

Logistics

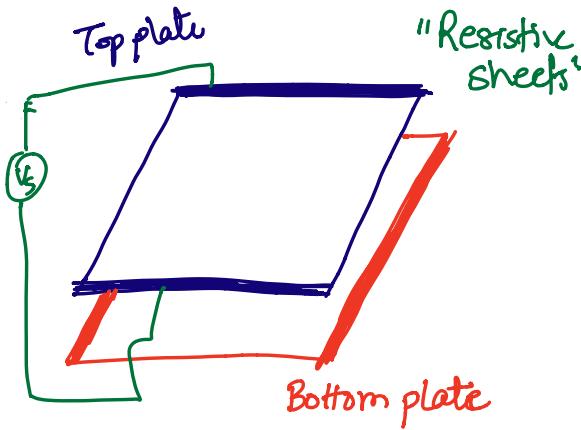
- MT scores released.
- OH today : 1-1 conversations
- Advising today 4pm.

Today:

- What does it mean to "make a model"?
- 2D Touchscreen recap.
- Equivalence: Resistance in series and parallel
- Superposition: Power of Module 1 in Module 2.
Linearity of Circuits.

- Page Rank
- Power of modeling

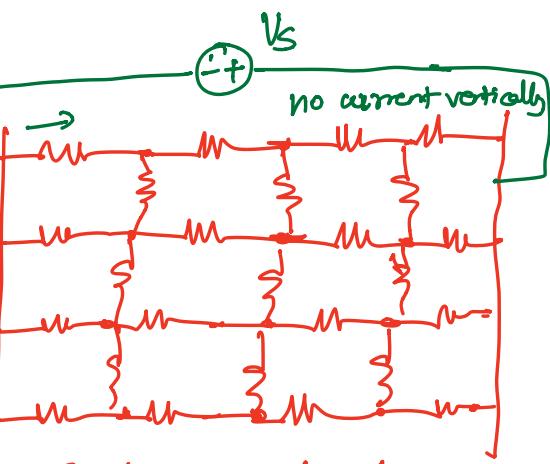
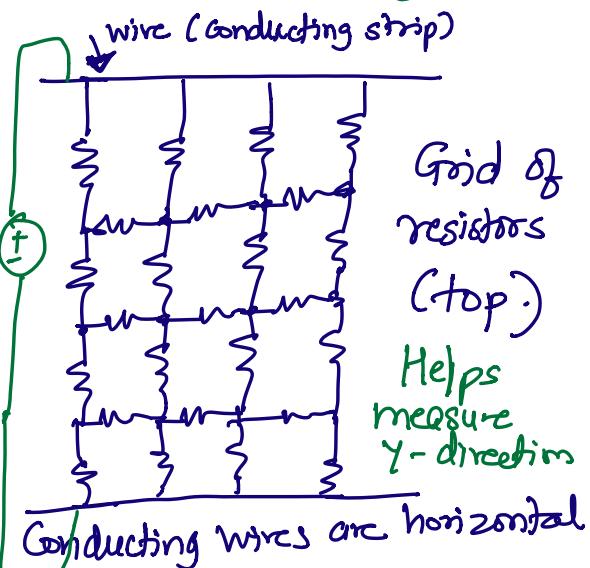
Physical Reality:
Two resistive sheets.



Dark shaded areas
/ OR /
are Conducting strips
like wires.

