ATOMIC EFFECTS IN COHERENT NEUTRINO SCATTERING

L.M. SEHGAL and M. WANNINGER

III. Physikalisches Institut, Technische Hochschule, D-5100 Aachen, Fed. Rep. Germany

Received 23 January 1986

The influence of atomic electrons on the coherent elastic scattering of low energy neutrinos is studied. For $Q^2R^2_{\mathrm{atom}} \lesssim 1$, significant interference occurs between scattering from the electrons and from the nucleus, leading to large differences between ν_e and ν_μ cross sections. In particular, ν_e scattering shows a sharp minimum at $Q^2 \approx (10\ \mathrm{keV})^2$ for ^{28}Si and ^{56}Fe. As a consequence, for neutrinos in the 10 keV range, the radiation pressure exerted on a medium is much greater for ν_μ than for ν_e. The exceptional case of hydrogen is discussed also.

The coherent scattering of weakly interacting neutral particles is of interest in several contexts: (a) as a possible basis for new types of detectors sensitive to temperature changes induced by the recoiling nucleus [1,2], (b) as a possible means for transfer of neutrino momentum to the outer envelope of a collapsing star (supernova) [3], and (c) as a possible mechanism for energy loss and trapping of "dark" matter in the earth or solar system [4].

We recently drew attention to an effect [5] that occurs in the coherent reaction $\nu_\ell + \mathrm{A} \to \nu_\ell + \mathrm{A}$ (where A is a nucleus) when the momentum transfer is $Q^2 \lesssim 4m_\ell^2$, m_ℓ being the mass of the charged lepton associated with the neutrino ν_ℓ. The charge radius of the neutrino, proportional to $\ln(m_\ell^2/M_\mathrm{W}^2)$, induces a radiative correction to the coherent amplitude, causing the cross sections for ν_e, ν_μ and ν_τ to differ. In this paper, we examine an effect that sets in when the momentum transfers are so low that $Q^2R^2_{\mathrm{atom}} \lesssim 1$, where R_{atom} is the radius of the target atom including the electronic shells. In such a circumstance it is appropriate to view the reaction as taking place on the atom as a whole (i.e. $\nu + \mathrm{atom} \to \nu + \mathrm{atom}$). Because of the fact that the coupling of ν_e to electrons is quite different from that of ν_μ or ν_τ, one may expect large differences in the behaviour of ν_e scattering compared to the other neutrino types. Such differences can be of importance in situations involving low-energy ($E_\nu \lesssim 100$ keV) neutrinos. (For an early discussion of coherent scattering from atoms, see ref. [6].)

We begin by writing down the basic formula for coherent scattering from a nucleus A. Neglecting terms of order E_ν/M (M being the nuclear mass), the differential cross section as a function of the scattering angle of the neutrino is

$$\mathrm{d}\sigma/\mathrm{d}\cos\theta = (G_\mathrm{F}^2 E_\nu^2/2\pi) \times [C_\mathrm{V}^2(1+\cos\theta) + C_\mathrm{A}^2(3-\cos\theta)]\,, \tag{1}$$

where C_V and C_A are Q^2-dependent matrix elements of the vector and axial vector neutral current charges. [Note that $Q^2 = 2E_\nu^2(1-\cos\theta)$.] for a nucleus of Z protons and N neutrons, one has, in the standard model,

$$C_\mathrm{V} = [Z(\tfrac{1}{2} - 2\sin^2\theta_\mathrm{w}) - \tfrac{1}{2}N]F_{\mathrm{nuc}}(Q^2)\,,$$

$$C_\mathrm{A} = \tfrac{1}{2}g_\mathrm{A}[(Z_+ - Z_-) - (N_+ - N_-)]F_{\mathrm{nuc}}(Q^2)\,. \tag{2}$$

