

Welcome to EECS 16A!

Designing Information Devices and Systems I



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Fall 2021

Module 2
Lecture 3
Power and Voltage/Current Measurements
(Note 13)



Last Lecture

- Solve circuits for the currents and node potentials
- Set up a matrix problem of the form $A \vec{x} = \vec{b}$

where

\vec{x} consists of the unknown currents and potentials

\vec{b} contains the independent current and voltage sources

A describes the relationship between them.

$$A \vec{x} = \vec{b}$$

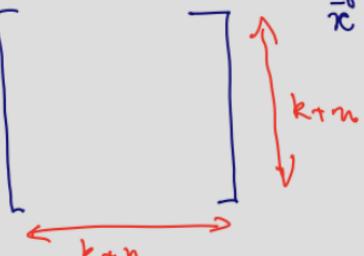
\vec{x} : unknowns

\vec{b} : knowns / constants

A : knowns / constants

$$\vec{x} = \begin{bmatrix} i_1 \\ \vdots \\ i_K \\ v_1 \\ \vdots \\ v_m \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_{K+m} \end{bmatrix}$$

$k+n$



Rules:

• KVL

• KCL

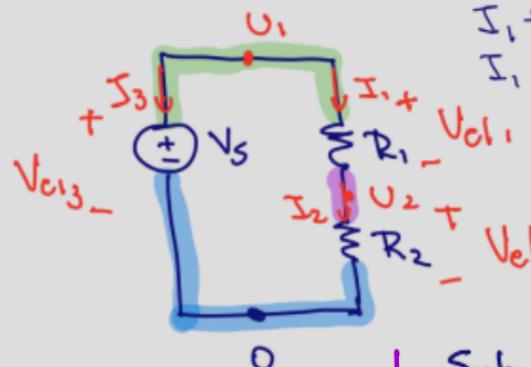
• Element definitions

• $I \propto V$ relationship

* Started Voltage Divider Analysis

Node Voltage Analysis – Voltage Divider

Step 7 – Use the IV relationships of each of the non-wire elements to fill in the remaining rows of A and



$$I_1 + I_3 = 0 \quad (1)$$

$$I_1 - I_2 = 0 \quad (2)$$

$$V_{el1} = U_1 - U_2$$

$$V_{el2} = U_2 - 0 = U_2$$

$$V_{el3} = U_1 - 0 = U_1$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}$$

Substitution:

$$\frac{V_{el1}}{E1_3} : U_1 - U_2 = R_1 \cdot I_1 \Rightarrow R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$
$$\frac{V_{el2}}{E1_2} : U_2 = R_2 I_2 \Rightarrow R_2 I_2 - U_2 = 0 \quad (4)$$

$$\frac{V_{el3}}{E1_3} : U_1 = V_s \quad (5)$$

Node Voltage Analysis – Voltage Divider

Step 8 – Solve the system of equations to determine values of unknown variables.

$$I_1 + I_3 = 0 \quad (1)$$

$$-I_1 + I_2 = 0 \quad (2)$$

$$R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$

$$R_2 I_2 - U_2 = 0 \quad (4)$$

$$U_1 = V_s \quad (5)$$

$$\begin{matrix} A & \xrightarrow{\text{A}} \\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ R_1 & 0 & 0 & -1 & 1 \\ 0 & R_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] & \left[\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{array} \right] \end{matrix}$$

