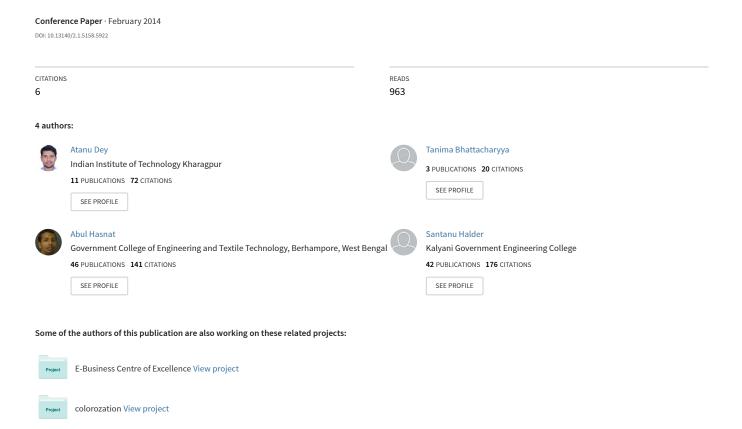
# A Fast FPGA based Architecture for Determining the Sine and Cosine Value





# A Fast FPGA based Architecture for Determining the Sine and Cosine Value

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Abstract— This paper aims to design a fast FPGA based architecture for determining the sine and cosine values using Taylor's Series. In this work, a pipelined architecture has been proposed for designing a flexible and scalable digital sine and cosine wave generator. An FPGA-Based architecture is implemented in VHDL, synthesized on Xilinx Spartan 3 xc3s200-5ft256 FPGA kit simulated on ModelSim 6.2c. The proposed architecture gives accuracy of the computed sine and cosine values up to 21st bits in all cases and up to 22 or 23 bits for some cases in six clock cycles only whereas CORDIC architecture needs 21 clocks for 21bit accuracy. Trigonometric waves is used in countless applications i.e. Software Defined Radio (SDR). The proposed system is able to operate at the frequency of 93.119 MHz.

Index Terms—FPGA, SDR, CORDIC, Sine, Cosine.

#### I. INTRODUCTION

There are plenty of applications which require digital wave generators. Wireless and mobile systems are among the fastest growing application areas; in particular, Software Defined Radio (SDR) is currently a focus of research and development. An SDR allows performing many functions based on a single hardware platform, thus highly reconfigurable resources for signal processing are needed, mainly for modulation and demodulation of digital signals. Fields of development are increasing every day with applications such as cell phones or military communication.

There is a widely used method, CORDIC [1-4] which is implemented in most of the application to generate digital sine and cosine waves. Although CORDIC algorithm computes trigonometric angle values in a simple and faster way but it needs n number of clock cycles to get accuracy up to n<sup>th</sup> bit [1-4]. So there is a need to develop a system which would give the better accuracy in less number of clock cycles. The proposed architecture offers an alternative, which takes 23 bits input (for single precision format) and generates Sine and Cosine values in six clock cycles only and gives accuracy up to 21 bits in all the cases (and for some cases 22 bits or 23 bits accuracy achieved) where as CORDIC needs 21 clock cycle to compute the same

accuracy. In the present work, The Taylor series [5-7] is implemented in VHDL based pipelined architecture is proposed which is synthesized in Xilinx Spartan 3 XC3S50-5PQ208 FPGA, simulated on the Modelsim 6.2c from Mentor Graphics Corporation.

This paper is organized as follows: Section II describes the existing CORDIC algorithm briefly. Section III briefly explains the Taylor series [5-7]. Section IV presents the top level design of proposed hardware. Section V depicts the proposed system architecture. Section VI shows the experimental results and finally section VII concludes and remarks about some of the aspects analyzed in this paper.

#### II. EXISTING METHODOLOGY

Coordinate Rotation Digital Computer (CORDIC) is a well known algorithm used to approximate iteratively some transcendental function [1-4].

Let the starting vector is defined as  $v_0 = (x_0, y_0)$  and after n iterations,  $v_0$  moves to  $v_n$  in anticlockwise direction by  $\theta^0$  as shown in figure 1.

