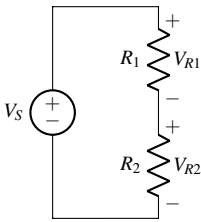
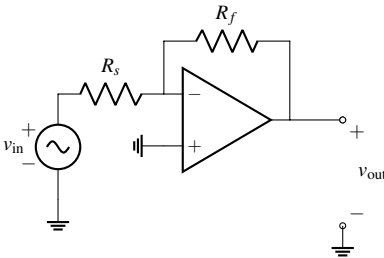
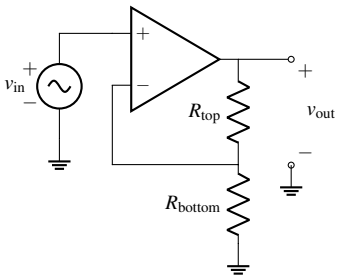
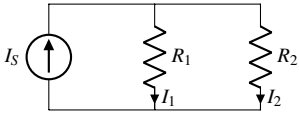
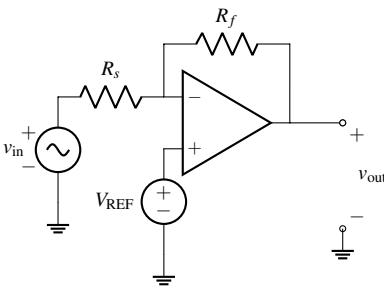
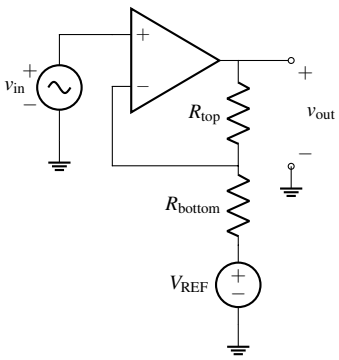
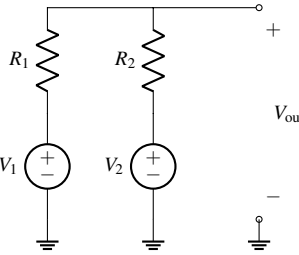
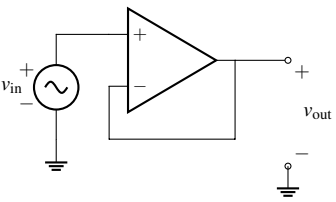


CSM 16A  
Fall 2020

Designing Information Devices and Systems I

Week 10

### Reference: Op-Amp Example Circuits

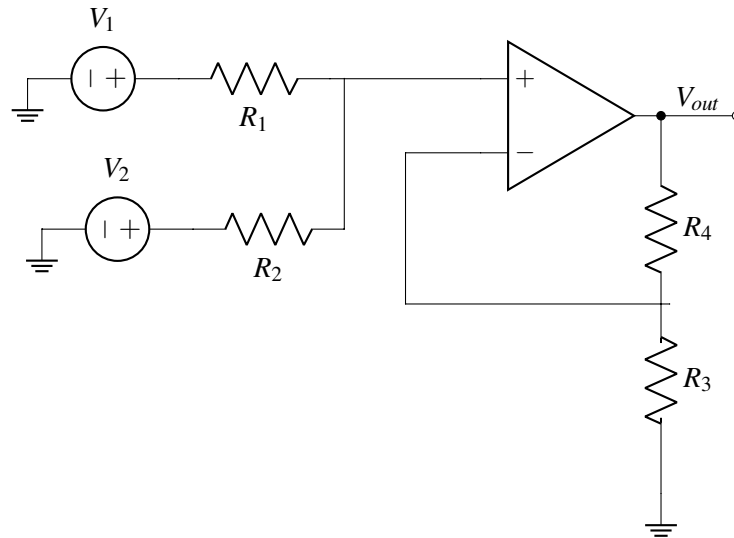
<p>Voltage Divider</p>  $V_{R_2} = V_S \left( \frac{R_2}{R_1 + R_2} \right)$	<p>Inverting Amplifier</p>  $v_{out} = v_{in} \left( -\frac{R_f}{R_s} \right)$	<p>Noninverting Amplifier</p>  $v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right)$
<p>Current Divider</p>  $I_1 = I_S \left( \frac{R_2}{R_1 + R_2} \right)$	<p>Inverting Amplifier with Reference</p>  $v_{out} = v_{in} \left( -\frac{R_f}{R_s} \right) + V_{REF} \left( \frac{R_f}{R_s} + 1 \right)$	<p>Noninverting Amplifier with Reference</p>  $v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left( \frac{R_{top}}{R_{bottom}} \right)$
<p>Voltage Summer</p>  $V_{out} = V_1 \left( \frac{R_2}{R_1 + R_2} \right) + V_2 \left( \frac{R_1}{R_1 + R_2} \right)$	<p>Unity Gain Buffer</p>  $v_{out} = v_{in}$	

