

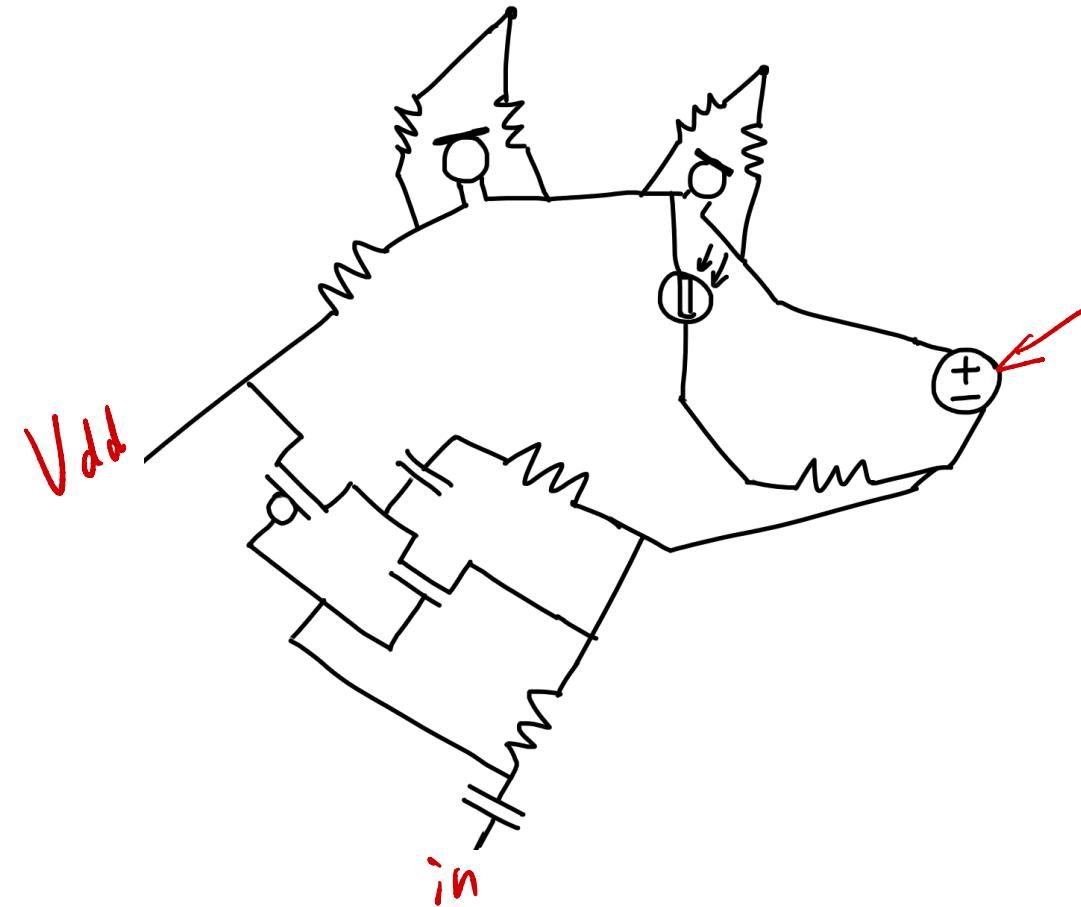
DOGGO 151/251A SP2022 Discussion 8

GSI: DIMA NIKIFOROV, YIKUAN CHEN



Agenda

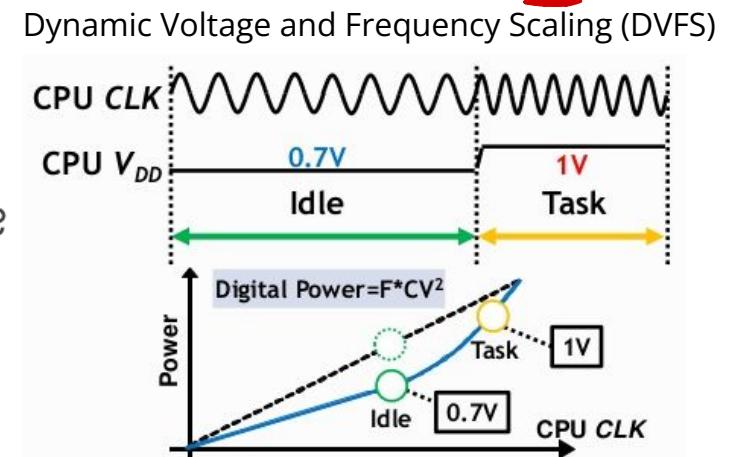
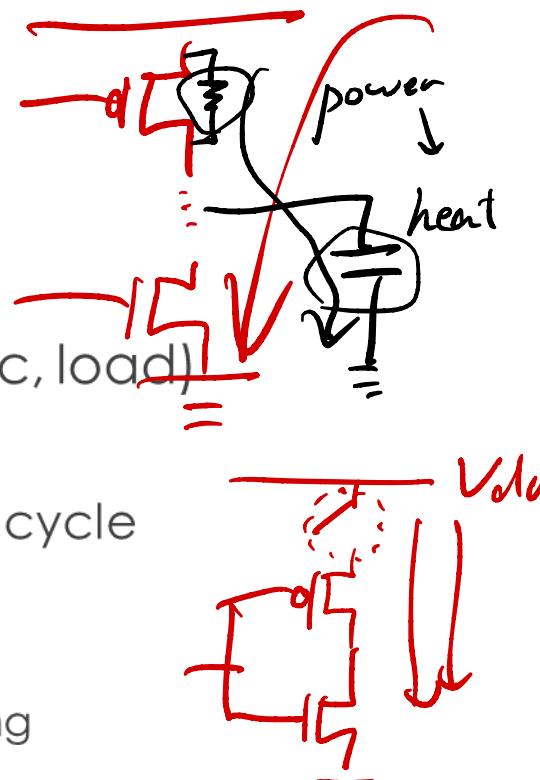
- Power/Energy
- Adders



