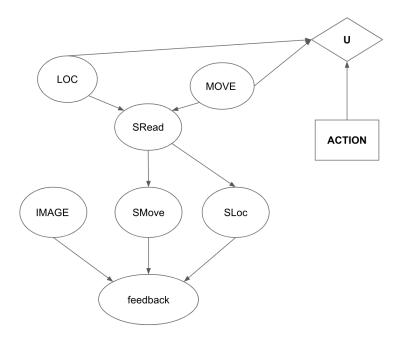
## Games and Utilities Solutions

## Q1. Vehicle Perception Indication

A vehicle is trying to identify the situation of the world around it using a set of sensors located around the vehicle.

Each sensor reading (SRead) is based off of an object's location (LOC) and an object's movement (MOVE). The sensor reading will then produce various values for its predicted location (SLoc) and predicted movement (SMove). The user will receive these readings, as well as the the image (IMAGE) as feedback.

(a) The vehicle takes an action, and we assign some utility to the action based on the object's location and movement. Possible actions are MOVE TOWARDS, MOVE AWAY, and STOP. Suppose the decision network faced by the vehicle is the following.



- (i) Based on the diagram above, which of the following could possibly be true?
  - $\blacksquare$  VPI (Image) = 0
  - $\square$  VPI (SRead) < 0
  - $\square$  VPI (SMove, SRead) > VPI (SRead)
  - $\mathbf{VPI}$  (Feedback) = 0
  - O None of the above

VPI(Image) = 0 because there is not active path connecting Image and U

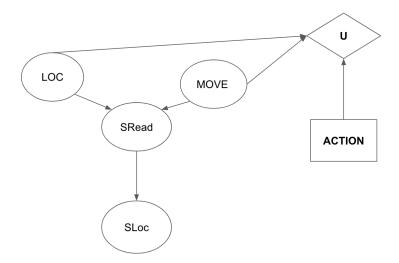
VPI cannot be negative, so option 2 is not selected.

 $VPI(SMove, SRead) = VPI(SMove \mid SRead) + VPI(SRead)$ , therefore we can cancel VPI(SRead) from both side, and it becomes asking if  $VPI(SMove \mid SRead) > 0$ . And we can see that cannot be true, because shading in SRead, there is no active path connecting SMove and U.

There is an active path connecting Feedback and U, therefore VPI(Feedback)  $\geq 0$ . It could still be 0 because active path only gives the possibility of > 0.

(ii)	Based on the diagram above, which of the following must necessarily be true?
	$\blacksquare$ VPI (Image) = 0
	$\square$ VPI (SRead) = 0
	■ VPI (SMove, SRead) = VPI (SRead)
	$\square$ VPI (Feedback) = 0
	O None of the above
	VPI(Image) = 0 because there is not active path connecting Image and U
	$\mathrm{VPI}(\mathrm{SRead})$ could be $>0$ because $\mathrm{SRead}\text{-}\mathrm{MOVE}\text{-}\mathrm{U}$ is an active path between $\mathrm{SRead}$ and $\mathrm{U}$
	$\label{eq:VPI} \text{VPI}(\text{SMove},  \text{SRead})  =  \text{VPI}(\text{SMove}      \text{SRead})  +  \text{VPI}(\text{SRead}),   \text{therefore we can cancel }  \text{VPI}(\text{SRead})$
	from both side, and it becomes asking if $VPI(SMove \mid SRead) == 0$ . And we can see that must true,
	because shading in SRead, there is no active path connecting SMove and U.
	VPI(Feedback) could be > 0 because feedback-SLoc-SRead-MOVE-U is an active path

Let's assume that your startup has less money, so we use a simpler sensor network. One possible sensor network can be represented as follows.



You have distributions of P(LOC), P(MOVE), P(SRead|LOC, MOVE), P(SLoc|SRead) and utility values U(a, l, m).

(b) Complete the equation for determining the expected utility for some ACTION a.

We can eliminate SRead and SLoc via marginalization, so they don't need to be included the expression

(c) Your colleague Bob invented a new sensor to observe values of *SLoc*.

- (i) Suppose that your company had no sensors till this point. Which of the following expression is equivalent to VPI(SLoc)?
  - $\blacksquare$  VPI(SLoc) = ( $\sum_{sloc} P(sloc) \text{ MEU}(SLoc = sloc)$ )  $\max_a \text{EU}(a)$
  - $\mathsf{VPI}(SLoc) = \mathsf{MEU}(SLoc) \mathsf{MEU}(\emptyset)$

  - O None of the above

Option 2 is correct by definition, and option 1 is the expanded version of option 2

- (ii) Gaagle, an established company, wants to sell your startup a device that gives you *SRead*. Given that you already have Bob's device (that gives you *SLoc*), what is the maximum amount of money you should pay for Gaagle's device? Suppose you value \$1 at 1 utility.
  - $\square$  VPI(SRead)
  - $\blacksquare$  VPI(SRead) VPI(SLoc)
  - $\square$  VPI(SRead, SLoc)
  - VPI(SRead, SLoc) VPI(SLoc)
  - O None of the above

Choice 4 is correct by definition

Choice 2 is true because  $VPI(SLoc \mid SRead) = 0$ , and thus

 $\label{eq:VPI} VPI(SRead) = VPI(SRead) + 0 = VPI(SRead) + VPI(SLoc \mid SRead) = VPI(SRead, SLoc), \ which makes choice 2 the same as choice 4$ 

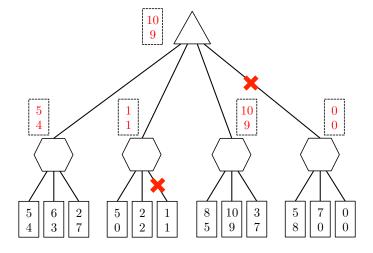
## Q2. Game Trees and Pruning

You and one of the 188 robots are playing a game where you both have your own score. In the leaf nodes of the game tree, your score is on top and the robot's score on the bottom.

- The maximum possible score for either player is 10.
- You are trying to maximize your score, and you do not care what score the robot gets.
- The robot is trying to minimize the absolute difference between the two scores. In the case of a tie, the robot prefers a lower score. For example, the robot prefers (5,3) to (6,3); it prefers (5,3) to (0,3); and it prefers (3,3) to (5,5).

The figure below shows the game tree of your max node followed by the robots nodes for your four different actions.

(a) Fill in the dashed rectangles with the pair of scores preferred by each node of the game tree.



- (b) You can save computation time by using pruning in your game tree search. On the game tree above, put an 'X' on line of branches that do not need to be explored. Assume that branches are explored from left to right.
- (c) You now have access to an oracle that tells you the order of branches to explore that maximizes pruning. On the copy of the game tree below, put an 'X' on line of branches that do not need to be explored given this new information from the oracle.

