EECS 16A     Designing Information Devices and Systems I
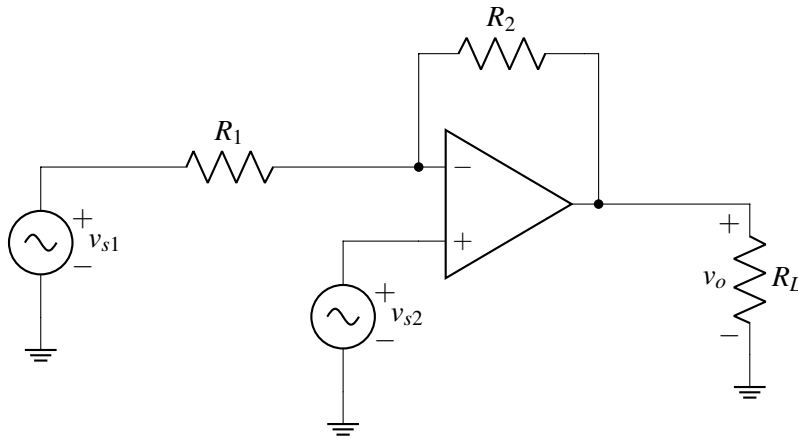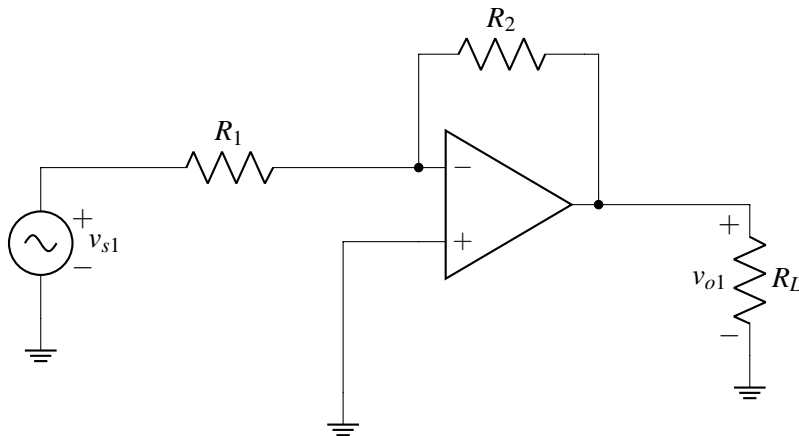Fall 2020                                                                    Discussion 12A

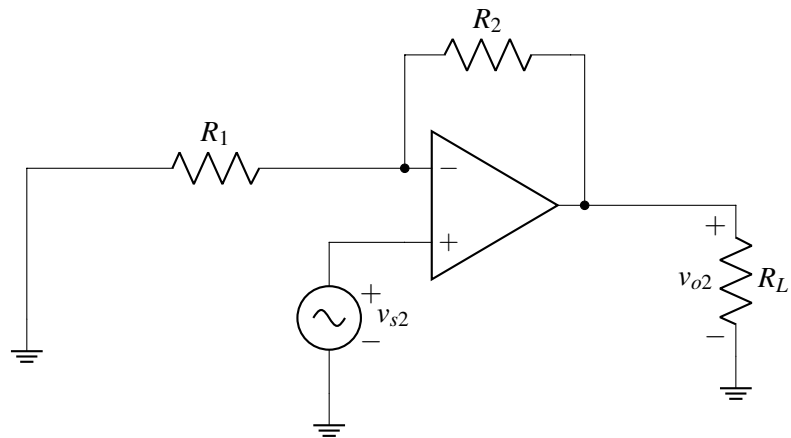## 1. Amplifier with Multiple Inputs

In this problem we will use superposition and the Golden Rules to find the output of the following op amp circuit with multiple inputs:
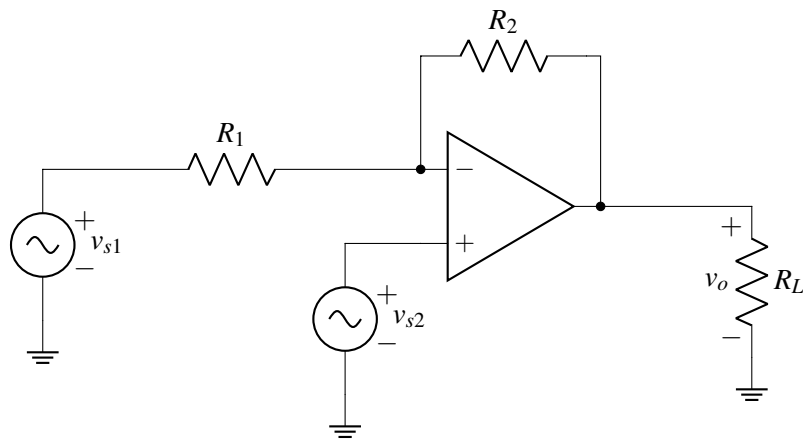


(a) First, let's turn off $v_{s2}$. Use the **Golden Rules** to find $v_{o1}$ for the circuit below.

