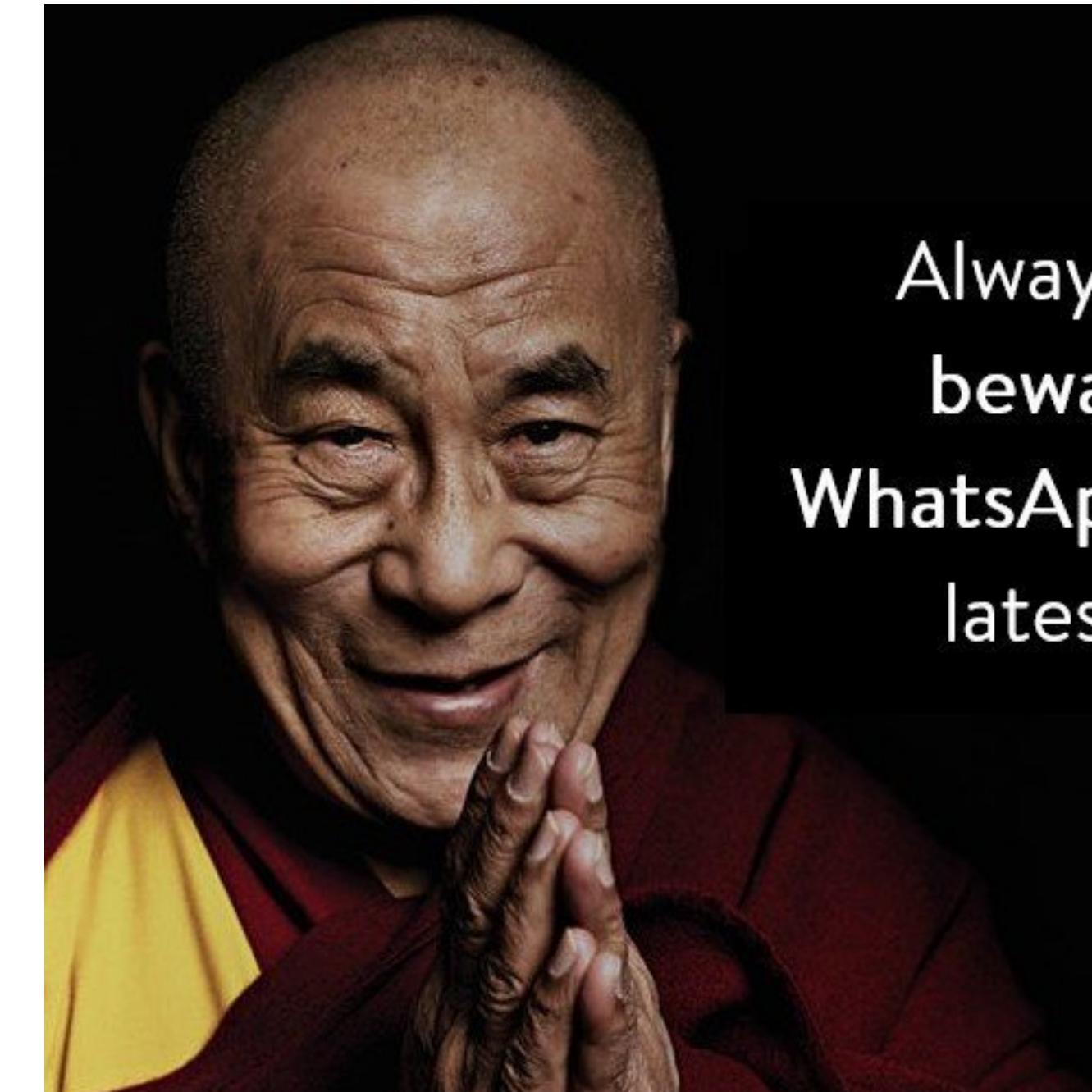


Crypto 4: Public Key



Always be true to yourself,
beware fake personas in
WhatsApp, and always download
latest software updates.

- The Dalai Lama

Administrivia!

- Project 1 due Friday at 11:59 PM Pacific
 - Reminder, you have slip days if you need them
- Homework 2 due Monday at 11:59 PM Pacific
 - Reminder, slip days do not apply
- **Reminder:**
 - Zoom chat for conversation
 - Zoom Q&A for Questions & Answers

Twitter Fight: Nick Vs Rust Rand_Core Random Number Generators

- Rust (well, the 3rd party library for it) has an interface for "secure" Random Number Generators... But they aren't actually secure!
- EG, "ChaCha8Rng"
 - A *reduced round* stream cipher!
 - That has no update() function: no way of adding in entropy after seeding
 - And seed() takes only 32B total (no combining entropy!)
 - Oh, and no rollback resistance either
- **NONE** of the "Secure" RNGs are actually cryptographically secure...
 - Because none accept and consume arbitrarily long seeds or have an update to mix in more entropy
- When I say ONLY use HMAC_DRBG, I mean it!
 - Use /dev/urandom and everything else you can think of to shove into HMAC_DRBG

And Vuln of the Day: CVE-2019-16303

- If you wrote an app in JHipster last year or before...
 - You probably want a password reset function...
 - Password reset generates "random" URLs
 - But of course, they used a bad RNG!
 - So generate a password request for your account
 - You get the RNGs state in the reset URL
 - Now you can generate more password resets...
 - And predict what the "random" URL is...
and take over any account you want!

Public Key...

- All our previous primitives required a "miracle":
 - We somehow have to have Alice and Bob get a shared k .
- Enter Public Key cryptography: the miracle of modern cryptography
 - How starting Friday, but ***what*** today
- Three primitives:
 - Public Key Agreement
 - Public Key Encryption
 - Public Key Signatures
- Based on some families of magic math...
 - For us, we will use some group-theory based primitives

Public Key Agreement

- Alice and Bob have a channel...
 - There may be an eavesdropper ***but not a manipulator***
 - The goal: Alice & Bob agree on a ***random*** value
 - This will be k for all subsequent communication
 - When done, the key is thrown away
 - Designed to prevent an attacker who later recovers Alice or Bob's long lived secrets from finding k .

Public Key Encryption

- Alice has ***two*** keys:
 - K_{pub} : Her public key, anyone can know
 - K_{priv} : Her private key, a deep dark secret
- Anyone has access to Alice's public key
- For anyone to send a message to Alice:
 - Create a random session key k
 - Used to encrypt the rest of the message
 - Encrypt k using Alice's K_{pub} .
- Only Alice can ***decrypt*** the message
 - The decryption function only works with K_{priv} !

Public Key Signatures

- Once again, Alice has *two* keys:
 - K_{pub} : Her public key, anyone can know
 - K_{priv} : Her private key, a deep dark secret
- She can sign a message
 - Calculate $H(M)$
 - $S(K_{priv}, H(M))$: Sign $H(M)$ with K_{priv} .
- Anyone can now verify
 - Recalculate $H(M)$
 - $V(K_{pub}, S(K_{priv}, H(M)), H(M))$: Verify that the signature was created with K_{priv}

Things To Remember...

- Public key is ***slow!***
 - Orders of magnitude slower than symmetric key
- Public key is based on delicate magic math
 - Discrete log in a group is the most common
 - RSA
 - Some new "post-quantum" magic...
- Some systems in particular are easy to get wrong
 - We will get to some of the epic crypto-fails later