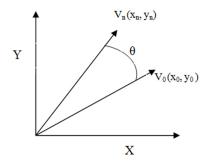


Figure 1. Anticlockwise rotation of a vector  $v_0$  to  $v_n$  by  $\theta^0$ 

So, the vector  $v_n = (x_n, y_n)$  can be represented by Eq. 1 and Eq. 2.

$$x_n = x_0 \cos\theta - y_0 \sin\theta \tag{1}$$

$$y_n = y_0 \cos\theta + x_0 \sin\theta \tag{2}$$

In each iteration i, the vector perform a micro-rotation by  $\theta_i$ , so the new vector is calculated with a similar function

$$x_{i+1} = x_i \cos \theta_{i+1} - y_i \sin \theta_{i+1}$$
 (3)

$$y_{i+1} = y_i \cos \theta_{i+1} + x_i \sin \theta_{i+1} \tag{4}$$

When the term  $\cos \theta_{i+1}$  is factorized, components in the vector are described by

$$x_{i+1} = \cos \theta_{i+1} (x_i - y_i \tan \theta_{i+1})$$
 (5)

$$y_{i+1} = \sin \theta_{i+1} (y_i + x_i \tan \theta_{i+1})$$
 (6)

 $\tan \theta_{i+1}$  is restricted to  $\pm 2^{-i}$ , so multiplication is converted in an arithmetic right shift. It is also useful to use the identity  $\cos \theta_{i+1} = \cos(\arctan 2^{-i})$  to define the next variables.

$$K_i = \cos(\arctan 2^{-i}) = 1/\sqrt{(1+2^{-2i})}$$
 (7)

$$d_i = \pm 1 \tag{8}$$

Cosine is an even function, therefore  $\cos(\alpha) = \cos(-\alpha)$ . So (5) and (6) can be transformed into

$$x_{i+1} = K_i(x_i - y_i d_i 2^{-i}) (9)$$

$$y_{i+1} = K_i (y_i + x_i d_i 2^{-i})$$
 (10)

Multiplication by  $K_i$  is avoided by considering it as a gain factor for all iterations. If n number of iterations are performed, then K is defined as the multiplication of every  $K_i$ .

$$K = \prod K_i = \prod I / \sqrt{(1 + 2^{-2i})}$$
 (11)

As the vector is initialized with constant K, the vector components of each the iteration are simplified to

$$x_{i+1} = (x_i - y_i d_i 2^{-i}) (12)$$

$$y_{i+1} = (y_i + x_i d_i 2^{-i})$$
(13)

On each iteration it is necessary to decide whether  $d_i = 1$  or  $d_i = -1$ . In order to make that decision, the difference between the desired angle and the current angle is used. So a new variable know as accumulator is defined as:

$$z_{i+1} = z_i - d_i \arctan 2^{-i} \tag{14}$$

The value of  $z_0$  is the angle for which sine and cosine are to be calculated. To know whether  $d_i$  should be positive or negative, the following rule is used:

$$d_i = -1 \qquad \sin z_i < 0$$
  

$$d_i = +1 \qquad \sin z_i \ge 0$$
(15)

The well known CORDIC by Volder [1], is a classical technique to compute both sine and cosine using such decomposition into micro-rotations. These rotations are chosen in such a way that the value can be computed using one adder or one subtractor and two shifters for each clock cycle.

In vectoring mode, coordinates  $(x_0, y_0)$  are rotated until  $y_0$  converges to zero. In rotation mode, initial vector  $(x_0, y_0)$  starts aligned with the x-axis and is rotated by an angle of  $\theta_i$  every cycle, so after n cycles,

 $\theta_i$  is obtained angle. Figure 2 shows the CORDIC pipeline architecture given by Esbetan O. Garcia et al[2].