$$I_1 = \frac{V_s}{R_1 + R_2}$$

$$I_2 = \frac{V_s}{R_1 + R_2}$$

$$I_3 = -\frac{V_s}{R_1 + R_2}$$

$$U_1 = V_s$$

$$U_2 = \frac{R_2}{R_1 + R_2} \cdot V_s$$

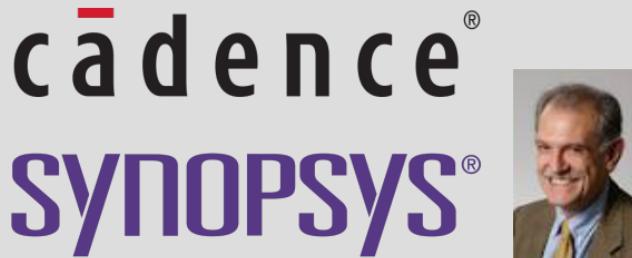
$\hookrightarrow \alpha L1$

α is an operator

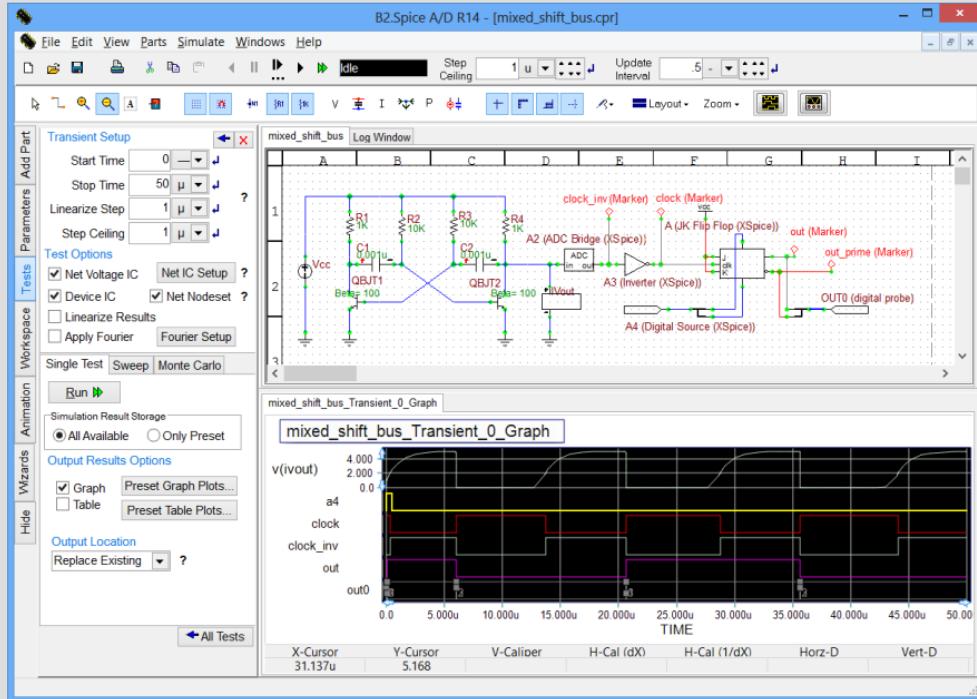
Electrical Circuit Analysis Algorithm (tool)

SPICE (Simulation Program with Integrated Circuit Emphasis): started as a student project at Berkeley!