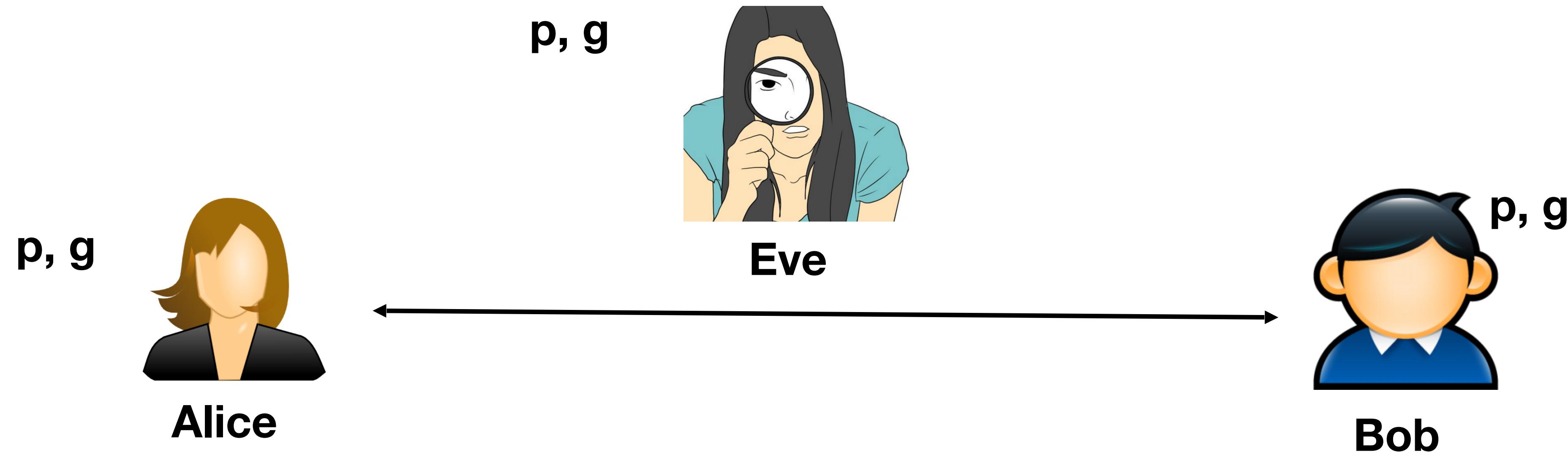
Our Roadmap For Public Key...

- Public Key:
 - Something **everyone** can know
- Private Key:
 - The secret belonging to a specific person
- Diffie/Hellman:
 - Provides key exchange with no pre-shared secret
- ElGamal & RSA:
 - Provide a message to a recipient only knowing the recipient's **public key**
- DSA & RSA signatures:
 - Provide a message that anyone can prove was generated with a **private key**

Diffie-Hellman Key Exchange

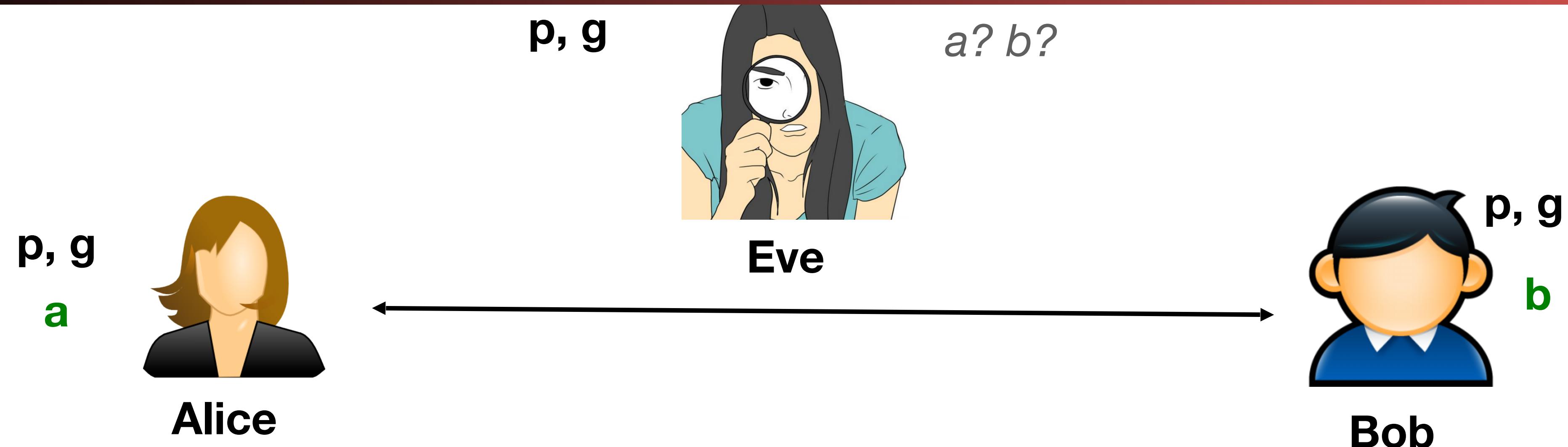
- What if instead they can somehow generate a random key when needed?
- Seems impossible in the presence of Eve observing all of their communication ...
 - How can they exchange a key without her learning it?
 - But: actually is possible using public-key technology
 - Requires that Alice & Bob know that their messages will reach one another without any meddling
 - Protocol: Diffie-Hellman Key Exchange (DHE)
 - The E is "Ephemeral", we use this to create a temporary key for other uses and then forget about it

