

1 Maximum Likelihood Estimation

Recall that a Geometric distribution is defined as the number of Bernoulli trials needed to get one success. $P(X = k) = p(1-p)^{k-1}$.

We observe the following samples from a Geometric distribution:

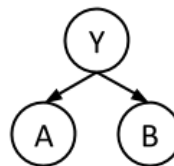
$x_1 = 5, x_2 = 8, x_3 = 3, x_4 = 5, x_5 = 7$

What is the maximum likelihood estimate for p ?

2 Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B . Y , A , and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A	1	1	1	1	0	1	0	1	1	1
B	1	0	0	1	1	1	1	0	1	1
Y	1	1	0	0	0	1	1	0	0	0



- (a) What are the maximum likelihood estimates for the tables $P(Y)$, $P(A|Y)$, and $P(B|Y)$?

Y	$P(Y)$
0	
1	

A	Y	$P(A Y)$
0	0	
1	0	
0	1	
1	1	

B	Y	$P(B Y)$
0	0	
1	0	
0	1	
1	1	

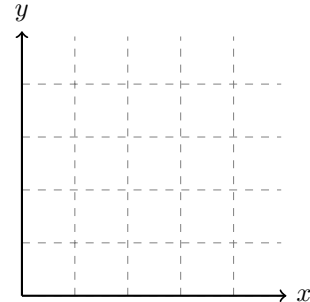
- (b) Consider a new data point ($A = 1, B = 1$). What label would this classifier assign to this sample?
- (c) Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for $P(A|Y)$ given Laplace Smoothing with $k = 2$.

A	Y	$P(A Y)$
0	0	
1	0	
0	1	
1	1	

3 Perceptron

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie:

#	Movie Name	A	B	Profit?
1	Pellet Power	1	1	-
2	Ghosts!	3	2	+
3	Pac is Bac	2	4	+
4	Not a Pizza	3	4	+
5	Endless Maze	2	3	-



- (a) Plot the data above and determine if the points are linearly separable.
- (b) Now you decide to use a perceptron to classify your data. Suppose you directly use the scores given above as features, together with a bias feature. That is $f_0 = 1$, $f_1 =$ score given by A and $f_2 =$ score given by B.

Run one pass through the data with the perceptron algorithm, filling out the table below. Go through the data points in order, e.g. using data point #1 at step 1.

step	Weights	Score	Correct?
1	$[-1, 0, 0]$	$-1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = -1$	yes
2			
3			
4			
5			

Final weights:

- (c) Have weights been learned that separate the data?
- (d) More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Circle the scenarios for which a perceptron using the features above can indeed perfectly classify movies which are profitable according to the given rules:
- (a) Your reviewers are awesome: if the total of their scores is more than 5, then the movie will definitely be profitable, and otherwise it won't be.
 - (b) Your reviewers are art critics. Your movie will be profitable if and only if each reviewer gives either a score of 2 or a score of 3.
 - (c) Your reviewers have weird but different tastes. Your movie will be profitable if and only if both reviewers agree.