MEE 423 Model Based Project FINAL REPORT

Door Mechanism System

İsmail Uğur Demiral 170412033

Mehmet Berke Parlat 160412014

Emin Yazar 180412058

Hasan Ünlü 180412035

Supervisor : Assist. Prof. Dr. Özgün BAŞER

1. Introduction

The objective of this project is to design a state-space controller for a door mechanism system powered by a DC motor. The system incorporates components such as a gears for motion control. This report covers the modeling, analysis, and controller design steps required to stabilize the system and ensure desired performance. MATLAB and Simulink tools are used to validate the design through simulations.

2. System Description

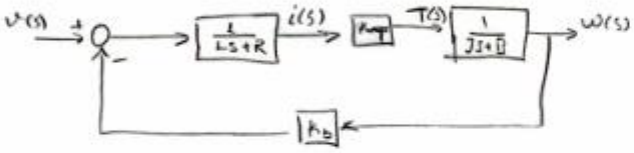
The door mechanism system is designed to automate the opening and closing process. It consists of:

- **DC Motor**: Converts electrical energy into rotational motion to drive the door.
- **Gear Mechanism**: Translates motor torque to the door with appropriate speed reduction.
- **System Overview**: A schematic diagram illustrates the interaction between components. The working principle involves controlling the motor's rotation to achieve precise door positioning.

Motivation for this system lies in its wide application in automated doors, enhancing convenience and security in various settings.

3. Mathematical Modeling The following steps outline the mathematical modeling process for the door mechanism system:

Step 1.Features of Our System



$L = 0,000632\ H$

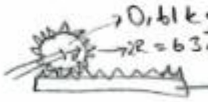
$R = 0,218\ \Omega$

$K_T = 0,242\ Nm/A$

$K_B = 0,242\ Vs/rad$

$J = 1,03 \cdot 10^{-4}\ kgm^2$

$B = \underline{0,2}68\ Nms/rad$



$\to 0,61\ kg - 40$ tooth

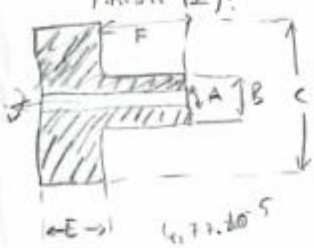
$\to 2R = 63,66\ mm \to R = 31,83\ mm = 0,03183\ m$

$\to m = 2,05\ kg$ - 200 tooth - 1m

Door weigth = 100 kg connected to rack

$M_{eq} = M + \frac{I}{R^2}$

Pinion (I):



$4,77 \cdot 10^{-5}$

$A = 12\ mm = 0,012m$

$B = 45\ mm = 0,045m$

$C = 63,66\ mm = 0,06366m$

$E = 18\ mm = 0,018m$

$F = 15\ mm = 0,015m$

$\text{Inertia}(I) = \frac{1}{2} m r^2$

$I = 3,1 \cdot 10^{-4}\ kgm^2$

$$m_{eq} = 102,05 + \underbrace{\frac{3,1 \cdot 10^{-4}}{(0,03183)^2}}_{0,305} = 102,355\ kg$$

$$\cancel{\frac{1}{2}} \underline{J_{eq}}\ w_{eq}^2 = \cancel{\frac{1}{2}} J_P\, w_P^2 + \frac{1}{2} J_m\, w_m^2 \qquad w_p = w_m$$