Energy

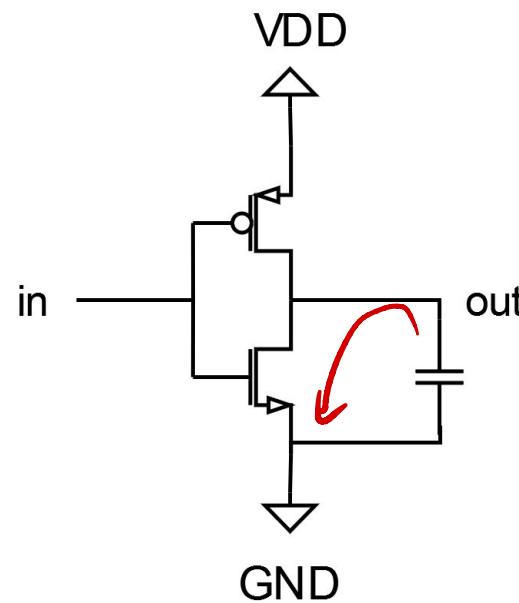
Power/Energy in Digital Circuits

- Fundamentally, charging/discharging capacitors (gate, parasitic, load) through resistances (PMOS, NMOS, wires)
 - Capacitors draw CV^2 joules from supply over 1 charge/discharge cycle
 - $\frac{1}{2}CV^2$ dissipated in PMOS as heat when charging
 - $\frac{1}{2}CV^2$ stored on capacitor, then dissipated in NMOS when discharging
- Dynamic power = $P_{\text{switching}} = \alpha CV^2 f$
 - How to minimize each term? $\uparrow \text{freq} \rightarrow V \downarrow$ slower clk
 - Minimizing which terms reduces total energy consumed?
- Static power = leakage \rightarrow wasted energy!