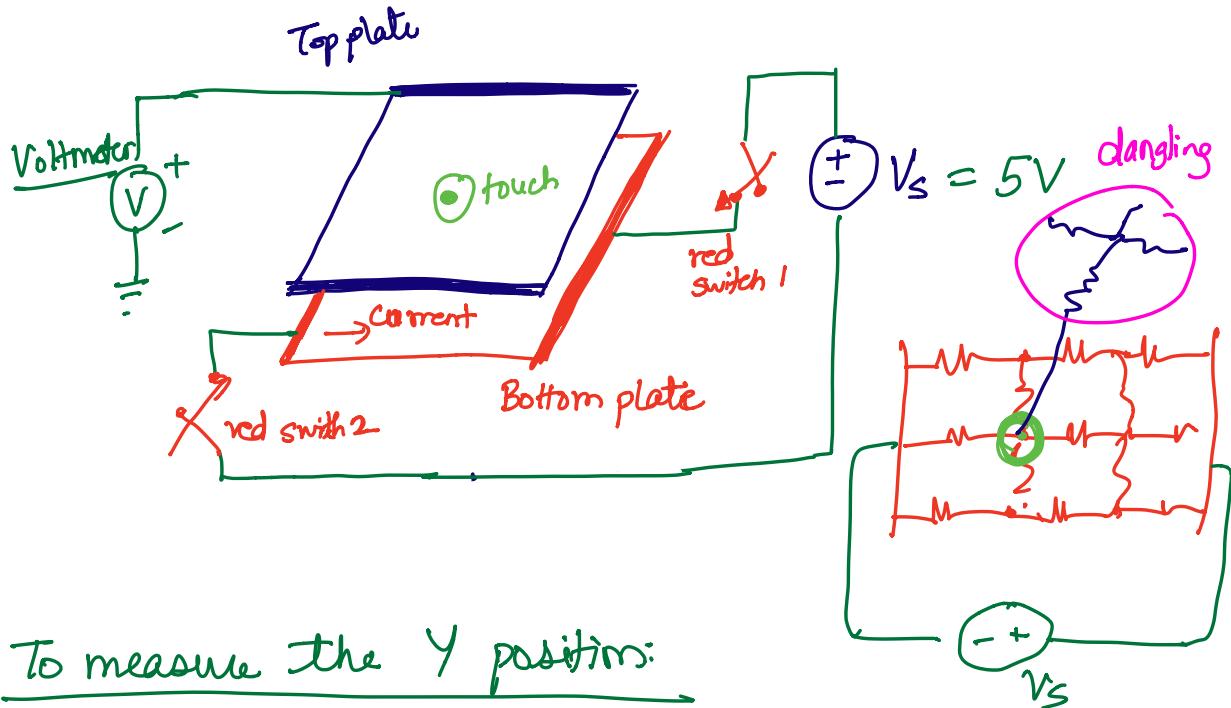
The grid of resistors is a MODEL for the physical reality of the continuous plate.

Model for reality.