## 1. Voltage Summers

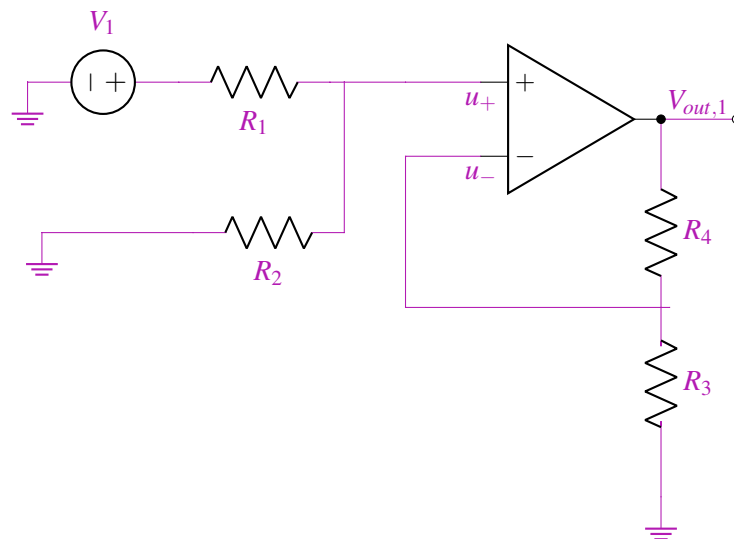
**Learning Goal:** This problem uses basic circuit analysis techniques to find the response of a summer circuit.

**Relevant Notes:** [Note 19](#) goes different op-amp circuit topology and corresponding derivations.

(a) Calculate  $V_{out}$  in terms of  $V_1$  and  $V_2$ . Assume that  $R_1 = R_2$ . Use superposition.



**Answer:** Let's first consider the case when only  $V_1$  is active. We deactivate voltage source  $V_2$  by replacing it with a wire:



We see that  $u_+$  is the middle node of a voltage divider with resistors  $R_1$  and  $R_2$ . We also know that input current  $I_+ = 0$ . So we can find  $u_+$  using:

$$u_+ = \frac{R_2}{R_1 + R_2} V_1.$$

Similarly  $u_-$  is the middle node of a voltage divider with resistors  $R_3$  and  $R_4$ . We know that input current  $I_- = 0$ . We can find  $u_-$  as:

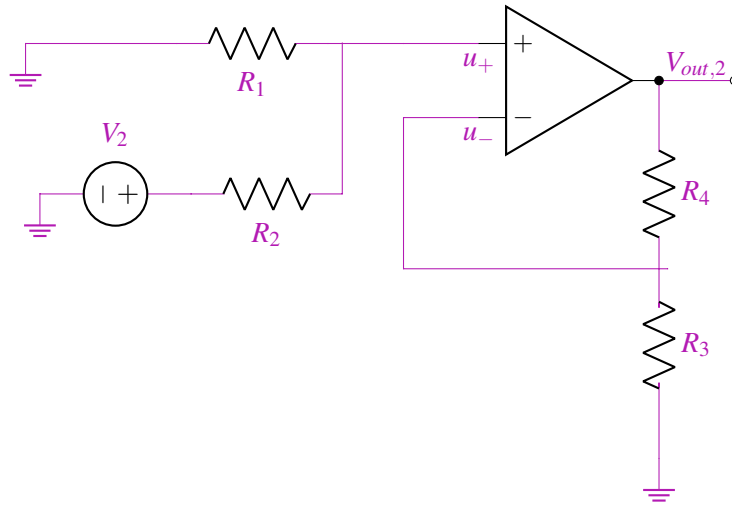
$$u_- = \frac{R_3}{R_3 + R_4} V_{out,1}.$$

