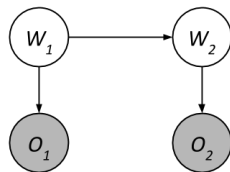


1 HMMs

Consider the following Hidden Markov Model.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1} W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

Suppose that we observe $O_1 = a$ and $O_2 = b$.

Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = a, O_2 = b)$ one step at a time.

(a) Compute $P(W_1, O_1 = a)$.

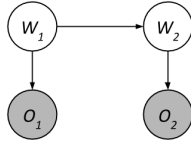
(b) Using the previous calculation, compute $P(W_2, O_1 = a)$.

(c) Using the previous calculation, compute $P(W_2, O_1 = a, O_2 = b)$.

(d) Finally, compute $P(W_2|O_1 = a, O_2 = b)$.

2 Particle Filtering

Let's use Particle Filtering to estimate the distribution of $P(W_2|O_1 = a, O_2 = b)$. Here's the HMM again.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1} W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

We start with two particles representing our distribution for W_1 .

$P_1 : W_1 = 0$

$P_2 : W_1 = 1$

Use the following random numbers to run particle filtering:

[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]

- (a) **Observe:** Compute the weight of the two particles after evidence $O_1 = a$.
- (b) **Resample:** Using the random numbers, resample P_1 and P_2 based on the weights.
- (c) **Predict:** Sample P_1 and P_2 from applying the time update.
- (d) **Update:** Compute the weight of the two particles after evidence $O_2 = b$.
- (e) **Resample:** Using the random numbers, resample P_1 and P_2 based on the weights.
- (f) What is our estimated distribution for $P(W_2|O_1 = a, O_2 = b)$?