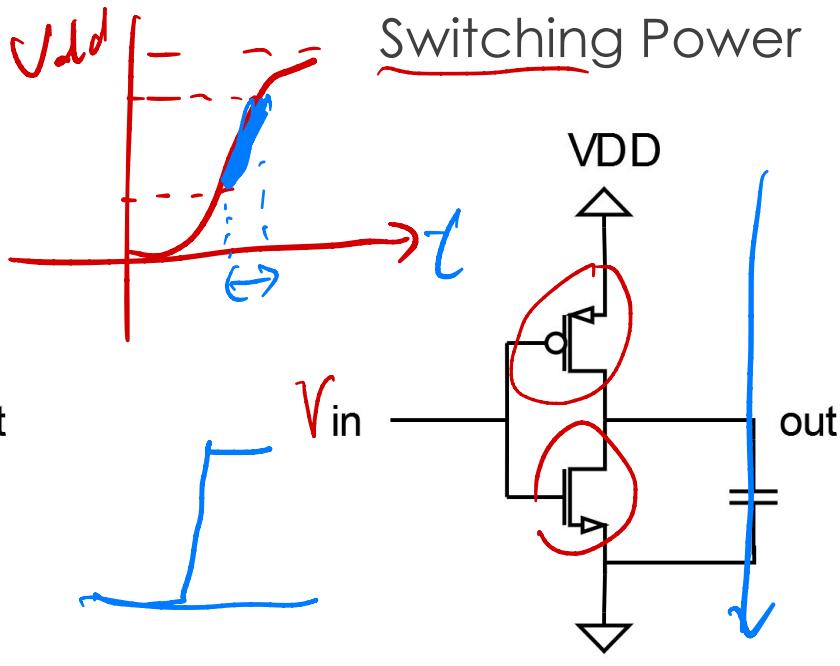
Power/Energy in Digital Circuits Causes

Dynamic Power



$$\frac{1}{2} C V^2$$

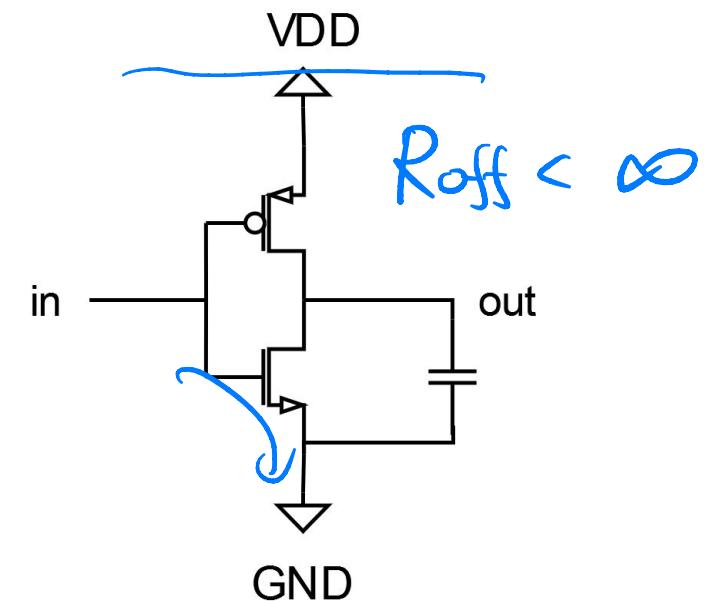
Switching Power



$$V_{dd} - V_{thp} > V > V_{th}$$

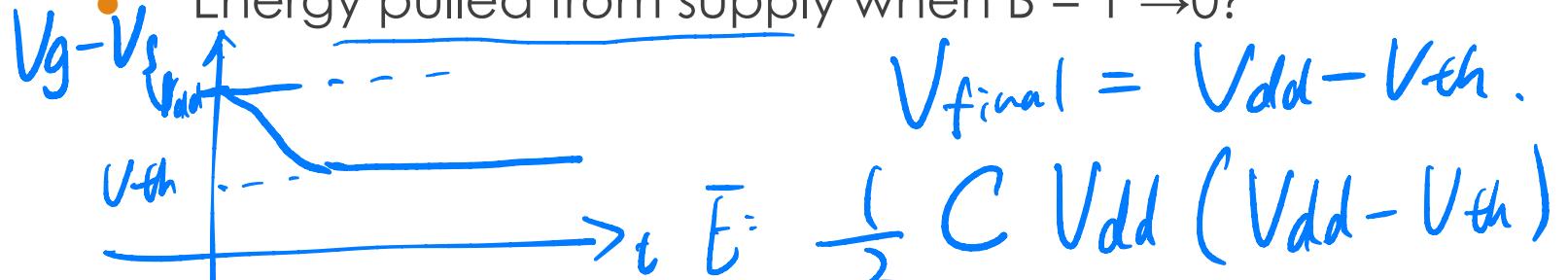
$$= \frac{IV}{R} = \frac{V^2}{R}$$

Leakage Power



Energy Example

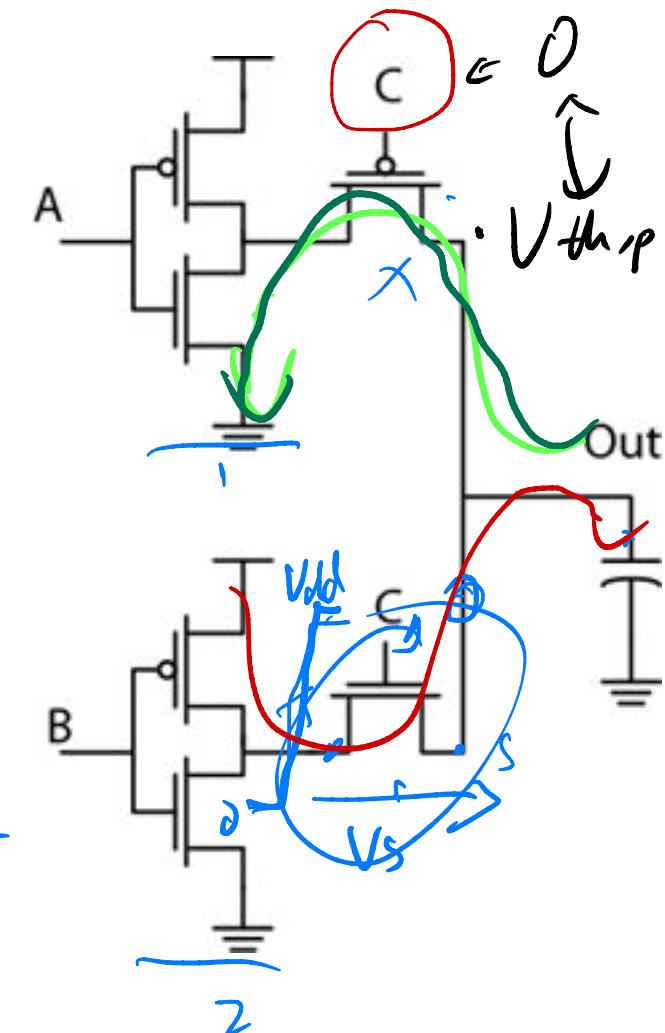
- Initially: $A = 1, C = 1, \text{Out} = 0$
- Energy pulled from supply when $B = 1 \rightarrow 0$?