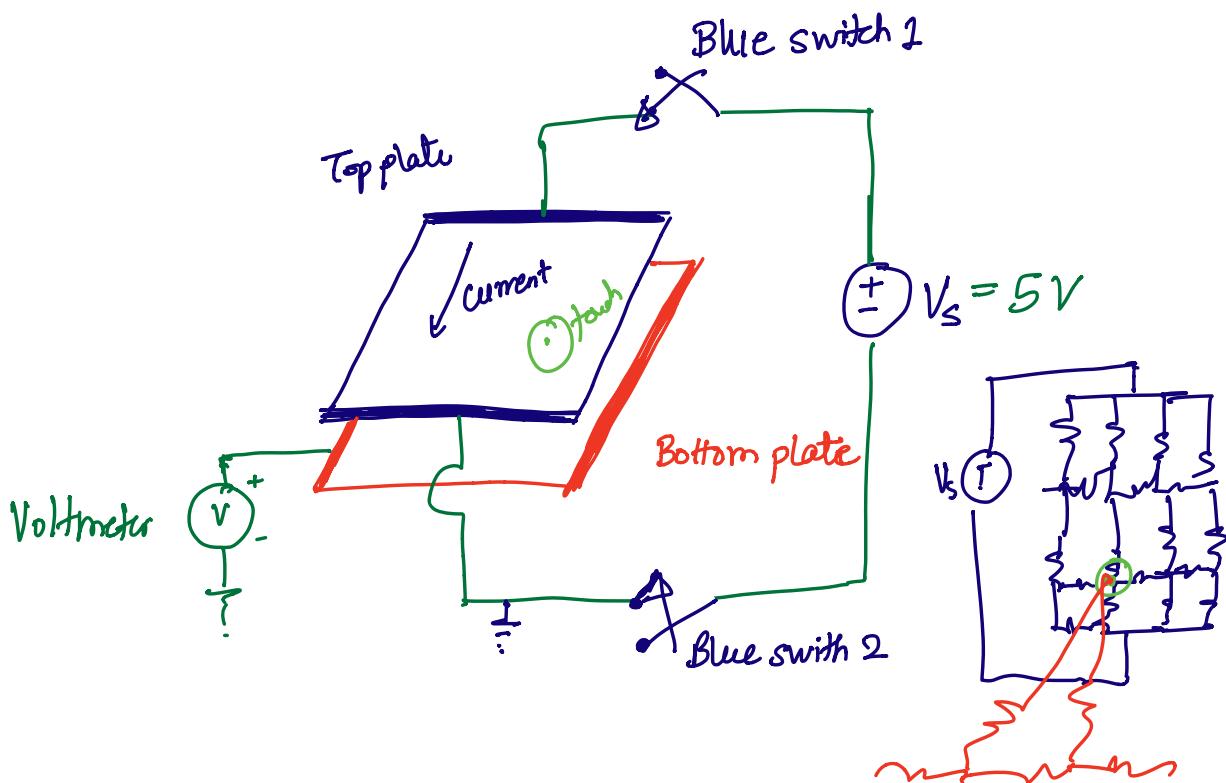


Helps measure the x direction.

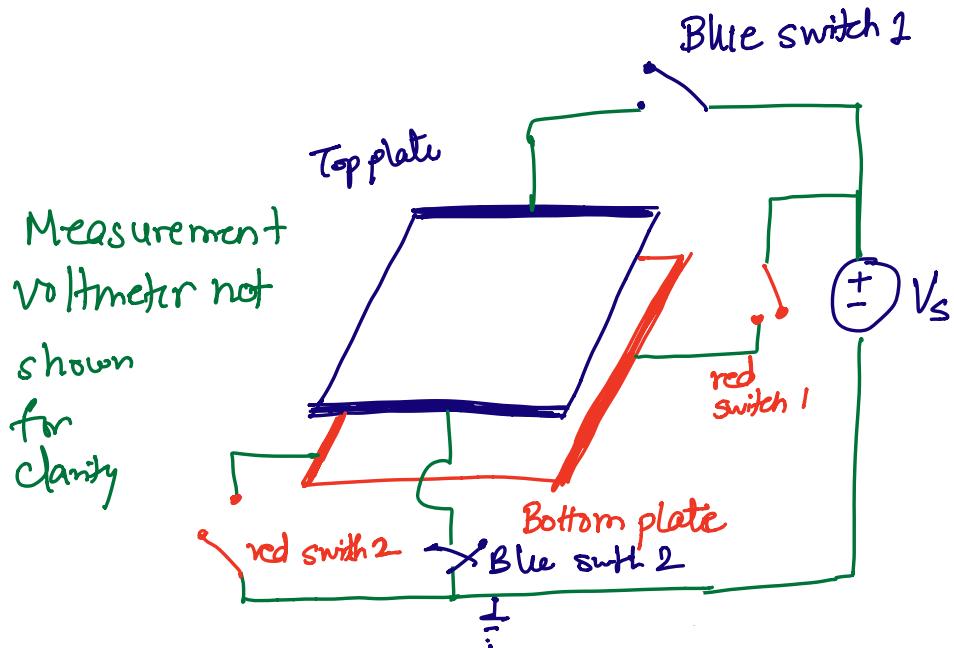
To measure the X position (horizontal position)



To measure the Y position:



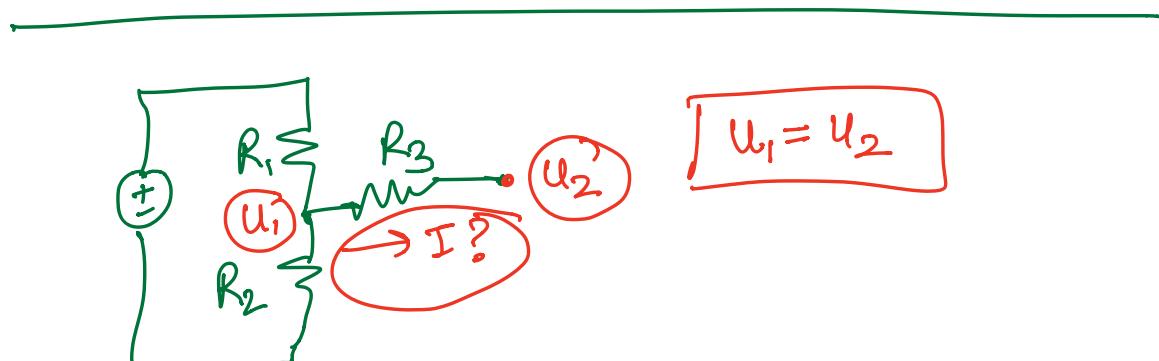
Putting it together:



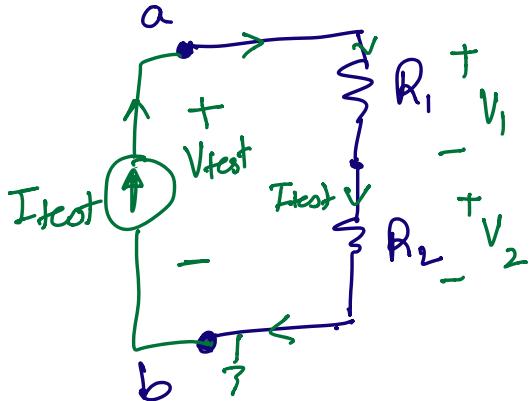
Dark shaded area: Conducting strip
— or —

Turn both blue switches on, red OFF

red OH blue OFF



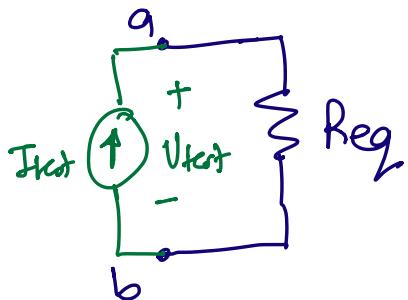
Equivalence : $I - V$ current and voltage at nodes a, b to be the same for both circuits.



$$V_1 = I_{\text{test}} \cdot R_1$$

$$V_2 = I_{\text{test}} \cdot R_2$$

$$\begin{aligned} V_{\text{test}} &= V_1 + V_2 \quad (\text{KVL}) \\ &= I_{\text{test}} \cdot R_1 + I_{\text{test}} \cdot R_2 \\ &= I_{\text{test}} \cdot (R_1 + R_2) \end{aligned}$$



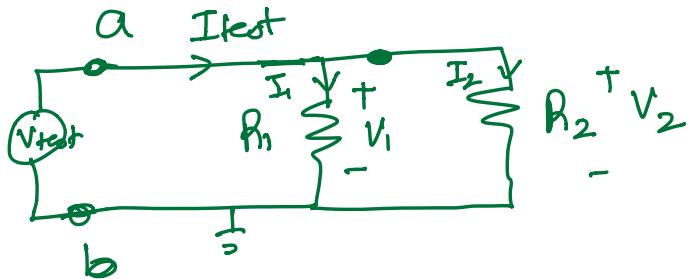
$$V_{\text{test}} = I_{\text{test}} \cdot (R_{\text{eq}})$$

$$\Rightarrow R_{\text{eq}} = R_1 + R_2$$

"Equivalence of circuits"

Resistors in series.

Equivalence 2 : Resistance in parallel.



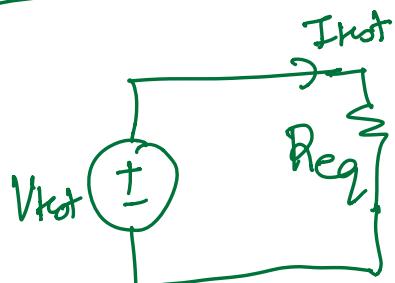
$$I_{\text{tot}} = I_1 + I_2.$$

$$V_1 = I_1 R_1, \quad , \quad V_2 = I_2 R_2$$

$$V_1 = V_2 = V_{\text{tot}}$$

$$V_{\text{tot}} = I_1 R_1, \quad , \quad V_{\text{tot}} = I_2 R_2$$

$$I_{\text{tot}} = I_1 + I_2 = \frac{V_{\text{tot}}}{R_1} + \frac{V_{\text{tot}}}{R_2} = V_{\text{tot}} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



