CS 70   Discrete Mathematics and Probability Theory

Spring 2021   HW 0

1   Administrivia

(a) Make sure you are on the course Piazza (for Q&A) and Gradescope (for submitting home-
    works, including this one). Find and familiarize yourself with the course website. What is its
    homepage's URL?

(b) Read the policies page on the course website.

   (i) What is the percentage breakdown of how your grade is calculated (please include both
        breakdowns)?
  (ii) How many discussions do you need to attend to get full credit for discussion attendance?
 (iii) Can you attend a section different from the one you signed up for?
 (iv) When are the Vitamins due?

Solution:

(a) The course website is located at http://www.eecs70.org/.

(b)   (i) Discussion attendance and vitamins are each worth 5% of your grade, homework is worth
           20% of your grade, the midterm is worth 30%, and the final is worth 40%.
      (ii) You must attend at least 13 discussions out of 27 to get full credit for discussion atten-
           dance.
     (iii) You are welcome to attend other discussion sections, but your attendance will not be
           counted for those sections.
     (iv) Vitamins are due every Friday at 10:00 PM, with a grace period until 11:59 PM.

2   Course Policies

Go to the course website and read the course policies carefully. Leave a followup in the Homework
0, Question 2 thread on Piazza if you have any questions. Are the following situations violations
of course policy? Write "Yes" or "No", and a short explanation for each.

(a) Alice and Bob work on a problem in a study group. They write up a solution together and
    submit it, noting on their submissions that they wrote up their homework answers together.

(b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.

(c) Erin comes across a proof that is part of a homework problem while studying course material. She reads it and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.

(d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.

(e) Heidi has completed her homework using LaTeX. Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution, and Irene uses it to write her own solution with a citation to Heidi.

(f) Joe found homework solutions before they were officially released, and every time he got stuck, he looked at the solutions for a hint. He then cited the solutions as part of his submission.

Solution:

(a) Yes, this is a violation of course policy. All solutions must be written entirely by the student submitting the homework. Even if students collaborate, each student must write a unique, individual solution. In this case, both Alice and Bob would be culpable.

(b) No, this is not a violation of course policy. While sharing *written solutions* is not allowed, sharing *approaches* to problems is allowed and encouraged. Because Carol only copied down *notes*, not *Dan's solution*, and properly cited Dan's contribution, this is an actively encouraged form of collaboration.

(c) No, this is not a violation of course policy. Using external sources to help with homework problems, while less encouraged than peer collaboration, is fine as long as (i) the student makes sure to understand the solution; (ii) the student uses understanding to write a new solution, and does not copy from the external source; and (iii) the student credits the external source. However, looking up a homework problem online is a violation of course policies; the correct course of action upon finding homework solutions online is to close the tab.

(d) Yes, this is a violation of course policy, and both Frank and Grace would be culpable. Even though Frank credits Grace, written solutions should never be shared in the first place, and certainly not copied down. This is to ensure that each student learns how to write and present clear and convincing arguments. To be safe, try not to let anybody see your written solutions at any point in the course—restrict your collaboration to *approaches* and *verbal communication*.

(e) Yes, this is a violation of course policy. Once again, a citation does not make up for the fact that written solutions should never be shared, in written or typed form. In this case, both Heidi and Irene would be culpable.

(f) Yes, this is a violation of course policy. Joe should not be reading solutions before they are officially released. Instead, Joe should ask for help when he is stuck through Piazza or Office Hours.

3 Use of Piazza

Piazza is incredibly useful for Q&A in such a large-scale class. We will use Piazza for all important announcements. You should check it frequently. We also highly encourage you to use Piazza to ask questions and answer questions from your fellow students.

(a) Navigate to the "Index" Piazza post, where you can find links to most resources in the course. Write down the Piazza post number for the Note 1 Thread. (When you see @x on Piazza, where x is a positive integer, then x is the post number of the linked post.)

(b) Read the Piazza Etiquette section of the course policies and explain what is wrong with the following hypothetical student question: "Can someone explain the proof of Theorem XYZ to me?" (Assume Theorem XYZ is a complicated concept.)

(c) When are the weekly posts released? Are they required reading?

Solution:

(a) The post number for the Note 1 Thread is 15.

(b) There are two things wrong with this question. First, this question does not pass the 5-minute test. This concept takes longer than 5 minutes to explain, and therefore is better suited to Office Hours. Second, this question does not hone in on a particular concept with which the student is struggling. Questions on Piazza should be narrow, and should include every step of reasoning that led up to the question. A better question in this case might be: "I understood every step of the proof of Theorem XYZ in Note 2, except for the very last step. I tried to reason it like this, but I didn't see how it yielded the result. Can someone explain where I went wrong?"

(c) The weekly posts are released every Sunday. They're required reading.

4 Discussion Assignment

Please confirm that you have signed up for one of the discussion section at `https://tinyurl.com/cs70-sp21-dis`. What is the name of your GSI and the time of your discussion section?

Solution: Ensure that they have signed up for a valid discussion section.

5 Academic Integrity

Please write or type out the following pledge in print, and sign it.

> I pledge to uphold the university's honor code: to act with honesty, integrity, and respect for others, including their work. By signing, I ensure that all written homework I submit will be in my own words, that I will acknowledge any collaboration or help received, and that I will neither give nor receive help on any examinations.

6 Propositional Practice

In parts (a)-(c), convert the English sentences into propositional logic. In parts (d)-(f), convert the propositions into English. In part (f), let $P(a)$ represent the proposition that a is prime.

(a) There is one and only one real solution to the equation $x^2 = 0$.

(b) Between any two distinct rational numbers, there is another rational number.