- Then, how much energy dissipated when $C = 1 \rightarrow 0$?

$$V_{cap} : V_{dd} - V_{th,n} \rightarrow \underline{\underline{V_{th,p}}}$$

$$\bar{E} : \frac{1}{2} C (V_{dd} - V_{th,n})^2 - \frac{1}{2} C V_{th,p}^2$$

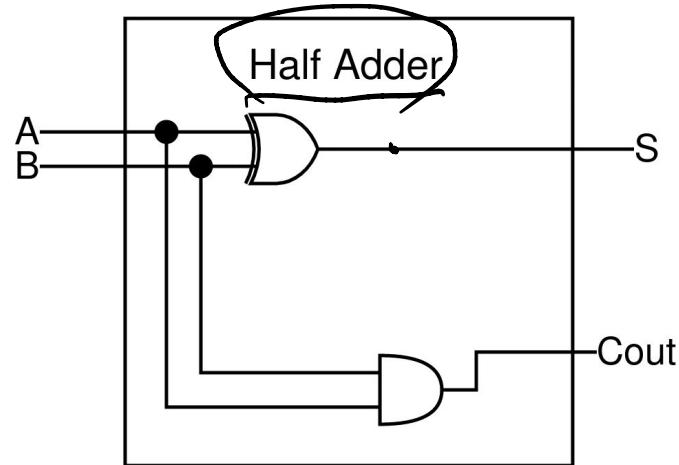


Adders

$$2 \overline{\downarrow} + 3 \overline{\uparrow} = \overline{\uparrow} 5$$

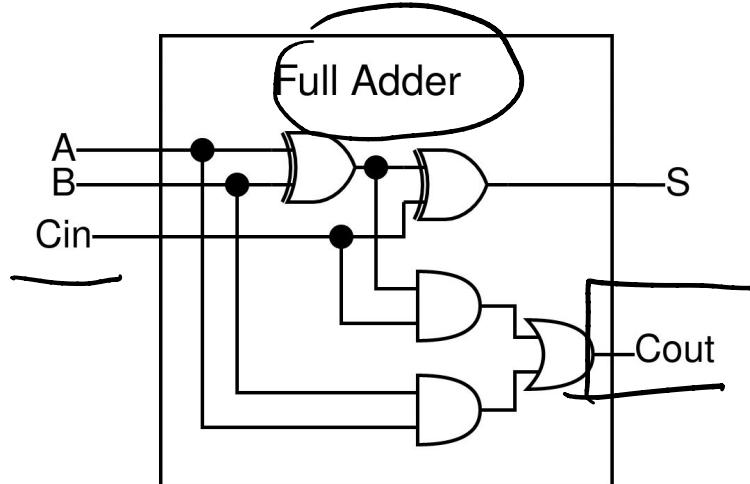
Adder Components

$$2 + 3 = 5$$



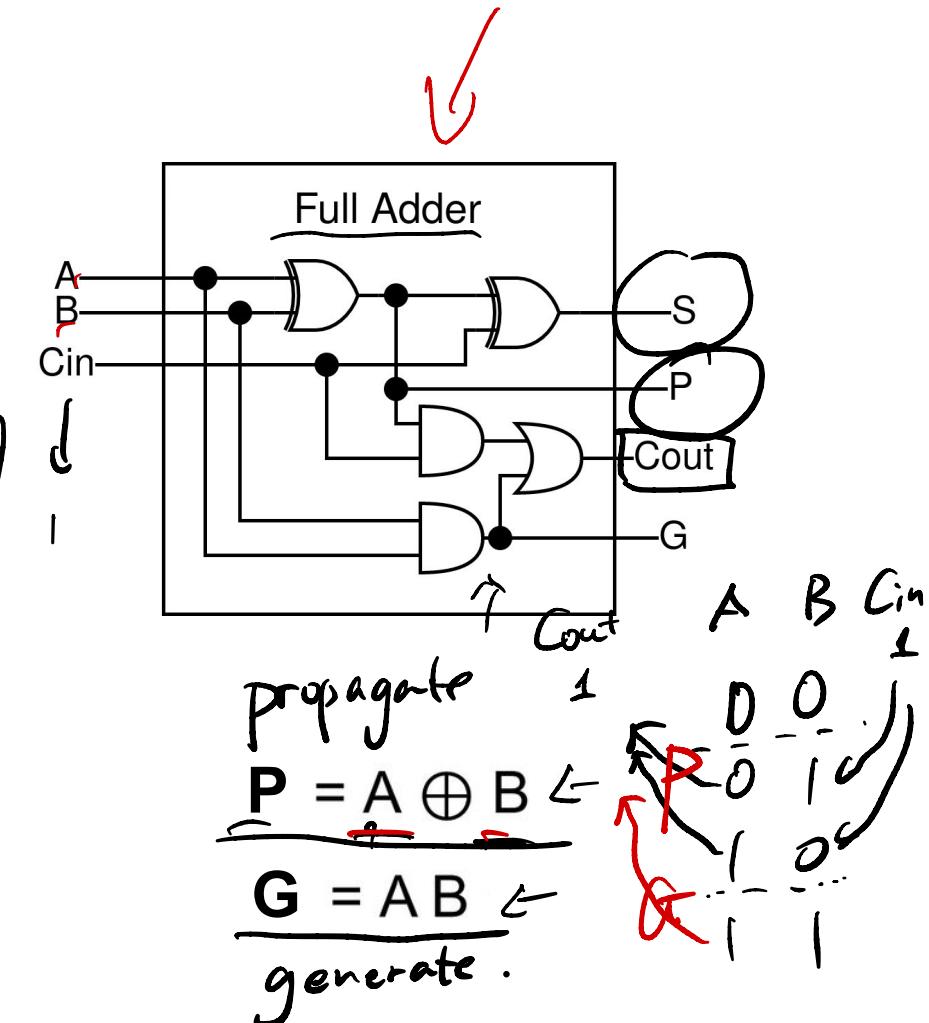
$$S = A \oplus B$$