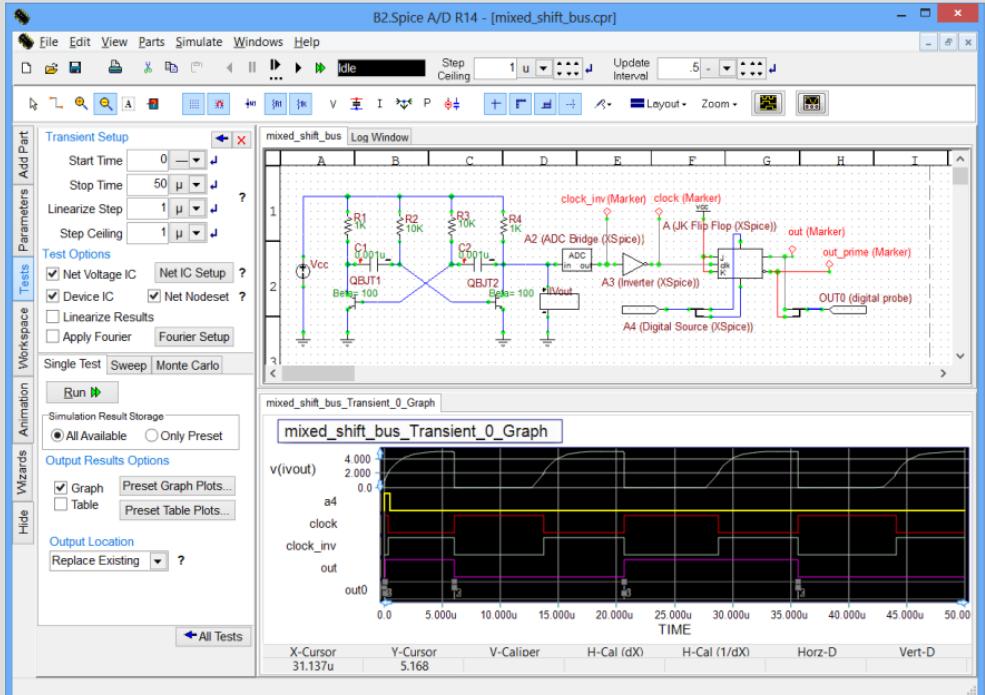
Now the basis for open-source electronic circuit simulation, to design and model device characteristics and check circuit boards



Prof. Alberto L. Sangiovanni-Vincentelli



Electrical Circuit Analysis Algorithm (tool)



How to think about Energy and Power in circuits?

Current: flow of charges (electrons moving from point A to B inside a material) $I = \frac{dQ}{dt}$

It takes **energy** to move charge from A \rightarrow B \Rightarrow Voltage

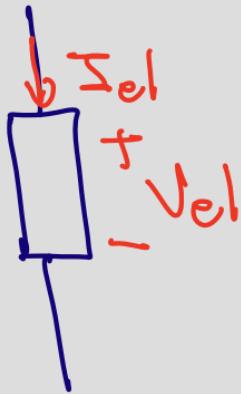
$$V_{AB} = \frac{dE}{dq}$$

Power: is the rate of change of energy

$$P = \frac{dE}{dt} \cdot \frac{dq}{dt} = V \cdot I$$
$$(V) \cdot (A) = (W)$$

Energy and Power

$$P_{el} = V_{el} \cdot I_{el}$$



if element is a resistor

$$P = V \cdot I = R \cdot I \cdot I = R \cdot I^2 \geq 0$$

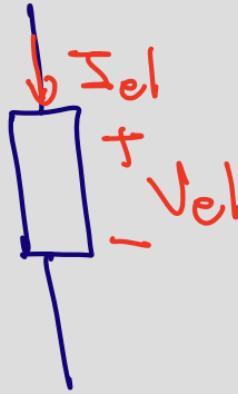
Power dissipated is positive

$$V_{el} = R \cdot I_{el}$$

$$I_{el} = \frac{V_{el}}{R}$$

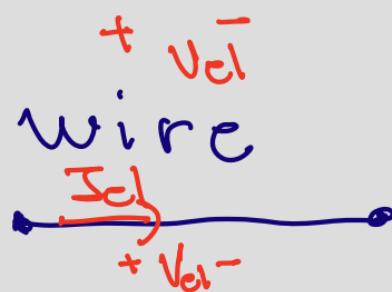
$$P = V \cdot I = V_{el} \cdot V_{el}/I = V_{el}^2/I \geq 0$$

