable.)

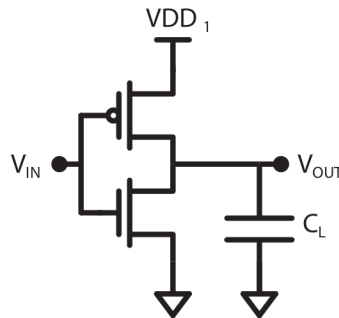
*Should I commit academic misconduct during this exam, let me receive a failing grade in EECS 16B or dismissal from the University.*

.....

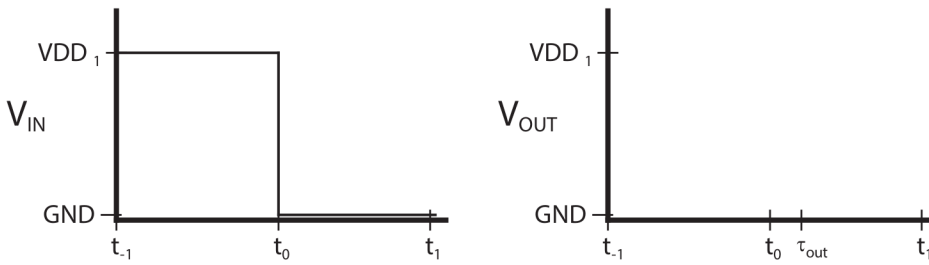
## 1 Digital logic

Designers of modern computer processors must navigate the trade-off between higher speed and lower power. Increasing the power supply voltage enables higher speed operation, but costs more power. Let's examine where the increased power draw and increased speed come from and where they do not. (Answers within 1-2 significant figures are correct if work is shown.)

- a) Let's examine an ideal inverter driving a signal line with capacitance  $C_L$ . Solve the first-order differential equation for  $V_{out}$ , given  $V_{in}(t < 0) = 1$  and  $V_{in}(t \geq 0) = 0$ , mosfet switch resistance  $R_n = R_p = 2\Omega$ , gate capacitance  $C_n = C_p = 5 \times 10^{-12}$  F, voltage threshold  $V_{TH} = 0.5VDD_1$  with  $VDD_1 = 1$  V, and signal line capacitance  $C_L = 2 \times 10^{-9}$  F.

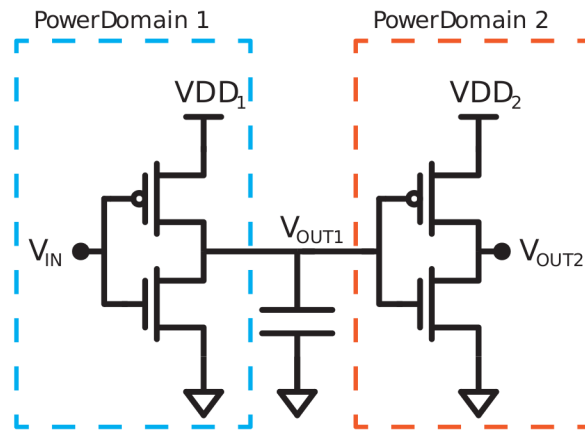


- b) Draw  $V_{\text{out}}$  for  $V_{\text{in}}$  as a periodic square wave with one period shown below. Indicate the value of  $V_{\text{out}}(\tau_{\text{out}})$  on the y-axis, where  $\tau_{\text{out}}$  is an arbitrary RC time constant of the output node.



- c) How much energy does  $C_L$  store just before time  $t_1$ , for  $\tau_{\text{out}} \ll (t_1 - t_0)$ ?