(c) If the square of an integer is greater than 4, that integer is greater than 2 or it is less than -2.

(d) $(\forall x \in \mathbb{R})\, (x \in \mathbb{C})$

(e) $(\forall x, y \in \mathbb{Z})(x^2 - y^2 \neq 10)$

(f) $(\forall x \in \mathbb{N})\, [\, (x > 1) \implies (\exists a, b \in \mathbb{N})\, ((a + b = 2x) \wedge P(a) \wedge P(b))\,]$

Solution:

(a) Let $p(x) = x^2$. The sentence can be read: "There is a solution x to the equation $p(x) = 0$, and any other solution y is equal to x". Or,
$$(\exists x \in \mathbb{R})\, ((p(x) = 0) \wedge ((\forall y \in \mathbb{R})(p(y) = 0) \implies (x = y))).$$

(b) The sentence can be read "If x and y are distinct rational numbers, then there is a rational number z between x and y." Or,
$$(\forall x, y \in \mathbb{Q})((x \neq y) \implies ((\exists z \in \mathbb{Q})(x < z < y \vee y < z < x))).$$
Equivalently,
$$(\forall x, y \in \mathbb{Q})((x = y) \vee (\exists z \in \mathbb{Q})(x < z < y \vee y < z < x)).$$
Note that $x < z < y$ is mathematical shorthand for $(x < z) \wedge (z < y)$, so the above statement is equivalent to
$$(\forall x, y \in \mathbb{Q})(x = y) \vee ((\exists z \in \mathbb{Q})\, ((x < z) \wedge (z < y)) \vee ((y < z) \wedge (z < x))).$$

(c) $(\forall x \in \mathbb{Z})\, ((x^2 > 4) \implies ((x > 2) \vee (x < -2)))$

(d) All real numbers are complex numbers.

(e) There are no integer solutions to the equation $x^2 - y^2 = 10$.

(f) For any natural number greater than 1, there are some prime numbers a and b such that $2x = a + b$.

　　In other words: Any even integer larger than 2 can be written as the sum of two primes.

　　Aside: This statement is known as Goldbach's Conjecture, and it is a famous unsolved problem in number theory (https://xkcd.com/1310/).

7　Implication

Which of the following assertions are true no matter what proposition Q represents? For any false assertion, state a counterexample (i.e. come up with a statement $Q(x,y)$ that would make the implication false). For any true assertion, give a brief explanation for why it is true.

(a) $\exists x \exists y Q(x,y) \implies \exists y \exists x Q(x,y)$.

(b) $\forall x \exists y Q(x,y) \implies \exists y \forall x Q(x,y)$.

(c) $\exists x \forall y Q(x,y) \implies \forall y \exists x Q(x,y)$.

(d) $\exists x \exists y Q(x,y) \implies \forall y \exists x Q(x,y)$.

Solution:

(a) True. There exists can be switched if they are adjacent; $\exists x, \exists y$ and $\exists y, \exists x$ means there exists x and y in our universe.

(b) False. Let $Q(x,y)$ be $x < y$, and the universe for x and y be the integers. Or let $Q(x,y)$ be $x = y$ and the universe be any set with at least two elements. In both cases, the antecedent is true and the consequence is false, thus the entire implication statement is false.

(c) True. The first statement says that there is an x, say x' where for every y, $Q(x,y)$ is true. Thus, one can choose $x = x'$ for the second statement and that statement will be true again for every y. Note: 4c and 4d are not logically equivalent. In fact, the converse of 4d is 4c, which we saw is false.

(d) False. Suppose Q is the statement "y is 5, and x is any integer". The antecedent is true when $y = 5$, but for $y \neq 5$, there is no x that will make it true.

8 Logical Equivalence?

Decide whether each of the following logical equivalence is correct and justify your answer.

(a) $\forall x \left(P(x) \wedge Q(x) \right) \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$

(b) $\forall x \left(P(x) \vee Q(x) \right) \equiv \forall x\, P(x) \vee \forall x\, Q(x)$

(c) $\exists x \left(P(x) \vee Q(x) \right) \equiv \exists x\, P(x) \vee \exists x\, Q(x)$

(d) $\exists x \left(P(x) \wedge Q(x) \right) \equiv \exists x\, P(x) \wedge \exists x\, Q(x)$

Solution:

(a) **Correct**.

Assume that the LHS is true. Then we know for an arbitrary x $P(x) \wedge Q(x)$ is true. This means that both $\forall x P(x)$ and $\forall x Q(x)$. Therefore the RHS is true. Now assume the RHS. Since for any x $P(x)$ and for any y $Q(y)$ holds, then for an arbitrary x both $P(x)$ and $Q(x)$ must be true. Thus the LHS is true.

(b) **Incorrect**. If $P(1)$ is true, $Q(1)$ is false, $P(2)$ is false and $Q(2)$ is true, the left-hand side will be true, but the right-hand side will be false.

(c) **Correct**

Assuming that the LHS is true, we know there exists some x such that one of $P(x)$ and $Q(x)$ is true. Thus $\exists x P(x)$ or $\exists x Q(x)$ and the RHS is true. To prove the other direction, assume the LHS is false. Then there does not exists an x for which $P(x) \vee Q(x)$ is true, which means there is no x for which $P(x)$ or $Q(x)$ is true. Therefore the RHS is false.

(d) **Incorrect**. If $P(1)$ is true and $P(x)$ is false for all other x, and $Q(2)$ is true and $Q(x)$ is false for all other x, the right hand side would be true. However, there would be no value of x at which both $P(x)$ and $Q(x)$ would be simultaneously true.