

EECS 151/251A
Spring 2021
Digital Design and
Integrated Circuits

Instructor:
Wawrzynek

Lecture 15 - Exam1 Review

Announcements

- Virtual Front Row for today 3/9:
 - James Shi
 - Parth Nobel
 - Zitao Fang
 - Daniel Guzman
- More questions/comments please!
- ***No class Tuesday. Exam 6-9PM***
- Homework #6 assignment solutions posted today - part of exam 1.
- No problem set due Monday.

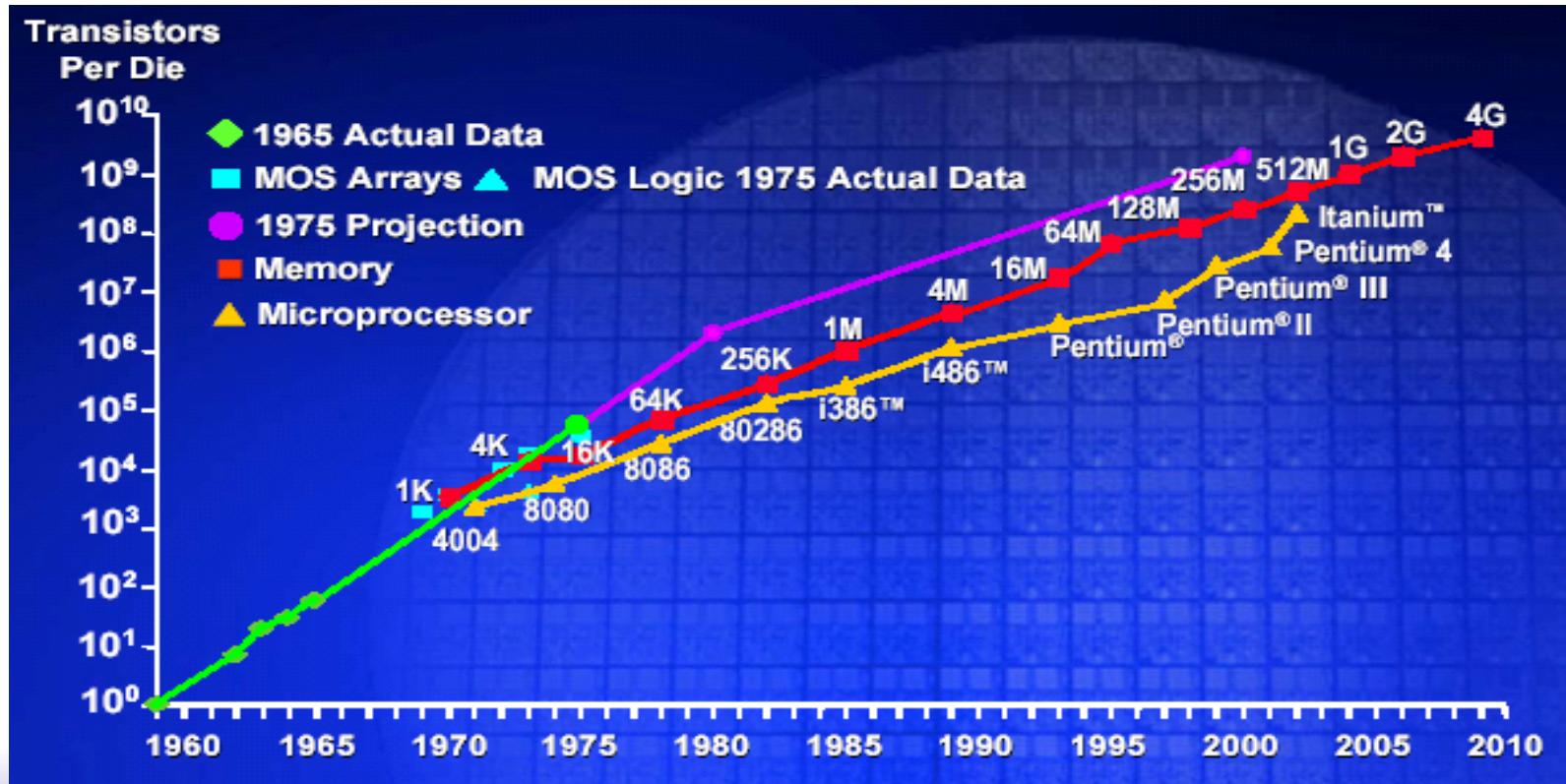
Exam 1 Details - Thur 3/11 6-9PM

- ❑ Open-book, notes, on-line references.
 - ❑ *Can't talk or communicate about the exam with anyone during the exam except for course staff.*
- ❑ Need to apply concepts from class and homework
 - ❑ you'll need to figure out what concept to apply and how
- ❑ Homework problems a good indicator of topic priority
- ❑ Previous exams posted, to understand my style
- ❑ zoom proctoring
 - ❑ answer your questions
 - ❑ help keep you honest
 - ❑ check piazza for details of procedure
- ❑ Remember, course *not curved*. (You do not need to cheat to be competitive)

Review with sample slides

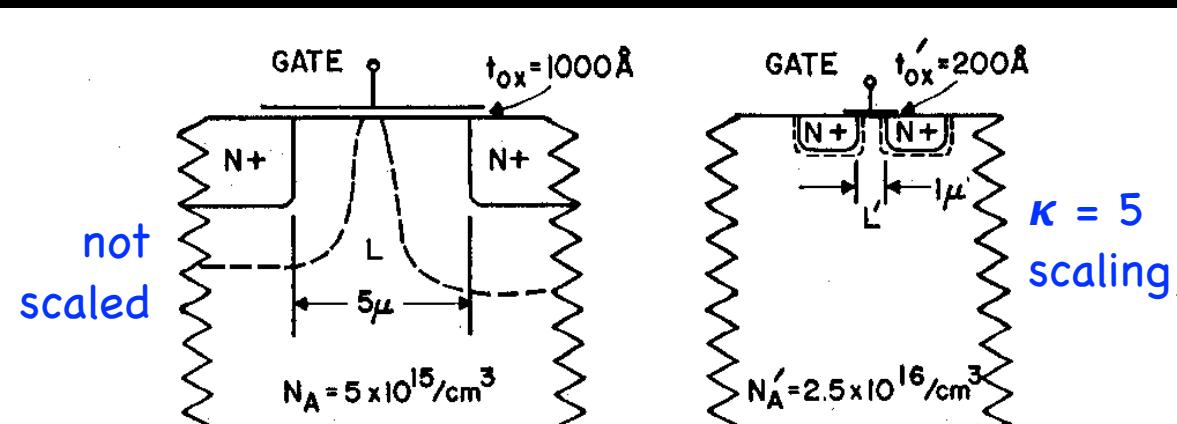
- Do not study only the following slides.
These are just representative of what you need to know.
- Go back and study the entire lecture.
- A list of important topics (more comprehensive than this set of slides) is posted on website:
 - “exam1-information.pdf”

Moore's Law – 2x transistors per 1-2 yr



Dennard Scaling

Things we do: scale dimensions, doping, Vdd.



What we get:
 κ^2 as many transistors
at the same power density!

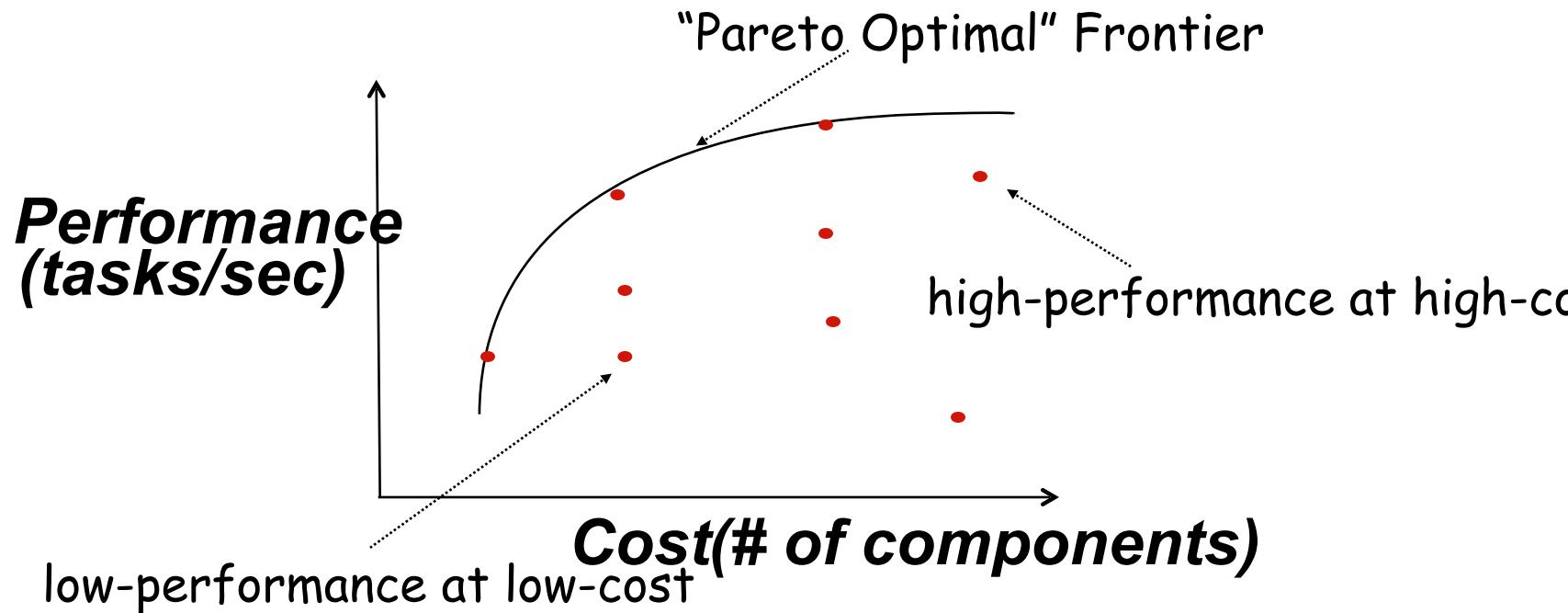
Whose gates switch κ times faster!

TABLE I
SCALING RESULTS FOR CIRCUIT PERFORMANCE

Device or Circuit Parameter	Scaling Factor
Device dimension t_{ox} , L , W	$1/\kappa$
Doping concentration N_a	κ
Voltage V	$1/\kappa$
Current I	$1/\kappa$
Capacitance $\epsilon A/t$	$1/\kappa$
Delay time/circuit VC/I	$1/\kappa$
Power dissipation/circuit VI	$1/\kappa^2$
Power density VI/A	1

Power density scaling ended in 2003
(Pentium 4: 3.2GHz, 82W, 55M FETs).

Design Space & Optimality



Cost

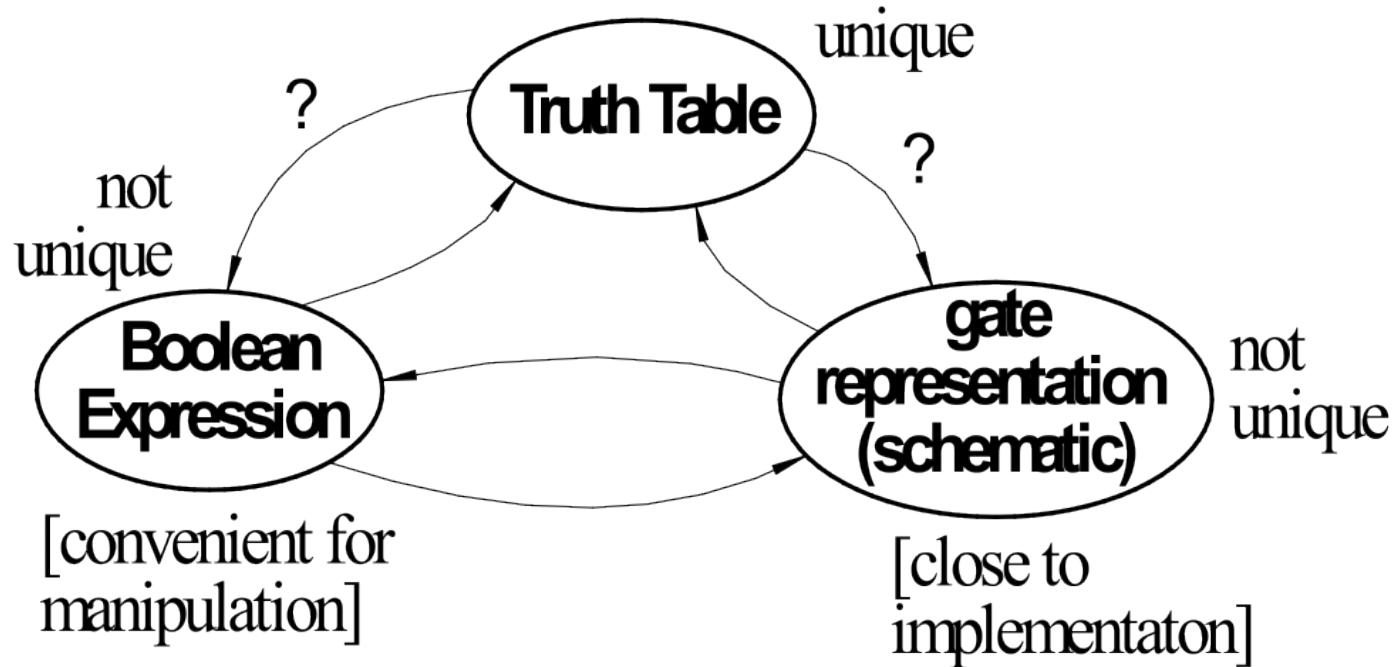
- **Non-recurring** engineering (NRE) costs
- Cost to develop a design (product)
 - Amortized over all units shipped
 - E.g. \$20M in development adds \$.20 to each of 100M units
- **Recurring** costs
 - Cost to manufacture, test and package a unit
 - Processed wafer cost is ~10k (around 16nm node) which yields:
 - 50-100 large FPGAs or GPUs
 - 200 laptop CPUs
 - >1000 cell phone SoCs

$$\text{cost per IC} = \text{variable cost per IC} + \frac{\text{fixed cost}}{\text{volume}}$$

$$\text{variable cost} = \frac{\text{cost of die} + \text{cost of die test} + \text{cost of packaging}}{\text{final test yield}}$$