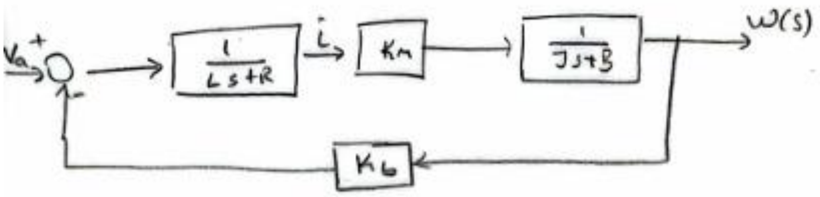
$$J_{eq} = J_p + J_m$$

$$J_{eq} = 3,1 \cdot 10^{-4} + 1,03 \cdot 10^{-4} = 4,13 \cdot 10^{-4}\ kg/m^2$$

```
Jm = 1.03 * 10^-4 ;% kgm^2
Jp = 3.10 * 10^-4 ;% kgm^2
Bm = 0.268;% Nms/rad
Brp = Bm * 80 / 100 ;

Jeq = Jm + Jp; % kgm^2
Beq = Brp + Bm; % Nms/rad % Friction considered as %80 of Bm and added to the Bm and found Beq
Kb = 0.242; % Vs/rad
Km = 0.242; % Nm/A
L = 0.000632; % Hennry
R = 0.218; % Ohm
```

Step 2. State Variables



$$V_a(t) - K_b\, w(t) = L\,\frac{di(t)}{dt} + R\, i(t)$$

$$T(t) = K_m\, i(t)$$

$$T(t) = J\,\frac{dw(t)}{dt} + B\, w(t)$$

<u>State variables</u> $\quad$ <u>Input</u>

$x_1 = w(t)$ $\quad$ $u = V_a(t)$,

$x_2 = i(t)$

$$K_m \cdot x_2 = J\dot{x}_1 + B x_1 \Rightarrow \dot{x}_1 = \frac{-B}{J} x_1 + \frac{K_m}{J} x_2$$

$$u - K_b x_1 = L\dot{x}_2 + R x_2 \Rightarrow \dot{x}_2 = -\frac{K_b}{L} x_1 - \frac{R}{L} x_2 + \frac{1}{L} u.$$

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} \frac{-B}{J} & \frac{K_m}{J} \\ -\frac{K_b}{L} & -\frac{R}{L} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

$$C = [1, 0] \quad , \quad D = 0$$

```
A = [-Beq/Jeq, Km/Jeq; -Kb/L, -R/L];
B = [0; 1/L];
C = [1, 0];
D = 0;

motor = ss(A,B,C,D);
gear_ratio = tf(1,[5,0]);
plant = motor * gear_ratio;
%step(plant)
A = plant.A;
B = plant.B;
C = [0,0,1];
D = 0;
```

```
plant =

  A =
            w       i       x
   w    -1168     586       0
   i   -382.9  -344.9   632.9
   x        0       0       0

  B =
         v
   w     0
   i     0
   x   0.5

  C =
       w  i  x
   y   0  0  1

  D =
       v
   y   0

Continuous-time state-space model.
```