Diffie-Hellman Key Exchange



1. Everyone agrees in advance on a well-known (large) prime p and a corresponding g : $1 < g < p-1$

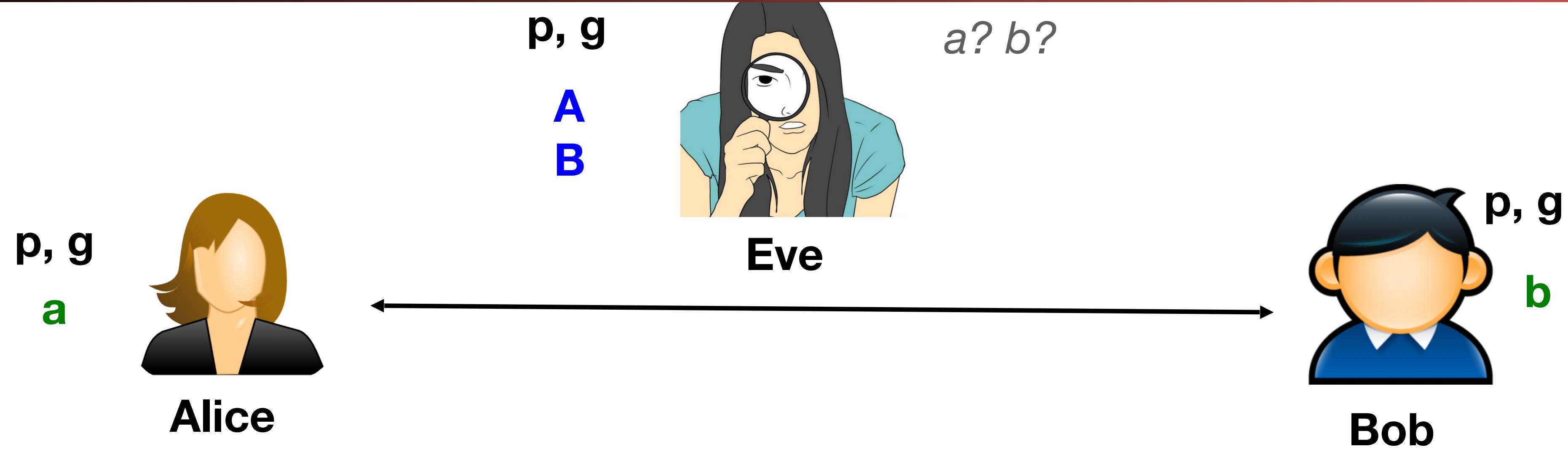
DHE



2. Alice picks random secret 'a': $1 < a < p-1$

3. Bob picks random secret 'b': $1 < b < p-1$

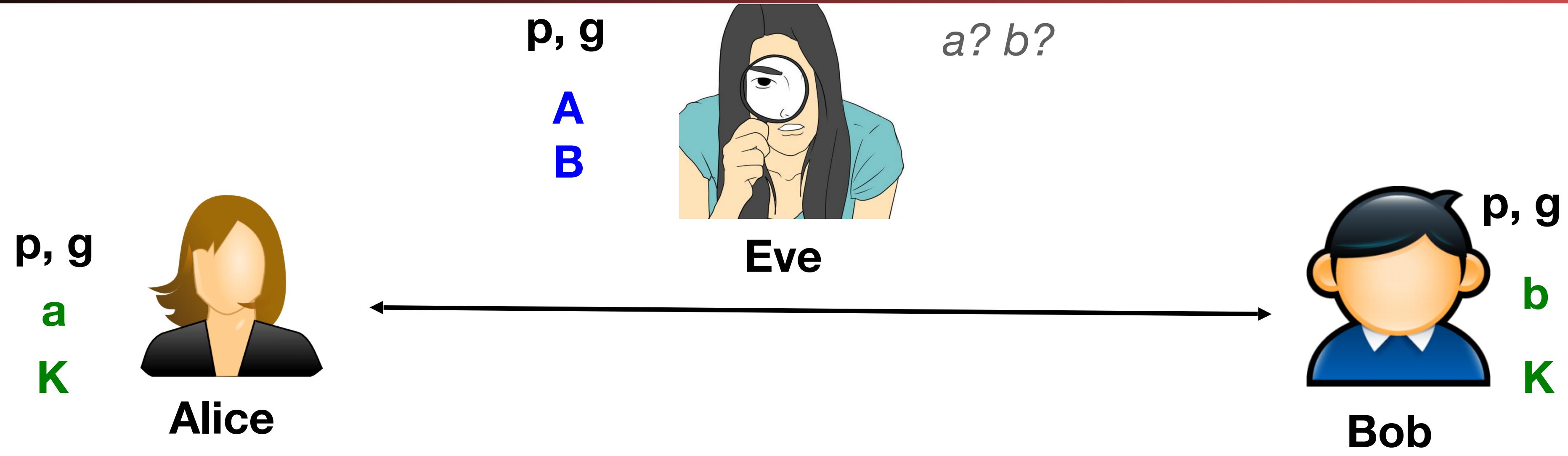
DHE



$$A = g^a \bmod p$$

4. Alice sends $A = g^a \bmod p$ to Bob $g^b \bmod p = B$
5. Bob sends $B = g^b \bmod p$ to Alice

DHE

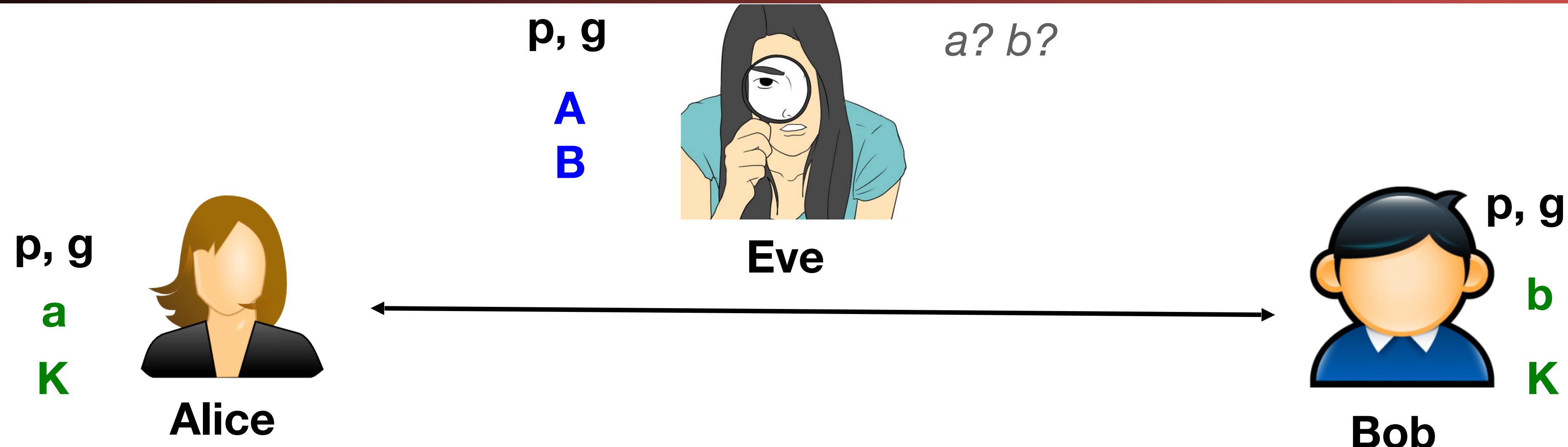


$$\begin{matrix} A = g^a \bmod p \\ B = g^b \bmod p \end{matrix}$$

6. Alice knows $\{a, A, B\}$, computes
 $K = B^a \bmod p = (g^b)^a = g^{ba} \bmod p$
7. Bob knows $\{b, A, B\}$, computes
 $K = A^b \bmod p = (g^a)^b = g^{ab} \bmod p$
8. K is now the shared secret key.

$$g^b \bmod p = B$$

DHE



While Eve knows $\{p, g, g^a \text{ mod } p, g^b \text{ mod } p\}$, believed to be **computationally infeasible** for her to then deduce $K = g^{ab} \text{ mod } p$.

She can easily construct $A \cdot B = g^a \cdot g^b \text{ mod } p = g^{a+b} \text{ mod } p$.
But computing g^{ab} requires ability to take *discrete logarithms* mod p .
Discrete log over the group defined by p and g **presumed** to be hard

This is Ephemeral Diffie/Hellman

- $K = g^{ab} \text{ mod } p$ is used as the basis for a "session key"
 - A symmetric key used to protect subsequent communication between Alice and Bob
 - In general, public key operations are vastly more expensive than symmetric key, so it is mostly used just to agree on secret keys, transmit secret keys, or sign hashes
 - If either a or b is random, K is random
- When Alice and Bob are done, they discard K, a, b
 - This provides ***forward secrecy***: Alice and Bob don't retain any information that a later attacker who can compromise Alice or Bob's secrets could use to decrypt the messages exchanged with K .
 - This is also why it is called "Ephemeral" D/H

Diffie Hellman is part of more generic problem

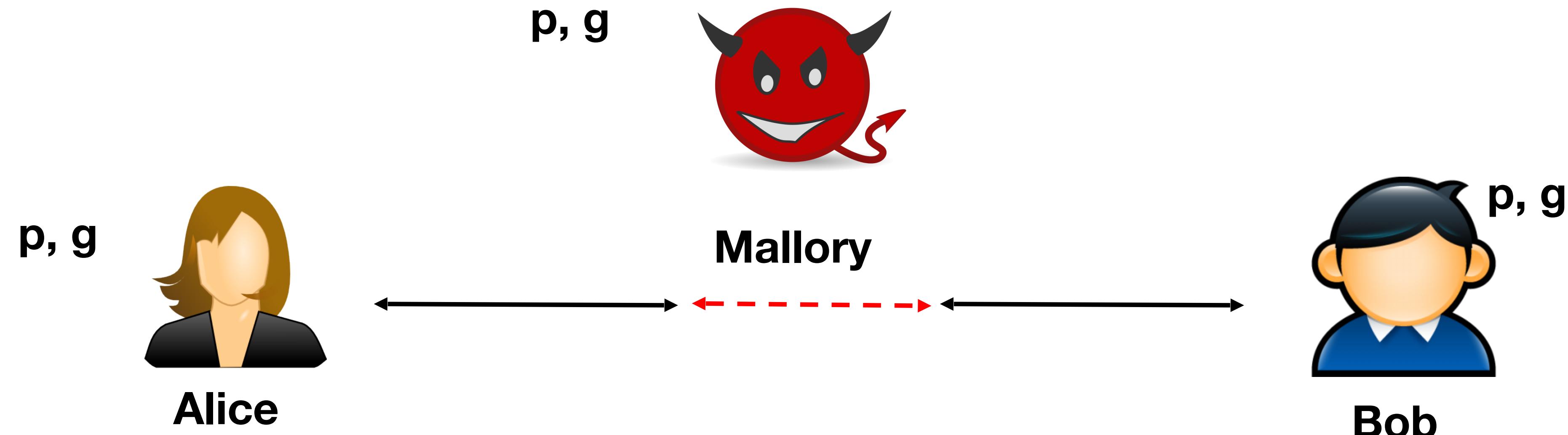
- This involved deep mathematical voodoo called "Group Theory"
 - Its actually done under a group G
- Two main groups of note:
 - Numbers mod p with generator g
 - Point addition in an elliptic curve C
 - Usually identified by number, eg. p256, p384 (NSA-developed curves) or Curve25519 (developed by Dan Bernstein, also 256b long)
- So EC (Elliptic Curve) == different group
 - Thought to be harder so fewer bits: 384b ECDHE ?= 3096b DHE
 - But still not as hard as AES: 128b AES ?= 256b ECDHE ?= 2048b DHE
 - But otherwise, its "add EC to the name" for something built on discrete log

But Its Not That Simple

- What if Alice and Bob aren't facing a passive eavesdropper
 - But instead are facing Mallory, an **active** Man-in-the-Middle
 - Mallory has the ability to change messages:
 - Can remove messages and add his own
 - Lets see... Do you think DHE will still work as-is?