(b) Now let's turn off $v_{o1}$. Use the **Golden Rules** to find $v_{o2}$ for the circuit below.

(c) Use **superposition** to find the output voltage $v_o$ for the circuit shown below.

**Reference: Inner products**
Let $\vec{x}$, $\vec{y}$, and $\vec{z}$ be vectors in real vector space $\mathbb{V}$. A mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on $\mathbb{V}$ if it satisfies the following three properties:

(a) Symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

(b) Linearity: $\langle \vec{x}, \vec{y}+\vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$

(c) Non-negativeness: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

We define the norm of $\vec{x} = [x_1, x_2, ..., x_n]^T$ as $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$.

2. **Mechanical Inner Products**

For the following pairs of vectors, find the Euclidean inner product $\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}$.

(a)

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
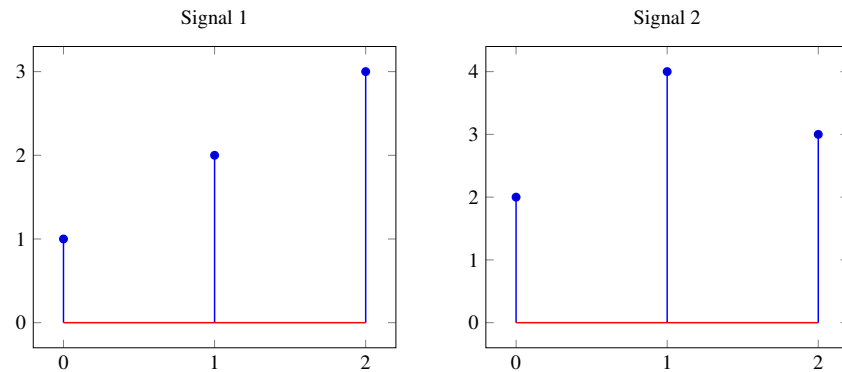
(b)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

### 3. Correlation

We are given the following two signals, $s_1[n]$ and $s_2[n]$ respectively.



Find the cross correlations, $\text{corr}_{s_1}(s_2)$ and $\text{corr}_{s_2}(s_1)$ for signals $s_[n]$ and $s_2[n]$. Recall

$$\text{corr}_x(y)[k] = \sum_{i=-\infty}^{\infty} x[i]y[i-k].$$

$$\text{corr}_{\vec{s_1}}(\vec{s_2})[k]$$

| $\vec{s_1}$ | 0 | 0 | 1 | 2 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s_2}[n+2]$ | | | | | | | |
| $\langle \vec{s_1}, \vec{s_2}[n+2] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s_1}$ | 0 | 0 | 1 | 2 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s_2}[n+1]$ | | | | | | | |
| $\langle \vec{s_1}, \vec{s_2}[n+1] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s_1}$ | 0 | 0 | 1 | 2 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s_2}[n]$ | | | | | | | |
| $\langle \vec{s_1}, \vec{s_2}[n] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s_1}$ | 0 | 0 | 1 | 2 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s_2}[n-1]$ | | | | | | | |
| $\langle \vec{s_1}, \vec{s_2}[n-1] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s_1}$ | 0 | 0 | 1 | 2 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s_2}[n-2]$ | | | | | | | |
| $\langle \vec{s_1}, \vec{s_2}[n-2] \rangle$ | + | + | + | + | + | + | = |

$$\text{corr}_{\vec{s_2}}(\vec{s_1})[k]$$

| $\vec{s}_2$ | 0 | 0 | 2 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s}_1[n+2]$ | | | | | | | |
| $\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s}_2$ | 0 | 0 | 2 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s}_1[n+1]$ | | | | | | | |
| $\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s}_2$ | 0 | 0 | 2 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s}_1[n]$ | | | | | | | |
| $\langle \vec{s}_2, \vec{s}_1[n] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s}_2$ | 0 | 0 | 2 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s}_1[n-1]$ | | | | | | | |
| $\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$ | + | + | + | + | + | + | = |

| $\vec{s}_2$ | 0 | 0 | 2 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\vec{s}_1[n-2]$ | | | | | | | |
| $\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$ | + | + | + | + | + | + | = |