$$C_o = AB$$

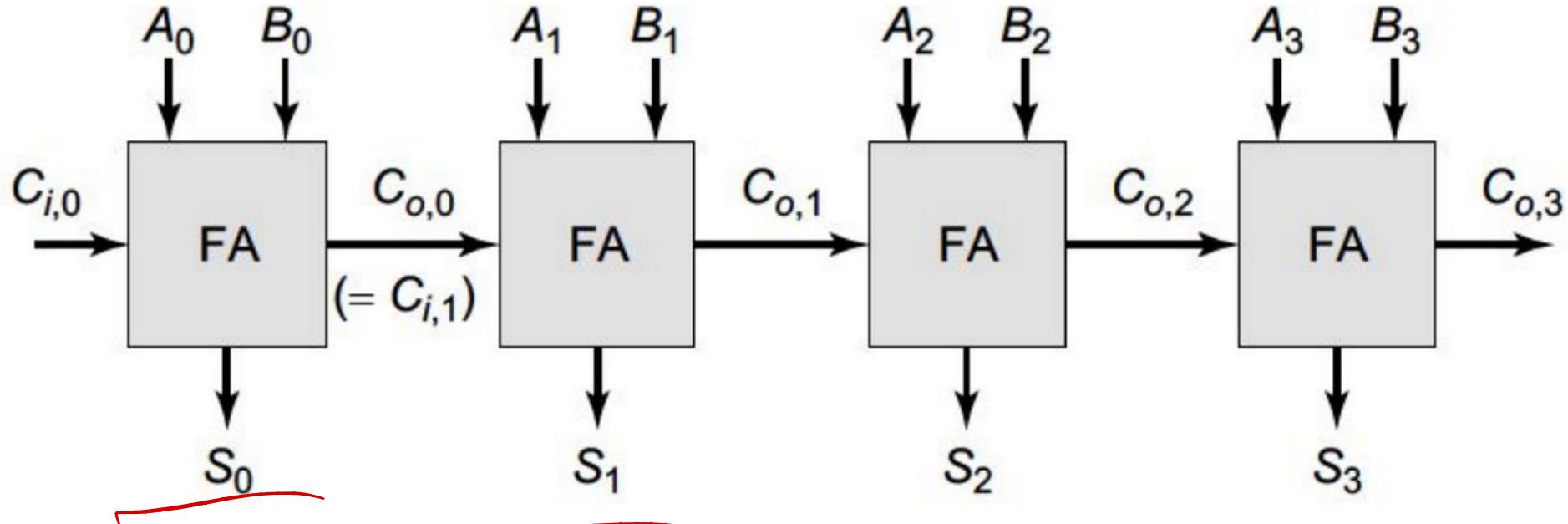


$$\underline{S = A \oplus B \oplus Ci}$$

$$C_o = AB + BCi + ACi$$

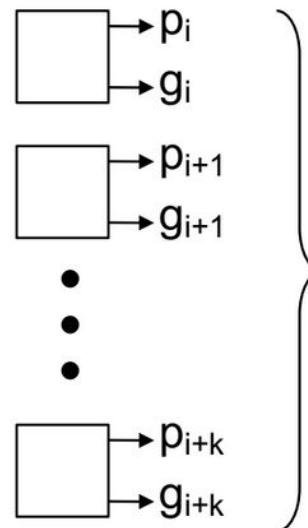


Ripple-Carry Adder



- Time Complexity? $O(N)$
- Area Complexity? $O(N)$

Carry-Lookahead Adder



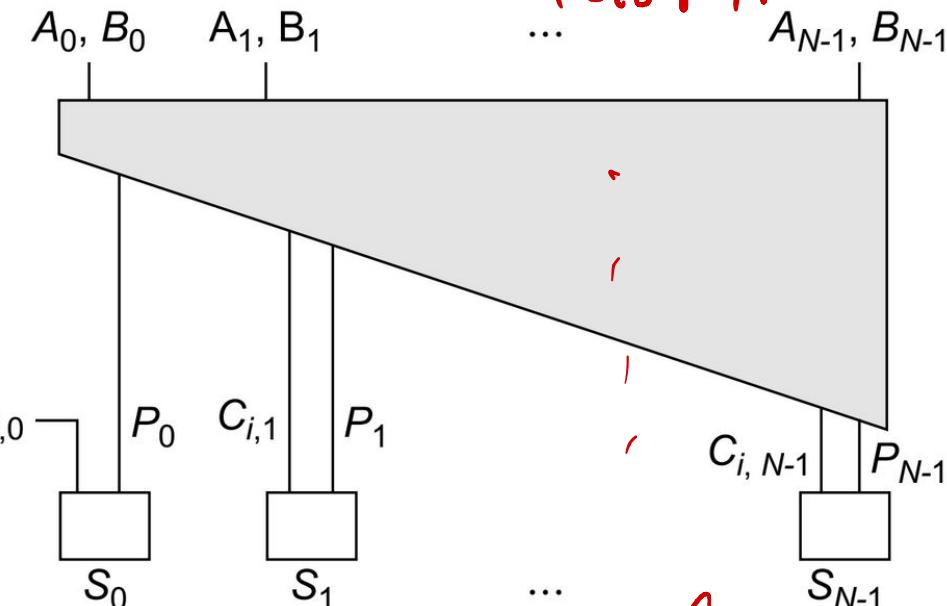
$$P = p_i p_{i+1} \dots p_{i+k}$$

$$G = g_{i+k} + p_{i+k}g_{i+k-1} + \dots + (p_{i+1}p_{i+2} \dots p_{i+k})g_i$$

\downarrow

in

D_7-D_1 10 bit adder



$$C_{o,k} = f(A_k, B_k, C_{o,k-1}) = G_k + P_k C_{o,k-1}$$

- ## • Time Complexity?

$\mathcal{O}(1) \subseteq N$ is big, 00
1. 0000

- ## • Area Complexity?

$O(N^2)$

N is big
not possible.
 $O(\log N)$

$$C_{o,k} = \alpha N^2$$

$$C_{10} = \frac{G_{10}}{G_8 P_9 P_{10}} + \cancel{\frac{G_9 P_{10}}{G_8 P_9 P_{10}}} - \dots$$

$$+ G_1 P_2 P_3 \dots P_{10}$$

$\#C_{in} P_0 P_1 P_2 \dots P_n$

Carry-Lookahead Tree Adder

- Time Complexity?

$$\mathcal{O}(\log_2 N)$$

- Area Complexity?