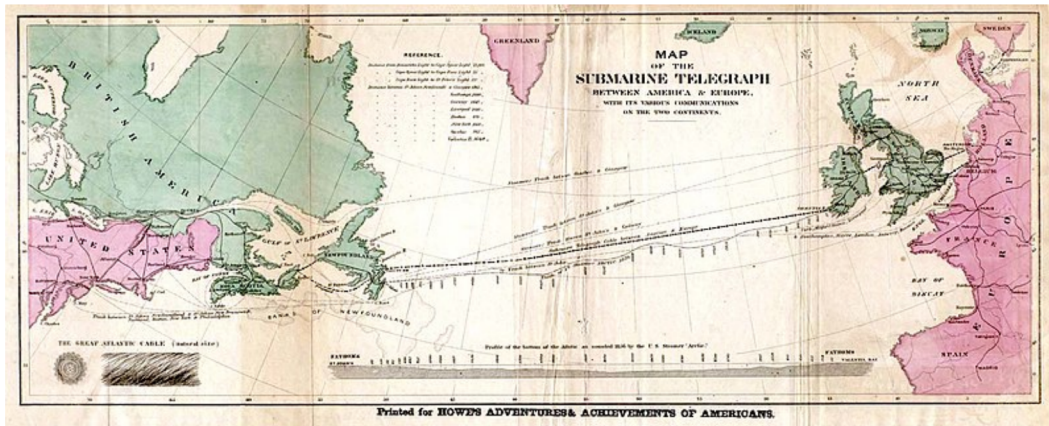
- d) Say  $V_{out1}$  is the input to a second power domain, where the inverters are connected to different VDDs. If each inverter's threshold is half of its respective VDD ( $V_{THX} = 0.5VDD_X$ ), how long does it take for  $V_{out1}$  to reach  $V_{TH2}$  and flip the second inverter if  $VDD_2 = 2VDD_1$ ?



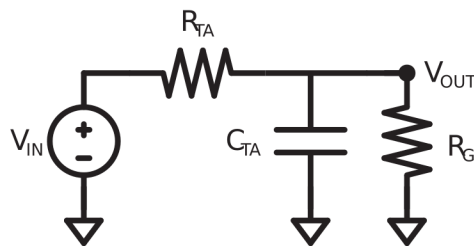
- e) Let  $\alpha$  be the ratio  $VDD_2/VDD_1$ . In the previous question,  $\alpha = 2$  wasn't a great value. Calculate a new  $\alpha$  so that the second inverter flips at  $t_{flip} = 3\tau_{out1}$ , where  $\tau_{out1}$  is the RC time constant related to node OUT1.

## 2 Problem 2

In 1858, the Atlantic Telegraph Company completed the first transatlantic telegraph cable. It took 16 hours to send Queen Victoria's inaugural 98 words, over a distance usually covered in 10 days. This would have been a success if the project hadn't been plagued with errors and the cable broken just days later. In this problem, you will use all of your modern engineering to understand all of things that went wrong. (Note, all of these assumptions are drastic over simplifications, but they are still a lot more than designers at the time knew!) (Answers within 1-2 significant figures are correct if work is shown.)



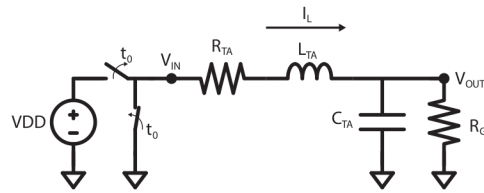
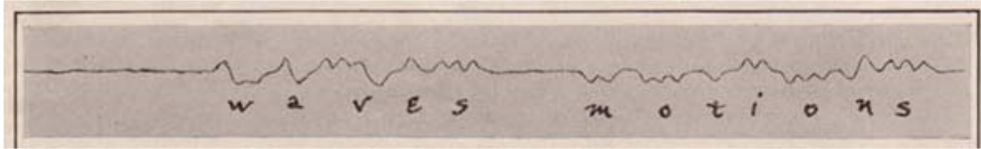
- a) While the concept of capacitance was understood at the time (Maxwell himself was consulting on the transatlantic cable) modeling or predicting capacitance was much more difficult. The designers had very little idea how slow they would have to send symbols until the cable was in place.