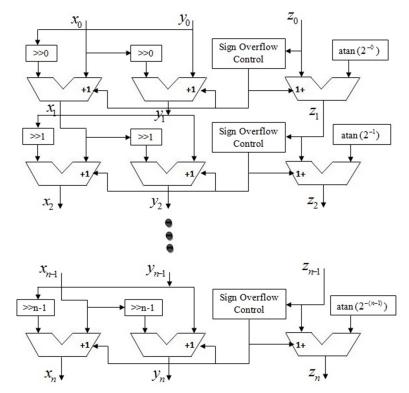


Figure 2. Pipelined CORDIC Architecture

Like CORDIC, proposed implementation does not need to have Cosine values stored in memory, instead it approximates the results, on each step of the iterative process. The proposed architecture may be extended for a specific resolution without a considerable increase in the number of components used.

### III. PROPOSED METHODOLOGY

Taylor series [5-7] is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. Sine and Cosine value is calculated using Eq. 16 and Eq. 17 respectively.

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} + \dots$$
 (16)

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{1!!} + \frac{x^{13}}{13!} - \dots$$
 (17)

Where  $-\infty < x < \infty$ 

As the present work takes into consideration tenth and eleventh power of x for cosine and sine values respectively, the Taylor series reduces to Eq. 18 and Eq. 19.

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$$
 (18)

$$\cos(x) = 1 - \frac{x^{2}}{2!} + \frac{x^{4}}{4!} - \frac{x^{6}}{6!} + \frac{x^{8}}{8!} - \frac{x^{10}}{10!}$$

$$\sin(x) = x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \frac{x^{7}}{7!} + \frac{x^{9}}{9!} - \frac{x^{11}}{11!}$$
(19)

As 1/(2!), 1/(4!), 1/(6!), 1/(8!) and 1/(10!) are constant so these values are stored in a look up table. The next section describes VHDL implementation of Eq. 18 and Eq.19.

#### IV. TOP LEVEL DESIGN

The top level design of proposed architecture is shown in Figure 3. The proposed architecture takes one 23bit value as input. Then the system calculates the Sine and Cosine value using the given input. Finally, the system generates two 23-bit values Sine and Cosine respectively. Fig. 3 shows the top level design of the proposed architecture.



Figure 3. The top level design of Proposed architecture

In the present work, if the input angle is less than 57.29 degree results to less than 1 in radian. Thus for any angle  $\theta$  which is less than 57.29 degree is directly converted into radian value and its 23 bit binary representation is used as input and as output Y, Z is considered as  $\sin(x)$ ,  $\cos(x)$  values respectively. But if the input angle x is greater than 57.29 degree, then simply (90 - x) is used as input angle and its 23 bit binary representation of the angle in radian is used as input to the system. Second input is set to one to indicate to alternate the output i.e. Y, Z is considered as cos(x), sin(x) respectively because sin(90 - x) = cos(x) or vice versa. This is done simply to avoid normalization of input and output values further i.e. if the input angle is less than 57.29 degree then its corresponding radian value lies in the range  $0 \le x < 1$ . Assuming decimal point on left side, corresponding binary value is given as input and output is also taken as the given binary string assuming decimal point lies in the left side. Thus no normalization is needed.

### V. System Architecture

The proposed system architecture is shown in Figure 4 which has a pipelined architecture, consisting two adder, three subtractor and five multiplier for sine and cosine angle calculation to get 21-bit accuracy all the times and 22 to 23 accuracy in some cases.

The various blocks of this architecture are described next. All the inputs in each and every block are taken in 23 bits and also all blocks forwards output in 23 bits.

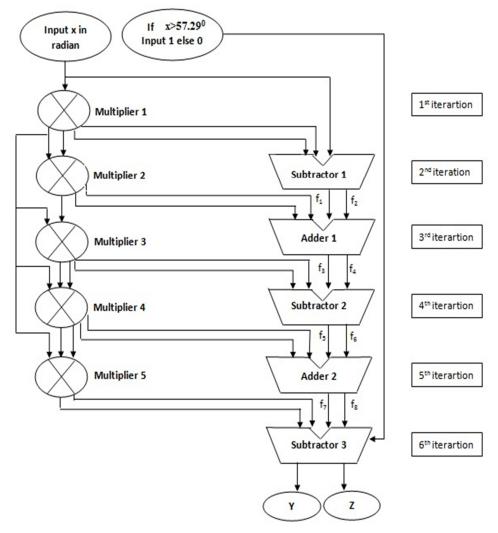


Figure 4. Proposed system architecture

# Multiplier 1:

Multiplier 2:

Multiplier1 unit takes angle as input (in radian) and calculates  $x^2$ ,  $x^3$ ,  $\frac{x^2}{2!}$  and  $\frac{x^3}{3!}$ . Multiplier1 forwards  $x^2$  value as input to all other multipliers. It also forwards  $x^3$  value as input to Multiplier 2 and also forwards  $\frac{x^2}{2!}$  and  $\frac{x^3}{3!}$  as inputs to Subtractor 1.

Multiplier 2 takes  $x^2$  and  $x^3$  as inputs and calculates  $x^4, x^5, \frac{x^4}{4!}$  and  $\frac{x^5}{5!}$ . Multiplier 2 forwards  $x^4$  and  $x^5$ 

values input to Multiplier 3 and also forwards  $\frac{x^4}{4!}$  and  $\frac{x^5}{5!}$  as inputs to Adder 1.

Multiplier 3:

Multiplier 3 takes  $x^4$  and  $x^5$  as inputs and calculates  $x^6, x^7, \frac{x^6}{6!}$  and  $\frac{x^7}{7!}$ . Multiplier 3 forwards  $x^6$  and  $x^7$  values input to Multiplier 4 and also forwards  $\frac{x^6}{6!}$  and  $\frac{x^7}{7!}$  as inputs to Subtractor 2.

# Multiplier 4:

Multiplier 4 takes  $x^6$  and  $x^7$  as inputs and calculates  $x^8, x^9, \frac{x^8}{8!}$  and  $\frac{x^9}{9!}$ . Multiplier 4 forwards  $x^8$  and  $x^9$  values input to Multiplier 5 and also forwards  $\frac{x^8}{8!}$  and  $\frac{x^9}{9!}$  as inputs to Adder 2.

Multiplier 5 takes  $x^8$  and  $x^9$  as inputs and calculates  $x^{10}$ ,  $x^{11}$ ,  $\frac{x^{10}}{10!}$  and  $\frac{x^{11}}{1!!}$ . Multiplier 5 forwards  $x^{10}$  and  $x^{11}$  values input to Multiplier 5 and also forwards  $\frac{x^{10}}{10!}$  and  $\frac{x^{11}}{1!!}$  as inputs to Subtractor 3.

Subtractor 1:

Multiplier 5:

It takes angle as direct input (in radian) and also two more inputs  $\frac{x^2}{2!}$  and  $\frac{x^3}{2!}$  and calculates  $f_1$  and  $f_2$  as follows

$$f_1 = x - \frac{x^3}{3!} \tag{18}$$

$$f_2 = 1 - \frac{x^2}{2!} \tag{19}$$

And forwards the outputs  $f_1$  and  $f_2$  as input to Adder 1.

# Adder 1:

It takes two inputs  $f_1$ ,  $f_2$  and calculates  $f_3$  and  $f_3$  as follows

$$f_3 = f_1 + \frac{x^5}{5!} \tag{20}$$

$$f_4 = f_2 + \frac{x^4}{4!} \tag{21}$$

And forwards the outputs  $f_3$  and  $f_4$  as input to Subtractor 2.

# Subtractor 2:

It take two inputs  $f_3$  and  $f_4$  and calculates  $f_5$  and  $f_6$  as follows

$$f_5 = f_3 - \frac{x^7}{7!} \tag{22}$$

$$f_6 = f_4 - \frac{x^6}{6!} \tag{23}$$

And forwards the outputs  $f_5$  and  $f_6$  as input to Adder 2.