$$4 + 2 + 1$$

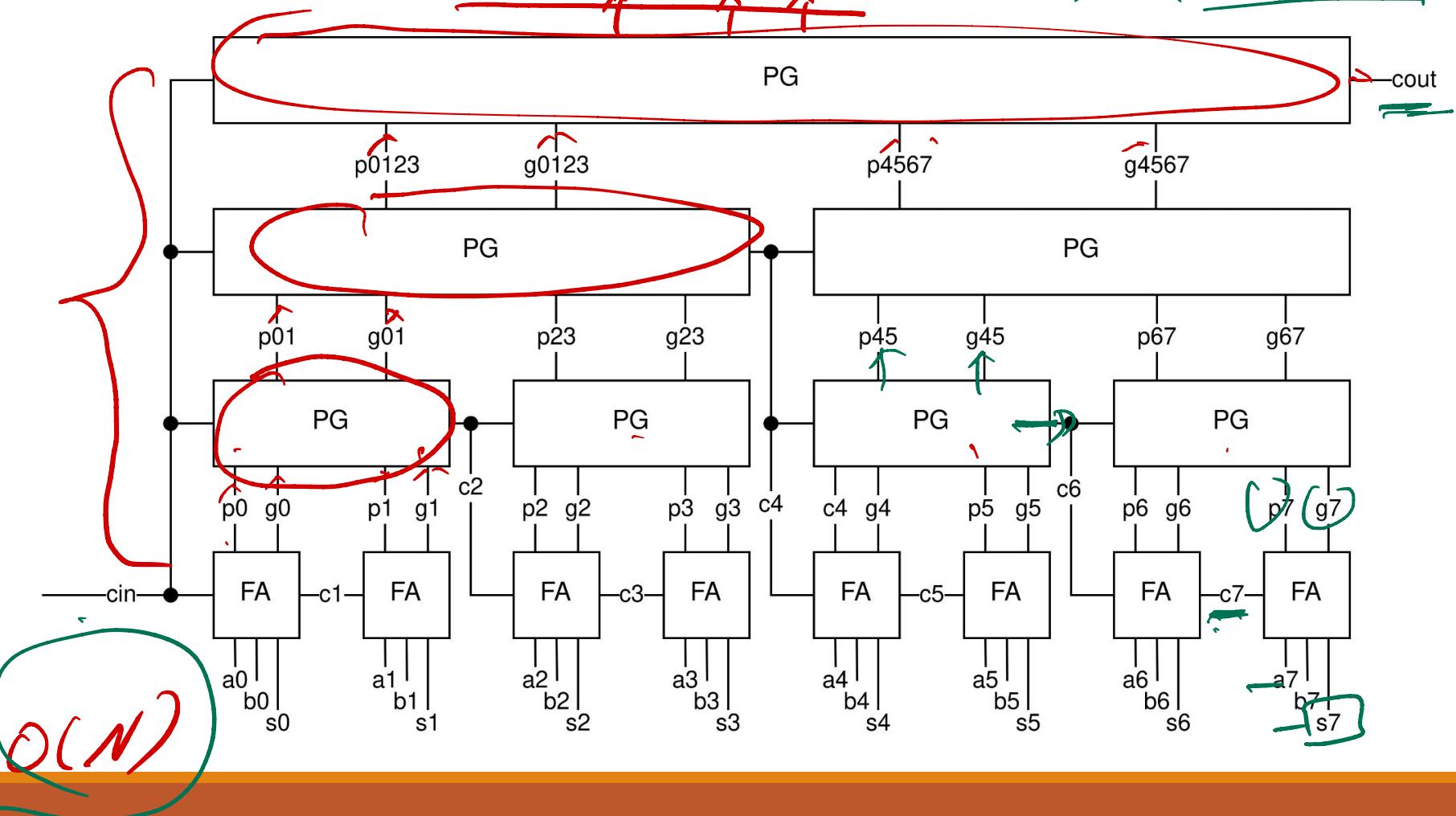
$$= 2^3 - 1$$

$$= \frac{2^{\log_2 8}}{8} - 1 \Rightarrow \mathcal{O}(N)$$

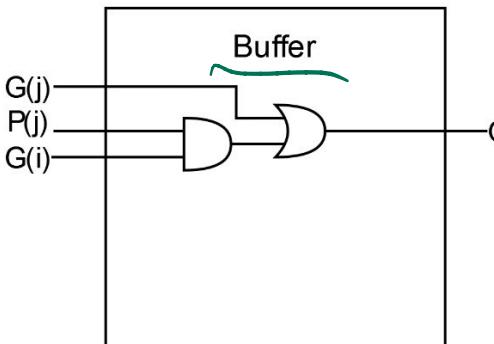
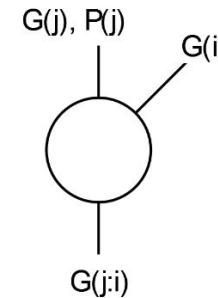
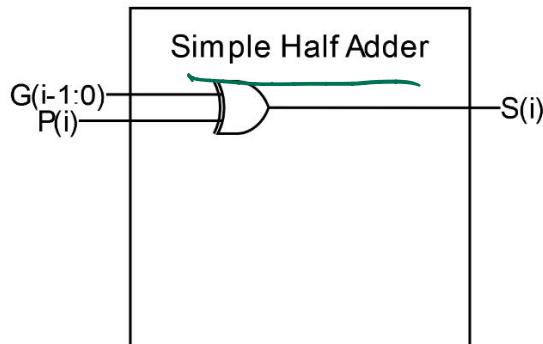
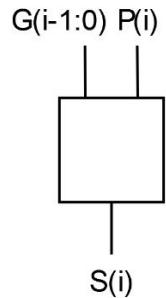
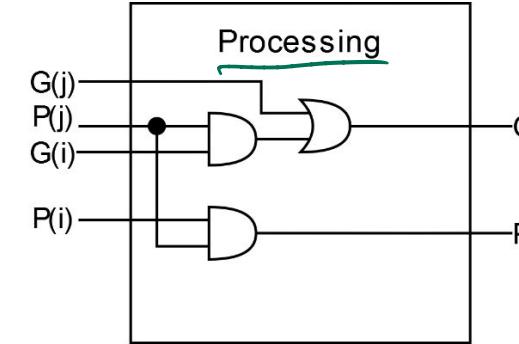
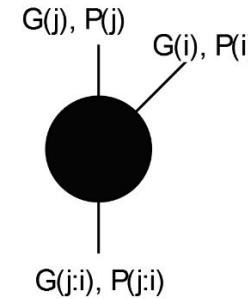
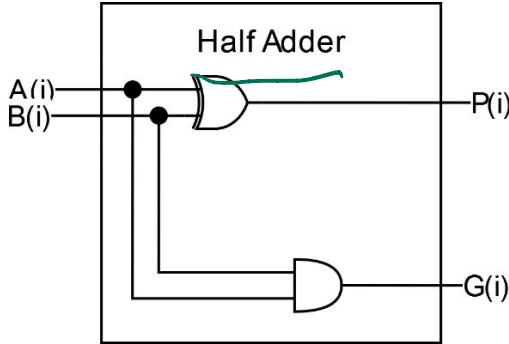
$P_{1:0} = \overline{P_1 \cdot P_0}$; Group Gen; Group Prop.