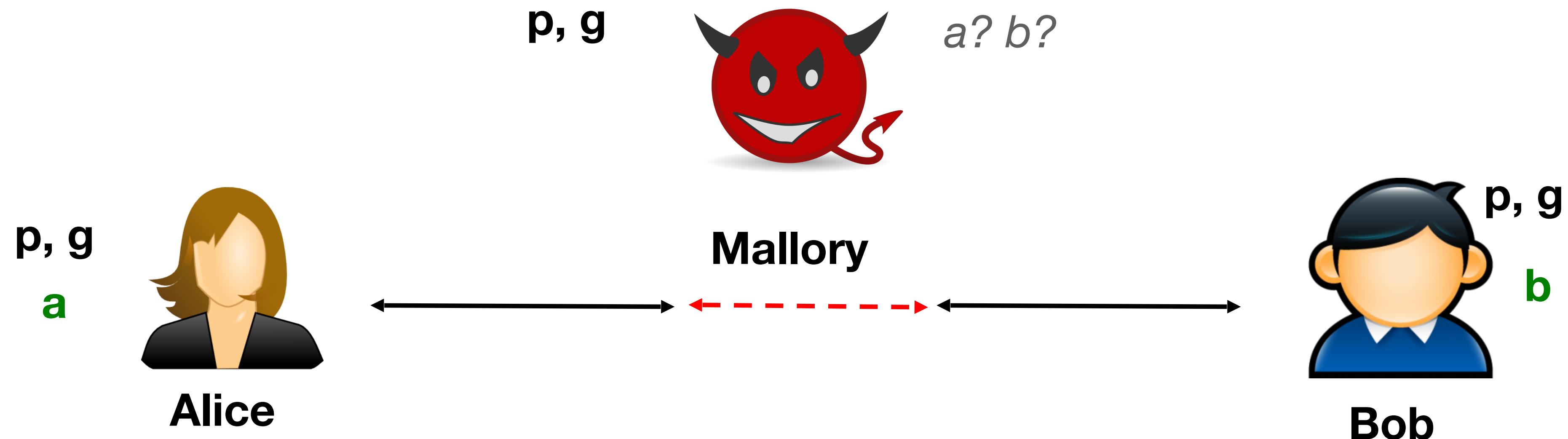


Attacking DHE as a MitM

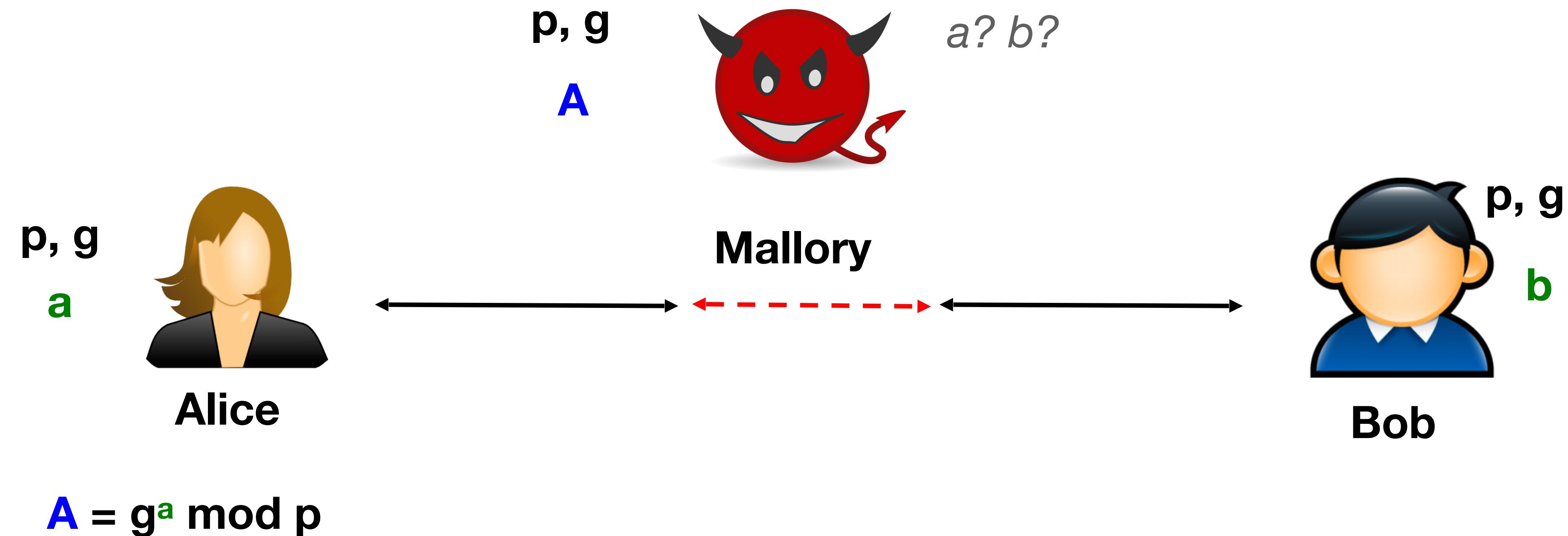


What happens if instead of Eve watching, Alice & Bob face the threat of a hidden Mallory (MitM)?