Relationship Among Representations

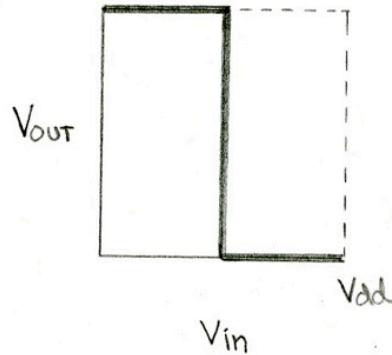
- * Theorem: Any Boolean function that can be expressed as a truth table can be written as an expression in Boolean Algebra using AND, OR, NOT.



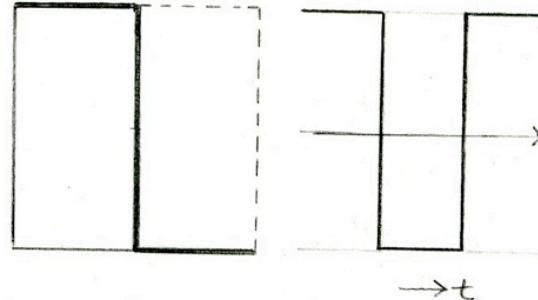
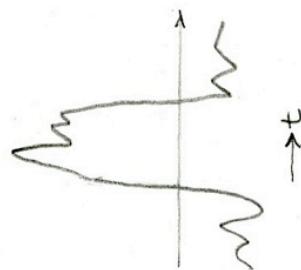
How do we convert from one to the other?

Inverter Example of Restoration

Example (look at 1-input gate, to keep it simple):



Idealize Inverter



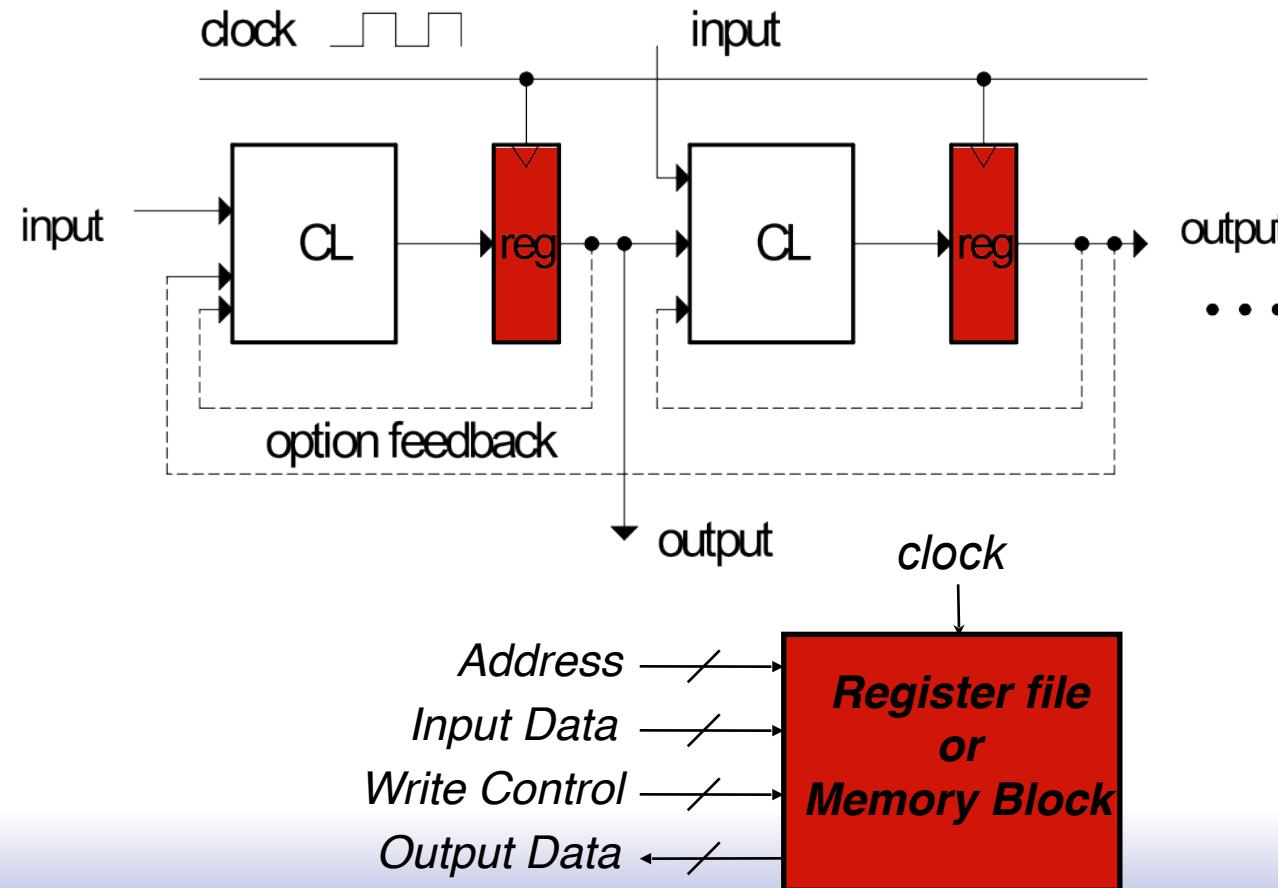
Actual Inverter

- Inverter acts like a “non-linear” amplifier
- The non-linearity is critical to restoration
- Other logic gates act similarly with respect to input/output relationship.

Register Transfer Level Abstraction (RTL)

Any synchronous digital circuit can be represented with:

- Combinational Logic Blocks (CL), plus
- State Elements (registers or memories)



- State elements are mixed in with CL blocks to control the flow of data.
- Sometimes used in large groups by themselves for “long-term” data storage.

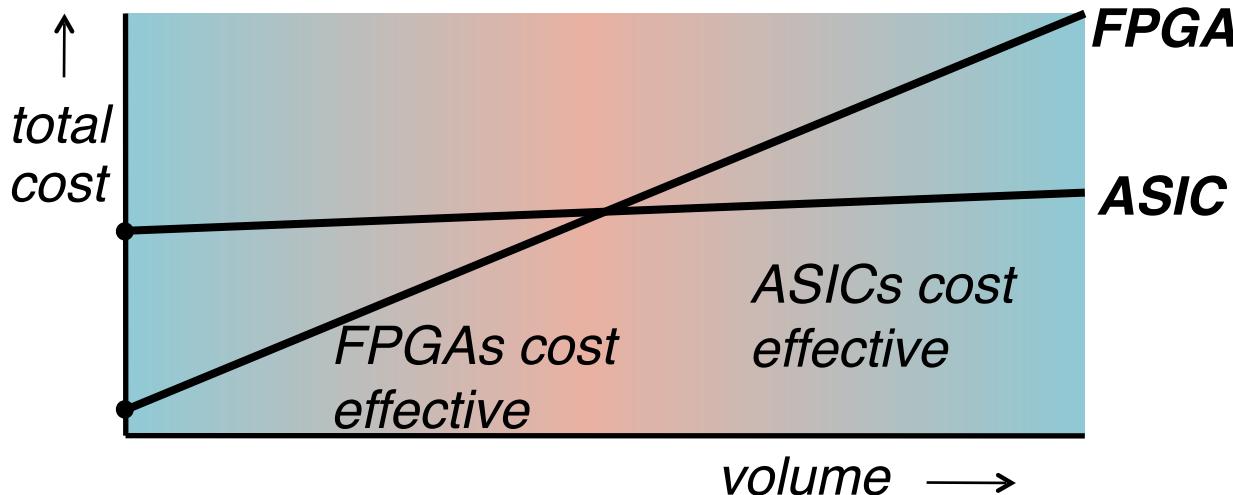
Implementation Alternative Summary

Full-custom:	All circuits/transistors layouts optimized for application.
Standard-cell:	Small function blocks/"cells" (gates, FFs) automatically placed and routed.
Gate-array (structured ASIC):	Partially prefabricated wafers with arrays of transistors customized with metal layers or vias.
FPGA:	Prefabricated chips customized with loadable latches or fuses.
Microprocessor:	Instruction set interpreter customized through software.
Domain Specific Processor:	Special instruction set interpreters (ex: DSP, NP, GPU).

These days, “ASIC” almost always means Standard-cell.

What are the important metrics of comparison?

FPGA versus ASIC



- **ASIC:** Higher NRE costs (10's of \$M). Relatively Low cost per die (10's of \$ or less).
- **FPGAs:** Low NRE costs. Relatively low silicon efficiency ⇒ high cost per part (> 10's of \$ to 1000's of \$).
- **Cross-over volume** from cost effective FPGA design to ASIC was often in the 100K range.

Hardware Description Languages

- Basic Idea:

- Language constructs describe circuits with two basic forms:
- **Structural descriptions:** connections of components. Nearly one-to-one correspondence to with schematic diagram.
- **Behavioral descriptions:** use high-level constructs (similar to conventional programming) to describe the circuit function.

- Originally invented for simulation.

- “logic synthesis” tools exist to automatically convert to gate level representation.
- High-level constructs greatly improves designer productivity.
- However, this may lead you to falsely believe that hardware design can be reduced to writing programs*

“Structural” example:

```
Decoder(output x0,x1,x2,x3;  
        inputs a,b)  
{  
    wire abar, bbar;  
    inv(bbar, b);  
    inv(abar, a);  
    and(x0, abar, bbar);  
    and(x1, abar, b );  
    and(x2, a,      bbar);  
    and(x3, a,      b );  
}
```

“Behavioral” example:

```
Decoder(output x0,x1,x2,x3;  
        inputs a,b)  
{  
    case [a b]  
        00: [x0 x1 x2 x3] = 0x8;  
        01: [x0 x1 x2 x3] = 0x4;  
        10: [x0 x1 x2 x3] = 0x2;  
        11: [x0 x1 x2 x3] = 0x1;  
    endcase  
}
```

Warning: this is a fake HDL!

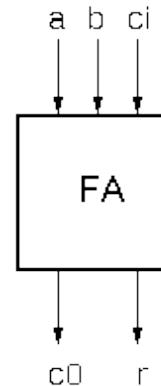
*Describing hardware with a language is similar, however, to writing a parallel program.