Here $g_\mathrm{A} = 1.25$, and $Z_\pm$ ($N_\pm$) denote the average numbers of protons (neutrons) in the nucleus with spin parallel (+) or antiparallel (−) to the nuclear spin. $F_{\mathrm{nuc}}(q^2)$ is the nuclear form factor (assumed to be the same for vector and axial vector charges) which is adequately represented by $\exp(-\tfrac{1}{6}R^2Q^2)$, R being the RMS radius. For even–even (spin zero) nuclei, the axial coupling C_A is zero. More generally, one expects $[C_\mathrm{A}/C_\mathrm{V}] \sim 1/A$, so that for most nuclei the

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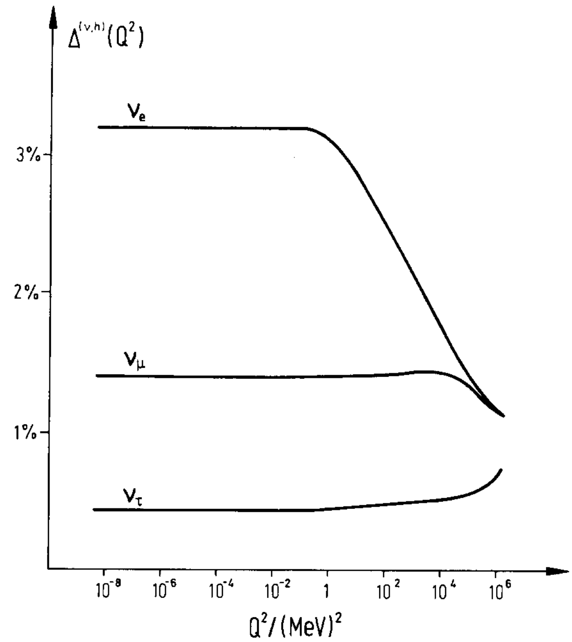


Fig. 1. Radiative correction to $\sin^2\theta_W$, defined by $\sin^2\theta_W[1 - \Delta^{\nu,h}(Q^2)]$ (after ref. [7]).

axial contribution in (1) may be neglected. (The exceptional case of hydrogen is discussed separately below.)

The effect of radiative corrections on the above cross section, discussed in ref. [5], is obtained by making the replacement

$$\sin^2\theta_W \to \sin^2\theta_W[1 - \Delta^{\nu,h}(Q^2)] , \tag{3}$$

where $\Delta^{\nu,h}(Q^2)$ can be computed from the work of ref. [7], and is plotted in fig. 1 for the three neutrino types. The dependence on neutrino flavour has its origin in the different neutrino charge radii. The absolute magnitude of $\Delta^{\nu,h}(Q^2)$ depends also on vacuum polarization effects due to fermion (quark and lepton) and boson loops. The values shown in fig. 1 were obtained keeping only the u and d quark contributions, and taking $\xi = m_H/m_Z = 1$. (An additional effect of the radiative corrections is to multiply the weak amplitude by an overall factor $\rho^{\nu,h}$ that is close to unity and the same for all neutrino flavours, which we neglect in the present considerations.)

We now consider the modification of eqs. (1) and (2) when one takes into account neutrino scattering from the complete atom, adding the effects of the nucleus and of the electrons. Assuming the Z electrons in the atom to be distributed according to a form factor $F_{el}(Q^2)$ [with $F_{el}(0) = 1$] the vector matrix element is modified to

$$C_V^{atom} = C_V + Z[\mp\tfrac{1}{2} + 2\sin^2\theta_W]F_{el}(Q^2), \tag{4}$$

where the + sign applies to ν_e scattering and the − sign to ν_μ (or ν_τ). Note that the standard model specifies the relative magnitude as well as the relative sign of the neutrino–electron and neutrino–nucleus amplitudes. To obtain the axial vector contribution, we note first that the axial charge associated with the electron cloud is

$$C_A^{el} = \mp\tfrac{1}{2}(L_+ - L_-)F_{el}(Q^2) , \tag{5}$$

where L_+ and L_- denote the numbers of electrons with spins parallel and antiparallel to the total electron spin, the + and − signs applying to $\nu = \nu_e$ and $\nu \neq \nu_e$, respectively. To the extent, however, that the spin of the electron cloud is uncorrelated with the spin of the nucleus (which is equivalent to the assumption that the hyperfine levels of the atom are degenerate and equally populated), the axial contributions of the nucleus and the electron add *incoherently*. Thus, following ref. [6], the axial piece of the cross section in eq. (1) is given by

$$(C_A^{atom})^2 = (C_A)^2 + (C_A^{el})^2 . \tag{6}$$

For atoms with even Z, one has $L_+ = L_-$ and so no axial contribution from electrons. For most atoms C_A^{atom} remains small compared to C_V^{atom}, unless the latter is diminished by a large cancellation.