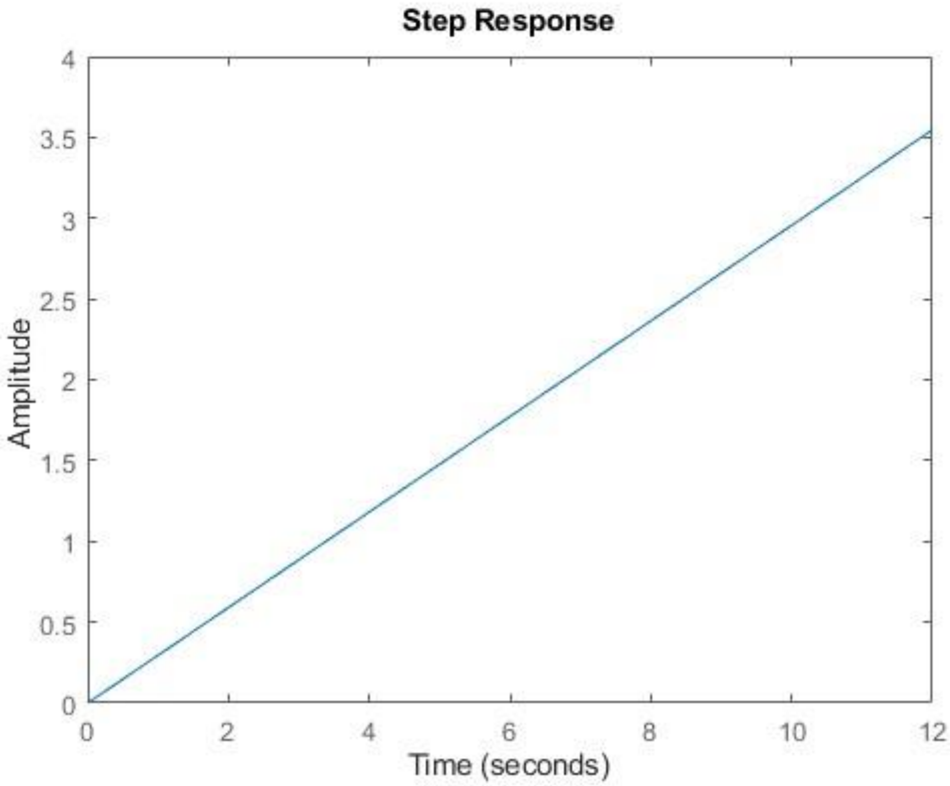


Figure 1. Step Response of Open Loop System

4. Control Design The control design process follows these steps:

1. **Pole Placement**:
   - Define control objectives, such as stability, overshoot minimization, and response time.
   - Select desired pole locations based on system performance requirements.
   - Design a state feedback control law u=−KXu = -KX to place the poles at desired locations.

Step 4 Our System’s Control Objectives

The settling time must be less 1 sec
max overshoot of the unit-step response less than %10

$$M_{ovp} = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \cdot 100 = 10\%$$

$$e^{\left(\frac{-\pi \xi}{\sqrt{1-\xi^2}}\right)} = 0,01$$

$$\frac{-\pi \xi}{\sqrt{1-\xi^2}} = \ln 0,01 \Rightarrow \frac{\pi^2 \xi^2}{1-\xi^2} = \underbrace{\ln^2 0,01}_{21,21}$$

$$\pi^2 \xi^2 = 21,21 - 21,21\, \xi^2$$

$$(\pi^2 + 21,21)\, \xi^2 = 21,21$$

$$\xi^2 = 0,685$$

$$\xi = 0,83$$

Settling time less than 1 sec

$$w_n = \frac{4}{0,82 \cdot w_n} = 1$$

$$w_n = 4,88 \text{ rad/s}$$

$$S_{1,2} = -\xi w_n \pm i\, w_n \sqrt{1-\xi^2}$$

$$= -0,82 \cdot 4,88 \pm i\, 4,88 \sqrt{1-0,82^2}$$

$$= -4,00 \pm i\, 2,79$$

$$S_3 = 150$$

$$(s+150) \cdot (s^2 + 7,78) = 0$$

$$s^3 + 150 s^2 + 7,78 s + 1167 = 0$$

$$s^3 + 1,513 \cdot 10^3 s^2 + 6,2727 \cdot 10^5 s = 0$$

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -6,2727 \cdot 10^5 & -1,513 \cdot 10^3 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$s^3 + (K_3 + 1513) s^2 + (K_2 + 627270) s + K_1 = s^3 + 150 s^2 + 7,78 s + 1167$$

$$K_3 = -1363$$

$$K_2 = -627262,22$$

$$K_1 = 1167$$

```
characteristicEquation = det(s * eye(size(A)) - A);
poles = eig(A);
%desired poles and new characteristic equation
F = [0 1 0;0 0 1;0 -6.2727e+05 -1.5130e+03];
G = [0;0;1];

DCE = det(s*eye(3)- F + G * [K1 K2 K3]);% = s^3 + 150s^2 + 7.78 s + 1167
%DCE = K1 + (627270 + K2)s + (K3 + 1513)s^2 + s^3
%K1 = 1167, K2 = -627262.22, K3 = -1363

ctrb_matrix = ctrb(plant);
iscontrollable = det(ctrb_matrix); % Different than zero which means controllable

pc = [-4+2.79i, -4-2.79i,-150];
K = place(A,B,pc);
Ntilda = rscale(plant, K);

Acl = A - B*K;
Bcl = B*Ntilda;
syscl = ss(Acl, Bcl, C, D);
step(syscl)
```