Energy and Power



$$P_{el} = V_{el} \cdot I_{el}$$

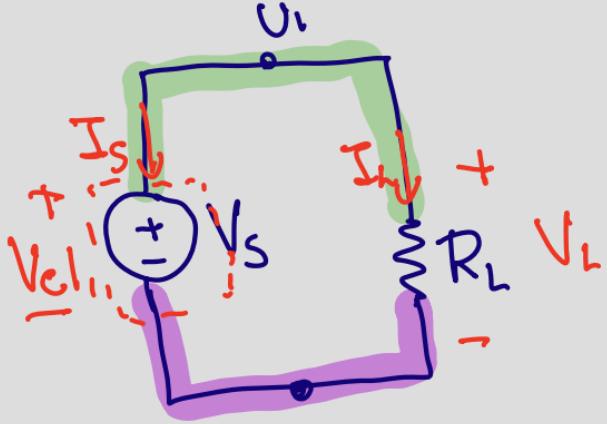
Open circuit



$$P_{el} = V_{el} \cdot I_{el} \cancel{> 0} = 0$$

$$\cancel{P_{el} = V_{el} \cdot I_{el} > 0} = 0$$

Example



Element 1
 $P_s = I_s \cdot V_{cl1}$ (def.)

$P_s = I_s \cdot V_s$

* Conservation
of Energy

KCL: $I_L + I_s = 0$

KVL: $V_L - 0 = V_L$

$V_L - 0 = V_{cl1} = V_s$

Power: Element L

$P_L = I_L \cdot V_L$ (def.)

$P_L = (-I_s) \cdot V_L$

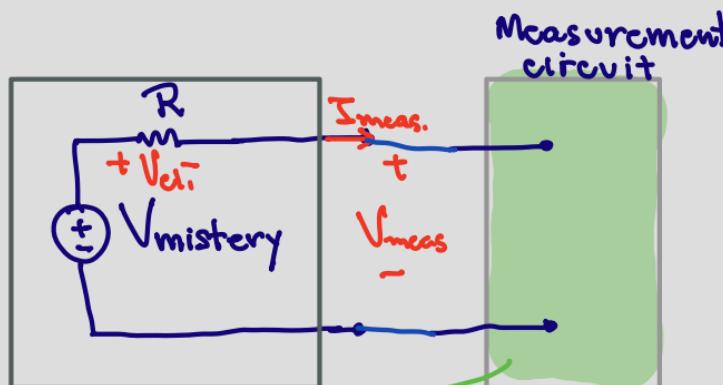
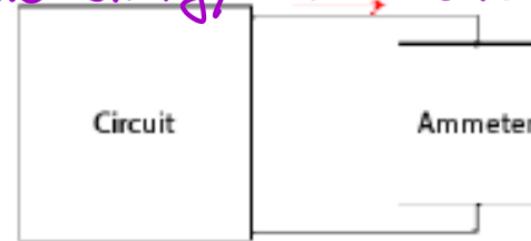
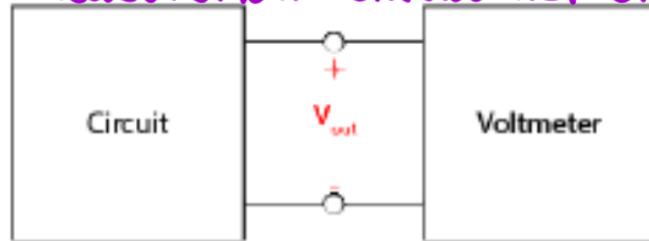
$P_L = (-I_s) \cdot V_s$

* $P_L = -P_s$

$P_L + P_s = 0$

How to measure Voltage and Current?

