SCATTERING OF A NEUTRINO ON A BOUND ELECTRON

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A fundamental inference of the current theory of weak interactions is the prediction of the possibility of the scattering of a neutrino or an antineutrino on an electron. The scattering cross section on a free electron has been obtained earlier, for example, in [1]. However, as the scattering process can be realized experimentally by directing a neutrino beam at some atoms, it is of interest to clarify what influence the fact that the electrons are bound has on the magnitude of the total scattering cross section. In this paper we discuss the scattering cross section of a neutrino beam on electrons in an atom in the 1–s state. The mass of the nucleus is taken to be infinite, as we neglect its recoil.

The probability amplitude of the scattering of a neutrino on a bound electron can be written in the form

$$A_{if} = \frac{G}{\sqrt{2}} \int [\bar{\psi}_{e_2}(x)\, O_\beta\, \psi_{e_1}(x)] \times [\bar{\psi}_{\nu_2}(x)\, O_\beta\, \psi_{\nu_1}(x)]\, d^4x, \tag{1}$$

Here G is the weak-interaction constant, $O_\beta = \gamma_\beta(1 + \gamma_5)$, $\psi_k(x)$ are the operators of single-particle states of the respective particles $k \equiv e_2, \nu_1, \nu_2$; $\psi_{e_1}(x)$ is the single-particle state of the bound electron. For light nuclei $\alpha Z \ll 1$ [2]

$$\psi_{e_1} = N\left(1 - \frac{i}{2}\alpha Z \gamma_4 \boldsymbol{\gamma} \frac{\mathbf{r}}{r}\right) e^{-\eta r} \cdot u_0 e^{-i\varepsilon_1 t}, \tag{2}$$

where $N = \sqrt{Z^3/\pi a_0^3}$, a_0 is the Bohr radius, u_0 is a bispinor with components $u_{s=1/2} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$; $u_{s=-1/2} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$,

$\eta = Z/a_0$, α is the fine-structure constant, and $\gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ and $\boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}$ are the Dirac matrices. The electron in the final state and the neutrino in the initial and the final state are described by the single-particle plane-wave functions:

$$\psi_\kappa(x) = \frac{1}{\sqrt{2\varepsilon_\kappa}} e^{ip_\kappa x} u_{\lambda_\kappa}(p_\kappa), \tag{3}$$

where p_k is the 4-momentum of a particle, u_{λ_k} are bispinors satisfying the Dirac equation, and λ_k is the polarization index of a particle. The normalization (3) corresponds to one particle per unit volume. In what follows we use as notation for the 4-moments of the particles q_1, p_2, q_2 for the initial neutriono, the final electron, and the final neutrino respectively. $q_1 = \{\omega_1 \mathbf{q}_1\}$, $q_2 = \{\omega_2 \mathbf{q}_2\}$, $p_2 = \{\varepsilon_2 \mathbf{p}_2\}$.

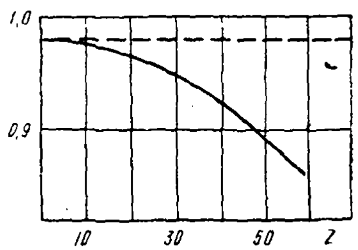

Fig. 1. Ratio of the scattering cross section on a bound electron to that on a free electron.

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Inserting the explicit forms of the functions into (1) and deriving the square of the modulus of the amplitude of the neutrino-electron scattering, we can obtain an expression for the probability of the process per unit time by the usual rules for averaging and the introduction of projection operators. The projection operator corresponding to the 1-s state is the five-component quantity $\Lambda_{1s} = 1/2\sum_{\lambda_e}\Phi_{e_1}\overline{\Phi}_{e_1}$, where Φ_{e_1} is the Fourier transform of the function (2), $\overline{\Phi} = \Phi^{+}\gamma_4$.

$$\Lambda_{1s} = \frac{32\pi a^3}{[1 + a^2t^2]^4}\,[\hat{\Lambda} + S], \tag{4}$$

$\hat{\Lambda} = \Lambda_\alpha\gamma_\alpha$, $a = 1/\eta$, $t = q_2 + p_2 - q_1$, Λ_α is a 4-vector with components $\{(1-t^2/4m_e^2), -t/m_e\}$, m_e is the mass of the electron, and S is the scalar part of the projection operator, equal to $(1 + t^2/4m_e^2)$. However, when the traces of the scalar part of Λ_{1s} are calculated, it turns out that they are equal to zero, so that we can drop S and regard Λ_{1s} as an ordinary 4-vector.

Taking account of (4), we can obtain the following expression for the differential probability of the process:

$$dW = \frac{16G^2a^5\delta(\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)}{\varepsilon_2\omega_2\omega_1(1 + a^2t^2)^4\pi^4}\,(q_2p_2)(q_2\Lambda)\,d\mathbf{q}_2d\mathbf{p}_2. \tag{5}$$

Direct integration with respect to $d\mathbf{q}_2$ and $d\mathbf{p}_2$ is difficult. In order to surmount this difficulty we can introduce a formal integration with respect to the vector $\mathbf{dt}$ in expression 5), using the property of the three-dimensional $\delta(\mathbf{t}-\mathbf{q}_2-\mathbf{p}_2+\mathbf{q}_1)$ function. Then, from the product of $\delta(\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)$ and $\delta(\mathbf{t}-\mathbf{q}_2-\mathbf{p}_2+\mathbf{q}_1)$ functions we can form a four-dimensional $\delta^4(t-q_2-p_2+q_1)$ function and, under the sign of integration with respect to dt, we can carry out the integration with respect to $d\mathbf{q}_2$ and $d\mathbf{p}_2$ using methods applicable to the calculation of cross sections of processes for free particles. The remaining integral with respect to $\mathbf{dt}$ is calculated without difficulty.

It is convenient to make a conversion to dimensionless coordinates in the following expressions, taking the mass of an electron as the unit of energy. As a result we obtain for the total scattering cross section of a neutrino on a bound electron ($\Omega_1 = \omega_1/m_e$)

$$\sigma = \frac{4G^2}{\pi}\,m_e^2\Omega_1^2\left[1 - \frac{1+3\pi}{4\pi}(\alpha Z)^2\right]. \tag{6}$$

As $Z \rightarrow 0$ (6) tends to the expression for the scattering of a neutrino on a free electron [3].

An analogous discussion of the scattering of an antineutrino on an electron leads to the following expression for the total cross section

$$\sigma = \frac{2G^2m_e^2\Omega_1}{3\pi}\left[1 - \frac{1}{(1+2\Omega_1)^3}\right]\left[1 - \frac{1+3\pi}{4\pi}(\alpha Z)^3\right]$$

Figure 1 shows the correction curve for the total scattering cross section of a neutrino or an electron as a function of the atomic number Z.

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