- i) Write the fully factored transfer function,  $H(j\omega)$ , in terms of the above variables for our model of the transatlantic cable.

- ii) The transatlantic cable could send Morse code at 6 words/hr and quality began to degrade at any faster frequency. Knowing this cutoff frequency and having our model of the cable allows us to estimate the capacitance of the cable. Solve for  $R_{TA}$  and  $C_{TA}$  if  $R_G = 1\ \Omega$ ,  $\omega_c = 0.05\ \text{rad/s}$ , and  $H(j0) = 1 \times 10^{-6}$ .

- b) A telegraph operator would send morse code symbols through a telegraph cable by connecting a switch between VDD or GND, so the input signal would look much like a square wave. The output signal, however, had a ripple (ringing) by the time it reached the other end of the cable. The effect of inductance on a telegraph line was rarely considered, until the transatlantic cable failed and a bunch of clever scientists wanted to know why.



- i) Find the symbolic values of matrix  $A$  in terms of  $L$  for the vector differential equation which describes our improved cable model with new values  $R_{TA} = 10,000 \Omega$ ,  $C_{TA} = 1 \times 10^{-6}$ , and  $R_G = 1 \Omega$ , with  $V_{OUT}(t_0) = 1$  and  $V_{IN}(t) = 0$  for all  $t$ .

$$\frac{d}{dt} \begin{bmatrix} V_{OUT} \\ I_L \end{bmatrix} = A \begin{bmatrix} V_{OUT} \\ I_L \end{bmatrix}$$

- ii) Choose any value of  $L$  that will cause ringing at the output of this cable for a square wave input. Justify your choice by solving for the eigenvalues of  $A$  with your chosen  $L$ .

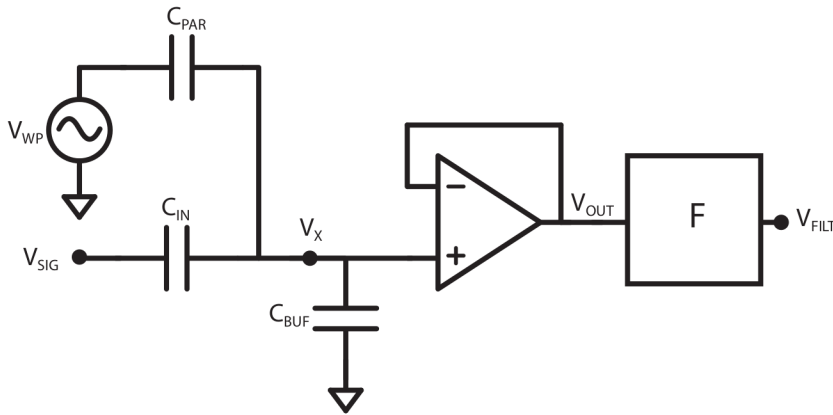


### 3 Problem 3

Any circuit designer who has spent enough time debugging in a lab will come across a mysterious 60 Hz signal. Why 60 Hz? Because that's the frequency of AC wall power. If you are not very careful, that frequency might sneak into your signal lines and create enormous interference. Let's examine how this happens. (Answers within 1-2 significant figures are correct if work is shown.)

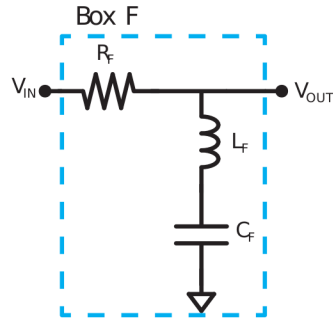
- a) You are given an approximate model of a signal line node with input voltage  $V_{\text{sig}}$  which goes through a unit buffer. There is some parasitic capacitance  $C_{\text{par}}$  that comes from sloppy wiring and a cheap power source that capacitively couples wall power to your signal node. So even when your signal is off, you're seeing a 60Hz sinusoid on your output node.