It is clear that the influence of electrons is largest when $Q^2 \lesssim 1/R_{atom}^2$ so that F_{el} is close to unity. Neglecting the axial contributions, one obtains from eqs. (1) and (4), in the limit $Q^2 = 0$,

$$\left.\frac{d\sigma}{d\cos\theta}\right|_{\theta=0} = \frac{G_F^2E_\nu^2}{2\pi} \times (Z - \tfrac{1}{2}N)^2 \quad \text{for } \nu_e ,$$

$$\times (-\tfrac{1}{2}N)^2 \quad \text{for } \nu_\mu, \nu_\tau , \tag{7}$$

independent of $\sin^2\theta_W$. Thus the forward cross section for ν_e differs from that of ν_μ or ν_τ, except in the case of an isoscalar target $N = Z$ when all three are equal. More generally, we note from eq. (4) that (for $N \geqslant Z$) the interference of the electron cloud and

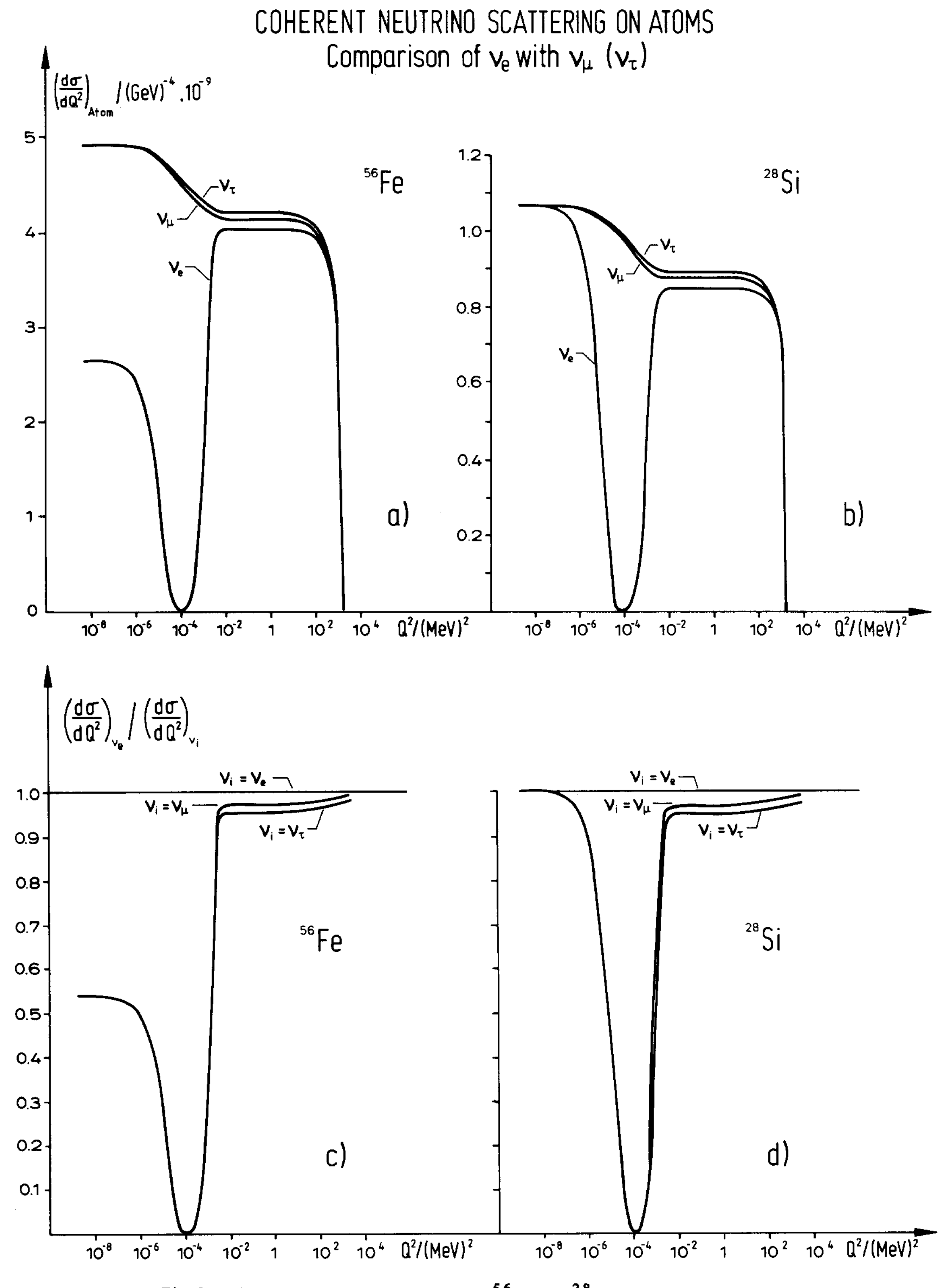


Fig. 2. Coherent neutrino scattering from ^{56}Fe and ^{28}Si atoms (E_ν = 20 MeV).

the nucleus is destructive for ν_e scattering and constructive for ν_μ or ν_τ. In the former case, the scattering amplitude passes through zero and changes sign as Q^2 varies from large to small values, producing a sharp dip in the differential cross section. Thus electrons tend to screen the weak charge of the nucleus as seen by a ν_e probe: the screening is complete at a value of Q^2 given by

$$F_{\mathrm{el}}(Q^2) = \frac{-(\frac{1}{2} - 2\sin^2\theta_\mathrm{w}) + \frac{1}{2}N/Z}{\frac{1}{2} + 2\sin^2\theta_\mathrm{w}} . \tag{8}$$