# Adder 2:

It takes two inputs  $f_5$ ,  $f_6$  and calculates  $f_7$  and  $f_8$  as follows

$$f_7 = f_5 + \frac{x^9}{9!} \tag{24}$$

$$f_8 = f_6 + \frac{x^8}{8!} \tag{25}$$

And forwards the outputs  $f_7$  and  $f_8$  as input to Subtractor 3.

Subtractor 3:

It take two inputs  $f_7$  and  $f_8$  and calculates  $f_9$  and  $f_{10}$  as follows

$$f_9 = f_7 - \frac{x^{11}}{1!!} \tag{26}$$

$$f_{10} = f_8 - \frac{x^{10}}{10!} \tag{27}$$

And finally the output  $f_9$  is the calculated value of  $\sin(x)$  and  $f_{10}$  as calculated value of  $\cos(x)$  if input angle in degree is less than 57.29 else  $f_9$  is the calculated value of  $\cos(x)$  and  $f_{10}$  as calculated value of  $\sin(x)$ . The system architecture shown in Fig. 4 is a pipeline architecture which computes sine and cosine values in only 6 clock cycle resulting faster computation.

### VI. EXPERIMENTAL RESULTS

The system for Sine Cosine value calculation is implemented using VHDL, synthesized for a Xilinx Spartan 3 xc3s200-5ft256 with simulation on the Modelsim 6.2c from Mentor Graphics Corporation. Experiment has

been conducted on different value in the range  $[0, \frac{\Pi}{2}]$ . The proposed architecture is capable to operate at the

clock frequency of 93.119 MHz and takes only 6 clock cycles to get the result in 23 bits. The device utilization summary for the proposed architecture for sine and cosine value computation is given in Table I. For comparison study of the proposed architecture with the standard CORDIC architecture, the device utilization summary of CORDIC architecture simulated on a Xilinx Spartan 3 xc3s200-5ft256 using Xilinx ISE 7.1 is also given in Table I.

 $TABLE\ I.\ COMPARISON\ DEVICE\ UTILIZATION\ SUMMARY\ OF\ THE\ PROPOSED\ ARCHITECTURE\ AND\ CORDIC$ 

	Proposed Method		CORDIC	
Parameter	Used	%	Used	%
Number of Slices	946	49	1075	55
Number of Flip Flops	1061	27	570	14
Number of 4 input LUTs	1411	36	1737	44
Number of bonded IOBs	69	39	43	24
Number of GCLKs	1	12	1	12
Maximum Frequency	93.119 MHz		124.67MHz	

Device utilization summary is given in Table I, it shows that the proposed architecture takes 129 (1075-946) number of Slices, 326 (1737-1411) number of 4 input LUTs less compared to that of CORDIC architecture. But the proposed architecture computes sine and cosine values up to 21<sup>st</sup> bit accuracy in only 6 clock cycle whereas CORDIC architecture will require 21 clock cycles to compute the same [1-3]. The FPGA based proposed system operates as fast as 93.119 MHz.

The proposed architecture has been tested using Modelsim 6.2c simulator and computation accuracy achieved up to 21<sup>th</sup> bits or more. Table II shows the computed sine and cosine values using the proposed architecture which clearly shows the accuracy up to 21<sup>th</sup> bits in only 6 clock cycles whereas to get accuracy up to 21<sup>th</sup> bits CORDIC architecture needs 21 clock cycles. Table II shows the computed sine and cosine values calculated using proposed method. The Figure 5(a) and 5(b) shows 23 bit, 22 bit and 21 bit accuracies given by the proposed system for sine and cosine values. For calculation of sine(x), the proposed system produces 23 bit, 22 bit and 21 bit accuracy for 53.33%, 81.66% and 100% cases respectively and for cosine(x), it produces 23 bit, 22 bit and 21 bit accuracy for 53.33%, 80%, and 100% cases respectively.

Figure 6 shows the snapshot of sample output which is simulated on ModelSim SE 6.2c where input angle (in radian) and output values are shown in red and yellow marked rectangles respectively.

