CS 61A Recursion and Midterm Review

Spring 2021 Lost 3: February 12, 2021 Solutions

1 Learning Goals

- Understand the general idea behind recursion
- Understand how to structure recursive functions
- Understand the general structure of counting problems and how to solve them
- Understand how to approach exam-level problems for various topics

2 Recursion Overview

2.1 What are three things you find in every recursive function?

1) Base Case(s)
2) Way(s) to reduce the problem into a smaller problem of the same type
3) Recursive case(s) that uses the solution of the smaller problem to solve the original (large) problem

2.2 When you write a Recursive function, you seem to call it before it has been fully defined. Why doesn't this break the Python interpreter?

When you define a function, Python does not evaluate the body of the function.

2.3 Below is a Python function that computes the nth Fibonacci number. Identify the three things it contains as a recursive function (from 1.1).

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

Domain is integers, range is integers.
Base Cases: if n == 0: ..., elif n == 1: ...
Finding Smaller Problems: finding fib(n - 1), fib(n - 2)
Recursive Case: when n is neither 0 nor 1, add together the fib(n - 1) and fib(n - 2) to find fib(n)

2.4 With the definition of the Fibonacci function above, draw out a diagram of the recursive calls made when **fib(4)** is called.

```
              fib(4)
              /    \
          fib(3)   fib(2)
         /    |     |    \
     fib(2) fib(1) fib(1) fib(0)
    /    |
fib(1)  fib(0)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

2.5 What does the following function **cascade2** do? What is its domain and range?

```
def cascade2(n):
    print(n)
    if n >= 10:
        cascade2(n//10)
        print(n)
```

Domain is integers, range is None. It takes in a number n and prints out n, then prints out n excluding the ones digit, then prints n excluding the hundreds digit, and so on, then back up to the full number.

3 Exam-Level Recursion + Lambda

3.1 **Fall 2016 Midterm 1, Question 5** An order 1 numeric function is a function that takes a number and returns a number. An order 2 numeric function is a function that takes a number and returns an order 1 numeric function. Likewise, an order n numeric function is a function that takes a number and returns an order $n - 1$ numeric function. The argument sequence of a nested call expression is the sequence of all arguments in all subexpressions, in the order they appear. For example, the expressionf(3)(4)(5)(6)(7)has the argument sequence 3, 4, 5, 6, 7.

Implement multiadder, which takes a positive integer n and returns an order n numeric function that sums an argument sequence of length n.

```
def multiadder(n):
    """Return a function that takes N arguments, one at a time, and adds them.
    >>> f = multiadder(3)
    >>> f(5)(6)(7)                # 5 + 6 + 7
    18
    >>> multiadder(1)(5)
    5
    >>> multiadder(2)(5)(6)       # 5 + 6
    11
    >>> multiadder(4)(5)(6)(7)(8) # 5 + 6 + 7 + 8
    26
    """

    assert n > 0


    if ________________________________________________________________________:


        return ________________________________________________________________


    else:


        return ________________________________________________________________
```

Complete the expression below by writing one integer in each blank so that the whole expression evaluates to 2016. Assume multiadder is implemented correctly.

```
def compose1(f, g):
    """Return the composition function which given x, computes f(g(x)).

    >>> add_one = lambda x: x + 1        # adds one to x
    >>> square = lambda x: x**2
    >>> a1 = compose1(square, add_one)   # (x + 1)ˆ2
    >>> a1(4)
    25
    >>> mul_three = lambda x: x * 3      # multiplies 3 to x
    >>> a2 = compose1(mul_three, a1)    # ((x + 1)ˆ2) * 3
    >>> a2(4)
    75
    >>> a2(5)
    108
    """
    return lambda x: f(g(x))
```

```
compose1(multiadder(_______)(1000), multiadder(_______)(10)(_______))(1)(2)(3)
```

```
def multiadder(n):
   """Return a function that takes N arguments, one at a time, and adds them.
    >>> f = multiadder(3)
    >>> f(5)(6)(7)                # 5 + 6 + 7
    18
    >>> multiadder(1)(5)
    5
    >>> multiadder(2)(5)(6)       # 5 + 6
    11
    >>> multiadder(4)(5)(6)(7)(8) # 5 + 6 + 7 + 8
    26
    """

    assert n > 0



    if n == 1:



        return lambda x: x
    else:


        return lambda a: lambda b: multiadder(n-1)(a+b)
```

compose1(multiadder(4)(1000), multiadder(3)(10)(1000))(1)(2)(3)

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

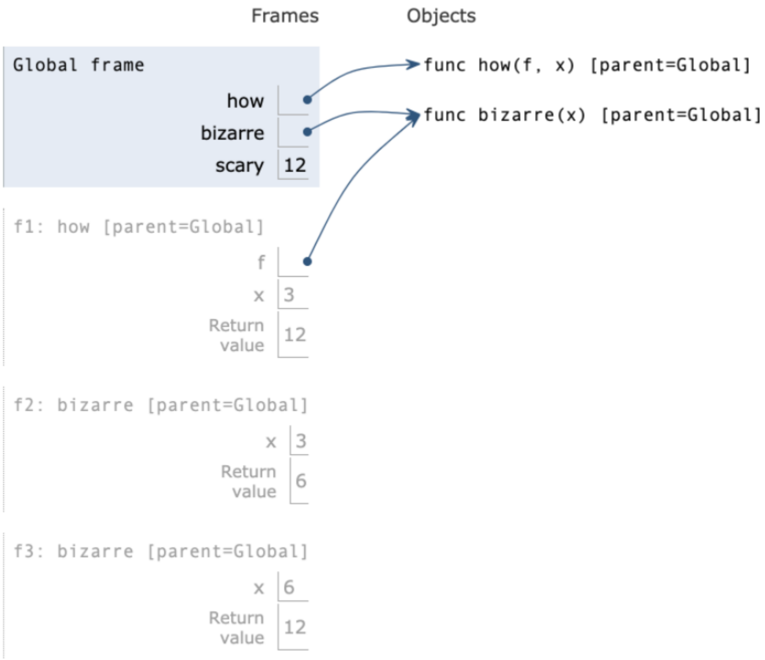
4 Reverse Environment Diagram Practice

4.1 Fill in the lines below so that the execution of the program would lead to the environment diagram below. You may not use any numbers in any blanks.

```
def how(f, x):
    return ________________

def bizarre(___):
    return 2 * _______

scary = ________(________, 3)
```


Frames
Objects
Global frame
how
bizarre
scary
12
func how(f, x) [parent=Global]
func bizarre(x) [parent=Global]
f1: how [parent=Global]
f
x
3
Return value
12
f2: bizarre [parent=Global]
x
3
Return value
6
f3: bizarre [parent=Global]
x
6
Return value
12

```
def how(f, x):
    return f(f(x))

def bizarre(x):
    return 2 * x

scary = how(bizarre, 3)
```

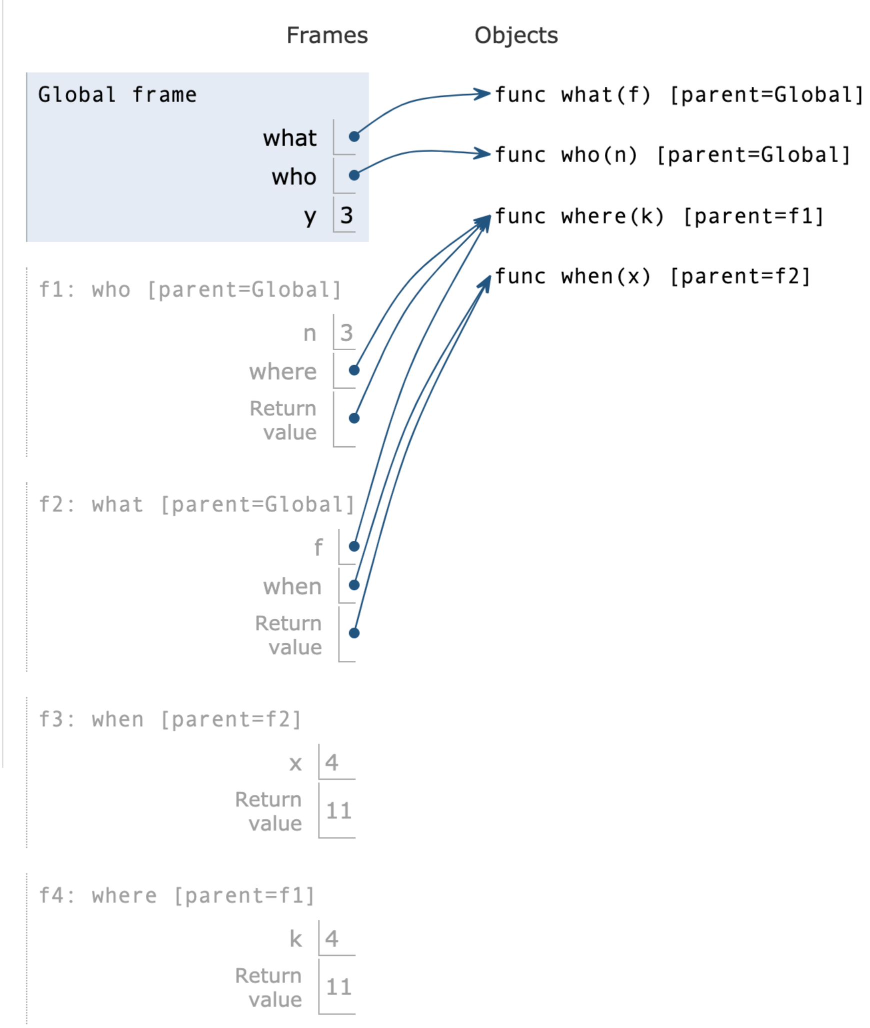
Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

4.2 Fill in the lines below so that the execution of the program would lead to the environment diagram below. You may not use any numbers in any blanks.

```
def what(_______):
    def _______(x):
        return ________
    return __________

def who(n):
    def __________(k):
        return 2 * k + n
    return ___________

y = 3
_____(____(____))(4)
```



```
def what(f):
    def when(x):
        return f(x)
    return when

def who(n):
    def where(k):
        return 2 * k + n
    return where

y = 3
what(who(y))(4)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.