The MitM Key Exchange

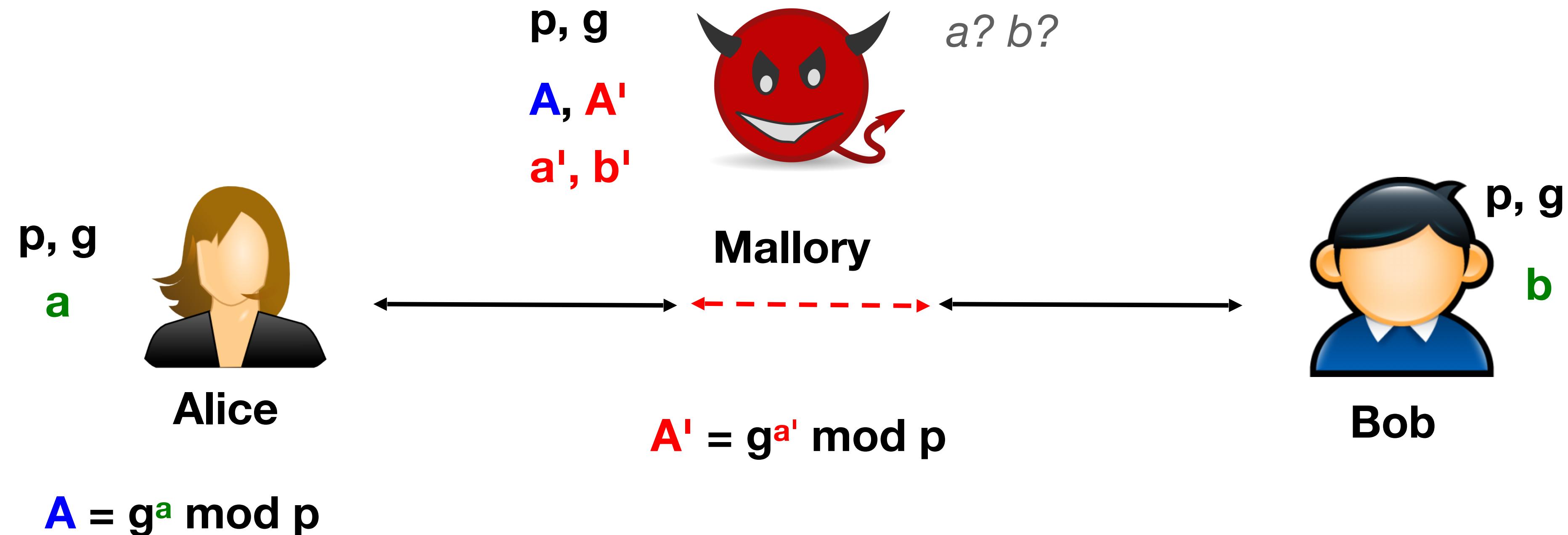


2. Alice picks **random** secret '**a1 < a < p-1**
3. Bob picks **random** secret '**b1 < b < p-1**

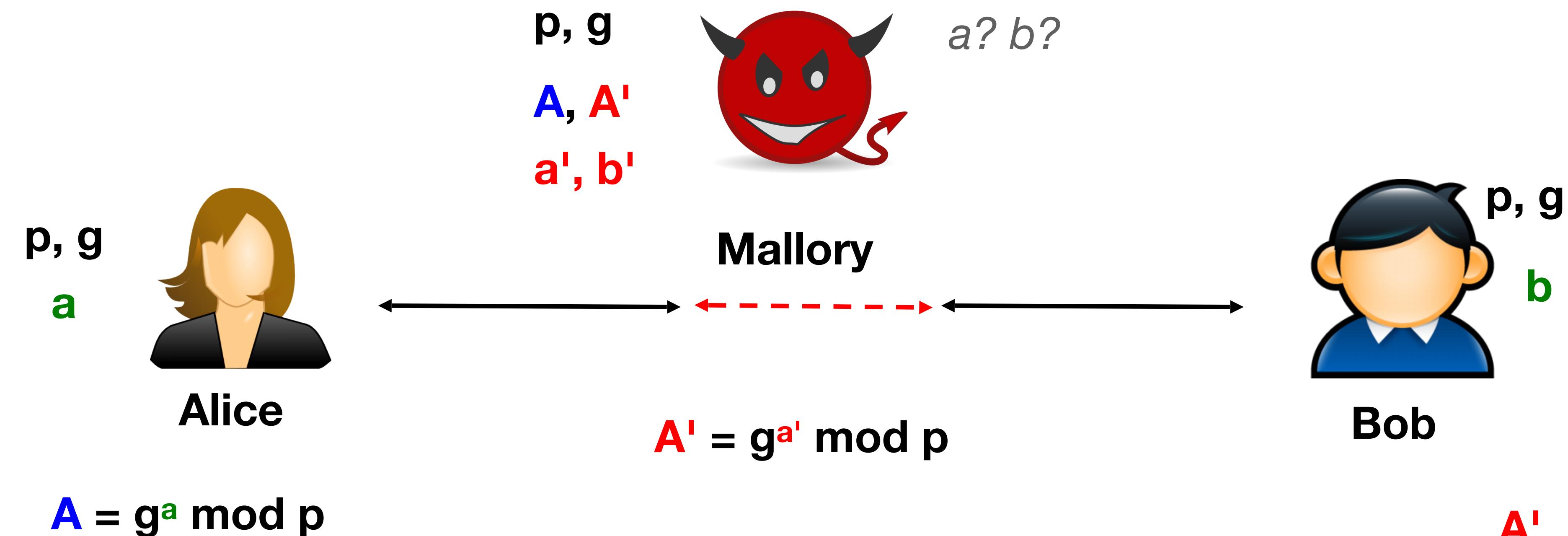


$$A = g^a \text{ mod } p$$

4. Alice sends $A = g^a \text{ mod } p$ to Bob
5. Mallory prevents Bob from receiving A



6. Mallory generates her own a', b'
7. Mallory sends $A' = g^{a'} \text{ mod } p$ to Bob

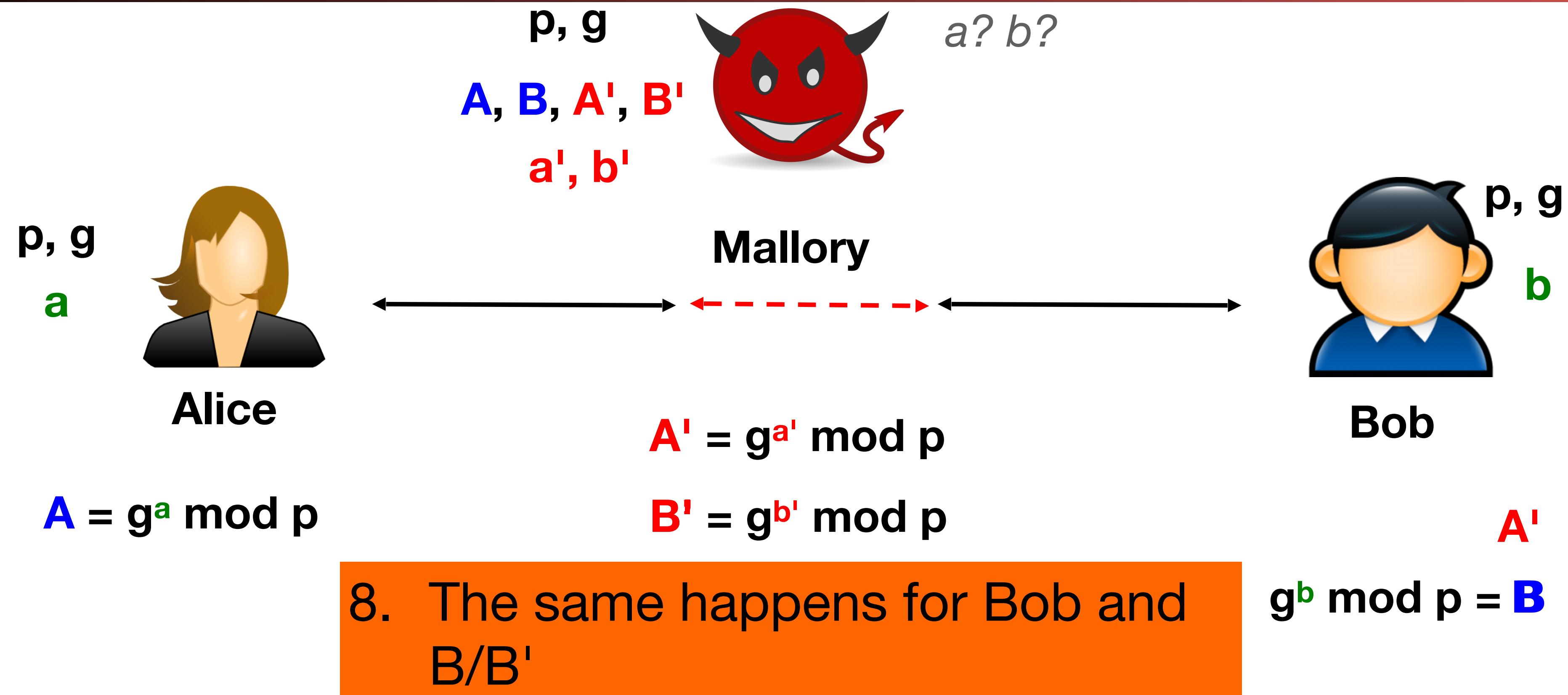


$$A = g^a \text{ mod } p$$

8. The same happens for Bob and B/B'

$$g^b \text{ mod } p = B$$

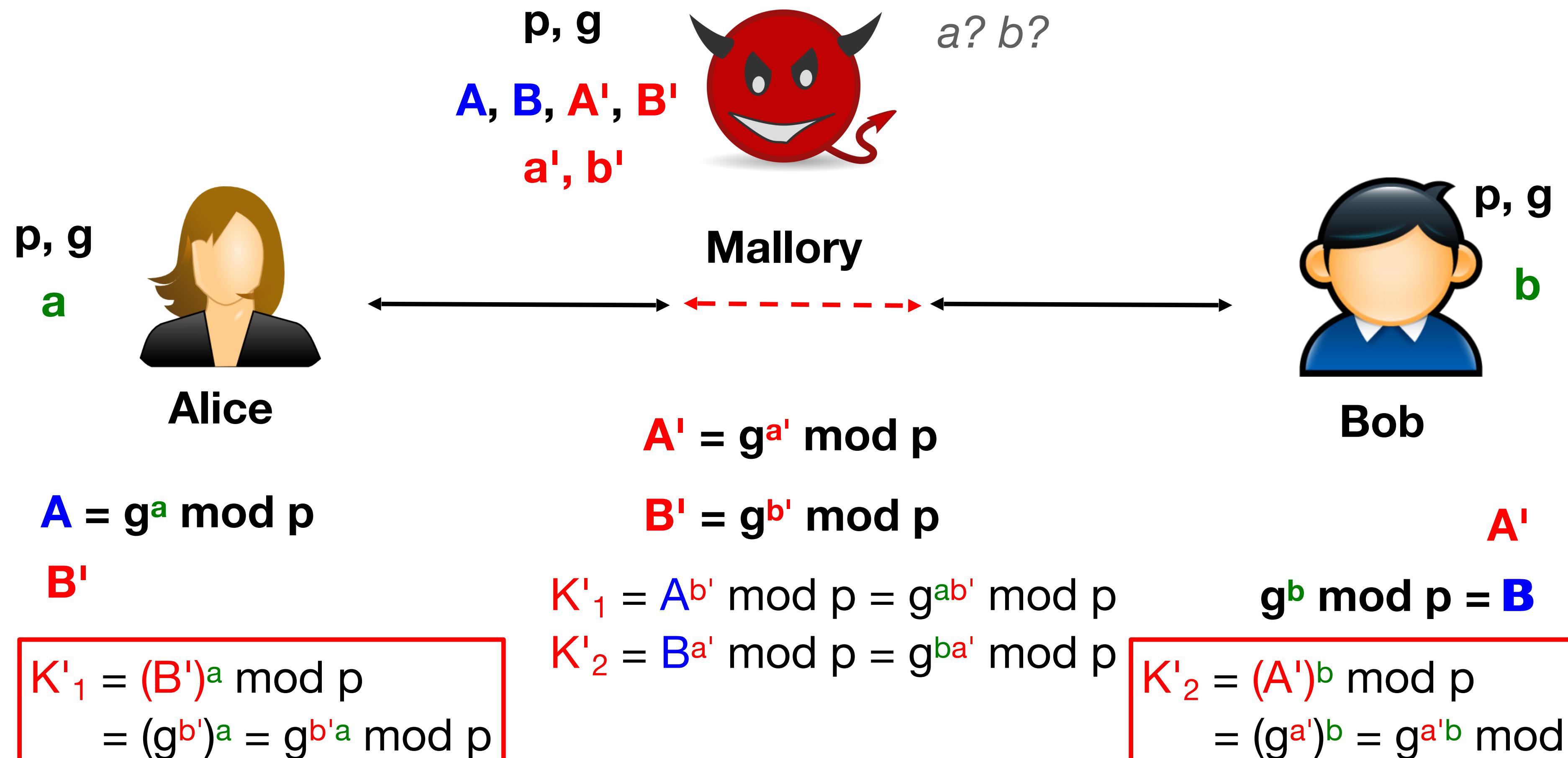
 A'



9. Alice and Bob now compute keys they share with ...
Mallory!

10. Mallory can relay encrypted traffic between the two ...

10'. Modifying it or making stuff up *however she wishes*



So We Will Want More...

- This is online:
 - Alice and Bob actually need to be active for this to work...
- So we want offline encryption:
 - Bob can send a message to Alice that Alice can read at a later date
- And authentication:
 - Alice can publish a message that Bob can verify was created by Alice later
 - Can also be used as a building-block for eliminating the MitM in the DHE key exchange:
Alice authenticates A, Bob verifies that he receives A not A'.

Public Key Cryptography #1:

RSA

- Alice generates two **large** primes, p and q
 - They should be generated randomly:
Generate a large random number and then use a "primality test":
A **probabilistic** algorithm that checks if the number is prime
- Alice then computes $n = p * q$ and $\Phi(n) = (p-1)(q-1)$
 - $\Phi(n)$ is Euler's totient function, in this case for a composite of two primes
- Choose random $2 < e < \Phi(n)$
 - e also needs to be relatively prime to $\Phi(n)$ but it can be small
- Solve for $d = e^{-1} \bmod \Phi(n)$
 - You can't solve for d without knowing $\Phi(n)$, which requires knowing p and q