Now we have to check if the circuit is in negative feedback. If we move the negative input of the op amp  $u_-$  upward,  $V_{out,1} = A(u_+ - u_-)$  moves downward and as a result  $u_- = \frac{R_3}{R_3 + R_4} V_{out,1}$  moves downward. So the result of the initial stimulus goes in the opposite direction of the initial stimulus, which is the requirement for negative feedback.

Since the circuit is in negative feedback, we can apply the second golden rule:

$$\begin{aligned} u_+ &= u_- \\ \Rightarrow \frac{R_2}{R_1 + R_2} V_1 &= \frac{R_3}{R_3 + R_4} V_{out,1} \\ \Rightarrow V_{out,1} &= \left(1 + \frac{R_4}{R_3}\right) \frac{R_2}{R_1 + R_2} V_1 \end{aligned}$$

Next we consider the case when only  $V_2$  is active. We deactivate voltage source  $V_1$  by replacing it with a wire:



Following a similar process we have:

$$\begin{aligned} u_+ &= \frac{R_1}{R_1 + R_2} V_2; \\ u_- &= \frac{R_3}{R_3 + R_4} V_{out,2}. \end{aligned}$$

(Note that  $u_+$  is now the voltage across  $R_1$ , while for the previous scenario  $u_+$  was the voltage across  $R_2$ .)

Using the second golden rule we have:

$$\begin{aligned} u_+ &= u_- \\ \Rightarrow \frac{R_1}{R_1 + R_2} V_2 &= \frac{R_3}{R_3 + R_4} V_{out,2} \\ \Rightarrow V_{out,2} &= \left(1 + \frac{R_4}{R_3}\right) \frac{R_1}{R_1 + R_2} V_2 \end{aligned}$$

Now using the superposition theorem, we can find  $V_{out}$ :

$$\begin{aligned} V_{out} &= V_{out,1} + V_{out,2} \\ \Rightarrow V_{out} &= \left(1 + \frac{R_4}{R_3}\right) \frac{R_2}{R_1 + R_2} V_1 + \left(1 + \frac{R_4}{R_3}\right) \frac{R_1}{R_1 + R_2} V_2 \end{aligned}$$

**Note that a similar expression can be reached by using the equations for the voltage summer and the non-inverting amplifier. Pattern matching this circuit with the voltage summer and the non-inverting amplifier and then employing the given equations is a valid technique to solve this problem.**

- (b) What values should we select for  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$  such that  $V_{out} = V_1 + 2V_2$ ?

**Answer:** From the last part, we have

$$V_{out} = \left(1 + \frac{R_4}{R_3}\right) \frac{R_2}{R_1 + R_2} V_1 + \left(1 + \frac{R_4}{R_3}\right) \frac{R_1}{R_1 + R_2} V_2$$

Comparing with  $V_{out} = V_1 + 2V_2$ , we have:

$$\left(1 + \frac{R_4}{R_3}\right) \frac{R_2}{R_1 + R_2} = 1 \quad (1)$$

$$\left(1 + \frac{R_4}{R_3}\right) \frac{R_1}{R_1 + R_2} = 2 \quad (2)$$

Dividing equation (1) with equation (2) we get:

$$\begin{aligned} \frac{R_2}{R_1} &= \frac{1}{2} \\ R_1 &= 2R_2 \end{aligned}$$

Now if we choose  $R_2 = 100\Omega$  and  $R_1 = 200\Omega$ , we have

$$\frac{R_2}{R_1 + R_2} = \frac{100}{200 + 100} = \frac{1}{3} \quad (3)$$

$$\frac{R_1}{R_1 + R_2} = \frac{200}{200 + 100} = \frac{2}{3} \quad (4)$$

Plugging in the values of equation (3) in equation (1) (or equation (4) in equation (2)), we have:

$$\begin{aligned} \left(1 + \frac{R_4}{R_3}\right) \frac{1}{3} &= 1 \\ 1 + \frac{R_4}{R_3} &= 3 \\ \frac{R_4}{R_3} &= 2 \\ R_4 &= 2R_3 \end{aligned}$$