Review - Ripple Adder Example

```
module FullAdder(a, b, ci, r, co);
    input a, b, ci;
    output r, co;

    assign r = a ^ b ^ ci;
    assign co = a&ci + a&b + b&ci;

endmodule
```



```
module Adder(A, B, R);
    input [3:0] A;
    input [3:0] B;
    output [4:0] R;

    wire c1, c2, c3;
    FullAdder
    add0(.a(A[0]), .b(B[0]), .ci(1'b0), .co(c1), .r(R[0]));
    add1(.a(A[1]), .b(B[1]), .ci(c1), .co(c2), .r(R[1]));
    add2(.a(A[2]), .b(B[2]), .ci(c2), .co(c3), .r(R[2]));
    add3(.a(A[3]), .b(B[3]), .ci(c3), .co(R[4]), .r(R[3]));

endmodule
```

A block diagram of a 4-bit Ripple Adder. It consists of four Full Adder (FA) modules connected in series. The first FA takes inputs *a₃* and *b₃*, and its carry output *c₃* is connected to the second FA's carry input. The second FA takes inputs *a₂* and *b₂*, and its carry output *c₂* is connected to the third FA's carry input. The third FA takes inputs *a₁* and *b₁*, and its carry output *c₁* is connected to the fourth FA's carry input. The fourth FA takes inputs *a₀* and *b₀*, and its carry output *c₀* is connected to the output *R*.

Example - Ripple Adder Generator

Parameters give us a way to generalize our designs. A module becomes a “generator” for different variations. Enables design/module reuse. Can simplify testing.

```
module Adder(A, B, R);  
    parameter N = 4;  
    input [N-1:0] A;  
    input [N-1:0] B;  
    output [N:0] R;  
    wire [N:0] C;  
  
    genvar i;  
  
    generate  
        for (i=0; i<N; i=i+1) begin:bit  
            FullAdder add(.a(A[i]), .b(B[i]), .ci(C[i]), .co(C[i+1]), .r(R[i]));  
        end  
    endgenerate  
  
    assign C[0] = 1'b0;  
    assign R[N] = C[N];  
endmodule
```

Declare a parameter with default value.

Note: this is not a port. Acts like a “synthesis-time” constant.

Replace all occurrences of “4” with “N”.

variable exists only in the specification - not in the final circuit.

Keyword that denotes synthesis-time operations

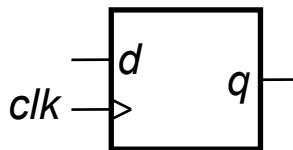
For-loop creates instances (with unique names)

```
Adder adder4 ( ... );  
Adder #(N(64)) adder64 ( ... );
```

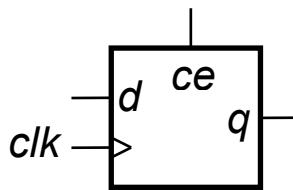
Overwrite parameter N at instantiation.

EECS151 Registers

- All registers are “N” bits wide - the value of N is specified at instantiation
- All positive edge triggered.

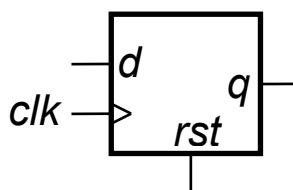


```
module REGISTER(q, d, clk);  
  parameter N = 1;
```



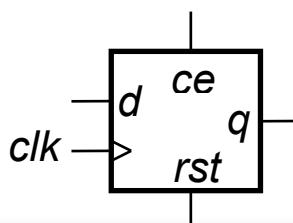
```
module REGISTER_CE(q, d, ce, clk);  
  parameter N = 1;
```

On the rising clock edge if clock enable (ce) is 0 then the register is disabled (it's state will not be changed).



```
module REGISTER_R(q, d, rst, clk);  
  parameter N = 1;  
  parameter INIT = 1b'0;
```

On the rising clock edge if reset (rst) is 1 then the state is set to the value of INIT. Default INIT value is all 0's.

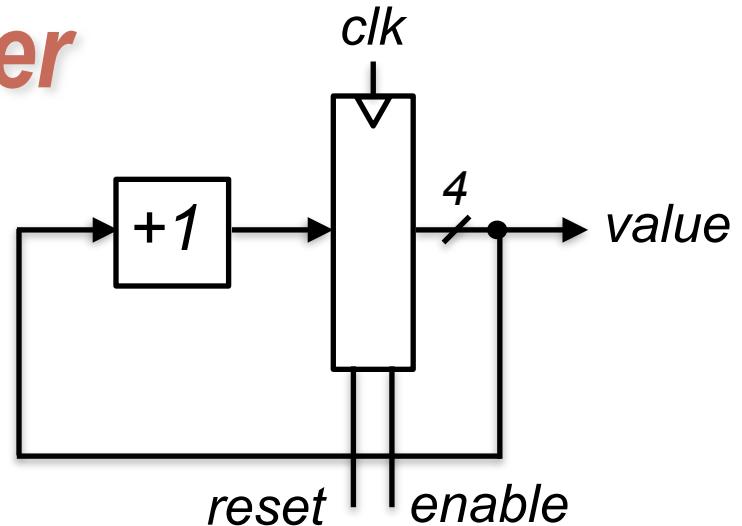


```
module REGISTER_R_CE(q, d, rst, ce, clk);  
  parameter N = 1;  
  parameter INIT = 1b'0;
```

Reset (rst) has priority over clock enable (ce).

4-bit wrap-around counter

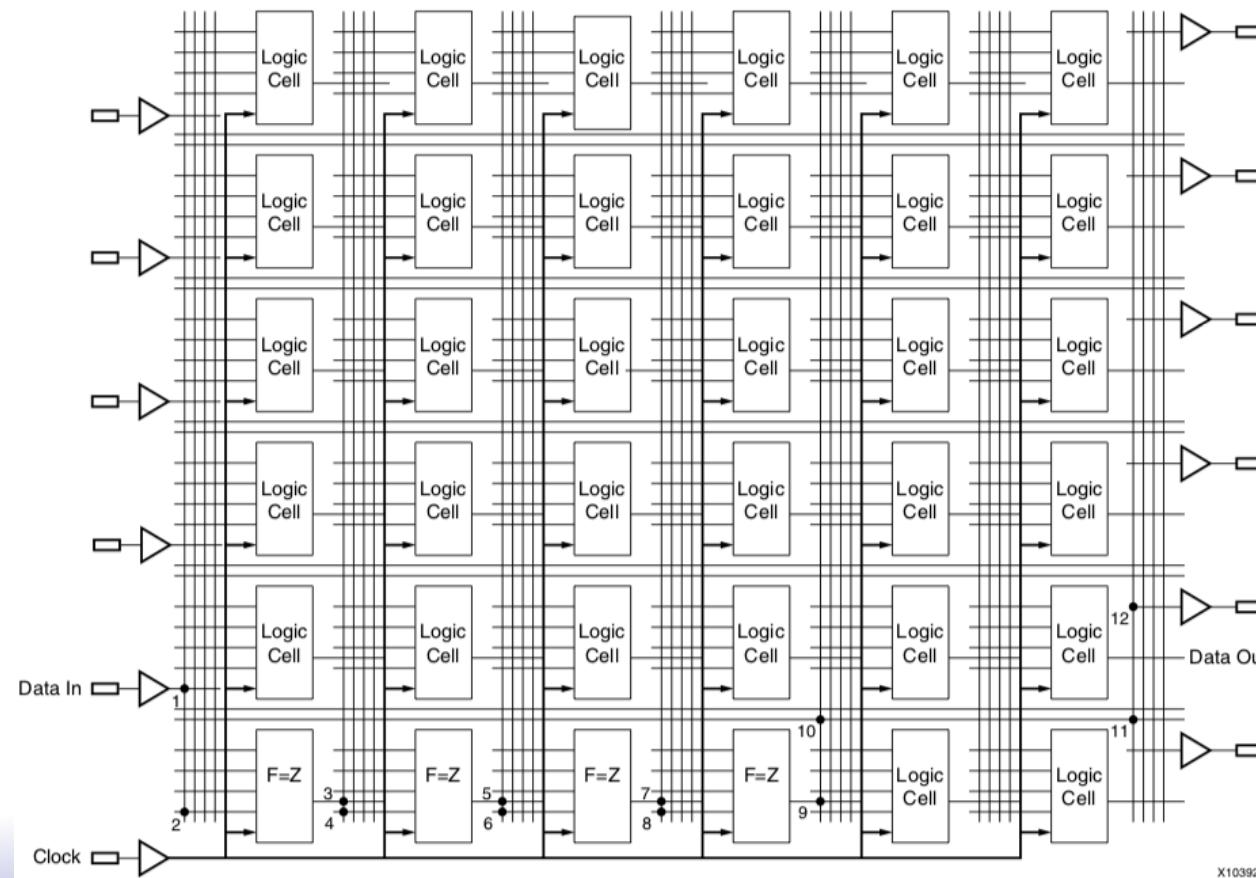
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
11, 12, 13, 14, 15, 0, 1, ...



```
module counter(value, enable, reset, clk);
    output [3:0] value;
    input      enable, reset, clk;
    wire [3:0]  next;
    REGISTER_R #(4) state (.q(value), .d(next), .rst(reset),
    assign next = value + 1;
endmodule // counter
```

FPGA Overview

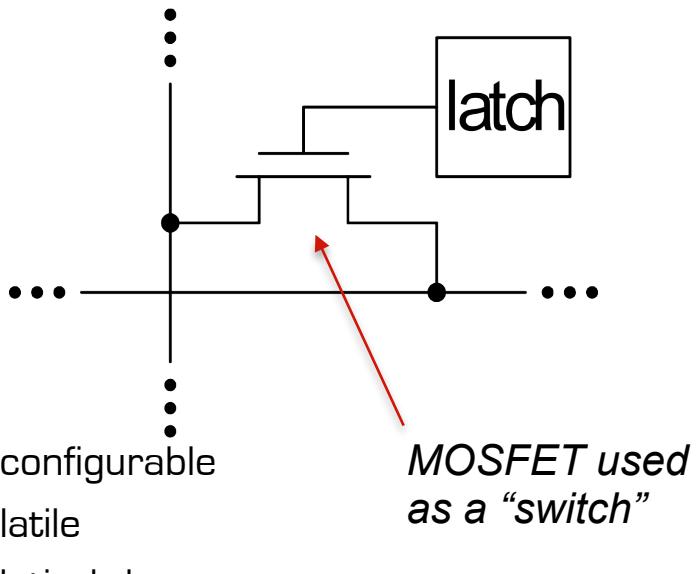
- Basic idea: two-dimensional array of logic blocks and flip-flops with a means for the user to configure (program):
 1. the interconnection between the logic blocks,
 2. the function of each block.



Simplified version of FPGA internal architecture

User Programmability

- Latch-based (Xilinx, Intel/Altera, ...)

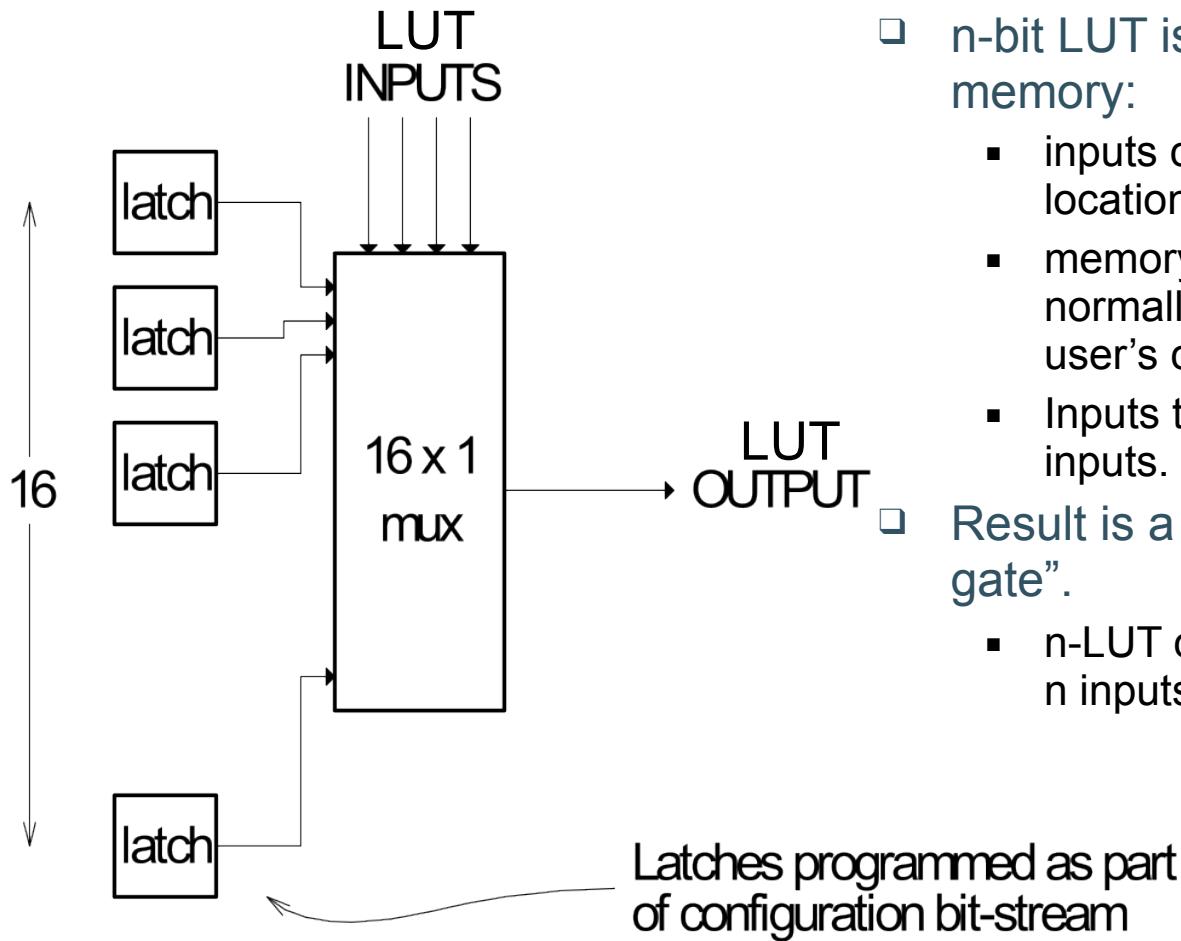


- Latches are used to:

1. control a switch to make or break cross-point connections in the interconnect
2. define the function of the logic blocks
3. set user options:
 - within the logic blocks
 - in the input/output blocks
 - global reset/clock

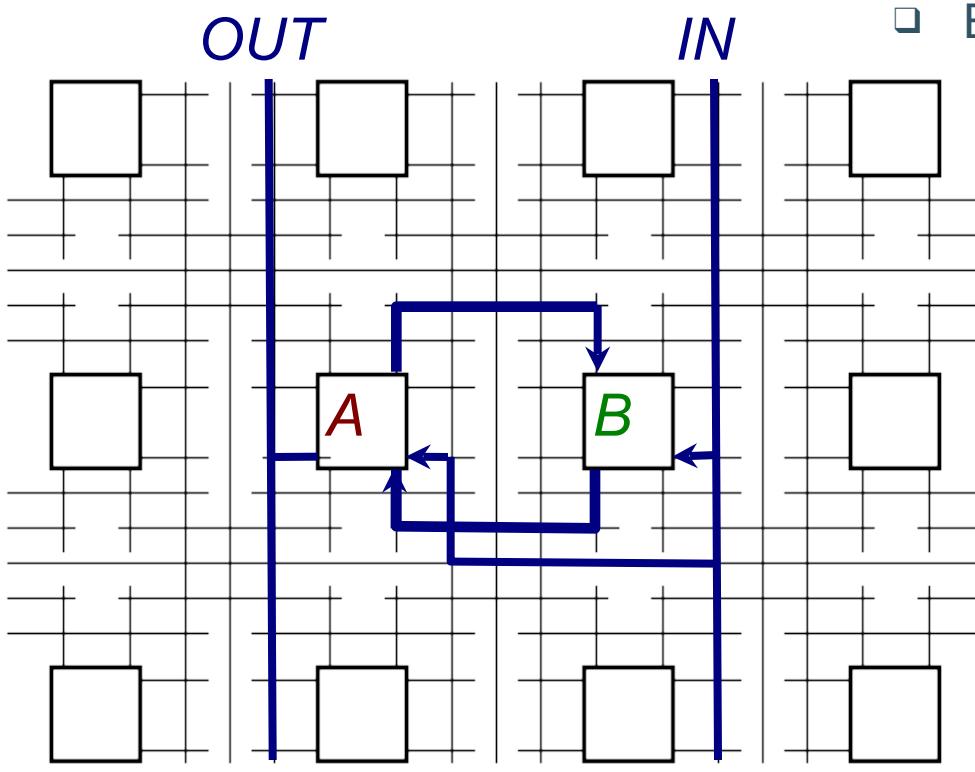
- “Configuration bit stream” is loaded under user control

4-LUT Implementation



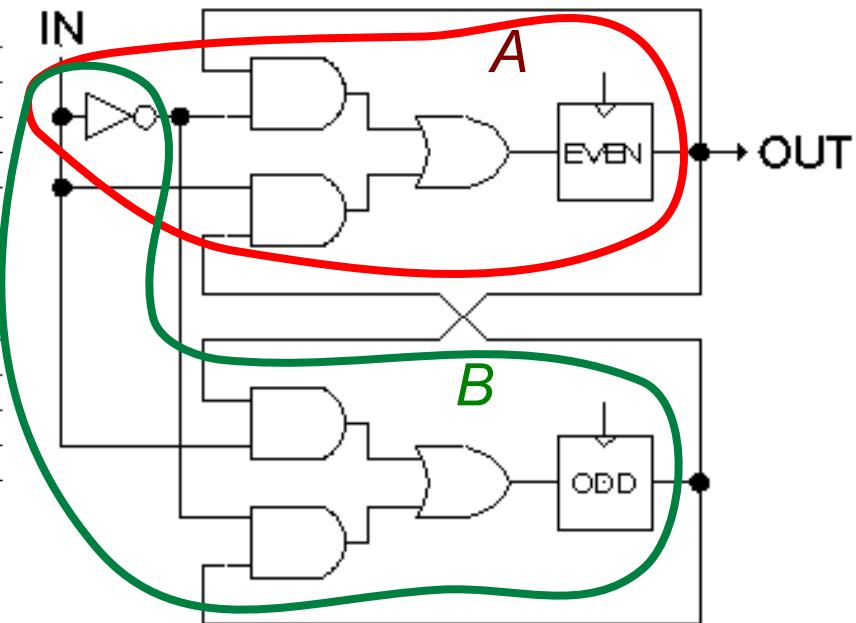
- n-bit LUT is implemented as a $2^n \times 1$ memory:
 - inputs choose one of 2^n memory locations.
 - memory locations (latches) are normally loaded with values from user's configuration bit stream.
 - Inputs to mux control are the CLB inputs.
- Result is a general purpose “logic gate”.
 - n-LUT can implement any function of n inputs!

Example Partition, Placement, and Route



Example Circuit:

- collection of gates and flip-flops



Two partitions. Each has single output, no more than 4 inputs, and no more than 1 flip-flop. In this case, inverter goes in both partitions.

Note: the partition can be arbitrarily large as long as it has not more than 4 inputs and 1 output, and no more than 1 flip-flop.

Some Laws of Boolean Algebra

Duality: A dual of a Boolean expression is derived by interchanging OR and AND operations, and 0s and 1s (literals are left unchanged).

$$\{F(x_1, x_2, \dots, x_n, 0, 1, +, \bullet)\}^D = \{F(x_1, x_2, \dots, x_n, 1, 0, \bullet, +)\}$$

Any law that is true for an expression is also true for its dual.

Operations with 0 and 1:

$$\begin{array}{ll} x + 0 = x & x * 1 = x \\ x + 1 = 1 & x * 0 = 0 \end{array}$$

Idempotent Law:

$$x + x = x \quad x \cdot x = x$$

Involution Law:

$$(x')' = x$$

Laws of Complementarity:

$$x + x' = 1 \quad x \cdot x' = 0$$

Commutative Law:

$$x + y = y + x \quad x \cdot y = y \cdot x$$

Algebraic Simplification

$$\begin{aligned} \text{Cout} &= a'bc + ab'c + abc' + abc \\ &= a'bc + ab'c + abc' + \textcolor{red}{abc} + abc \\ &= a'bc + \textcolor{red}{abc} + ab'c + abc' + \textcolor{red}{abc} \\ &= [\textcolor{red}{a'} + a]bc + ab'c + abc' + abc \\ &= [\textcolor{red}{1}]bc + ab'c + abc' + abc \\ &= bc + ab'c + abc' + \textcolor{red}{abc} + abc \\ &= bc + ab'c + \textcolor{red}{abc} + \textcolor{red}{abc}' + \textcolor{red}{abc} \\ &= bc + \textcolor{red}{a(b' + b)c} + abc' + abc \\ &= bc + \textcolor{red}{a[1]c} + abc' + abc \\ &= bc + ac + ab[c' + c] \\ &= bc + ac + ab[\textcolor{red}{1}] \\ &= bc + ac + ab \end{aligned}$$

Canonical Forms

- Standard form for a Boolean expression - unique algebraic expression directly from a true table (TT) description.
- Two Types:
 - * **Sum of Products (SOP)**
 - * **Product of Sums (POS)**
- Sum of Products (disjunctive normal form, minterm expansion). Example:

Minterms	a	b	c	f	f'
$a'b'c'$	0	0	0	0	1
$a'b'c'$	0	0	1	0	1
$a'bc'$	0	1	0	0	1
$a'bc$	0	1	1	1	0
$ab'c'$	1	0	0	1	0
$ab'c$	1	0	1	1	0
abc'	1	1	0	1	0
abc	1	1	1	1	0

One product (and) term for each 1 in f:
 $f = a'b'c + ab'c' + ab'c + abc' + abc$
 $f' = a'b'c' + a'b'c + a'bc'$

What is the cost?

Karnaugh Map Method

- Adjacent groups of 1's represent product terms

a	0	1	
b	0	0	1
1	0	1	1

$$f = a$$

a	0	1	
b	0	1	1
1	0	0	0

$$g = b'$$

ab	00	01	11	10	
c	0	0	0	1	0
1	0	1	1	1	1

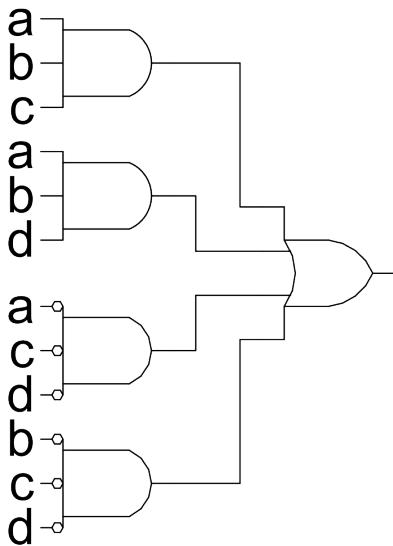
$$\text{cout} = ab + bc + ac$$

ab	00	01	11	10	
c	0	0	0	1	1
1	0	0	1	1	1

$$f = a$$

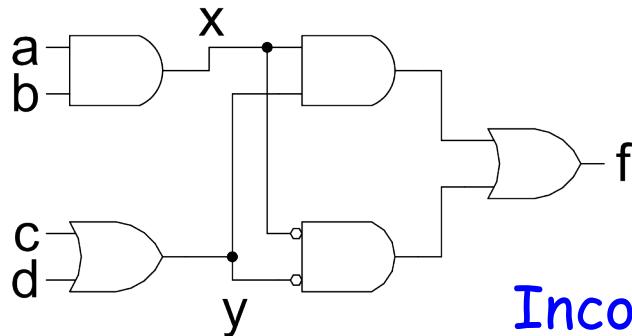
Multi-level Combinational Logic

Another Example: $F = abc + abd + a'c'd' + b'c'd'$



$$\text{let } x = ab \quad y = c+d$$

$$f = xy + x'y'$$



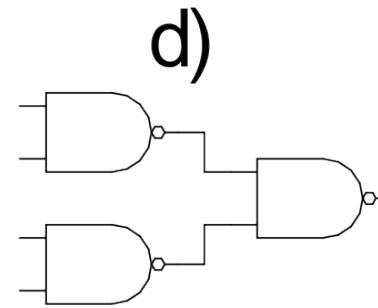
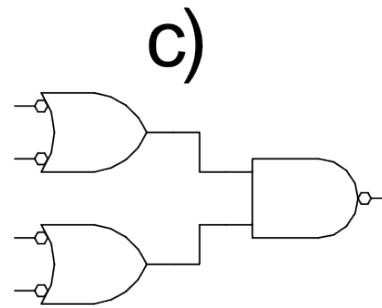
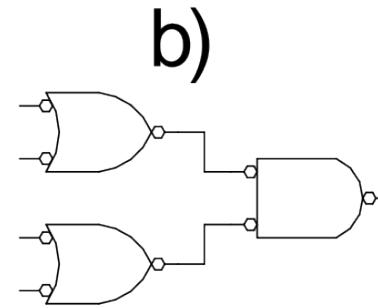
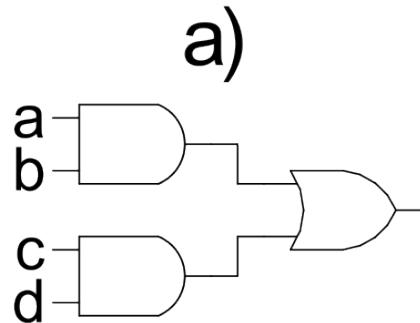
Incorporates fanout.