- n, e are public, d, p, q , and $\Phi(n)$ are secret

RSA Encryption

- Bob can easily send a message m to Alice:
 - Bob computes $c = m^e \bmod n$
 - Without knowing d , it is believed to be intractable to compute m given c , e , and n
 - But if you can get p and q , you can get d :
 - It is ***not known*** if there is a way to compute d without also being able to factor n , but it is known that if you can factor n , you can get d .
 - And factoring is ***believed*** to be hard to do
- Alice computes $m = c^d \bmod n = m^{ed} \bmod n$
- Time for some math magic...

RSA Encryption/Decryption, con't

- So we have: $D(C, K_D) = (M^{e \cdot d}) \bmod n$
- Now recall that d is the multiplicative inverse of e , modulo $\phi(n)$, and thus:
 - $e \cdot d = 1 \bmod \phi(n)$ (by definition)
 - $e \cdot d - 1 = k \cdot \phi(n)$ for some k
- Therefore $D(C, K_D) = M^{e \cdot d} \bmod n = (M^{e \cdot d - 1}) \cdot M \bmod n$
 $= (M^{k\phi(n)}) \cdot M \bmod n$
 $= [(M^{\phi(n)})^k] \cdot M \bmod n$
 $= (1^k) \cdot M \bmod n \quad \text{by Euler's Theorem: } a^{\phi(n)} \bmod n = 1$
 $= M \bmod n = M$

(believed) Eve can recover M from C iff Eve can factor $n=p \cdot q$

But It Is Not That Simple...

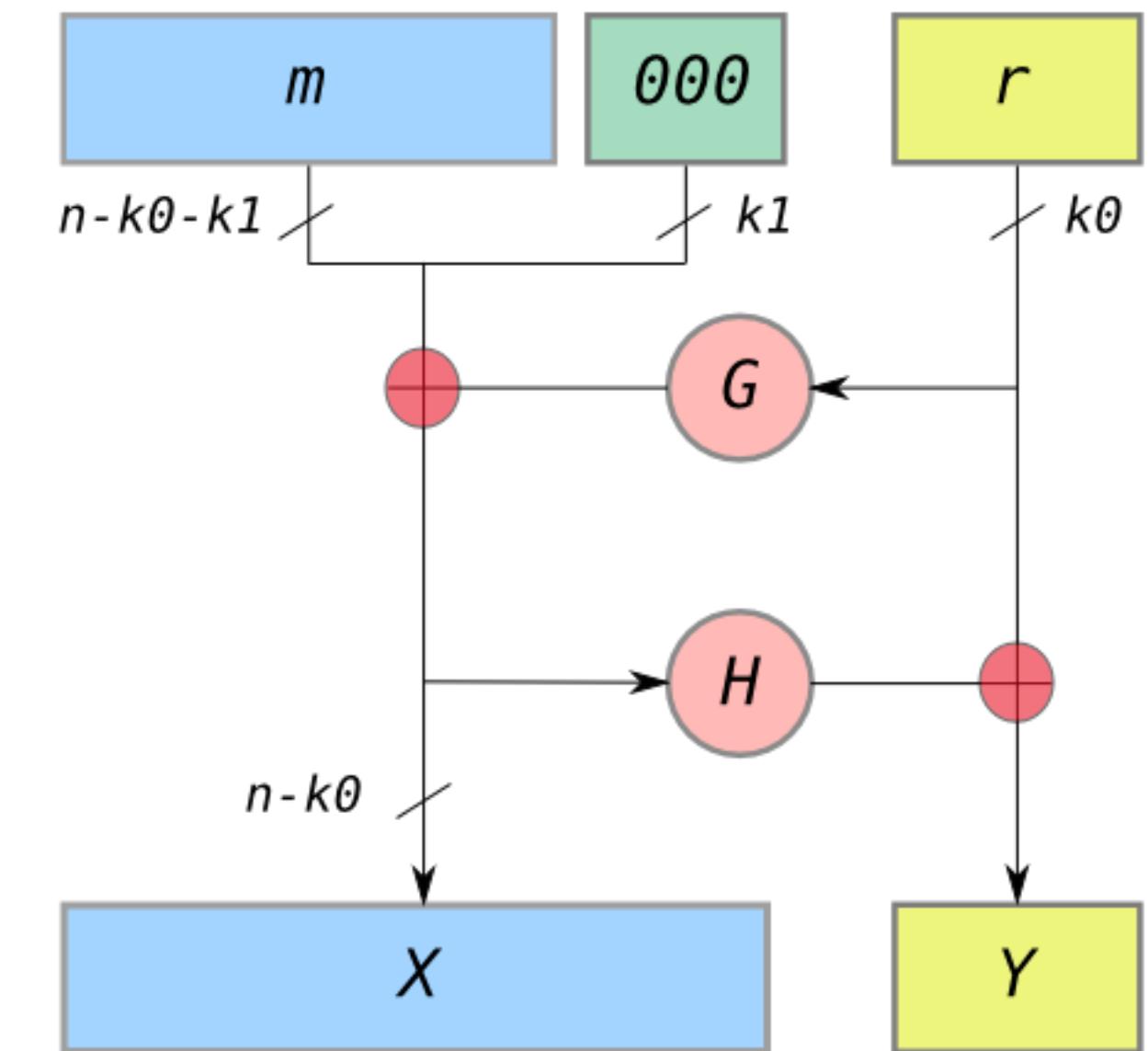
- What if Bob wants to send the same message to Alice twice?
 - Sends $m^{e_a} \bmod n_a$ and then $m^{e_a} \bmod n_a$
 - Oops, not IND-CPA!
- What if Bob wants to send a message to Alice, Carol, and Dave:
 - $m^{e_a} \bmod n_a$
 - $m^{e_b} \bmod n_b$
 - $m^{e_c} \bmod n_c$
 - This ends up leaking information an eavesdropper can use **especially** if $3 = e_a = e_b = e_c$!
- Oh, and problems if both **e** and **m** are small...
- As a result, you **can not** just use plain RSA:
 - You need to use a "padding" scheme that makes the input random but reversible