Measurement should not change the energy of the circuit



Must behave as
an open-circuit

$$\text{Goal : } V_{\text{meas}} = V_{\text{mystery}}$$

$$V_{\text{el}} = I_{\text{meas}} \cdot R$$

$$\text{KVL : } V_{\text{mystery}} - V_{\text{el}} - V_{\text{meas}} = 0$$

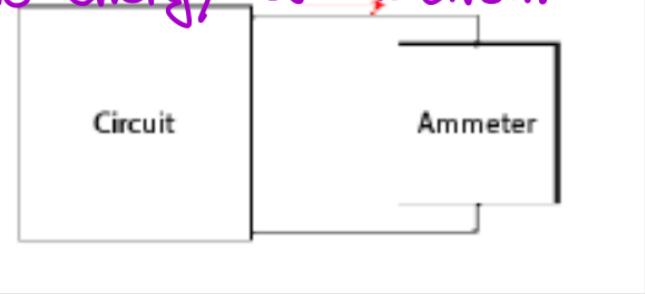
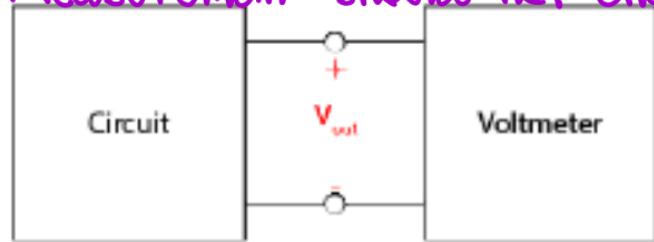
$$V_{\text{mystery}} = V_{\text{el}} + V_{\text{meas}}$$

$$V_{\text{mystery}} = I_{\text{meas}} \cdot R + V_{\text{meas}}$$

$$V_{\text{mystery}} = V_{\text{meas}} \quad |_{\text{if } I_{\text{meas}} = 0}$$

How to measure Voltage and Current?

Measurement should not change the energy of the circuit



Task / Goal :

Measure $I_{mystery}$

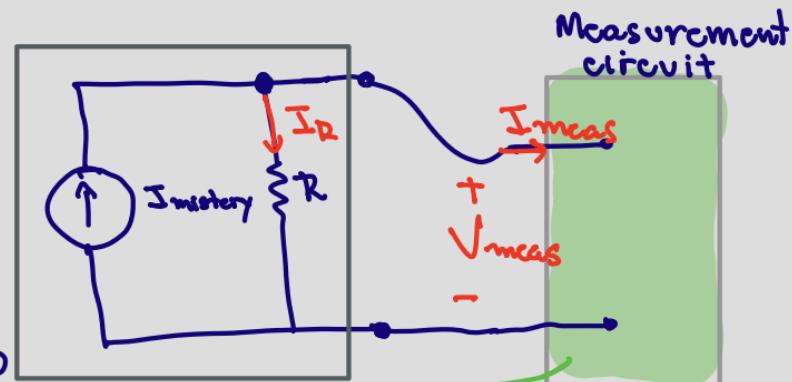
$$KCh: I_{mystery} = I_R + I_{meas}$$

$$I_{mystery} = I_{meas}$$

if $I_R=0$

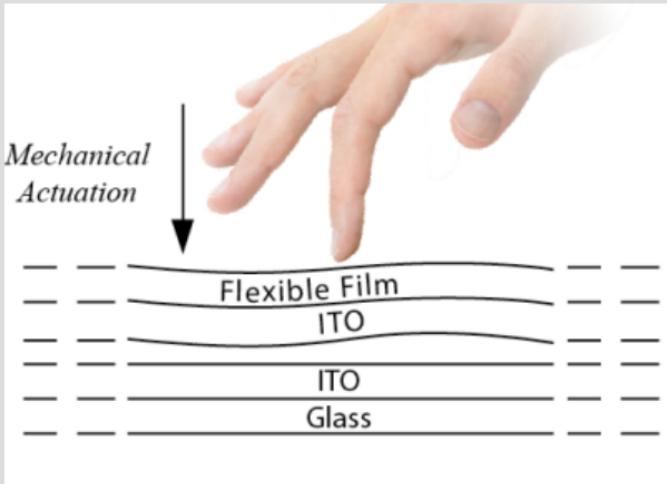
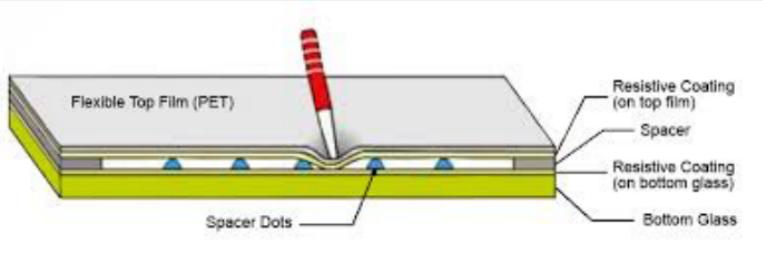
$$I_R = \frac{V_{meas}}{R}$$