The wall power voltage is  $V_{\text{wp}}(t) = 120 \cos(\omega_{\text{wp}} t)$ . If the desired signal is  $V_{\text{sig}}(t) = 50 \cos(\omega_{\text{sig}} t)$  where  $\omega_{\text{sig}} = 2\pi 1000$  (rad/sec) design a passive filter for box  $F$  that will attenuate a 60 Hz tone by  $\frac{1}{10}$  or more, without significantly attenuating  $V_{\text{sig}}$ . You need only write the factored form transfer function,  $H_F(j\omega)$ , and justify why it fulfills the design specifications.



b) Say, now, that  $V_{\text{sig}}(t) = 0.5 \cos(\omega_{\text{sig1}} t) + 0.5 \cos(\omega_{\text{sig2}} t)$  where  $\omega_{\text{sig1}} = 2\pi 1000$  (rad/sec) and  $\omega_{\text{sig2}} = 2\pi 5$  (rad/sec). We would need a filter which is the opposite of a bandpass filter, one which lets through low and high signals, but removes middle frequencies. Such a filter is called a notch filter.

i) Explain what is happening to the impedance of the following notch filter at  $\omega_{\text{wp}} = 2\pi 60$  if  $L_F$  and  $C_F$  are in resonance.



ii) Calculate  $C_F$  for the notch filter shown below if the only available inductor and resistor are  $L_F = 1 \times 10^{-6}$  H and  $R_F = 1000 \Omega$  and we wish to remove  $\omega_{\text{wp}}$ .

#### 4 Diagonalization I

In this problem all matrices are over the complex numbers. You may use the fact that if  $x$  is a complex number, there is a complex number  $\sqrt{x}$  such that  $(\sqrt{x})^2 = x$ .

- a) State a basis in which  $A$  has a diagonal representation, and give coordinates for  $A$  relative to this basis. If you are not able to do so, explain why.

$$A = \begin{pmatrix} \alpha & -\beta & 0 \\ \beta & \alpha & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- b) State the eigenvalues of  $A$  in the simplest form possible. If you are not able to do so, explain why.

$$A = \begin{pmatrix} a^2 & \sqrt{2}ab + b^2j \\ \sqrt{2}ab - b^2j & -a^2 \end{pmatrix}$$

c) State the eigenvalues of  $A$  in polar form. If you are not able to do so, explain why.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & k \\ k & 0 & 0 & 0 & 0 \\ 0 & k & 0 & 0 & 0 \\ 0 & 0 & k & 0 & 0 \\ 0 & 0 & 0 & k & 0 \end{pmatrix}$$

## 5 Diagonalization II

- a) If  $T \in \mathbb{C}^{m \times n}$  is a matrix, then  $T^*T$  has nonnegative real eigenvalues. How does the construction of the SVD of  $T$  rely on this fact? (Length: 1 sentence)
- b) Does there exist a matrix  $A \in \mathbb{C}^{2 \times 2}$  that is *not* diagonalizable? If so, give and explain an example. If no, explain why not. (Length: 1–2 sentences)
- c) Does there exist a matrix  $A \in \mathbb{C}^{n \times n}$  that does not have an eigenvector? If so, give and explain an example. If not, explain why not. (Length: 1–2 sentences)
- d) We proved that a matrix  $A \in \mathbb{C}^{n \times n}$  satisfying  $A = A^*$  can be factored  $A = PDP^*$ , where  $P$  is unitary and  $D$  is diagonal. How did we use mathematical induction in the process? (Length: 1 sentence for base case, 1 sentence for inductive step)
- e) Answer True/False: a matrix of noisy instrument measurements is more likely to have high rank than low rank.

## 6 PCA

Answer whether each scatter plot resembles the scatter plot of the projections of a dataset onto its first two principal components, in which the first principal component is shown on the horizontal axis and the second is shown on the vertical axis. (True/False: *True* if it does, *False* if it doesn't)

If not possible, state that there is not enough information to do so. Justify each answer with 1–2 sentences.