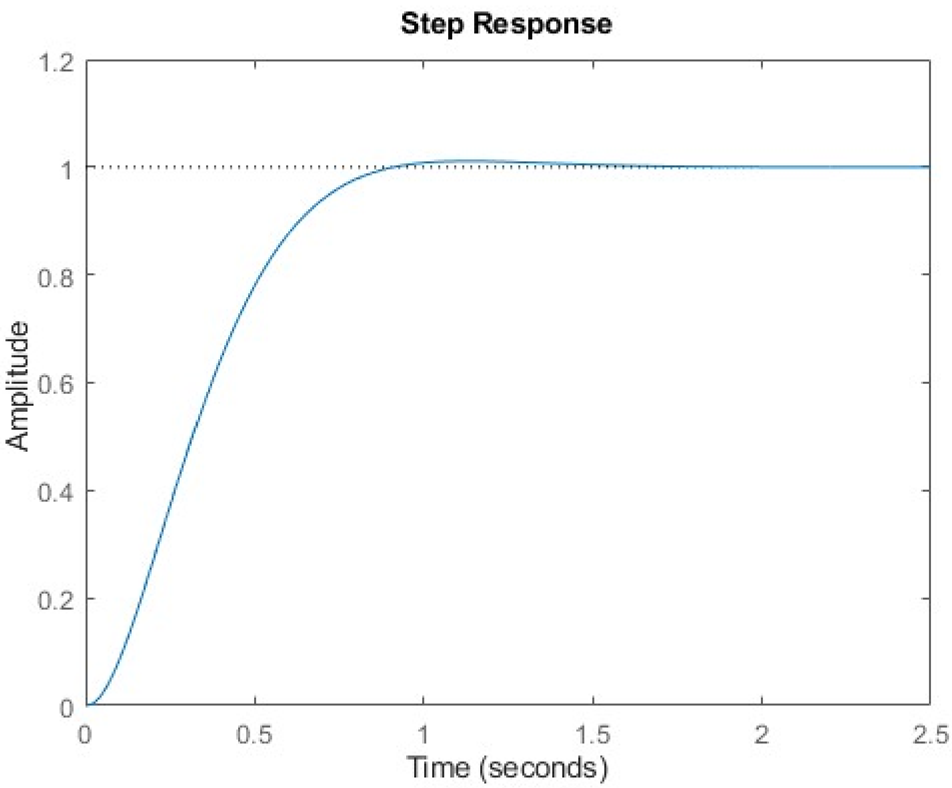


Figure 2. Step Response of Closed Loop System With Pole Placement

Step 5

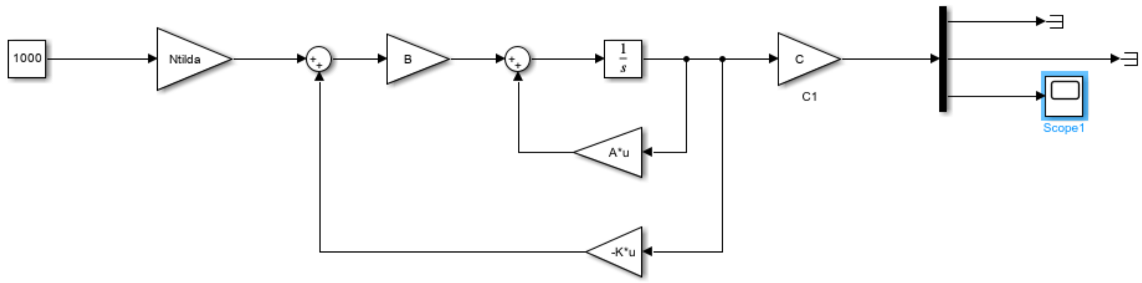


Figure 3. Simulink block diagram of Closed Loop System

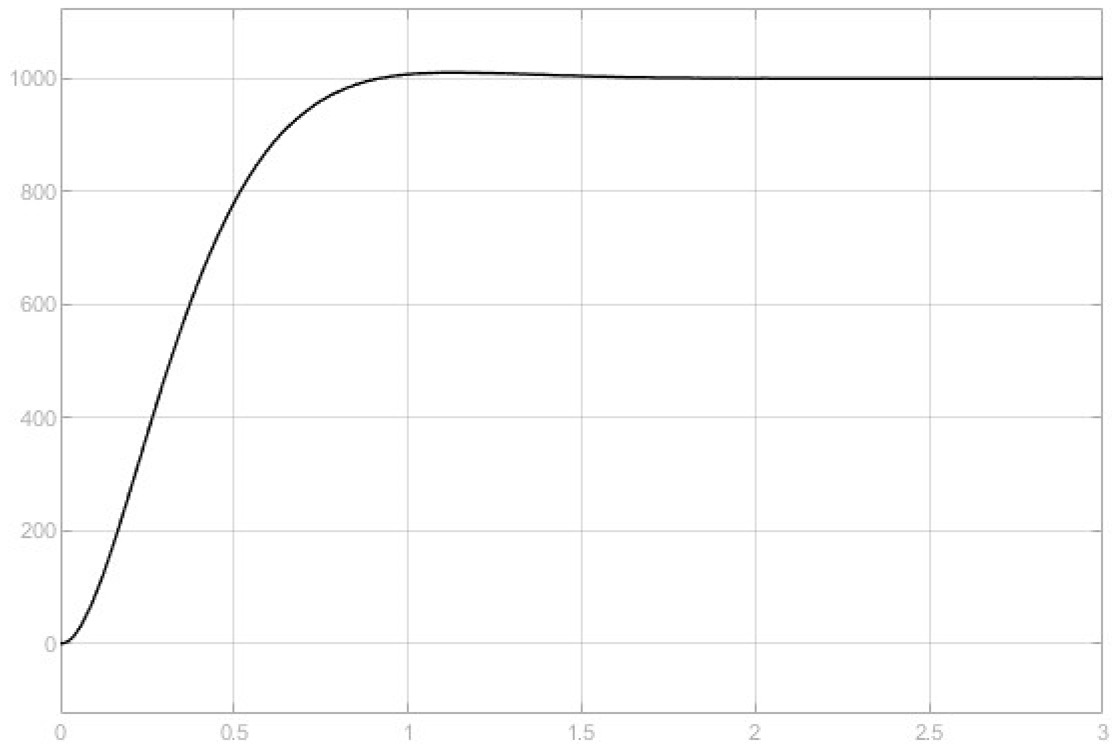


Figure 4. Output of Closed Loop System

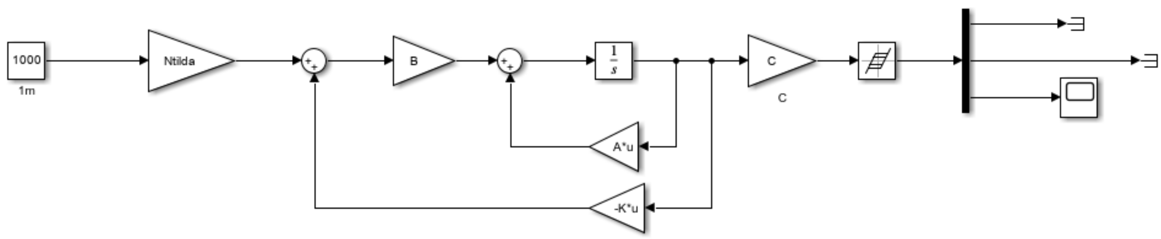


Figure 5. Simulink block diagram of Closed Loop System with nonlinearity

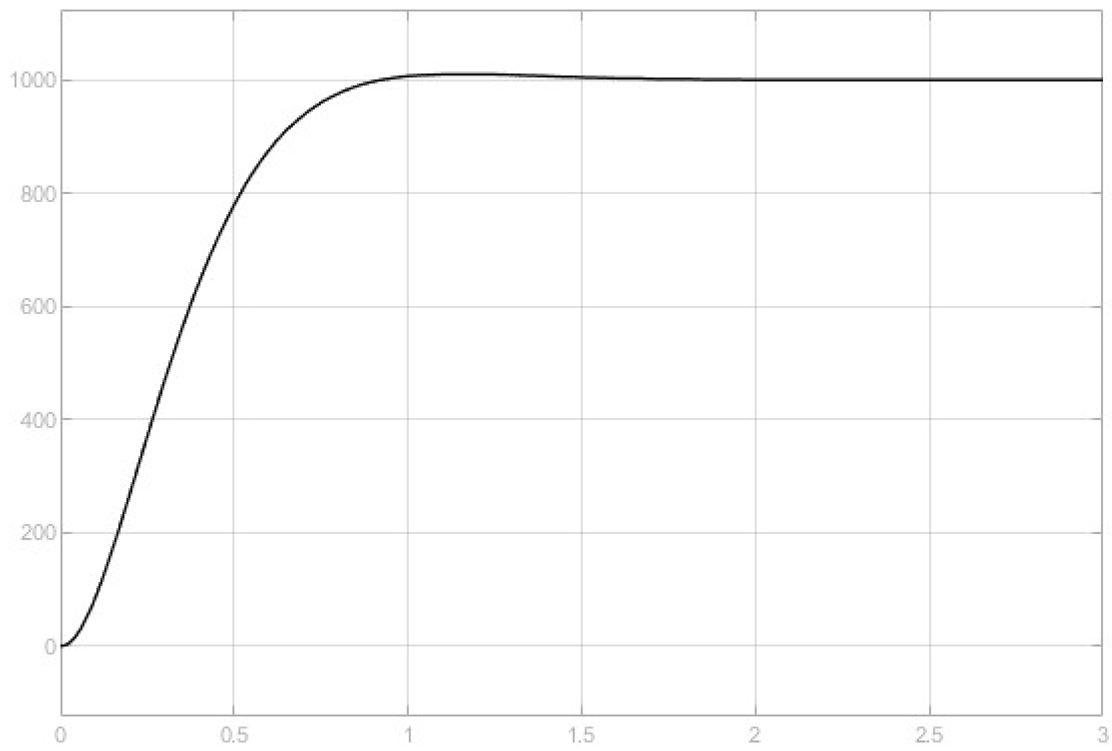


Figure 6. Output of Closed Loop System with nonlinearity

2. **Integral Control Design**:
    - Introduce an integral action to eliminate steady-state error.

Step 6. Integral Action Design

$$A = \begin{bmatrix} -1,168\cdot10^3 & 585,9664 & 0 \\ -382,9114 & -344,9367 & 632,9114 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0,5 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad D = 0$$

$$z = y - r = cx - r$$

$$\dot{\underline{x}} = (A - \underline{b}K_p)\underline{x} - \underline{b}K_i z$$

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A - \underline{b}K_p & -\underline{b}K_i \\ \underline{c} & 0 \end{bmatrix} \begin{bmatrix} \underline{x} \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r$$

$$\bar{A} = \begin{bmatrix} A & \underline{0} \\ -\underline{c} & 0 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} \underline{b} \\ 0 \end{bmatrix}$$

```
%Integral Action
Abar = [plant.A, zeros(3,1);-plant.C, 0];
Bbar = [plant.B;0];
newcharacteristicEquation = det(s * eye(size(Abar)) - Abar - Bbar * [K1 K2 K3 K4]);
eig(Abar);
pc_integral = [-4+2.79i, -4-2.79i,-150 ,-200];
Kint = acker(Abar, Bbar, pc_integral);
F = Kint(1:3); % Closed loop coefficient
H = Kint(4);% Integral gain
```