$$I_R=0; \text{ is } V_{meas}=0$$

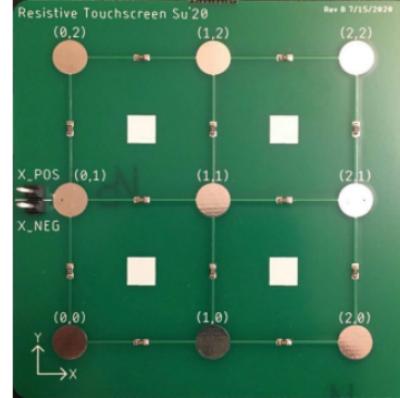


Must behave as a wire

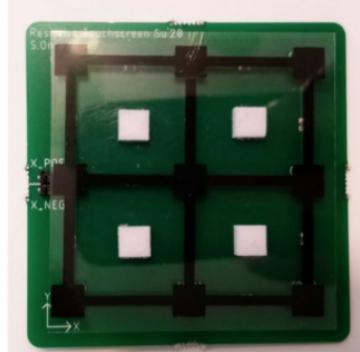
Resistive Touch Screen



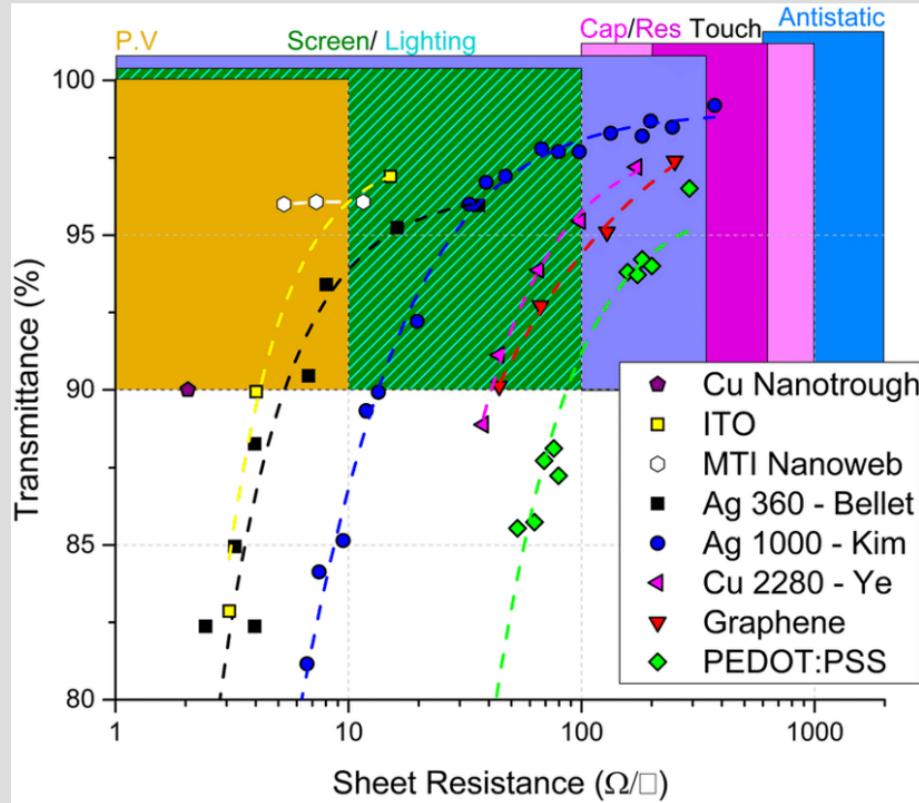
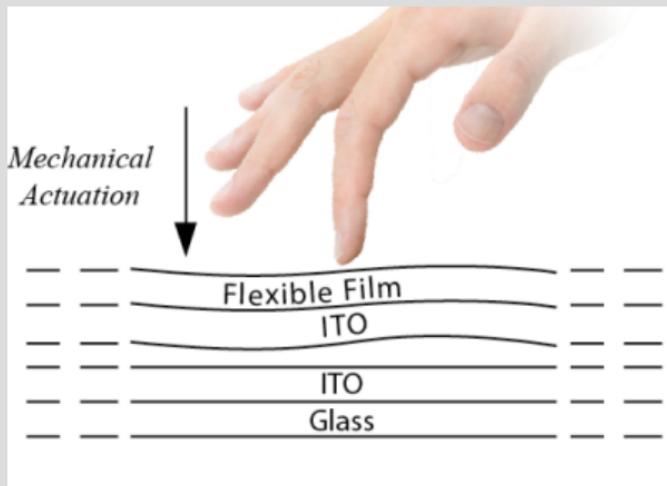
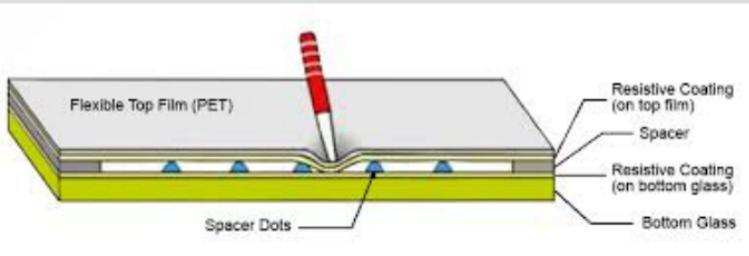
Bottom Layer: Resistive Layer