TABLE II. EXPERIMENTAL RESULTS FOR BOTH SINE AND COSINE VALUES

Sl	Angle (Degre e)	Angle in Radian (23-bit)*	Sine(x)*	Sine(x) (Decimal)	Cosine(x)*	Cosine(x) (Decimal)
01	0.20	00000000111001001100 001	000000001110010011000 01	0.00349065	111111111111111111001 101	0.99999390
02	2.60	00001011100111011110 110	000010111001110011101 00	0.04536298	111111111111111001000 101	0.99897062
03	3.750	00010000110000010101 001	000100001011111001000 10	0.06540298	111111111011100111010 111	0.99785892
04	5.000	00010110010101110001 100	000101100100111111011 00	0.08715581	11111111000001101001 111	0.99619469
05	5.458	00011000011000101111 011	000110000101100110000 11	0.09511606	11111110110101101101 111	0.99542125
06	6.743	00011110001000001100 010	000111100000111011111 01	0.11741607	11111110001110101010 111	0.99308276
07	7.85	00100011000100101111 101	001000101111011011101 01	0.13658011	11111101100110011101 111	0.99062902
08	8.59	00100110011000010110 011	001001100011110010100 01	0.14936277	11111101001000001101 101	0.98878246
09	10.372	00101110010101111011 000	001011100001011100000 00	0.18003846	11111011110100010001 111	0.98365957
10	12.895	00111001100111011000 101	001110010010000101011 00	0.22316505	11111001100010110011 101	0.97478067
11	13.9475	00111110010100010110 101	001111011101101000101 00	0.24103271	11111000011100111100 111	0.97051699
12	15.39	01000100110000110110 000	010000111111000001110 11	0.26538784	11110110110100011111 111	0.96414173
13	16.595	01001010001001011010 110	010010010001110101100 10	0.28560473	11110101010101100100 010	0.95834732
14	17.80	01001111100001111111 101	010011100100001000001 11	0.30569530	111100111011111101100 000	0.95212939
15	19.03	01010101000001101110 000	010100110111100011100 01	0.32606318	11110010000000100101 010	0.94534778
16	20.382	01011011000100010101 000	010110010010100010111 00	0.34827757	111011111111110001110 010	0.93739128
17	22.75	01100101101001011110 000	0110001011111111101111 10	0.38671096	11101100000101010101 111	0.92220097
18	24.00	01101011001110111010 100	0110100000011111111100 11	0.40673664	11101001110111100001 111	0.91354545
19	26.67	01110111001010011010 011	011100101110011111101 01	0.44885116	11100100110000110110 011	0.89360652
20	28.335	01111110100110100001 110	011110011000000100010 11	0.47462597	11100001010100111111 100	0.88018758
21	29.23	10000010100110011101 010	011111010000001001010 10	0.48831653	11011111011001110001 001	0.87266651
22	30.00	10000110000010101001 001	10000000000000000000000000000000000000	0.50000000	11011101101100111101 011	0.86602540
23	31.57	10001101000011100101 110	100001100000011010110 11	0.52353987	11011010000111001100 000	0.85200117
24	32.75	10010010010101000001 001	100010100111110101001 11	0.54097447	11010111010011100101 010	0.84103901
25	35.50	10011110100111011001 001	100101001010100011110 10	0.58070283	11010000011010011110 000	0.81411551
26	37.257	10100110011101110100 001	1001101011111101011101 10	0.60539111	110010111110000011111 001	0.79592804
27	38.82	10101101011100110000 111	101000000111101011101 11	0.62687581	11000111011101000101 110	0.77911907
28	39.73	10110001100000111110 111	101000111010000010101 11	0.63917058	11000100111000010111 001	0.76906499
29	42.365	10111101010010011110 010	101011001000000110000 10	0.67385104	10111101001001100110 011	0.73886711
30	45.00	11001001000011111101 101	101101010000010011110 01	0.70710678	10110101000001001111 010	0.70710678
31	47.283	11010011010000110011 000	101111000001011000101 11	0.73471334	10101101101010100010 110	0.67837769