RSA-OAEP

(Optimal asymmetric encryption padding)

- A way of processing m with a hash function & random bits
 - Effectively "encrypts" m replacing it with $X = [m, 0\dots] \oplus G(r)$
 - G and H are hash functions (EG SHA-256)
 - k_0 = # of bits of randomness, $\text{len}(m) + k_1 + k_0 = n$
 - Then replaces r with $Y = H(G(r) \oplus [m, 0\dots]) \oplus R$
 - This structure is called a "Feistel network":
 - It is always designed to be reversible.
Many block ciphers are based on this concept applied multiple times with G and H being functions of k rather than just fixed operations
- This is more than just block-cipher padding (which involves just adding simple patterns)
 - Instead it serves to both pad the bits and make the data to be encrypted "random"



But Its Not That Simple...

Timing Attacks

Computer Science 161 Fall 2020

Weaver

- Using normal math, the ***time*** it takes for Alice to decrypt **c** depends on **c** and **d**
 - Ruh roh, this can leak information...
 - More complex RSA implementations take advantage of knowing **p** and **q** directly... but also leak timing
- People have used this to guess and then check the bits of **q** on OpenSSL
 - <http://crypto.stanford.edu/~dabo/papers/ssl-timing.pdf>
 - And even more subtle things are possible...

```
x = c
for j = 1 to n
    x = mod(x2, N)
    if dj == 1 then
        x = mod(xC, N)
    end if
next j
return x
```



SwiftOnSecurity
@SwiftOnSecurity

Following



So How to Find Bob's Key?

- Lots of stuff later, but for now...
The Leap of Faith!
- Alice wants to talk to Bob:
 - "Hey, Bob, tell me your public key!"
 - Now on all subsequent times...
 - "Hey, Bob, tell me your public key", and check to see if it is different from what Alice remembers
 - Works assuming the ***first time*** Alice talks to Bob there isn't a Man-in-the-Middle
 - ssh uses this

RSA Signatures...

- Alice computes a hash of the message $H(m)$
 - Alice then computes $s = (H(m))^d \text{ mod } n$
- Anyone can then verify
 - $v = s^e \text{ mod } m = ((H(m))^d)^e \text{ mod } n = H(m)$
- Once again, there are "F-U"s...
 - Have to use a proper encoding scheme to do this properly and all sort of other traps
 - One particular trap: a scenario where the attacker can get Alice to repeatedly sign things (an "oracle")



But Signatures Are Super Valuable...

- They are how we can prevent a MitM!
- If Bob knows Alice's key, and Alice knows Bob's...
 - How will be "next time"
- Alice doesn't just send a message to Bob...
 - But creates a random key k ...
 - Sends $E(M, K_{\text{sess}})$, $E(K_{\text{sess}}, B_{\text{pub}})$, $S(H(M), A_{\text{priv}})$
- Only Bob can decrypt the message, and Bob can verify the message came from Alice
 - So Mallory is SOL!

RSA Isn't The Only Public Key Algorithm

- Isn't RSA enough?
 - RSA isn't particularly compact or efficient: dealing with 2000b (comfortably secure) or 3000b (NSA-paranoia) bit operations
 - Can we get away with fewer bits?
 - Well, Diffie-Hellman isn't any better...
 - But ***elliptic curve*** Diffie-Hellman is
- RSA also had some patent issues
 - So an attempt to build public key algorithms around the Diffie-Hellman problem

El-Gamal

- Just like Diffie-Hellman...
 - Select p and g
 - These are public and can be shared:
Note, they need to be carefully considered how to create p and g ...
Math beyond the level of this class
- Alice chooses x randomly as her private key
 - And publishes $h = g^x \text{ mod } p$ as her public key
- Bob, to encrypt m to Alice...
 - Selects a *random* y , calculates $c_1 = g^y \text{ mod } p$, $s = h^y \text{ mod } p = g^{xy} \text{ mod } p$
 - s becomes a shared secret between Alice and Bob
 - Maps message m to create m' , calculates $c_2 = m' * s \text{ mod } p$
- Bob then sends $\{c_1, c_2\}$

El-Gamal Decryption

- Alice first calculates $s = c_1^x \bmod p$
 - Then Alice calculates $m' = c_2 * s^{-1} \bmod p$
 - Then Alice calculates the inverse of the mapping to get m
- Of course, there are problems...
 - Attacker can always change m' to $2m'$
 - What if Bob screws up and reuses y ?
 - $c_2 = m_1' * s \bmod p$
 $c_2' = m_2' * s \bmod p$
 - Ruh roh, this leaks information:
 $c_2 / c_2' = m_1' / m_2'$
 - So if you know m_1 ...



In Practice: Session Keys...

- You use the public key algorithm to encrypt/agree on a session key..
 - And then encrypt the real message with the session key
 - You **never** actually encrypt the message itself with the public key algorithm
 - Often a set of keys: encrypt and MAC keys that are separate in each direction
- Why?
 - Public key is **slow**... Orders of magnitude slower than symmetric key
 - Public key may cause weird effects:
 - EG, El Gamal where an attacker can change the message to **$2m$** ...
 - If m had meaning, this would be a problem
 - But if it just changes the encryption and MAC keys, the main message won't decrypt

DSA Signatures...

- Again, based on Diffie-Hellman
 - Two initial parameters, **L** and **N**, and a hash function **H**
 - **L** == key length, eg 2048
 - **N <= len(H)**, e.g. 256
 - An N-bit prime **q**, an L-bit prime **p** such that **p - 1** is a multiple of **q**, and **g = h^{(p-1)/q} mod p** for some arbitrary **h** ($1 < h < p - 1$)
 - $\{p, q, g\}$ are public parameters
 - Alice creates her own random private key **x < q**
 - Public key **y = g^x mod p**

Alice's Signature...

- Create a random value $k < q$
 - Calculate $r = (g^k \bmod p) \bmod q$
 - If $r = 0$, start again
 - Calculate $s = k^{-1} (H(m) + xr) \bmod q$
 - If $s = 0$, start again
 - Signature is $\{r, s\}$ (Advantage over an El-Gamal signature variation: Smaller signatures)
- Verification
 - $w = s^{-1} \bmod q$
 - $u_1 = H(m) * w \bmod q$
 - $u_2 = r * w \bmod q$
 - $v = (g^{u_1} y^{u_2} \bmod p) \bmod q$
 - Validate that $v = r$

But Easy To Screw Up...

- **k** is not just a nonce... It must be random and **secret**
 - If you know **k**, you can calculate **x**
 - And even if you just reuse a random **k**...
for two signatures **s_a** and **s_b**
 - A bit of algebra proves that $k = (H_A - H_B) / (s_a - s_b)$
 - A good reference:
 - How knowing **k** tells you **x**:
<https://rdist.root.org/2009/05/17/the-debian-pgp-disaster-that-almost-was/>
 - How two signatures tells you **k**:
<https://rdist.root.org/2010/11/19/dsa-requirements-for-random-k-value/>



And *NOT* theoretical: Sony Playstation 3 DRM

- The PS3 was designed to only run ***signed*** code
 - They used ECDSA as the signature algorithm
 - This prevents unauthorized code from running
 - They had an ***option*** to run alternate operating systems (Linux) that they then removed
- Of course this was catnip to reverse engineers
 - Best way to get people interested: ***remove*** Linux from a device...
- It turns out one of the key authentication keys used to sign the firmware...
 - Ended up reusing the same k for multiple signatures!