Top Layer: Flexible Resistive Layer

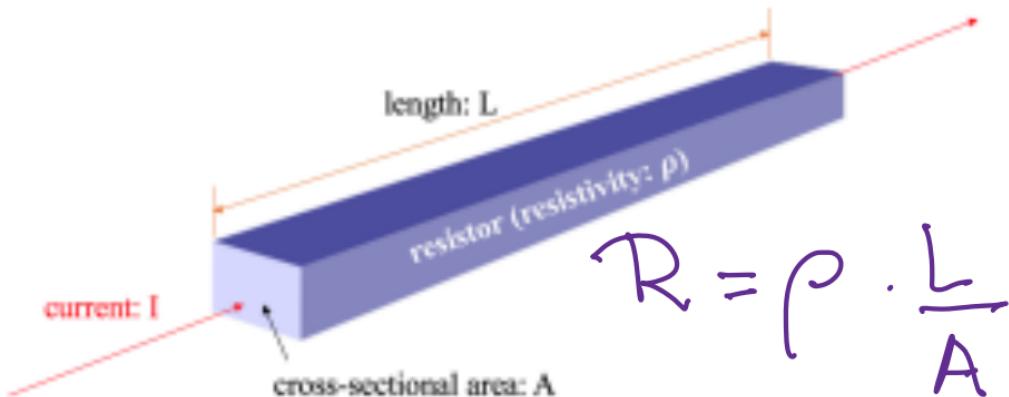


Resistive Touch Screen



Resistance, Resistivity, Conductivity – Properties of Materials

Material	Electrical characteristics	
	Electrical Resistivity ($\Omega \times \text{cm}$)	Electrical Conductivity ($\Omega^{-1} \times \text{cm}^{-1}$)
Cu	0.034×10^{-5}	29×10^5
Fe	32.54×10^{-5}	0.031×10^5
Ag	0.36×10^{-5}	2.8×10^5
Al	0.03×10^{-5}	33.3×10^5
Ni	0.046×10^{-5}	21.7×10^5
Cu-Fe	33.37×10^{-5}	0.030×10^5
Cu-Ag	2.71×10^{-5}	0.37×10^5
Al-Ni	0.564×10^{-5}	1.77×10^5