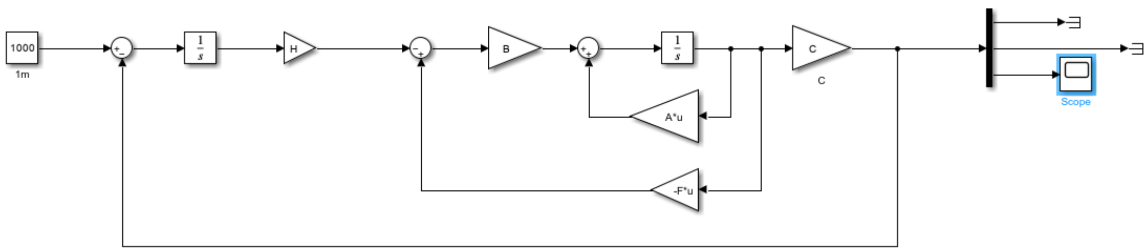


Figure 7. Simulink block diagram of Closed Loop Integral Action System

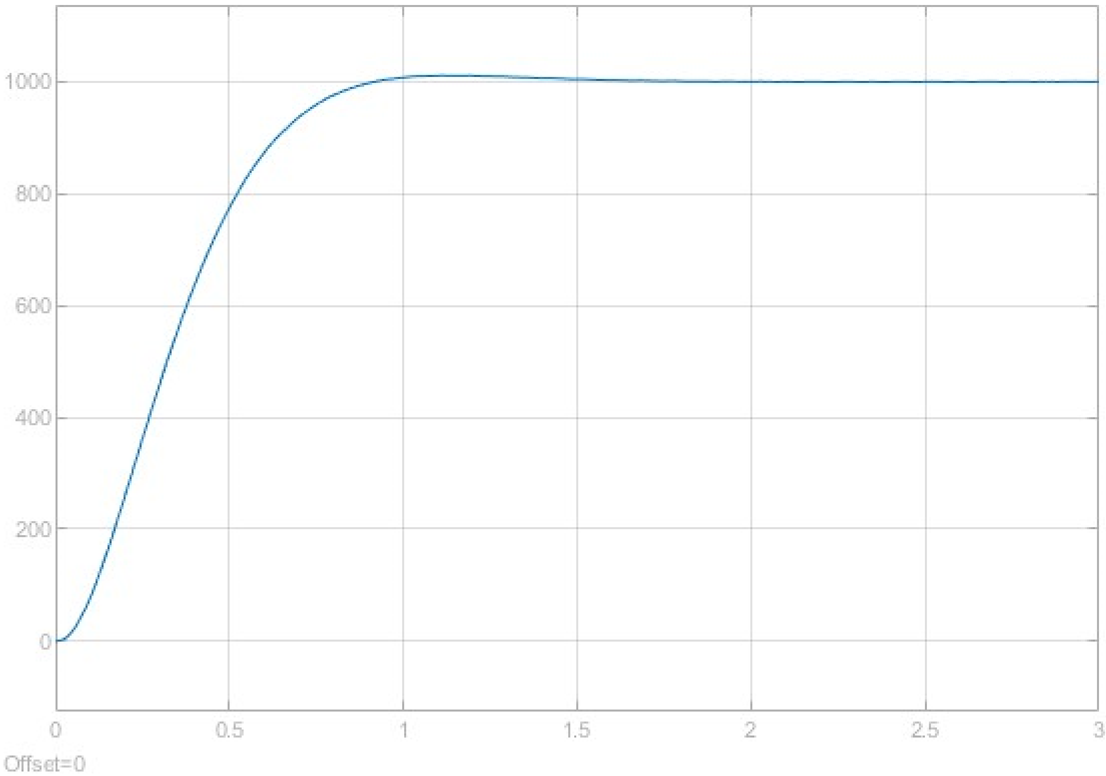


Figure 8. Output of Closed Loop Integral Action System with nonlinearity

5. Simulation and Results

The following simulation steps are conducted to evaluate system performance. Simulation results, including graphs and performance metrics, are compared to demonstrate improvements achieved through the designed controller.

6. Conclusion

This project has successfully demonstrated the design and analysis of a state-space controller for a door mechanism system. The integration of state feedback and integral control has achieved key objectives, including system stability, minimized overshoot, and elimination of steady-state error. The simulations have shown that the designed controller significantly improves the system's response compared to its open-loop behavior.

The use of pole placement techniques allowed for precise control of dynamic behavior, while the incorporation of an integral action ensured zero steady-state error. These results validate the theoretical design and highlight the effectiveness of state-space methods in control system engineering.