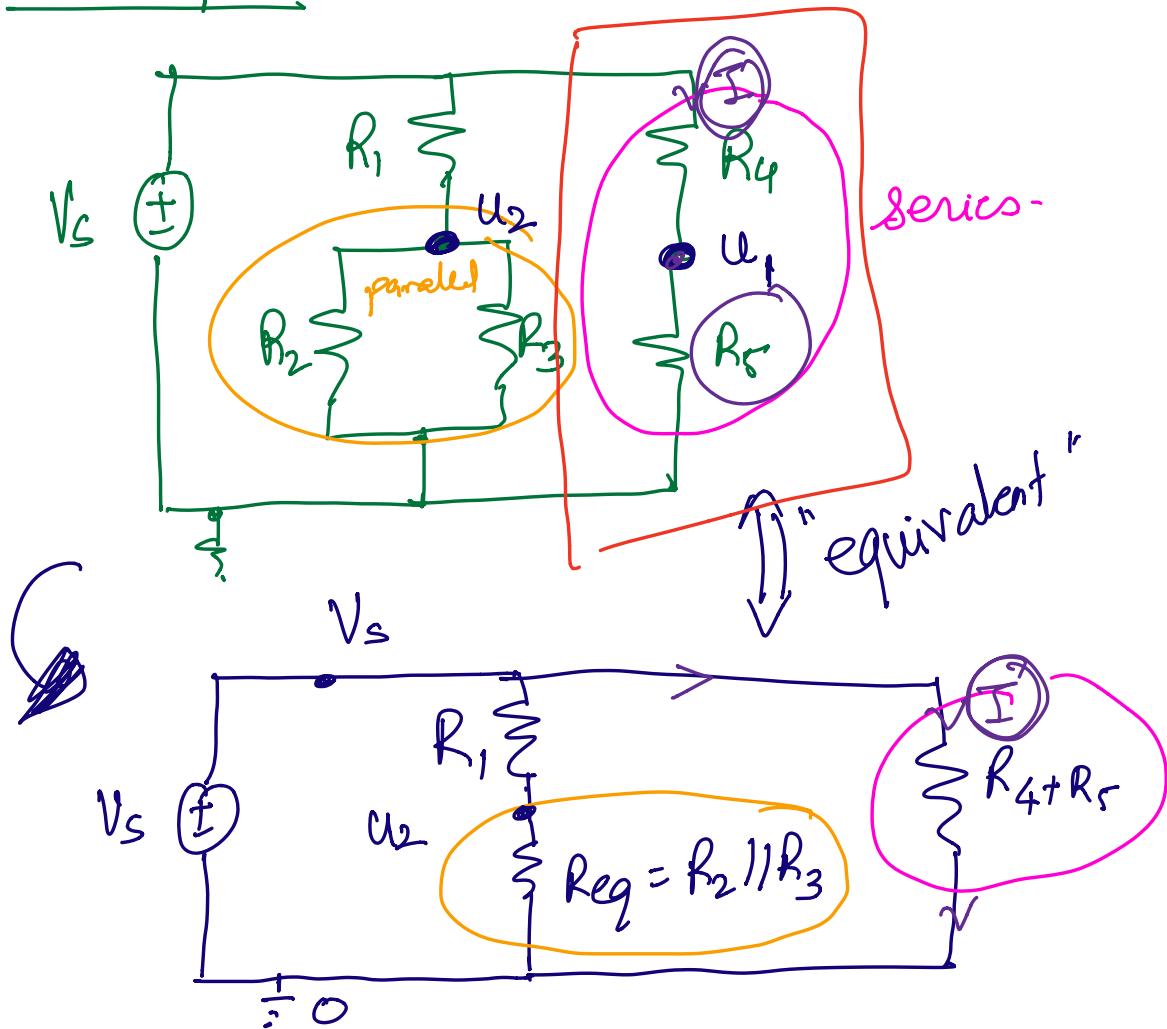
$$\bullet V_{\text{tot}} = I_{\text{tot}} \cdot R_{\text{eq}}$$

$$= I_{\text{tot}} = \frac{V_{\text{tot}}}{R_{\text{eq}}}$$

$$\Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{Resistance in parallel}$$

Notation: $R_{eq} = R_1 \parallel R_2$

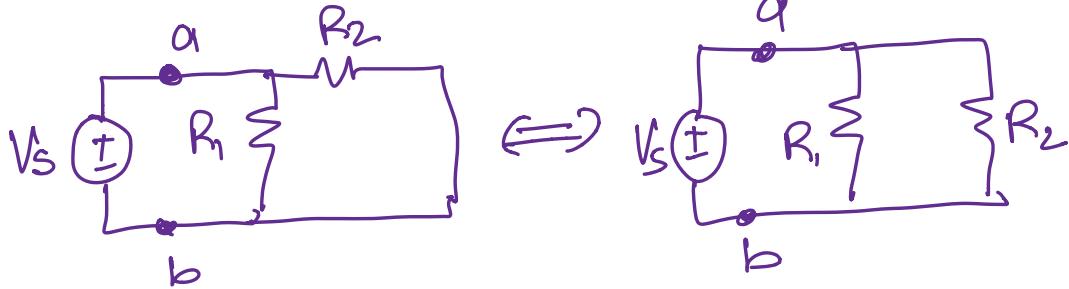
Example:



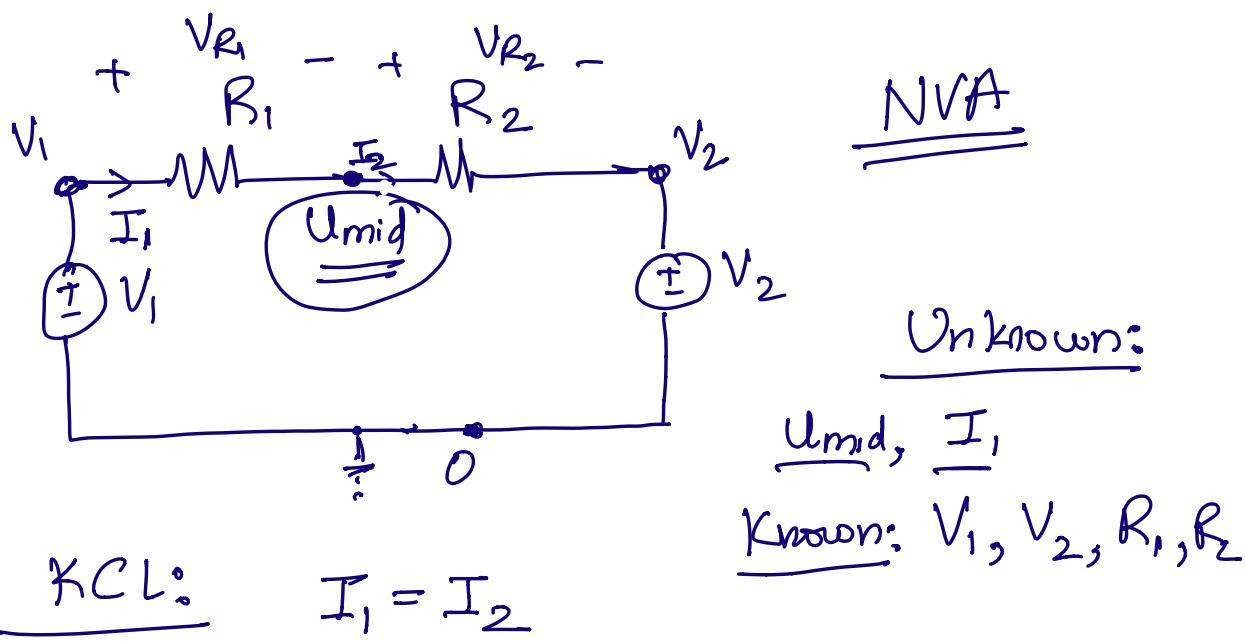
What happened to U_1 ?

$$U_2 = \frac{R_2 \parallel R_3}{R_1 + (R_2 \parallel R_3)} \cdot V_s$$

$$V_S = I(R_4 + R_5)$$



Superposition : Power of the linearity of circuits



Element eqns: $V_{R_1} = I_1 R_1$, $V_{R_2} = I_2 R_2$

Substitute:

$$V_{R_1} = V_1 - U_{mid} = I_1 R_1 \quad (1)$$

$$V_{R_2} = U_{mid} - V_2 = I_2 R_2 = I_1 R_1 \quad (2)$$

(1) \Rightarrow

$$U_{mid} + I_1 R_1 = V_1$$

$$I_1 = \frac{V_1 - V_2}{R_1 + R_2}$$

(2) $U_{mid} - I_2 R_2 = V_2$

$$U_{mid} = \frac{R_1 V_2 + R_2 V_1}{R_1 + R_2}$$

$$\vec{x} = \begin{bmatrix} U_{mid} \\ I_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix} \begin{bmatrix} U_{mid} \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\underbrace{\quad}_{A}$

$$\vec{x} = A^{-1} \vec{b}$$