For an isoscalar nucleus, the right hand side of eq. (8) is $2\sin^2\theta_\mathrm{w}/(\frac{1}{2} + 2\sin^2\theta_\mathrm{w}) \approx 1/2$. Thus the minimum of $\mathrm{d}\sigma/\mathrm{d}Q^2$ occurs at a value of Q^2 where the atomic form factor $F_{\mathrm{el}}(Q^2)$ has dropped to about 1/2. In contrast to ν_e scattering, the interference of electrons and nucleus in ν_μ (or ν_τ) scattering is constructive (for $\sin^2\theta_\mathrm{w} < 1/4$) and remains small because the electron amplitude is proportional to $(-\frac{1}{2} + 2\sin^2\theta_\mathrm{w})$. Thus only a minor enhancement of the cross section at low Q^2 occurs in this case.

As an illustration of the above remarks, we have plotted in fig. 2 the behaviour of coherent scattering of 20 MeV neutrinos from two typical atoms, ^{56}Fe and ^{28}Si. The former has $N = 30, Z = 26$, while the latter has an isoscalar nucleus $N = Z = 14$. As seen from the figure, for Q^2 in the domain $R^2_{\mathrm{nuc}} \ll 1/Q^2 \ll R^2_{\mathrm{atom}}$, the cross sections for ν_e, ν_μ and ν_τ are closely similar, differing only through the charge radius effect [which causes $\mathrm{d}\sigma/\mathrm{d}Q^2$ for ν_μ (ν_τ) to be 4% (6%) higher than for ν_e for $Q^2 \approx 1\ \mathrm{MeV}^2$]. For lower values of Q^2, the electron shell begins to play a role, causing the ν_e cross section to plunge to a minimum while producing a minor enhancement in the ν_μ or ν_τ cases. For $Q^2 = 0$, the cross sections take the limiting values given in eq. (7). In obtaining the curves in fig. 2, we used the electronic form factors given in ref. [8]. The position of the dip in $\mathrm{d}\sigma/\mathrm{d}Q^2$ is approximately $Q^2 = (10\ \mathrm{keV})^2$ for both ^{28}Si and ^{56}Fe.