No convenient hand methods exist for multi-level logic simplification:

- a) CAD Tools use sophisticated algorithms and heuristics
Guess what? These problems tend to be NP-complete
- b) Humans and tools often exploit some special structure (example adder)

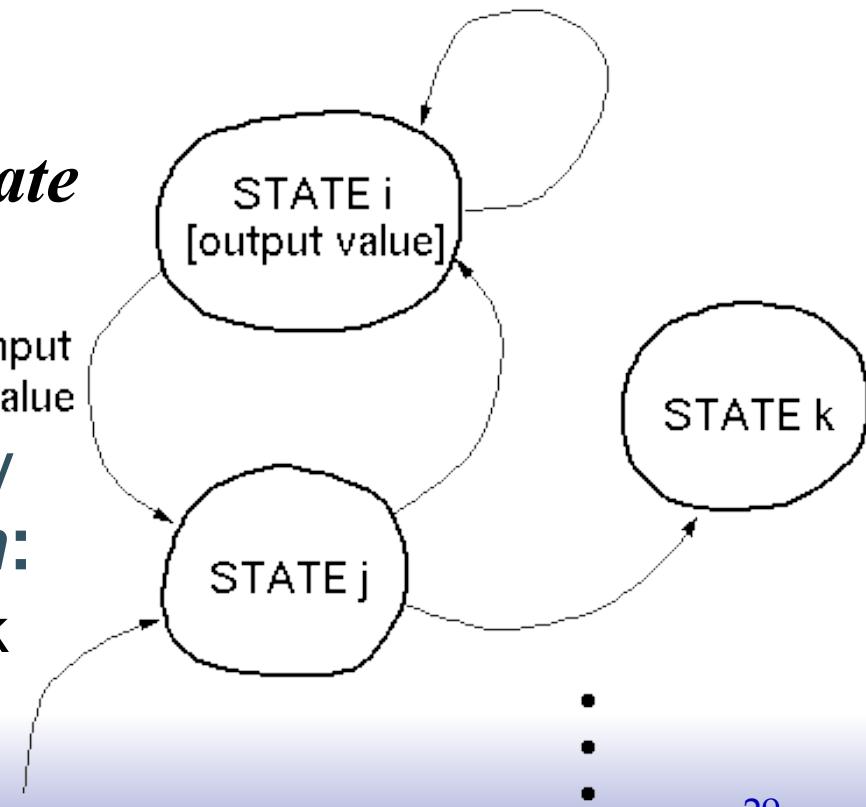
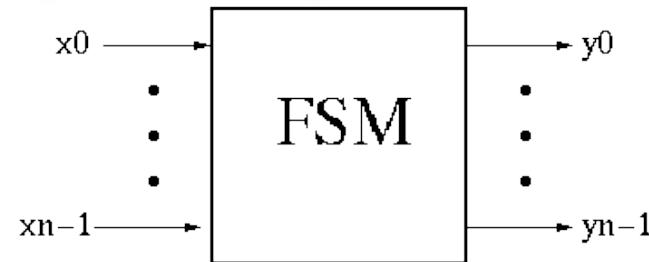
NAND-NAND & NOR-NOR Networks

- Mapping from AND/OR to NAND/NAND



Finite State Machines (FSMs)

- **FSM** circuits are a type of *sequential circuit*:
 - output depends on present *and* past inputs
 - effect of past inputs is represented by the current **state**
- Behavior is represented by **State Transition Diagram**:
 - traverse one edge per clock cycle.



Formal Design Process (3,4)

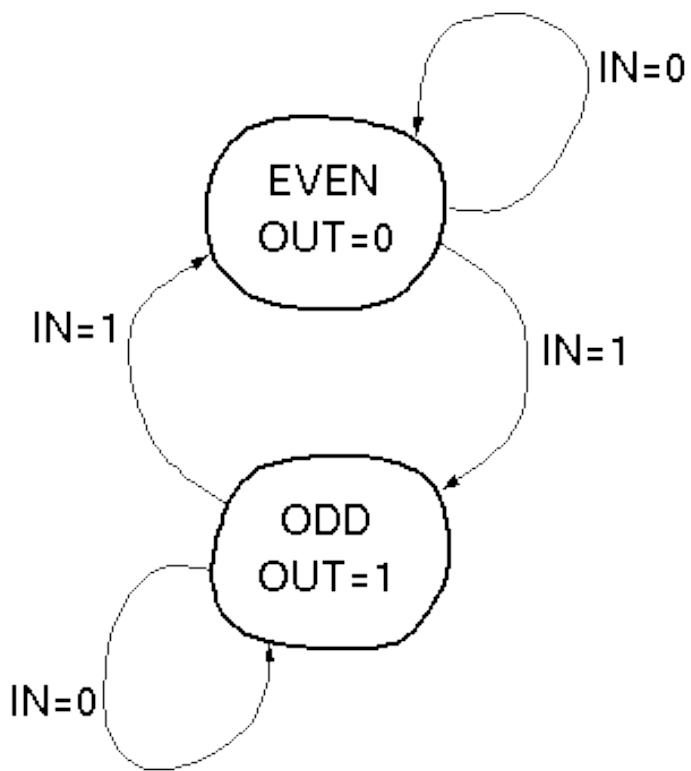
State Transition Table:

present state	OUT	IN	next state
EVEN	0	0	EVEN
EVEN	0	1	ODD
ODD	1	0	ODD
ODD	1	1	EVEN

Invent a code to represent states:

Let 0 = EVEN state, 1 = ODD state

present state (ps)	OUT	IN	next state (ns)
0	0	0	0
0	0	1	1
1	1	0	1
1	1	1	0



Derive logic equations from table (how?):

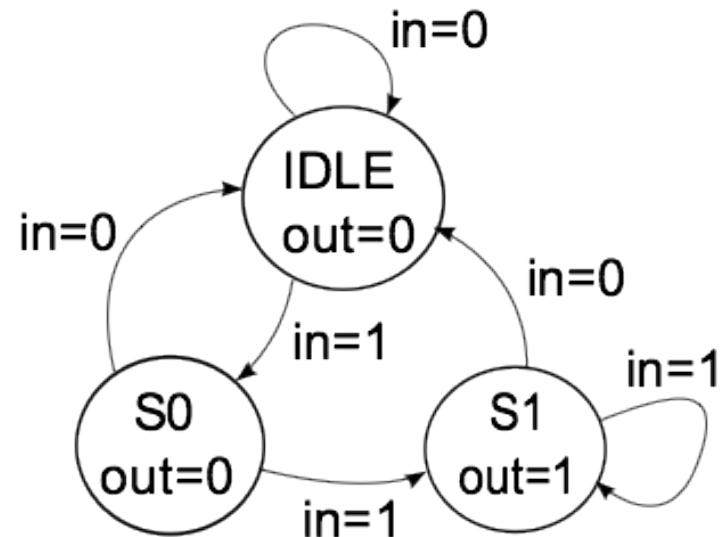
$$OUT = PS$$

$$NS = PS \text{ xor } IN$$

FSM CL block rewritten

```
always @* * for sensitivity list
begin
    next_state = IDLE;
    out = 1'b0;
    case (state)
        IDLE : if (in == 1'b1) next_state = S0;
        S0   : if (in == 1'b1) next_state = S1;
        S1   : begin
            out = 1'b1;
            if (in == 1'b1) next_state = S1;
        end
        default: ;
    endcase
end
Endmodule
```

Normal values: used unless specified below.

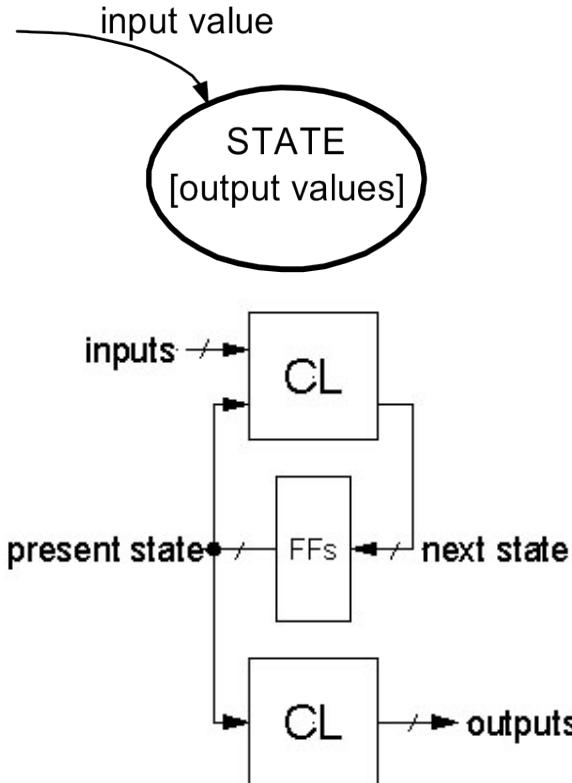


Within case only need to specify exceptions to the normal values.

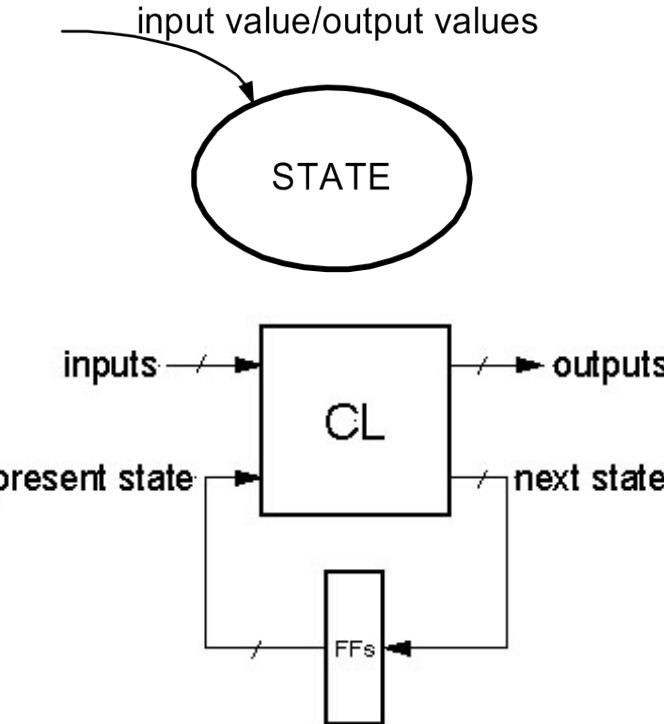
Note: The use of “blocking assignments” allow signal values to be “rewritten”, simplifying the specification.