$$C_{out,b} = G_{b:0} + \underline{P_{b:0} \cdot C_{0,in}}$$

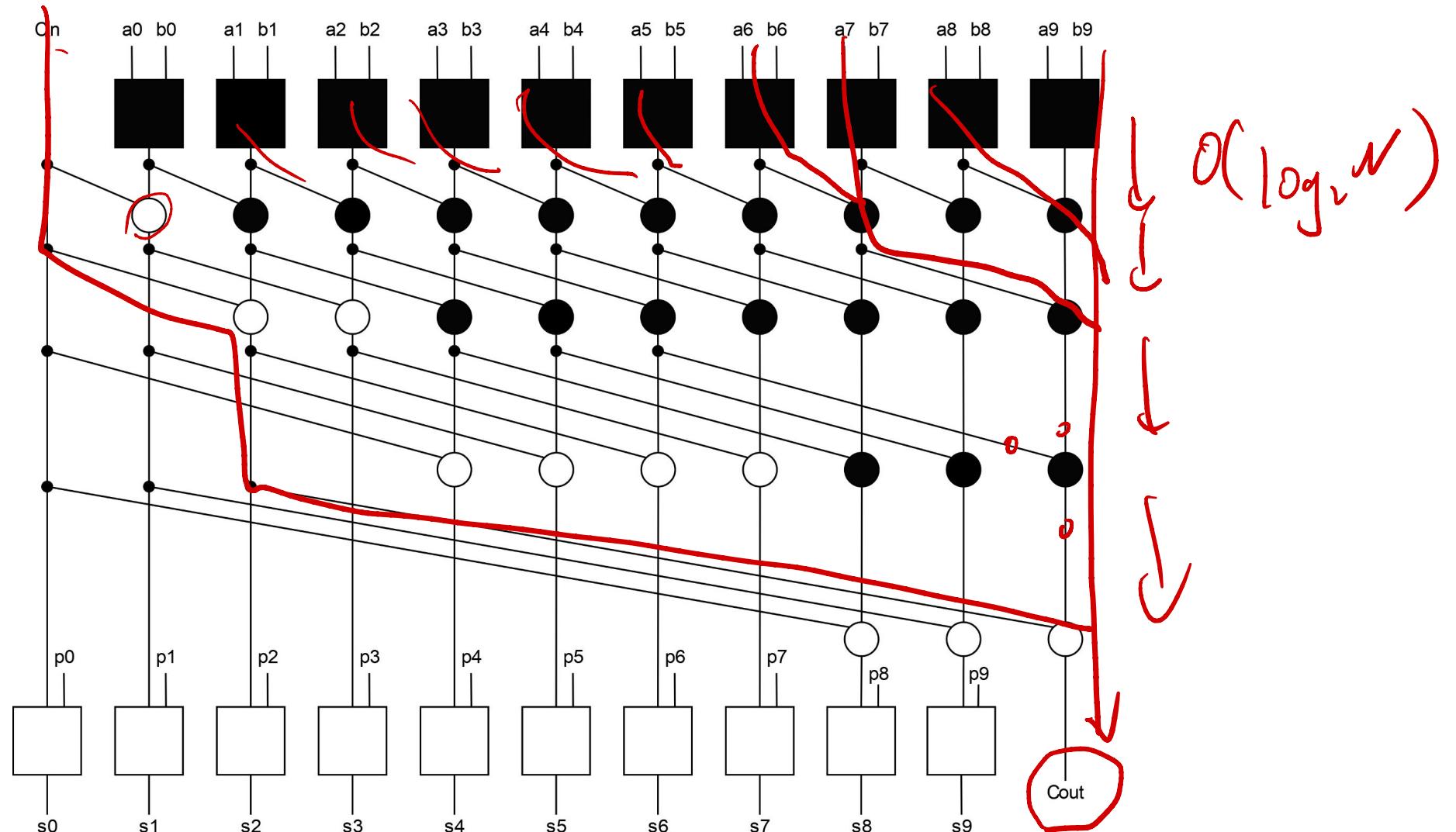
$$P_{1:0} = P_1 \cdot P_0, G_{1:0} = G_1 + P_1 \cdot G_0, \rightarrow C_{out1} = \underline{G_{1:0}} + \underline{P_{1:0} \cdot C_{0,in}}$$



Kogge-Stone Tree Adder (Components)



Kogge-Stone Tree Adder



Doggo Adder



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Questions?