The small momentum transfers ($Q^2 \sim 100\ \mathrm{keV}^2$) at which atomic effects are significant correspond to very low recoil energies ($E_{\mathrm{recoil}} = Q^2/2M \approx 10^{-3}$ eV for $A = 50$), which are below the threshold of detectability even in the "cold" detectors described in ref. [1]. However, the effects discussed above could have consequences in astrophysical contexts involving neutrinos of energy $\lesssim$100 keV. As one illustration, we consider the radiation pressure exerted on an atomic medium by a neutrino beam which is incident normally [9]. The pressure is

$$P = (\mathrm{Flux}) \cdot (D) \cdot \int_{-1}^{+1} \mathrm{d}\cos\theta \, \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} E_\nu (1 - \cos\theta) , \tag{9}$$

D being the depth of the medium. Using the results for $\mathrm{d}\sigma/\mathrm{d}\cos\theta$ obtained above, we have computed the difference in the radiation pressures of ν_e and ν_μ or ν_τ, expressed through the ratio

$$[P(\nu_i) - P(\nu_e)] / [P(\nu_i) + P(\nu_e)] ,$$

with $i = \mu, \tau$. This ratio is plotted in fig. 3 for ^{28}Si and ^{56}Fe, and shows that for $E_\nu \sim 10$ keV the pressure exerted by ν_μ or ν_τ is an order of magnitude higher than for ν_e.

Finally, as an example of an atomic system in which the axial contribution is significant, we consider neutrino scattering from atomic hydrogen. The vector coupling is given by

$$C_\mathrm{V}(^1\mathrm{H}) = (\tfrac{1}{2} - 2\sin^2\theta_\mathrm{w}) F_{\mathrm{P,V}}(Q^2) + [\mp\tfrac{1}{2} + 2\sin^2\theta] F_{1\mathrm{s}}(Q^2) , \tag{10}$$

where $F_{\mathrm{P,V}}$ denotes the vector form factor of the proton and $F_{1\mathrm{s}}(Q^2)$ that of the electron in the 1s orbit. (Notice that this coupling is small for ν_μ or ν_τ scattering. Further, for ν_e scattering, unlike the case of $N \geqslant Z$ nuclei, the proton and electron charges interfere constructively.) In accordance with eq. (6), the axial contribution (assuming uncorrelated electron and proton spins) is

$$[C_\mathrm{A}(^1\mathrm{H})]^2 = [\tfrac{1}{2} g_\mathrm{A} F_{\mathrm{P,A}}(Q^2)]^2 + [\mp\tfrac{1}{2} F_{1\mathrm{s}}(Q^2)]^2 , \tag{11}$$

where $F_{\mathrm{P,A}}$ now denotes the axial vector form factor of the proton. The axial piece is the same for all neutrino types, and is the major contribution for ν_μ and ν_τ scattering. The behaviour of neutrino scattering from atomic hydrogen is depicted in fig. 4, and obviously contrasts with the situation for heavier atoms ($N \geqslant Z$) plotted in fig. 2. Notable is the absence of any deep minimum in the ν_e cross section, which is, in fact, the dominant cross section at small angles. Observe also the peak in the cross section for backward scattering ($Q^2 = Q^2_{\mathrm{max}}$) which is the characteristic hallmark of the axial contribution.