We can choose  $R_3 = 100\Omega$  and  $R_4 = 200\Omega$ , so that the output voltage is:

$$V_{out} = \left(1 + \frac{200}{100}\right) \frac{100}{200 + 100} V_1 + \left(1 + \frac{200}{100}\right) \frac{200}{200 + 100} V_2 = (1 + 2) \frac{1}{3} V_1 + (1 + 2) \frac{2}{3} V_2 = V_1 + 2V_2.$$

## 2. Multi-stage Amplifier

**Learning Goal:** The objective of this problem is to understand how multiple stages of op-amp circuits can be used to achieve a specific circuit gain.

**Relevant Notes:** [Note 19 Section 19.5](#) goes over inverting and non-inverting amplifiers.

- (a) What is the range of values that we can scale  $V_{in}$  by when using a non-inverting op amp? (What are possible values for the gain?)

**Answer:** Recall that when using a non-inverting op amp, the equation for  $V_{out}$  is given by

$$V_{out} = V_{in} \left( 1 + \frac{R_2}{R_1} \right)$$

where  $R_2$  is the upper resistor and  $R_1$  is the lower one (connected to ground). The circuit gain  $G$  is represented by

$$G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$$

We can choose any values for resistance from  $[0, \infty)$ ; then, the minimum gain would be 1 if we chose  $R_2 = 0$ ; the maximum gain approaches infinity as we choose some very large  $R_2$  and/or a very small  $R_1$ . Hence, the range of gains is from 1 to infinity, and our range of values for  $V_{out}$  is  $[V_{in}, \infty)$ .

- (b) What is the range of values that we can scale  $V_{in}$  by when using an inverting op amp? (What are the possible values for the gain?)

**Answer:** Recall that when using an inverting op amp, the equation for  $V_{out}$  is given by

$$V_{out} = -V_{in} \frac{R_2}{R_1}$$

. The circuit gain is represented by

$$G = -\frac{R_2}{R_1}$$

.

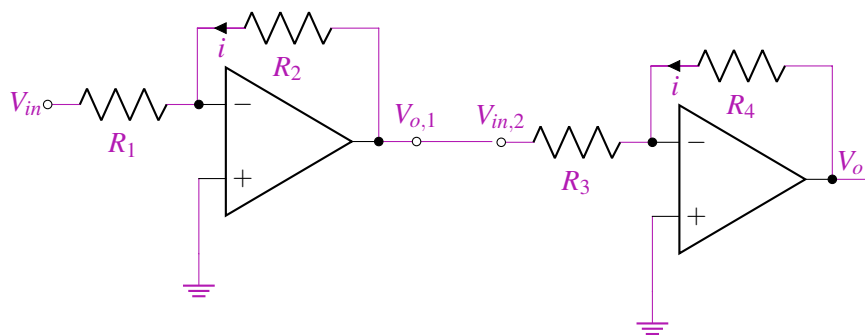
Again, we can choose any values for resistance from  $[0, \infty)$ ; then, the minimum absolute value scaling would be 0 if we chose  $R_2 = 0$ ; the maximum absolute value scaling approaches infinity as we choose some very large  $R_2$  and/or a very small  $R_1$ . Hence, the range of gains is from 0 to  $-\infty$ , and our range of values for  $V_{out}$  is also  $(-\infty, 0]$ .

- (c) Can you design a single inverting/ non-inverting amplifier with circuit gain  $G = 0.5$ ? If not, what range of gain values is not reachable using a single inverting op amp or a single non-inverting op amp?

**Answer:** From part (a), we found that a non-inverting op amp can only reach gain values from  $[1, \infty)$ . From part (b), we found that an inverting op amp can only reach gain values from  $(-\infty, 0]$ . So using only a single op-amp, we cannot reach values between  $(0, 1)$ , specifically  $G = 0.5$ .

- (d) How would you construct a circuit using inverting/ non-inverting amplifiers so that the overall circuit gain is  $G = 0.5$ ?