$$R = \rho \cdot \frac{L}{A}$$

Note 12

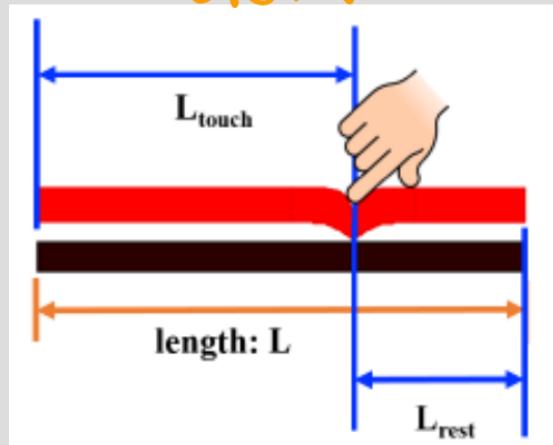
- longer the wire \rightarrow the more E is lost
- Wide wires \rightarrow lower resistance
- Wire properties depend on materials choice.

ρ = resistivity
(property of materials)

$\frac{L}{A}$ ∴ geometric parameters
(property of the wire)

Resistive Touch Screen

Problem: to find the location of touch.



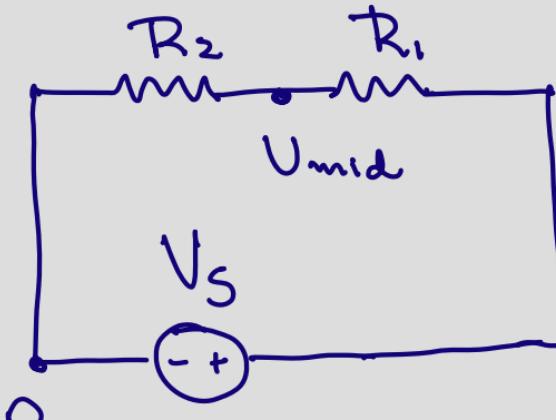
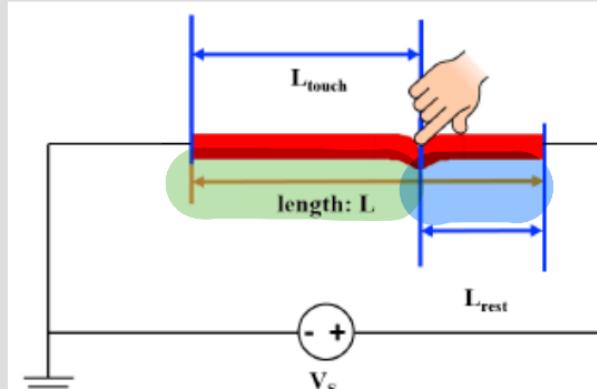
Go from **mechanical** to
electrical quantity.

Want to measure $\frac{h_{\text{touch}}}{L}$

h_{touch} is unknown

Resistive Touch Screen – First model

$V_{mid} = ?$



$$V_{mid} = \frac{R_2}{R_2 + R_1} \cdot V_s \quad (\text{Voltage Divider})$$



$$R_1 = \rho \cdot \frac{h_{rest}}{A}$$

$$R_2 = \rho \cdot \frac{h_{touch}}{A}$$

$$V_{mid} = \frac{\rho \cdot h_{touch}/A}{\rho \cdot h_{touch}/A + \rho \cdot h_{rest}/A} \cdot V_s$$

$$V_{mid} = \frac{h_{touch}}{L_{touch} + L_{rest}} \cdot V_s = \frac{h_{touch}}{L} \cdot V_s$$