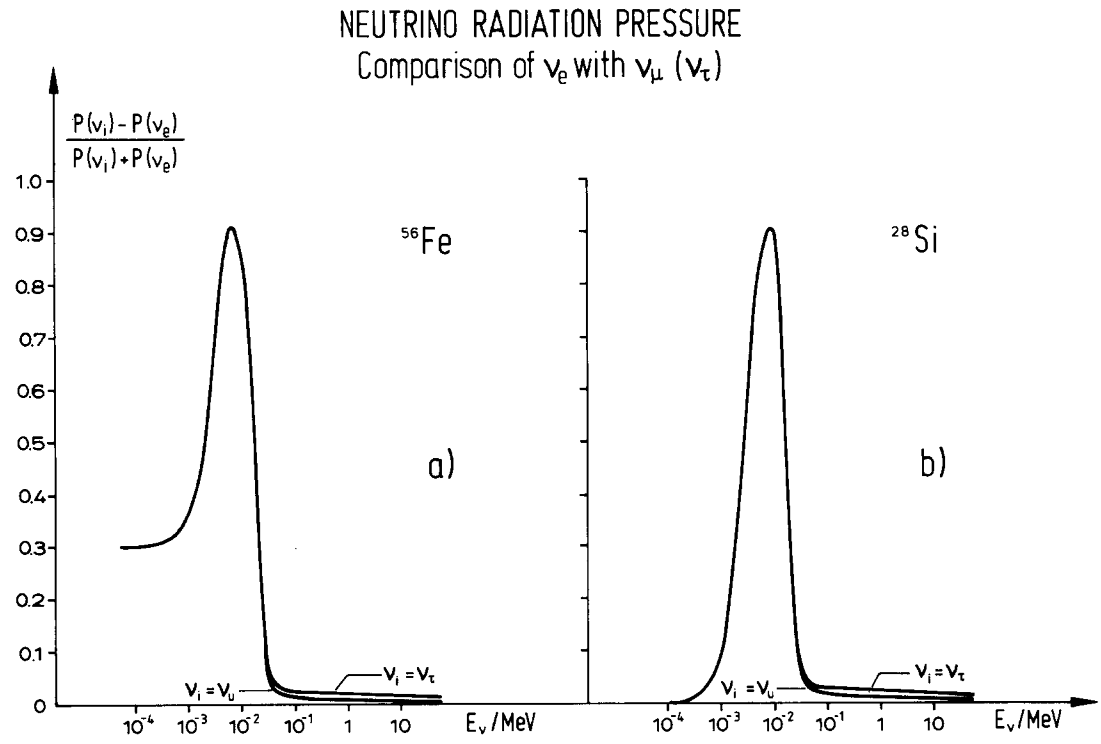
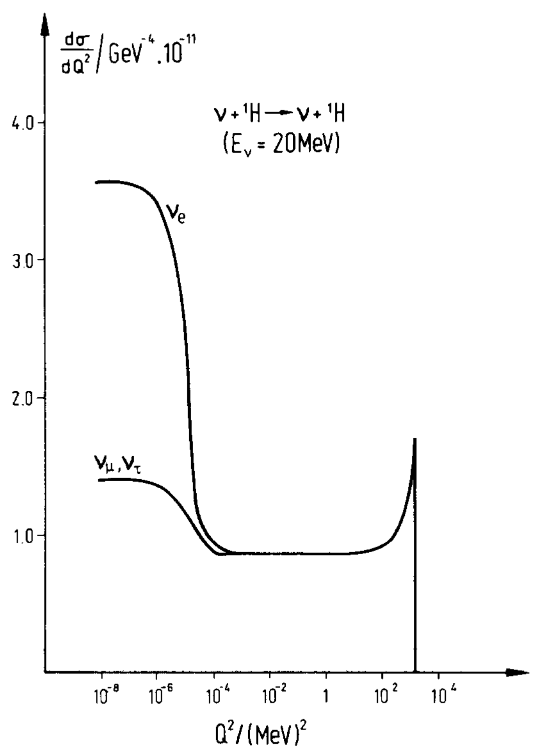


Fig. 3. Radiation pressure of neutrinos on a medium containing ^{28}Si or ^{56}Fe atoms. Comparison of ν_e with ν_μ (ν_τ).



It is curious that coherent neutrino scattering from an atom is practically the only reaction that is sensitive to the relative sign of the neutrino–electron and neutrino–nucleon couplings. A closely connected effect which also involves the coherent superposition of nuclear and electron amplitudes is the index of refraction experienced by a neutrino in a material medium [10].

We wish to thank the Bundesministerium für Forschung und Technologie for support of this research. One of us (M.W.) wishes to thank the Bavarian state for the award of a stipend, and Professor R. Rodenberg for kind encouragement.

Fig. 4. Coherent neutrino scattering from atomic hydrogen (E_ν = 20 MeV).

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