**Answer:** We can use two inverting op-amps to achieve overall gain  $G = 0.5$ . Say the first op-amp has gain  $G_1$  and the second op-amp  $G_2$ , so  $G = G_1 G_2$ . From part (b), these gains can take on values from  $(-\infty, 0]$ , so multiplying them together helps reach  $G = 0.5$ .



For the first inverting op-amp, we have  $G_1 = -\frac{R_2}{R_1}$ , and for the second,  $G_2 = -\frac{R_4}{R_3}$ . Plugging these into the equation for  $G$ :

$$G = G_1 G_2 = \frac{R_2 R_4}{R_1 R_3} = 0.5$$

$$R_1 R_3 = 2 R_2 R_4$$

To reach our desired gain, we can pick any combination of  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$  that satisfies this equation. One such solution is  $R_1 = 200\Omega$  and  $R_2 = R_3 = R_4 = 100\Omega$ .

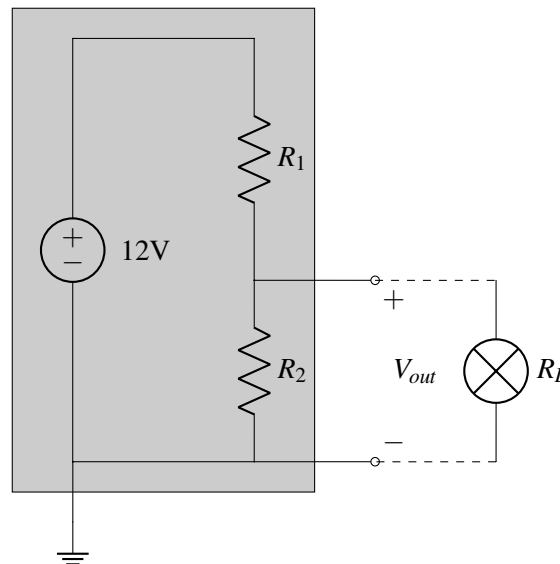
### 3. Op Amps as Buffers

**Learning Goal:** This problem helps understand the operating principle of an op-amp buffer and how it helps with loading.

**Relevant Notes:** [Note 19 Section 19.7](#) goes through different op-amp circuit topologies and corresponding derivations.

Now we will revisit a problem that you might have seen before, with our new knowledge of op-amps. We have access to a circuit inside a 'black box' as shown below, with two terminals coming out of it.

- (a) We need a voltage of 6V to power a light bulb with resistance  $R_L$ . Design  $R_1$  and  $R_2$  inside the black box so that the voltage across  $R_2$  is exactly equal to this required voltage **when the bulb is not connected**; i.e.  $V_{R_2} = V_{out} = 6V$ .



**Answer:** The voltage across  $R_2$  is given by

$$V_{R_2} = \frac{R_2}{R_1 + R_2} \times 12V$$

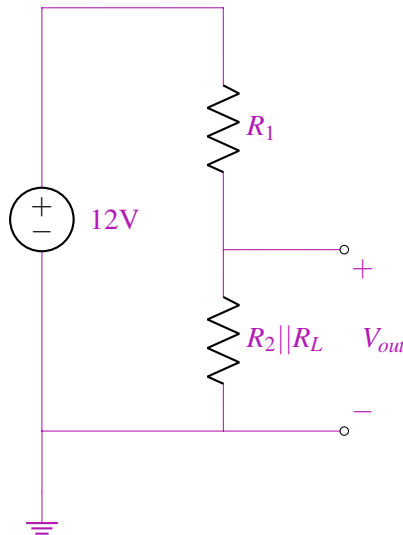
If we set  $V_{R_2} = 6V$ , we get

$$\begin{aligned} 6V &= \frac{R_2}{R_1 + R_2} \times 12V \\ \implies R_1 &= R_2 \end{aligned}$$

For example, we can choose  $R_1 = R_2 = 1k\Omega$ .

- (b) Now let us connect the bulb  $R_L$  across  $R_2$ . What is the voltage across  $R_1$ ,  $R_2$  and  $R_L$  when the bulb is connected when  $R_L = R_2$ ? Use the values of  $R_1$  and  $R_2$  from the last part. Will the light bulb turn on? What happens if  $R_L = 2R_2$ ?