FSM Recap

Moore Machine

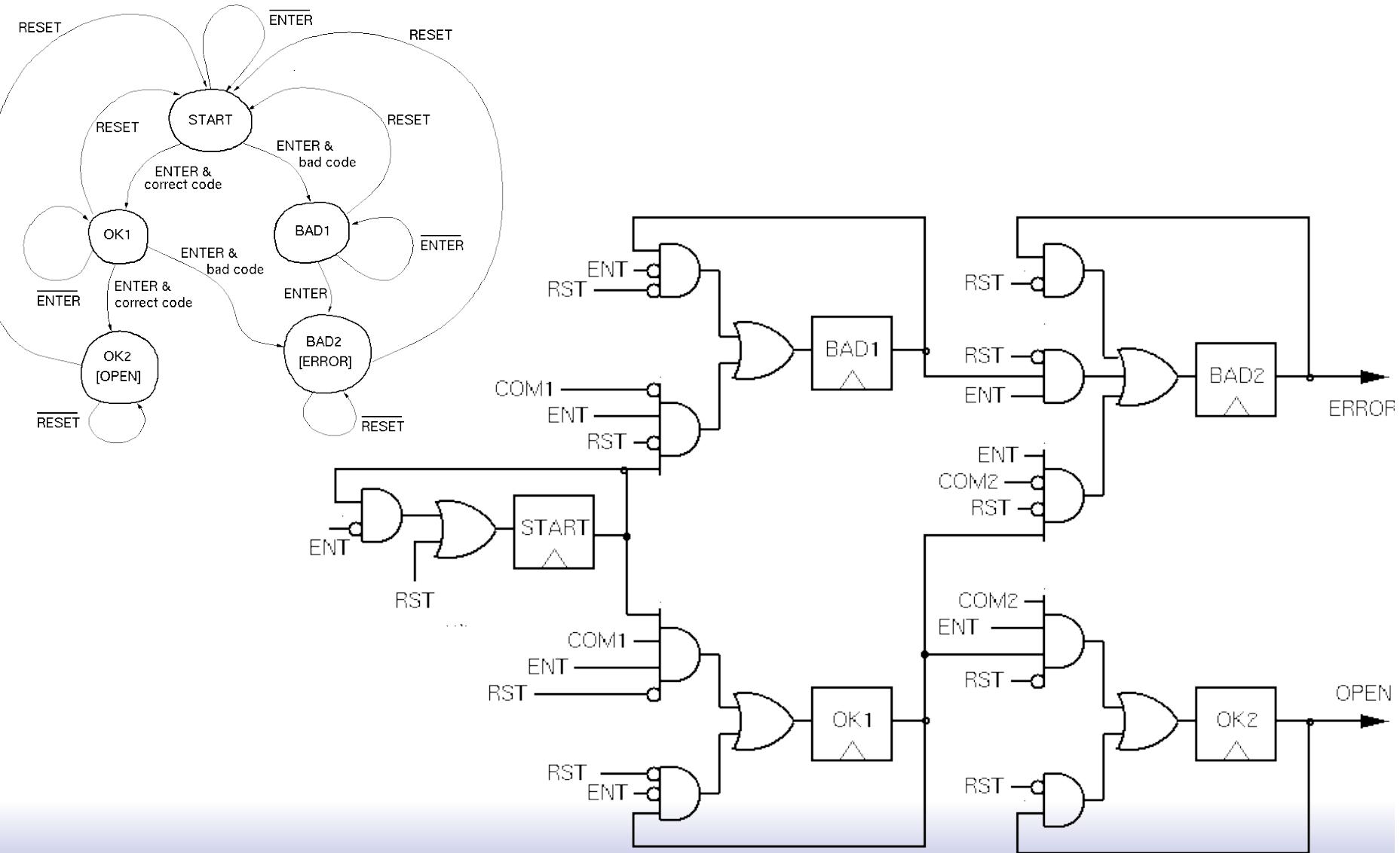


Mealy Machine



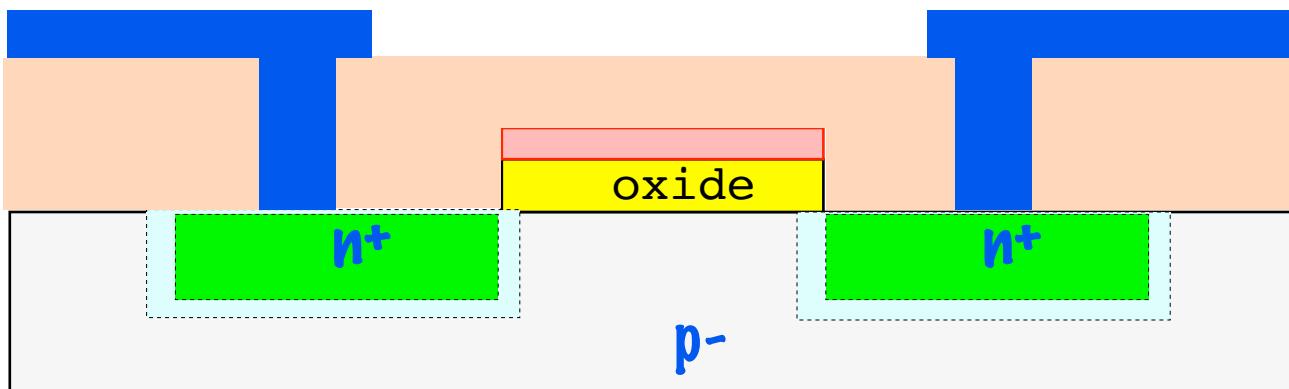
Both machine types allow one-hot implementations.

One-hot encoded combination lock



Final product ...

Vd

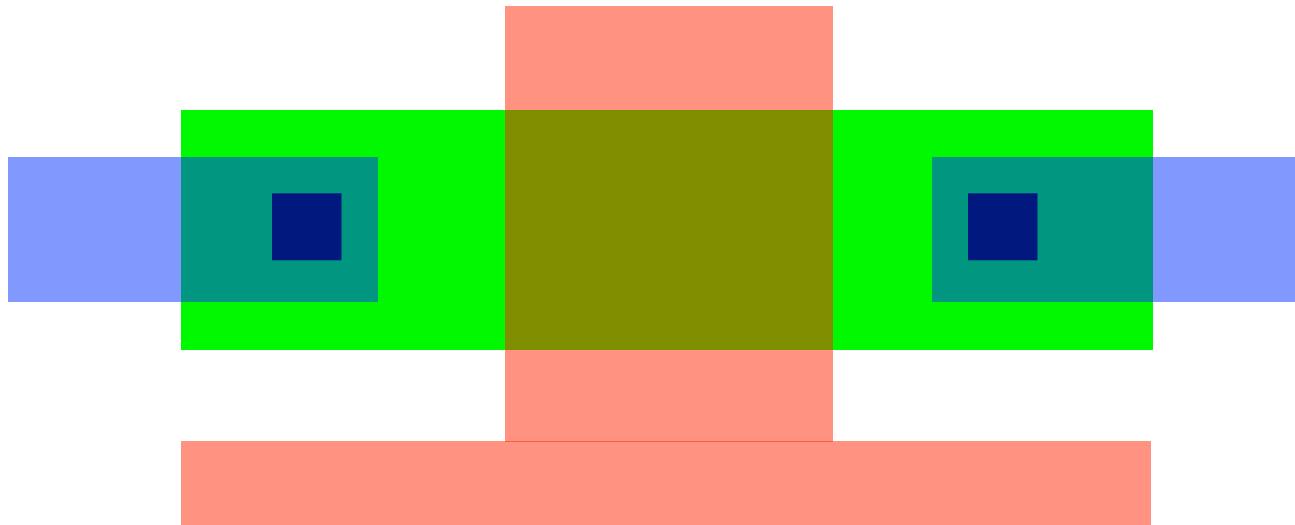


Vs

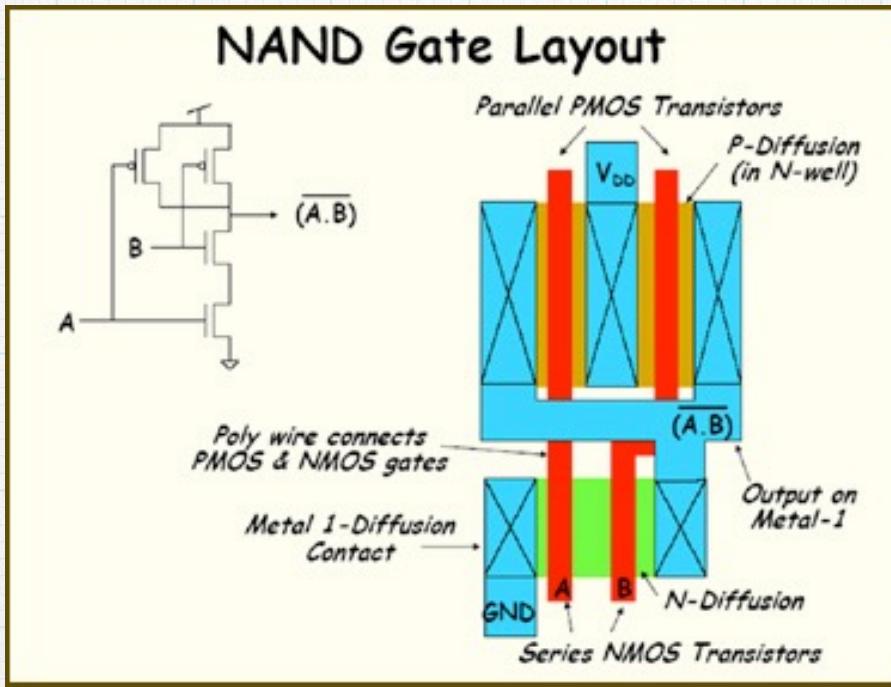
"The planar process"

Jean Hoerni,
Fairchild
Semiconductor
1958

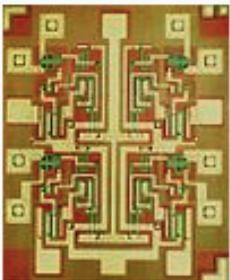
Top-down view:



Physical Layout



- ▶ How do transistor circuits get “laid out” as geometry?
- ▶ What circuit does a physical layout implement?
- ▶ Where are the transistors and wires and contacts and vias?



MOSFET scaling
(process nodes)

10 μm – 1971
6 μm – 1974
3 μm – 1977
1.5 μm – 1981
1 μm – 1984
800 nm – 1987
600 nm – 1990
350 nm – 1993
250 nm – 1996
180 nm – 1999
130 nm – 2001
90 nm – 2003
65 nm – 2005
45 nm – 2007
32 nm – 2009
22 nm – 2012
14 nm – 2014
10 nm – 2016
7 nm – 2018
5 nm – 2020

Future
3 nm – ~2022
2 nm – >2023

State of the art

* From Wikipedia

► 7nm

As of September 2018, mass production of 7 nm devices has begun. The first mainstream 7 nm mobile processor intended for mass market use, the [Apple A12 Bionic](#), was released at their September 2018 event. Although [Huawei](#) announced its own 7 nm processor before the Apple A12 Bionic, the Kirin 980 on August 31, 2018, the [Apple A12 Bionic](#) was released for public, mass market use to consumers before the Kirin 980. Both chips are manufactured by [TSMC](#). On July 7, 2019, AMD officially launched their [Ryzen](#) 3000 series of central processing units, based on the TSMC 7 nm process and [Zen 2](#) microarchitecture.

► 5nm

In October 2019, TSMC started sampling 5nm [A14 processors for Apple](#). In December 2019, TSMC announced an average yield of ~80%, with a peak yield per wafer of >90% for their 5nm test chips with a die size of 17.92 mm². In mid 2020 TSMC claimed its (N5) 5nm process offered 1.8x the density of its 7nm N7 process, with 15% speed improvement or 30% lower power consumption; an improved sub-version (N5P) was claimed to improve on N5 with +5% speed or -10% power.^[19]

On October 13, 2020, Apple announced a new [iPhone 12](#) lineup using the [A14](#), together with the [Huawei Mate 40](#) lineup using the [HiSilicon Kirin 9000](#), which were the first devices to be commercialized on TSMC's 5nm node. Later, on November 10, 2020, Apple also revealed three new Mac models using the [Apple M1](#), another 5nm chip. According to Semianalysis, the A14 processor has a transistor density of 134 million transistors per mm².

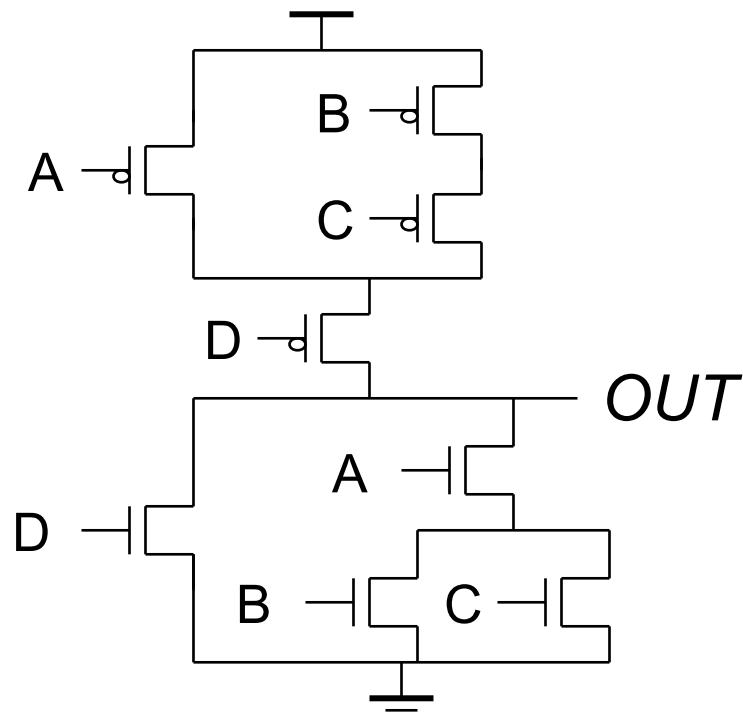
► 3nm

As of 2019, [Intel](#), [Samsung](#), and [TSMC](#) have all announced plans to put a 3 nm semiconductor node into commercial production. Samsung's 3 nm process is based on [GAAFET](#) (gate-all-around field-effect transistor) technology, a type of [multi-gate MOSFET](#) technology, while TSMC's 3nm process will still use [FinFET](#) (fin field-effect transistor) technology,^[1] despite TSMC developing GAAFET transistors.