And ***NOT*** Theoretical: Android RNG Bug + Bitcoin

- OS Vulnerability in 2013
Android "SecureRandom" wasn't actually secure!
 - Not only was it low entropy, it would occasionally return the **same value multiple times**
- Multiple Bitcoin wallet apps on Android were affected
 - "Pay B Bitcoin to Bob" is signed by Alice's public key using ECDSA
 - Message is broadcast publicly for all to see
 - So you'd have cases where "Pay B to Bob" and "Pay C to Carol" were signed with the same k
 - **So of course** someone scanned for *all* such Bitcoin transactions



And *Still* Happens!

Chromebook

- Chromebooks have a built in U2F "Security key"
 - Enables signatures using 256b ECDSA to validate to particular websites
- There was a bug in the secure hardware!
 - Instead of using a random k that was 256b long, a bug caused it to be 32b long!
 - So an attacker who had a signature could simply try all possible k values!
- Fortunately in this case the damage was slight: this is for authenticating to a single website: each site used its own private key
- But still...
- <https://www.chromium.org/chromium-os/u2f-ecdsa-vulnerability>



So What To Use?

- Paranoids like me:
Good libraries and use the parameters from NSA's CNSA suite
 - Open algorithms approved for Top Secret communication
 - Better yet, libraries that implement full protocols that use these under the hood!
- Symmetric cipher: AES: 256b
 - CFB mode, thankyouverymuch. Counter mode and modes which include counter mode can DIAF...
- Hash function: SHA-384
 - Use HMAC for MAC
- RSA: 3072b
- Diffie/Hellman: 3072b
- ECDH/ECDSA: P-384
 - But really, this is extra paranoid, 2048b RSA/DH, 256b EC, 128b AES, SHA-256 excellent in practice

How Can We Communicate With Someone New?

- Public-key crypto gives us amazing capabilities to achieve confidentiality, integrity & authentication without shared secrets ...
- But how do we solve MITM attacks?
- How can we trust we have the true public key for someone we want to communicate with?
- Ideas?

Trusted Authorities

- Suppose there's a party that everyone agrees to trust to confirm each individual's public key
 - Say the Governor of California
 - Issues with this approach?
 - How can everyone agree to trust them?
 - Scaling: huge amount of work; single point of failure ...
 - ... and thus Denial-of-Service concerns
 - How do you know you're talking to the right authority??

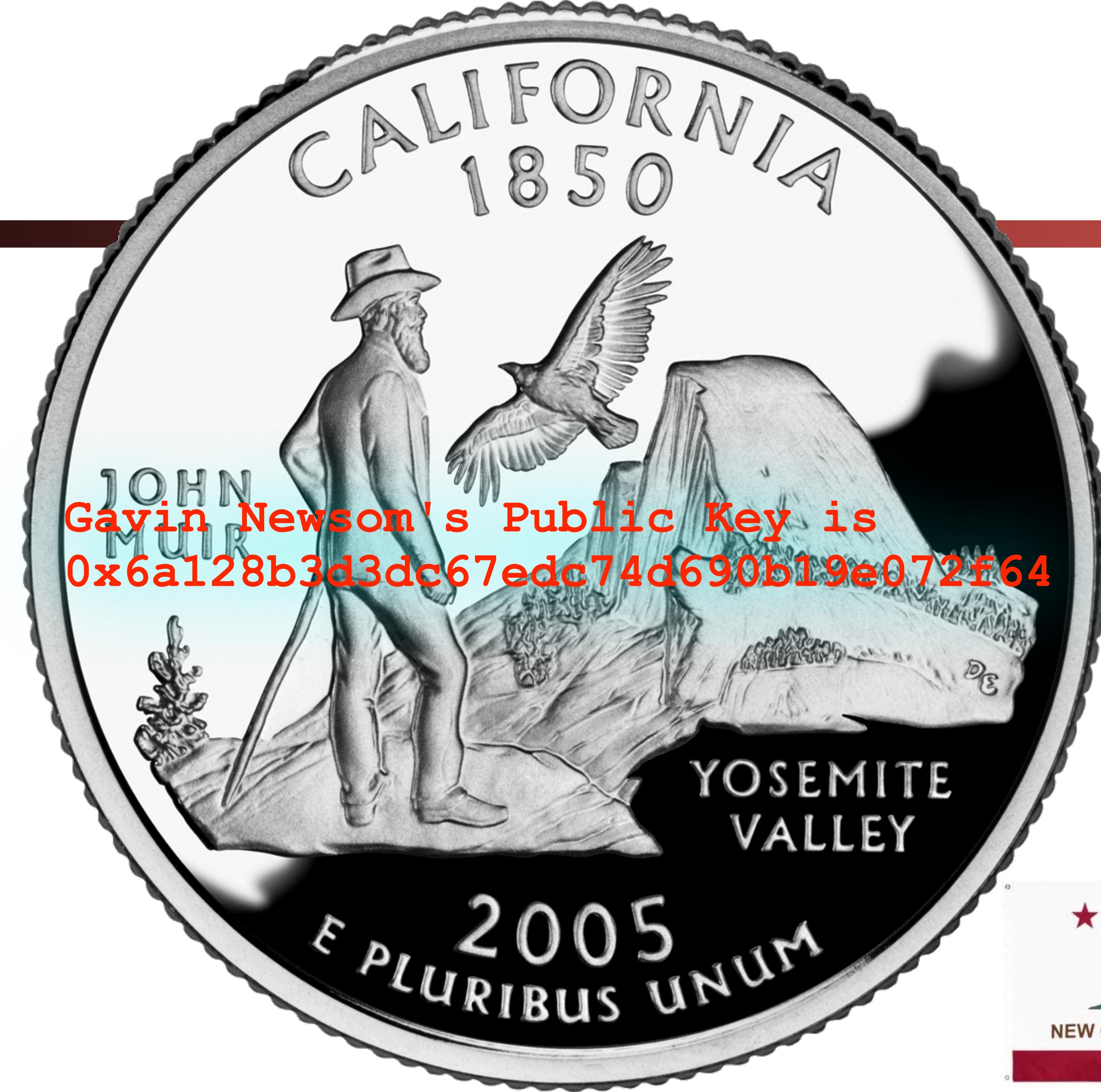


Trust Anchors

- Suppose the trusted party distributes their key so everyone has it ...







Trust Anchors

- Suppose the trusted party distributes their key so everyone has it ...
- We can then use this to bootstrap trust
 - As long as we have confidence in the decisions that that party makes

Digital Certificates

- Certificate (“cert”) = signed claim about someone’s public key
 - More broadly: a signed *attestation* about some claim
- Notation:
 - $\{ M \}_K$ = “message M encrypted with public key k”
 - $\{ M \}_{K^{-1}}$ = “message M signed w/ private key for K”
- E.g. M = “Nick's public key is $K_{\text{Nick}} = 0xF32A99B\dots$ ”
Cert: M,
 $\{ \text{“Nick's public key ... } 0xF32A99B\dots \text{”} \}_{K^{-1}\text{Gavin}}$
 $= 0x923AB95E12\dots 9772F$

Certificate



Gavin Newsom hereby asserts:

Nick's public key is $K_{Nick} = 0xF32A99B...$

The signature for this statement using

K^{-1}_{Gavin} is $0x923AB95E12\ldots9772F$

Certificate



Gavin Newsom hereby asserts:

Nick's public key is $K_{Nick} = 0xF32A99B...$

The signature for this statement using

K^{-1} This is **$0x923AB95E12\ldots9772F$**

Certificate



Gavin Newsom hereby asserts:

Nick's public key is $K_{Nick} = 0xF32A99B...$

The signature for is computed over all of this

K_{Gavin}^{-1} is **$0x923AB95E12\ldots9772F$**

Certificate



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Nick's public key is $K_{Nick} = 0xF32A99B...$

The signature for this statement using

K^{-1}_{Gavin} is $0x923AB95E12\ldots9772F$

and can be
validated using:

Certificate



This:

Gavin Newsom hereby asserts:

Nick's public key is K_{Nick}

The signature for this statement is

K_{Gavin}^{-1} is **0x923AB95**

