EECS 16A   Designing Information Devices and Systems I
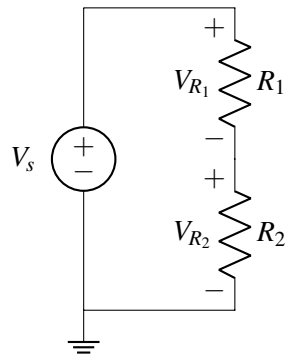## Spring 2022   Discussion 7B
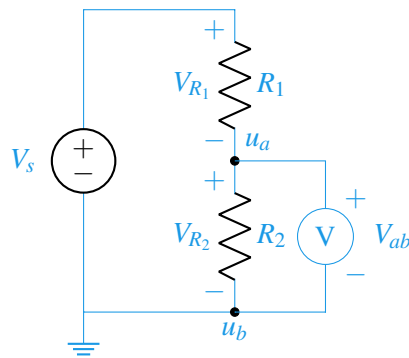
1. Volt and Ammeter

(a) For the voltage divider below, how would we connect a voltmeter to the circuit to measure the voltage V_{R_2}?


Answer: We connect our voltmeter to the voltage divider such that the voltage across the voltmeter nodes is equal to the voltage we want to measure.

i.e. $V_{ab} = u_a - u_b = V_{R_2}$

(b) What would happen if we accidentally connected an ammeter in the same configuration instead? Assume our ammeter is ideal.

Answer: An ideal ammeter behaves like a wire or a short between two nodes as depicted below:
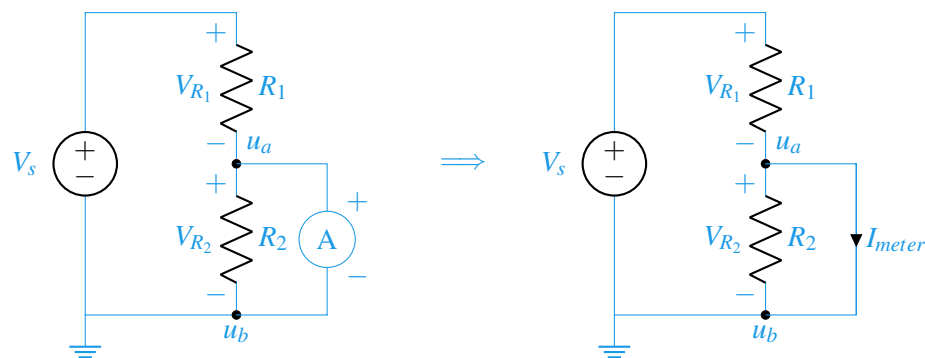
When we have a short, or a new lower resistance path for the current to travel, current will choose to take the path of least resistance. In this case, the ideal wire has no resistance so all of the current leaving R_1 will flow through the wire instead of R_2.
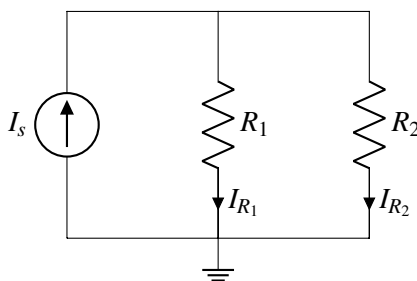
Mathematically, we can see this because this wire has now combined the nodes u_a and u_b into one node. In other words, $u_a = u_b$. We can use this to show that no current is flowing through R_2.

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_a - u_b}{R_2} = \frac{0}{R_2} = 0$$

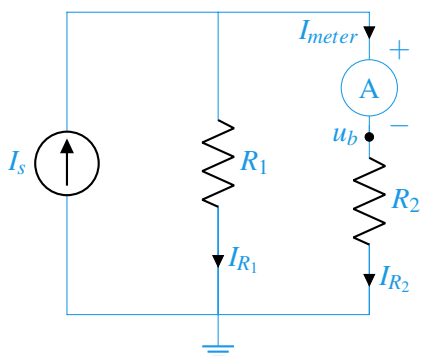Therefore, an equivalent circuit can be drawn as shown below:
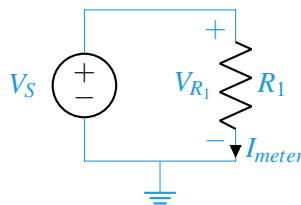
(c) For the current divider below, how would we connect an ammeter to the circuit to measure the current I_{R_2}?