32   50.009					1		1
33   31.22   001	32	50.009			0.76614540		0.64266727
35   54.50   011   101000001000101   0.81411551   10010100101101000111110   0.58070233     36   56.789   1111101101111000101   1101010000111101000   0.83665917   01001000001101111101   0.58070233     37   58.43   100011000001101001   110111010000111001100	33	51.252			0.77990633		0.62589624
35   36   36   36   36   36   37   38   38   38   38   38   38   38	34	52.00			0.78801063		0.61566147
36   56.789	35	54.50			0.81411551		0.58070283
38   60.00   100001100001010101   1101110110110110101   0.8602540   10000000000000000000000000000000000	36	56.789	11111101101111000101	110101100010111101001	0.83665917	10001100001101111010	0.54772386
38   60.00	37	58.43			0.85200117		0.52353987
10	38	60.00			0.86602540		0.50000000
10	39	61.665			0.88018758		0.47462597
41	40	63.33			0.89360652		0.44885116
42   67.25   0.00	41	66.00			0.91354545		0.40673664
44   70.97   0101010100000110111   1111001000000100101   0.94534778   100   0.3482/75/1     45   72.20   01001111100001111111   11110011011101	42	67.25	000	11	0.92220097		0.38671096
44   70.97   000   10   0.945347/8   001   0.32606318     45   72.20   0100111110000111111   1111001101101101000   0.95212939   111   0.00011100000000   0.30569530     46   73.405   01001010001010101   111101010101010000   0.95834732   010010010001110110   0.28560473     47   74.61   01000101100011010   111101010100011111   0.96414173   011   0.26538784     48   76.0525   0011111001010001010   1111100011110011   0.97051699   0011110110110100010   0.24103271     49   77.105   0011100110111011   1111100110001011   0.97478067   001111001000001010   0.22316505     50   79.628   001011100101011101   11111011010000001010   0.98878246   001001100001110000   0.18003846     51   81.41   0010011001100101   1111110100110010101   0.998878246   00100110010101010   0.14936277     52   82.15   0010001100010101   111111010011001101   0.99062902   0010001011110110   0.13658011     53   82.50   001000011000010100   11111101001110101   0.999062902   001000011010100001   0.13052619     54   83.257   0001110001010010   111111100011010101   0.99938276   00011000001101010   0.09511606     55   84.542   0001000011000010100   11111110001010101	43	69.618			0.93739128		0.34827757
46	44	70.97			0.94534778		0.32606318
Table   Tabl	45	72.20			0.95212939		0.30569530
48	46	73.405	110	10	0.95834732		0.28560473
48	47	74.61			0.96414173		0.26538784
49         77.105         101         01         0.9/4/8067         100         0.22316305           50         79.628         001011100101111011         111110111101000100011         0.98365957         00101110000101110000         0.18003846           51         81.41         0010011001010010010111         111111011001100110111         0.98878246         00100110001111001010         0.14936277           52         82.15         00100011000100101111         11111101100111011101         0.99062902         001000101111011101110         0.13658011           53         82.50         001000110000011000         111111101110111101010         0.99144486         001000001101010101010         0.13052619           54         83.257         0001111000100001100         111111101101011010101         0.99308276         00011100000111011010         0.11741607           55         84.542         0001100001100101111         111111110100001101011         0.99542125         00011000011100110011001         0.09511606           56         85.00         000100101001011110011         11111111111011100110101         0.99785892         0001000011110111001         0.06540298           58         87.40         00000001110011100110111001         11111111111111111111100111         0.999998617         000000000111001100100         0.00534	48	76.0525			0.97051699		0.24103271
50         79.628         000         11         0.98365957         000         0.18003846           51         81.41         0010011000100101010         111111010010000010111         0.98878246         00100110001110011100101         0.14936277           52         82.15         00100011000100101111         111111011001100111011         0.99062902         001000010111011010101         0.13658011           53         82.50         0010000110000010100         111111101011010110101         0.99144486         00100001110100001         0.13052619           54         83.257         0001110001000001000         11111110001101010101         0.99308276         0001111000001101111         0.11741607           55         84.542         00011000011000101111         111111110110101010111         0.99542125         00011000011001011001         0.09511606           56         85.00         000101000011000001         111111111011100110011         0.99619469         0001001000111001111110         0.08715574           57         86.25         000100001110011101110111         111111111111111111001110010         0.99785892         0001000011110011100110         0.06540298           58         87.40         00000000111001110010         111111111111111111111100111         0.99998617         000000000111100110010         0.005235	49	77.105			0.97478067		0.22316505