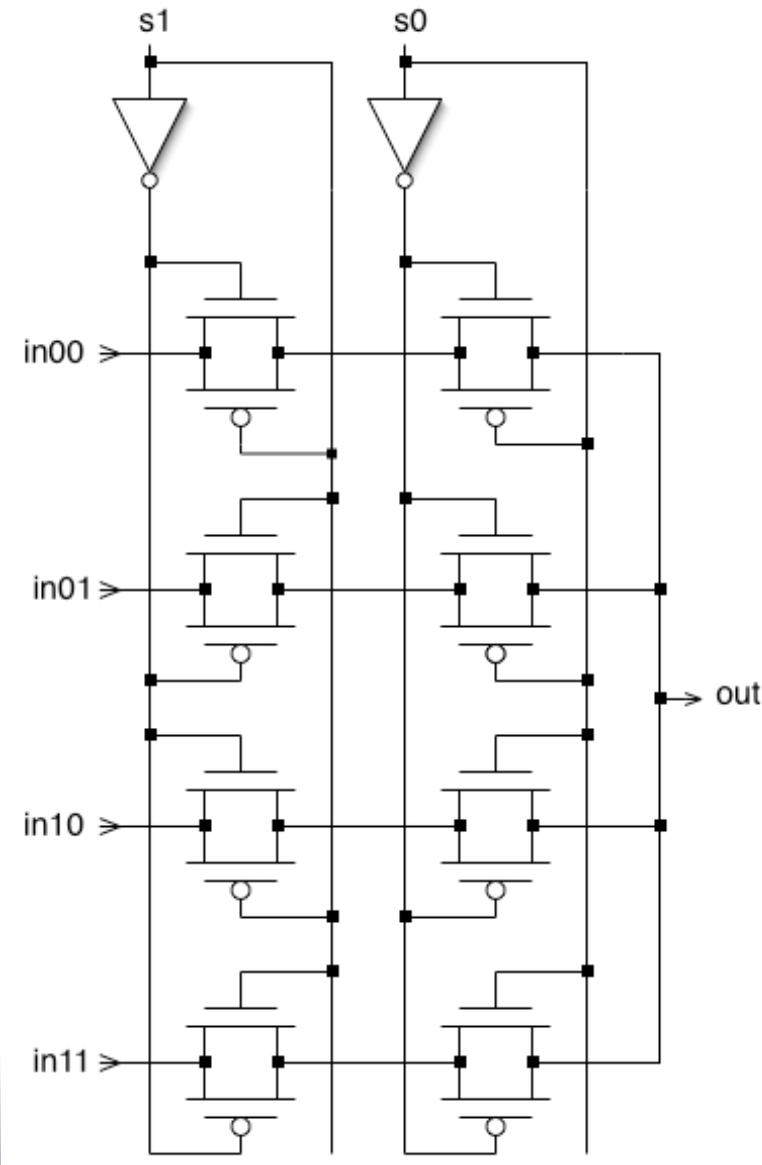
Complex CMOS Gate

$$OUT = \overline{D + A \cdot (B + C)}$$

$$OUT = \overline{D \cdot A + B \cdot C}$$



4-to-1 Transmission-gate Mux

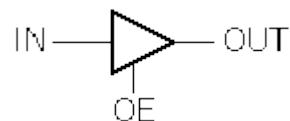


- The series connection of pass-transistors in each branch effectively forms the AND of s_1 and s_0 (or their complement).
- Compare cost to logic gate implementation

Any better solutions?

Tri-state Buffers

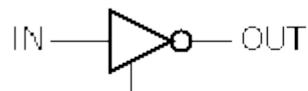
Tri-state Buffer:



OE	IN	OUT
0	0	Z
0	1	Z
1	0	0
1	1	1

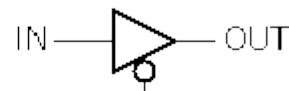
"high
impedance" (output
disconnected)

Variations:



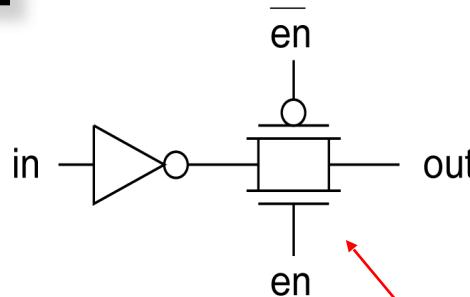
OE	IN	OUT
0	-	Z
1	0	1
1	1	0

Inverting buffer

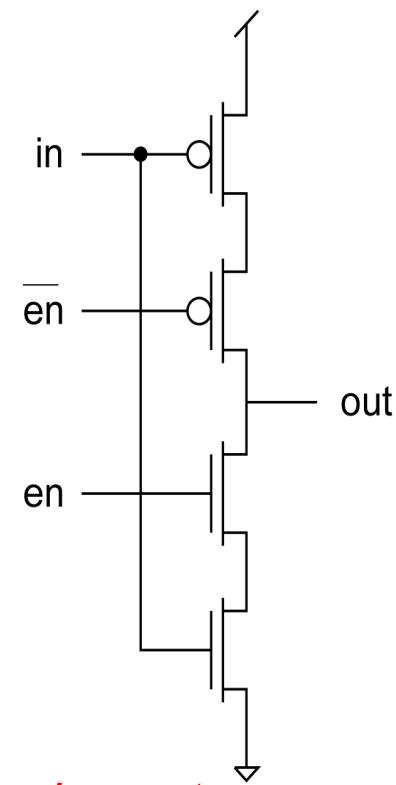


OE	IN	OUT
0	0	0
0	1	1
1	-	Z

Inverted enable

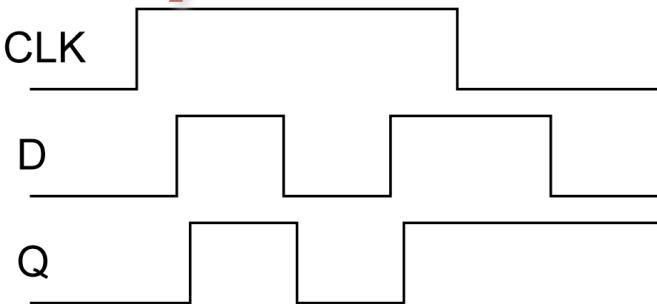
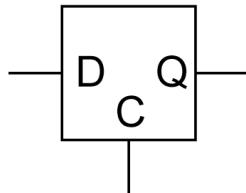


transmission gate
useful in
implementation

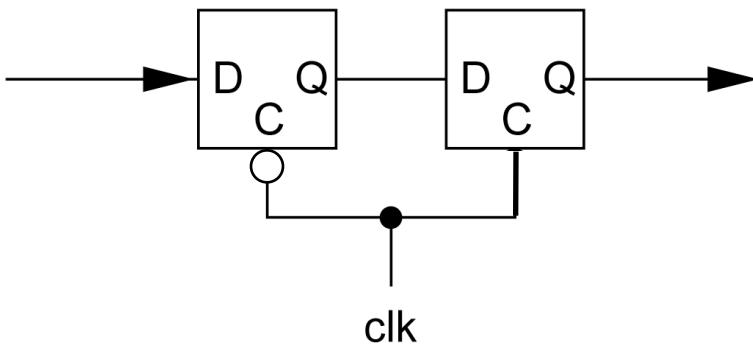


Latches and Flip-flops

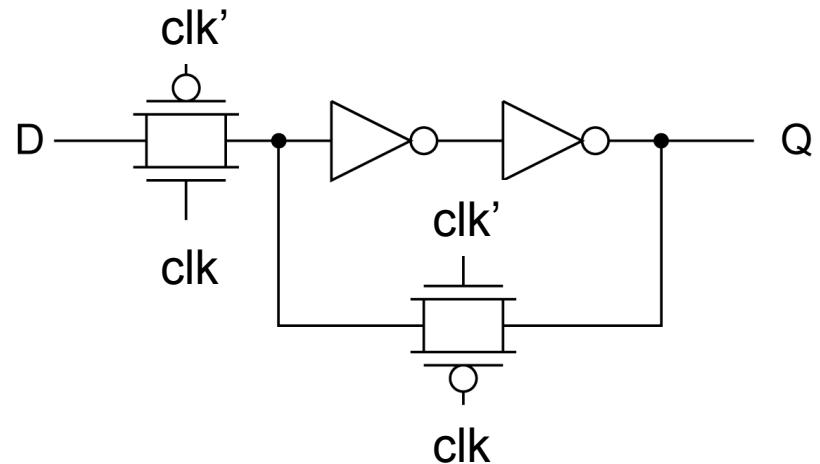
Positive Level-sensitive *latch*:



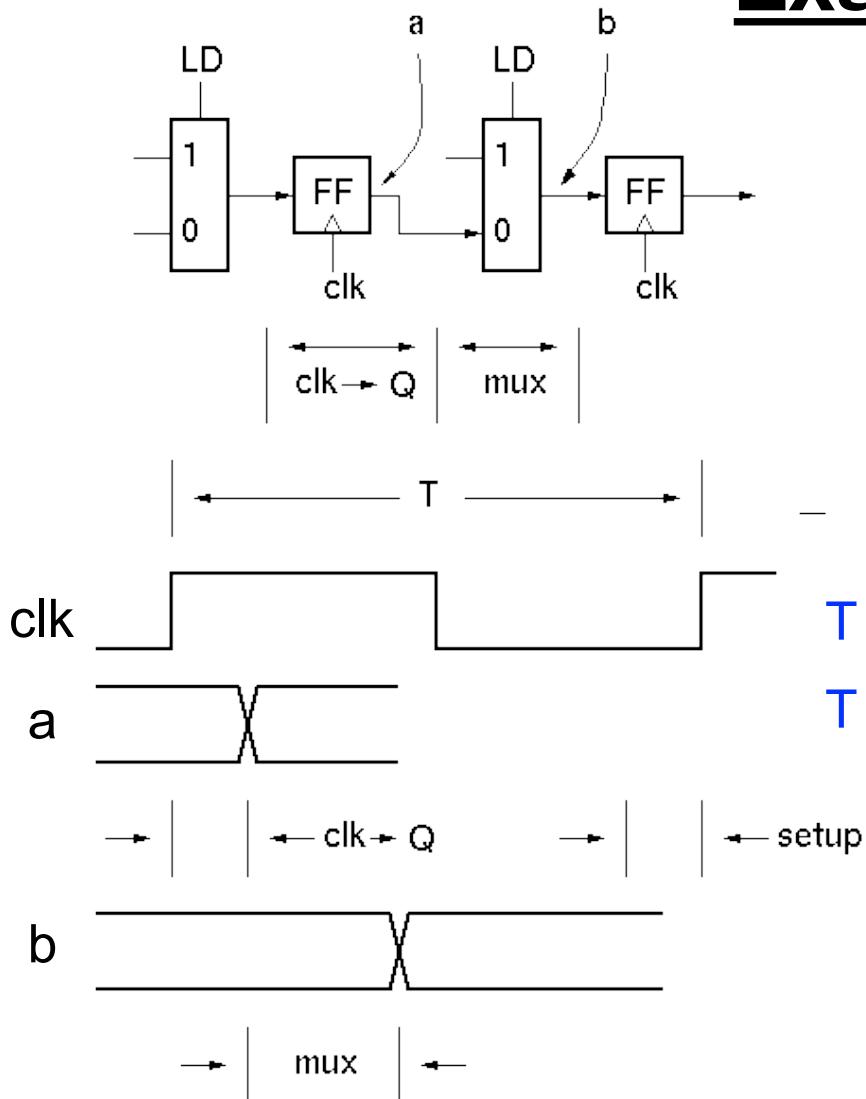
Positive Edge-triggered **flip-flop**
built from two level-sensitive
latches:



Latch Implementation:



Example



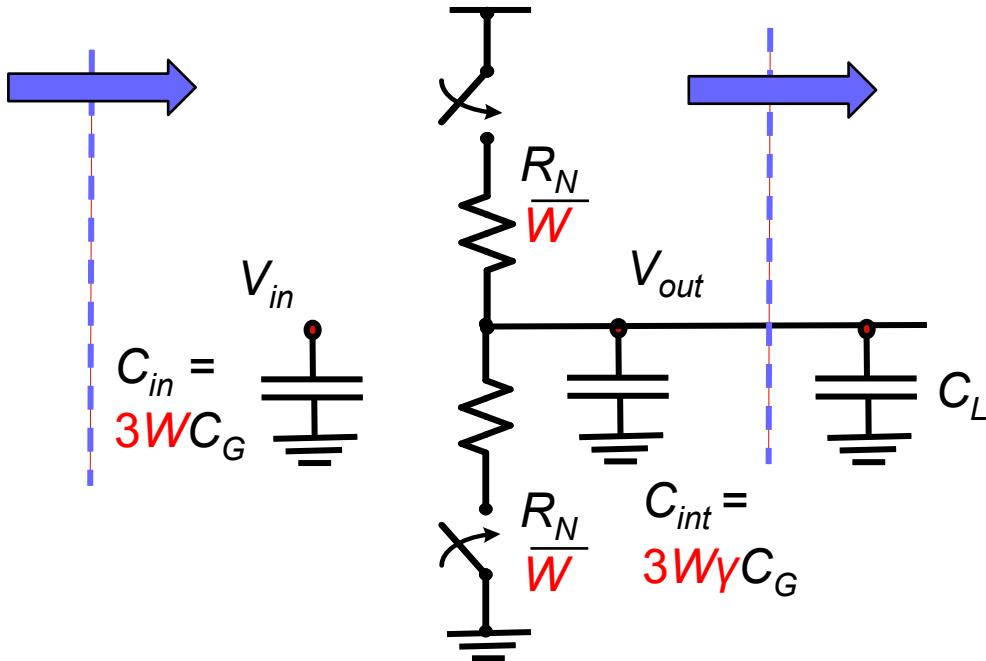
Parallel to serial
converter circuit

$$T \geq \text{time}(\text{clk} \rightarrow \text{Q}) + \text{time}(\text{mux}) + \text{time}(\text{setup})$$

$$T \geq \tau_{\text{clk} \rightarrow \text{Q}} + \tau_{\text{mux}} + \tau_{\text{setup}}$$

Inverter with Load Capacitance

$$t_p = 0.69(R_N/W)(C_{int} + C_L)$$



$$= 0.69(R_N/W)(3W\gamma C_G + C_L)$$

$$= 0.69(3\gamma R_N C_G) \left(1 + \frac{C_L}{\gamma C_{in}}\right)$$

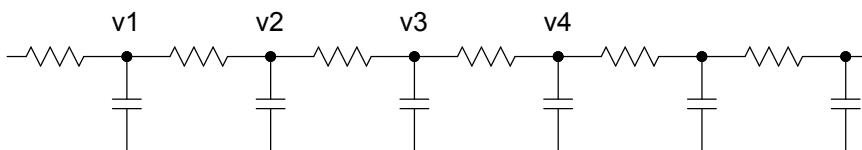
$$= t_{inv}(1 + \frac{C_L}{\gamma C_{in}}) = t_{p0}(1 + f/\gamma)$$

$f = \text{fanout} = \text{ratio between load and input capacitance of gate}$

Wire Delay

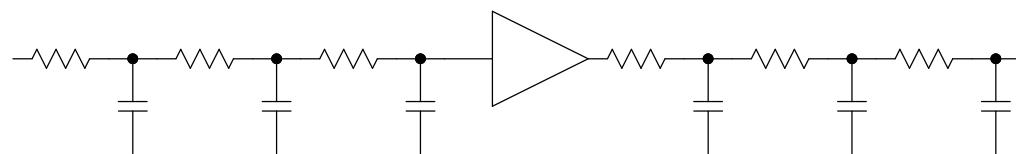
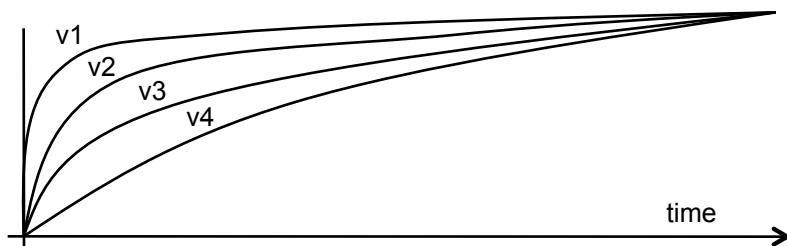
- Even in those cases where the transmission line effect is negligible:

- Wires posses distributed resistance and capacitance



- Time constant associated with distributed RC is proportional to the square of the length

- For **short wires** on ICs, resistance is insignificant (relative to effective R of transistors), but C is important.
 - Typically around half of C of gate load is in the wires.
- For **long wires** on ICs:
 - busses, clock lines, global control signal, etc.
 - Resistance is significant, therefore distributed RC effect dominates.
 - signals are typically “rebuffed” to reduce delay:



How to retime logic

Critical path is 5.
We want to improve
it without changing
circuit semantics.

Circles are combinational
logic, labelled with delays.

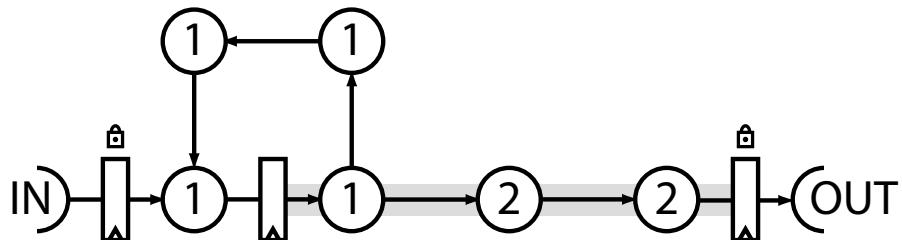


Figure 1: A small graph before retiming. The nodes represent logic delays, with the inputs and outputs passing through mandatory, fixed registers. The critical path is 5.

Add a register, move
one circle.
Performance
improves by 20%.

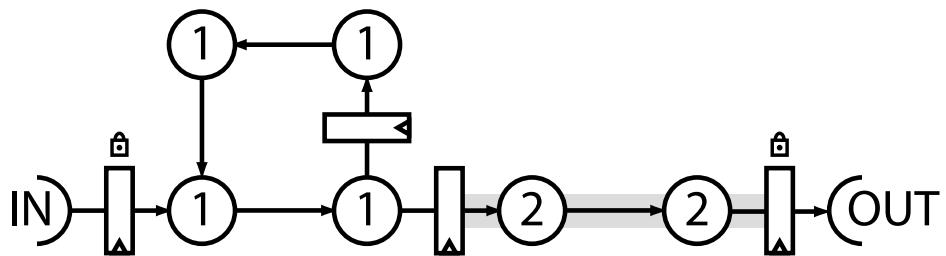
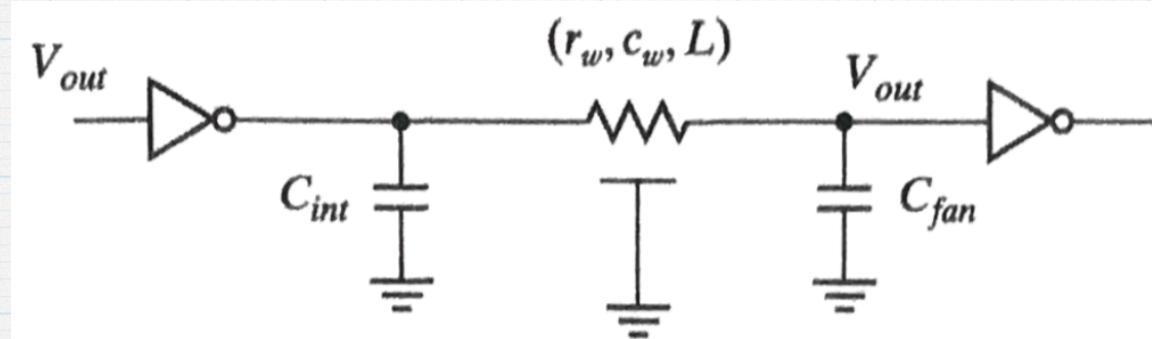


Figure 2: The example in Figure 1 after retiming. The critical path is reduced from 5 to 4.

Logic Synthesis tools can do this in
simple cases.

Gate Driving long wire and other gates

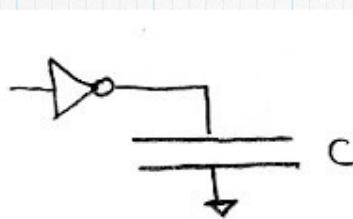


$$R_w = r_w L, \quad C_w = c_w L$$

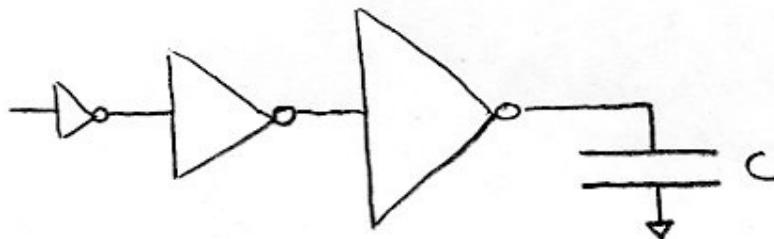
$$\begin{aligned} t_p &= 0.69R_{dr}C_{int} + 0.69R_{dr}C_w + 0.38R_wC_w + 0.69R_{dr}C_{fan} + 0.69R_wC_{fan} \\ &= 0.69R_{dr}(C_{int} + C_{fan}) + 0.69(R_{dr}c_w + r_wC_{fan})L + 0.38r_wc_wL^2 \end{aligned}$$

Driving Large Loads

- ▶ Large fanout nets: clocks, resets, memory bit lines, off-chip
- ▶ Relatively small driver results in long rise time (and thus large gate delay)

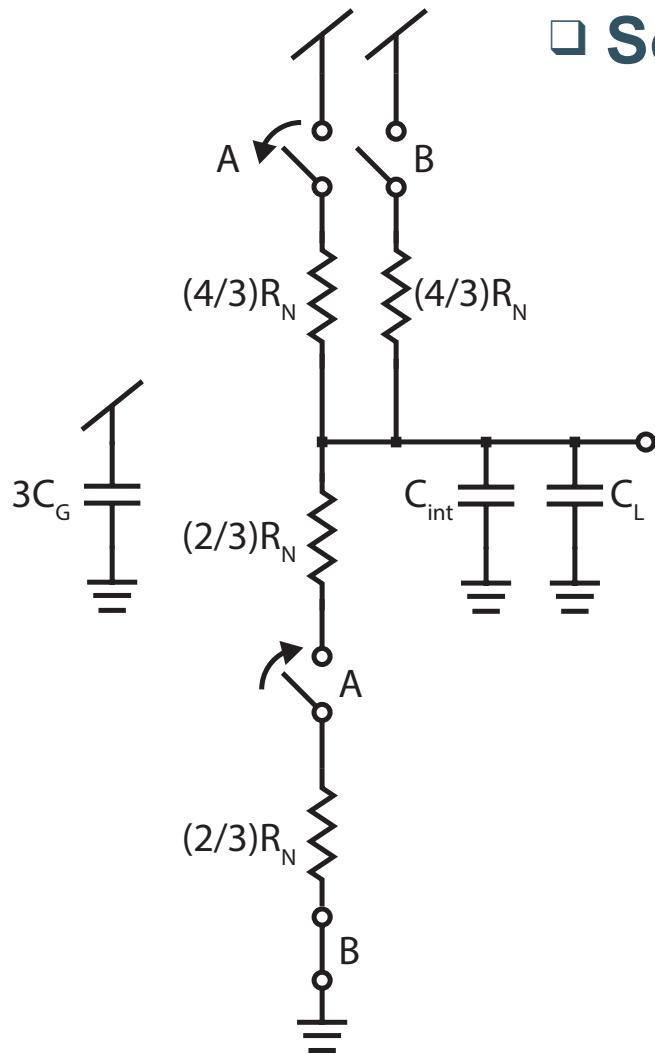


Staged Buffers



- ▶ How to optimally scale drivers?
- ▶ Optimal trade-off between delay per stage and total number of stages?

2-input NAND Gate



□ Solve:

$$C_{int} = (6/2)C_D + (3/2)C_D = (9/2)C_D$$

$$\begin{aligned}
 t_p &= 0.69 \cdot 2 \left(\frac{2R_N}{3W} \right) (C_{int} + C_L) \\
 &= 0.69 \left(\frac{4R_N}{3W} \right) \left(\frac{9}{2}\gamma WC_G + C_L \right) \\
 &= 0.69 \left(\frac{R_N}{W} \right) \left(6\gamma WC_G + \frac{4}{3}C_L \right) \\
 &= 0.69 \left(\frac{R_N}{W} \right) 3\gamma WC_G \left(2 + \frac{4C_L}{3\gamma (3WC_G)} \right) \\
 &= [0.69 \cdot 3R_N\gamma C_G] \left(2 + \frac{4C_L}{3\gamma C_{IN}} \right) \\
 &= t_{p0} \left(2 + \frac{4f}{3\gamma} \right)
 \end{aligned}$$