Answer: We connect our ammeter to the current divider such that the current going through the ammeter is equal to the current we want to measure. By doing a KCL at node u_b we can see that $I_{meter} = I_{R_2}$.

(d) What would happen if we accidentally connected a voltmeter in that configuration instead? Assume the voltmeter is ideal.

Answer: An ideal voltmeter behaves like an open circuit as depicted below:
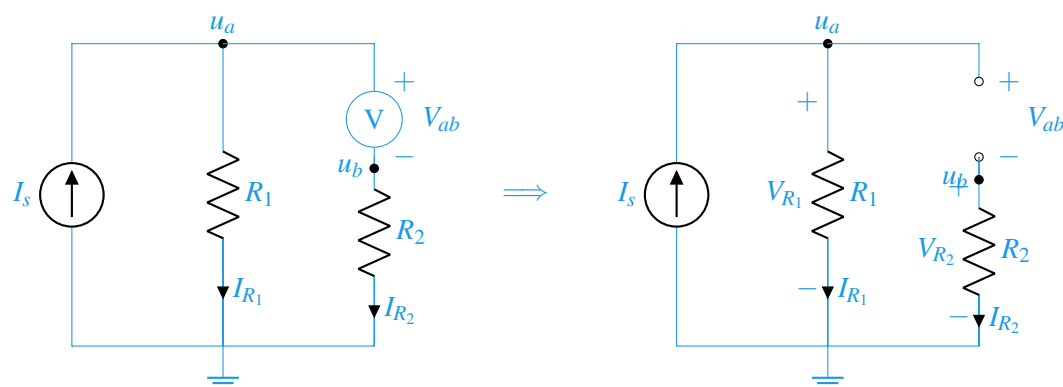
The open circuit creates a dead end and prevents any current from flowing through therefore $I_{R_2} = 0$. As a result, we will notice that the node u_b is now equal to the ground node.
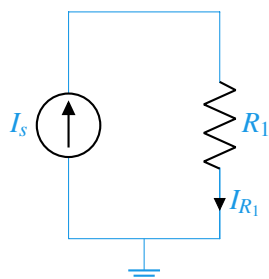
$$V_{R_2} = I_{R_2}R_2 = 0 = u_b - 0$$

$$u_b = 0$$

With this knowledge, we can conclude that the voltmeter will actually read the voltage across the resistor R_1.

$$V_{ab} = u_a - u_b = u_a - 0 = V_{R_1}$$

Since $I_{R_2} = 0$ the resistor R_2 has no effect on our circuit and an equivalent circuit can be drawn below:
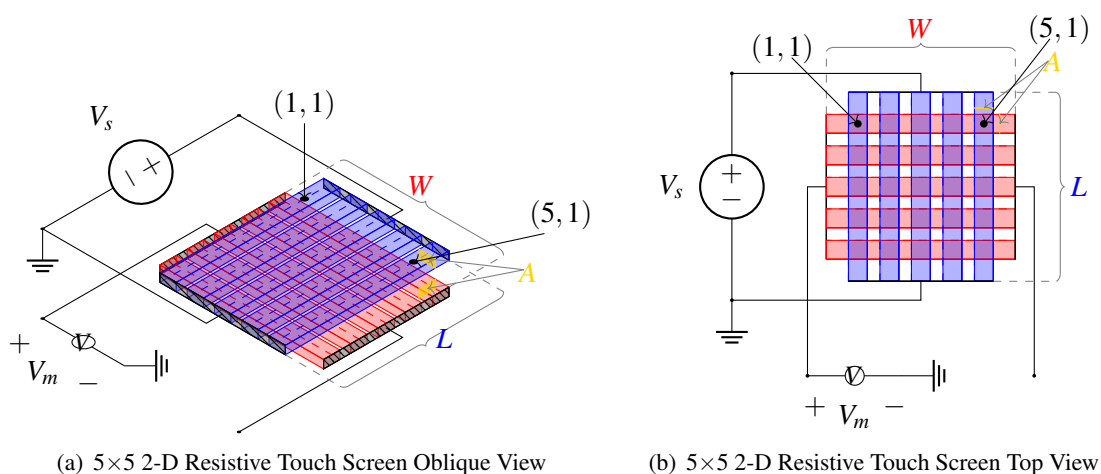