**Answer:** When we connect the bulb across  $R_2$ , the equivalent circuit is the following:



The voltage  $V_{out}$  given by

$$V_{out} = \frac{R_2 || R_L}{R_1 + R_2 || R_L} \times 12V$$

When  $R_L = R_2$ :

$$R_2 || R_L = R_2 || R_2 = \frac{R_2}{2}.$$

Hence  $V_{out}$  is:

$$V_{out} = \frac{R_2 || R_L}{R_1 + R_2 || R_L} \times 12V = \frac{\frac{R_2}{2}}{R_1 + \frac{R_2}{2}} \times 12V = \frac{\frac{R_2}{2}}{R_2 + \frac{R_2}{2}} \times 12V = 4V < 6V$$

So the bulb will not turn on.

When  $R_L = 2R_2$ :

$$R_2 || R_L = R_2 || 2R_2 = \frac{2R_2}{3}.$$

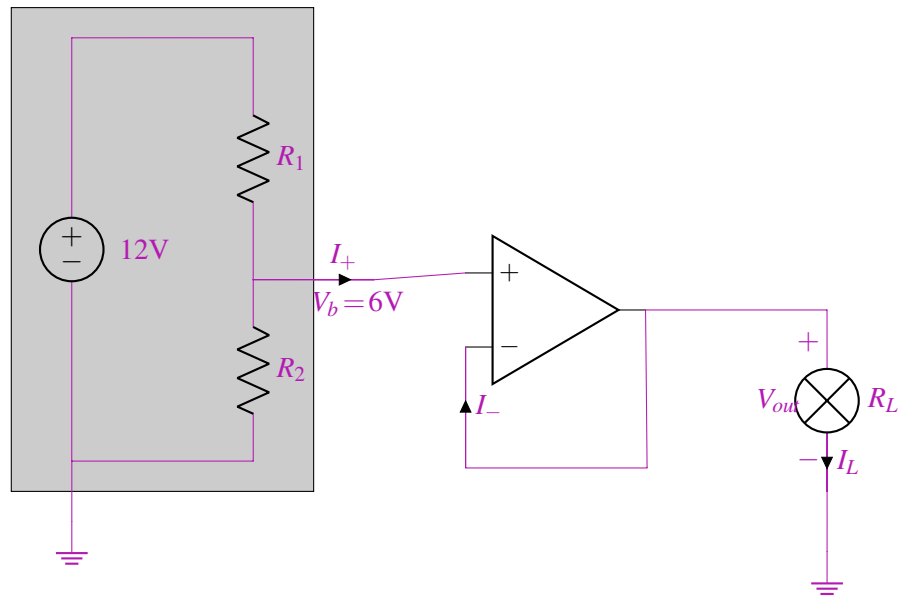
Hence  $V_{out}$  is:

$$V_{out} = \frac{R_2 || R_L}{R_1 + R_2 || R_L} \times 12V = \frac{\frac{2R_2}{3}}{R_1 + \frac{2R_2}{3}} \times 12V = \frac{\frac{2R_2}{3}}{R_2 + \frac{2R_2}{3}} \times 12V = 4.8V < 6V$$

Although,  $V_{out}$  is now closer to 6V, the bulb will still not turn on.

- (c) Using your knowledge of op-amps, how could you resolve this issue of  $V_{out}$  changing based on the value of  $R_L$ ? Think about how you might use an op-amp buffer.

**Answer:** We can introduce a 'buffer' op-amp as shown below:



A buffer op amp effectively 'decouples' its input and output, It does this by preventing the bulb from drawing current from the black box circuit. Remember that the input currents of an ideal op-amp:

$$I_+ = I_- = 0,$$

while the output current  $I_L$  can adjust itself to any extent, depending on the demand placed by  $V_{out}$ .

Let us use nodal analysis and the golden rules to formally solve this circuit. Firstly, we observe that the op-amp is in negative feedback configuration. Using the golden rules, we have that  $V_+ = V_- = V_b = 6V$ . Also, because the feedback connection is a short,  $V_{out} = V_-$ . Therefore,  $V_{out} = V_b = 6V$ . This is exactly what we want! The voltage across  $R_L$  equals  $V_{out} = 6V$ , which is above the required voltage.