2. Resist the Touch

(a) 5×5 2-D Resistive Touch Screen Oblique View

(b) 5×5 2-D Resistive Touch Screen Top View

Figure 1: $N \times N$ Resistive Touch Screen, $N = 5$

(a) 2×2 2-D Resistive Touch Screen Oblique View
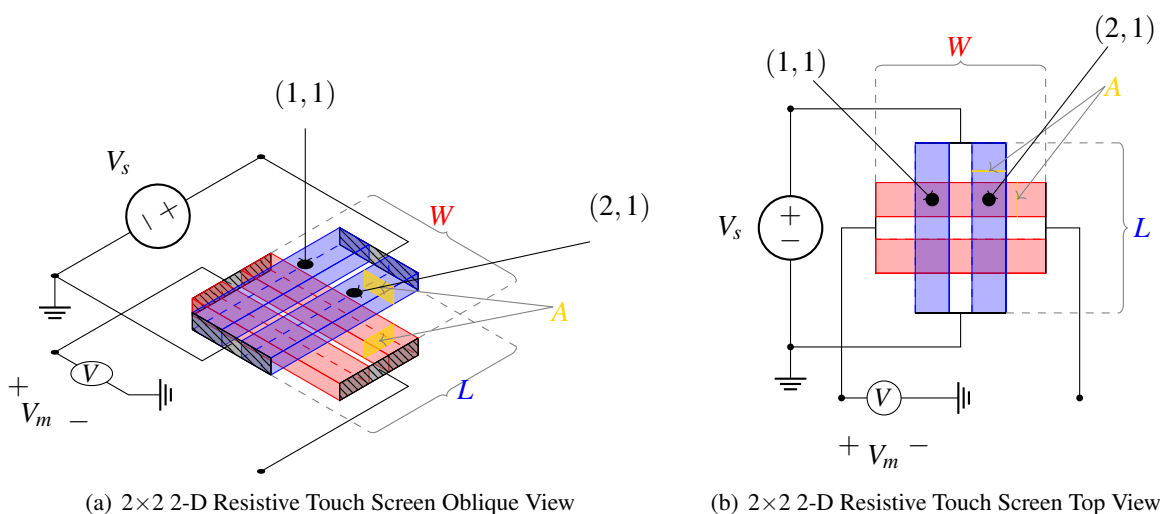

(b) 2×2 2-D Resistive Touch Screen Top View

Figure 2: 2×2 Resistive Touch Screen

In this question we will be re-examining the 2-dimensional resistive touchscreen. This touchscreen is slightly different from the one shown in lecture and more like the one we will be examining in lab.

The touchscreen has length L and width W and is composed of a rigid bottom-layer and a flexible top-layer. Instead of having two continuous resistive sheets on the top and bottom layers, this is a simpler implementation with N vertical strips of conductive material in the top layer and N horizontal strips of conductive material in the bottom layer. The strips of a single layer are all connected by an ideal conducting plate on each side. All strips have resistivity, ρ, and cross-sectional area, A.

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally, and that the upper left touch point in Figure 1(b) is position $(1,1)$, and the upper right touch point is $(N,1)$. The spacing between the strips in the top layer is $\frac{W}{N+1}$, and the spacing between the strips in the bottom layer is $\frac{L}{N+1}$.

(a) Find the resistance R_y for a single vertical blue strip and R_x for a single horizontal red strip as a function of the screen dimensions W and L, the strip resistivity ρ, and the cross-sectional area A.

   The equation for resistance is $R = \frac{\rho l}{A}$

Therefore for the bottom red horizontal resistive strips we have, $R_x = \frac{\rho W}{A}$.

For the top blue vertical resistive strips, $R_y = \frac{\rho L}{A}$.

(b) Consider a 2×2 example for the touchscreen circuit, as shown in Figure 2.

Assume that we connect a voltage source V_s between the top and bottom terminals of the blue strips, and a voltmeter V_m to one of the left or right terminals as depicted in the diagram.

If $V_s = 3\,\text{V}$, $R_x = 2000\,\Omega$, and $R_y = 2000\,\Omega$, draw the equivalent circuit for when the point $(2,2)$ is pressed and solve for the measured voltage, V_m, with respect to ground.

Since all of the resistive strips are equally spaced, the resistor above point $(2,2)$ on the vertical blue strip becomes $\rho \frac{\frac{2L}{3}}{A} = \frac{2}{3}\rho\frac{L}{A} = \frac{2}{3}R_y$ and the resistor below point $(2,2)$ on the vertical strip becomes $\rho \frac{\frac{L}{3}}{A} = \frac{1}{3}\rho\frac{L}{A} = \frac{1}{3}R_y$.

The bottom layer red resistors, although they must be drawn in the equivalent circuit, do not affect the measured voltage, V_m, as there are open circuits, leading to no currents through those resistors, and therefore no voltage drops over either of them.
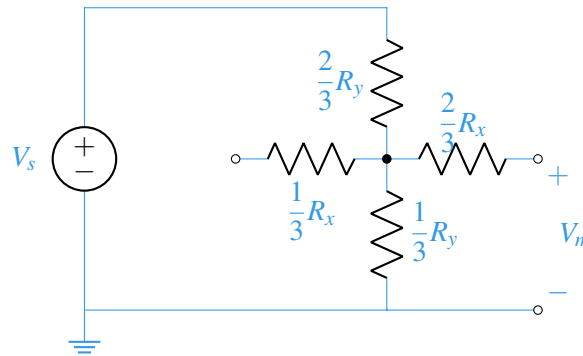
Observing that the rightmost top layer blue resistive strip forms a voltage divider, and remembering that there is no voltage drop across the dangling R_x resistors, we can determine V_m using the voltage divider equation.

Therefore, $V_m = V_{(2,2)} = V_s \dfrac{\frac{1}{3}R_y}{\frac{1}{3}R_y + \frac{2}{3}R_y} = \frac{1}{3}V_s = 1\,\text{V}.$

(c) Suppose a touch occurs at coordinates (i, j) for an arbitrary $N \times N$ touchscreen, and the voltage source and meter are connected as in the figures. A 5×5 example is shown in Figure 1(b). Find an expression for V_m as a function of V_s, N, i, and j. Again, the upper left corner is the coordinate $(1,1)$ and the upper right coordinate is $(N,1)$.

The voltage does not depend on the x coordinate, as the meter is connected to the red dangling resistors along the horizontal. We will only be able to detect changes in the y coordinate. If the touch point occurs at (i, j), the i-th blue vertical bar from the left will be split into lengths of $L_{top} = \frac{j}{N+1}L$ and $L_{bottom} = \frac{N+1-j}{N+1}L$ at the j-th touch point from the top. The voltage divider takes the voltage over the bottom resistor, so we will see $V_m = \frac{L_{bottom}}{L}V_s = \frac{\frac{N+1-j}{N+1}L}{L}V_s = \frac{N+1-j}{N+1}V_s$