51         81.41         011         01         0.988/8246         001         0.1493027/           52         82.15         00100011000100101111         111111011001100110111         0.99062902         001000101111011101110         0.13658011           53         82.50         001000011000001000         11111110011101011010         0.99144486         0010000011100100001         0.13052619           54         83.257         0001110001000001100         1111111100111010101         0.99308276         000111000001110111         0.11741607           55         84.542         000110000110001111         1111111101010101011         0.99542125         0001100001100101000         0.09511606           56         85.00         00010110010101110001         111111111001110011         0.99619469         00010100011100101111101         0.08715574           57         86.25         000100001110011011101         11111111111111101110011001         0.99785892         0001000011000111111100110         0.06540298           58         87.40         0000000111001101110010         11111111111111111111100111         0.99998617         0000000011100110011001         0.00523596           60         89.80         00000000011100100100         11111111111111111111111111111110011         0.99999300         00000000011100010100         0.0	50	79.628			0.98365957		0.18003846
52         82.15         101         11         0.99062902         101         0.13658011           53         82.50         00100001100000110010         1111111011100111101010         0.99144486         0010000101101010101010         0.13052619           54         83.257         00011110001000001100         11111111000111010101         0.99308276         0001111000011101111         0.11741607           55         84.542         000110000110010111         11         0.99542125         001         001         0.09511606           56         85.00         000101100101110001         11111111000001101011         0.99619469         000101000011001111101         0.08715574           57         86.25         000100001110011101110         111111111111111101110011001         0.99785892         0001000011110011100110         0.06540298           58         87.40         0000101110011100110         11111111111111111110011001         0.99987062         000010011100111001110         0.04536298           59         89.70         0000000011100101010100         111111111111111111111111111111111111	51	81.41			0.98878246		0.14936277
53         82.50         010         10         0.99144488         101         0.13052619           54         83.257         00011110001000001100         11111111000111010101         0.99308276         0001111000011101111         0.11741607           55         84.542         0001100001100101111         11111111000001101011         0.99542125         0001100001100110011001         0.09511606           56         85.00         0001011001010110001         11111111100000110011         0.99619469         0001011001001111101         0.08715574           57         86.25         000100001000001001         1111111110111001101         0.99785892         0001000011110011001         0.06540298           58         87.40         000010110011101110         1111111111111111100111         0.99897062         00000101110011100110         0.04536298           59         89.70         0000000011100101110010         111111111111111111111110011         0.99998617         0000000011100101110010         0.00523596           60         89.80         000000000111001001100         111111111111111111111111111110011         0.99999300         00000000111001001100         0.00349065	52	82.15	101		0.99062902		0.13658011
54         83.257         010         11         0.99308276         101         0.11741607           55         84.542         0001100001100101111         111111101101011011011         0.99542125         0001100001010011001000         0.09511606           56         85.00         0001011001011110001         11111111100000110001         0.99619469         00010110010011111101         0.08715574           57         86.25         00010000110000011001         111111111011100111001         0.99785892         00010000101111100100         0.06540298           58         87.40         0000101110011101110         111111111111111111100011         0.99897062         00001011100111001110         0.04536298           59         89.70         000000001010101110010         111111111111111111111111111111111111	53	82.50			0.99144486		0.13052619
55         84.542         011         11         0.99542125         011         0.09511606           56         85.00         000101100101110001         111111111000001101011         0.99619469         00010110010011111101         0.08715574           57         86.25         00010000110000010101         111111111011100111011         0.99785892         000100001111111001100         0.06540298           58         87.40         0000101110011101110         1111111111111111110011         0.99897062         00001011100111001110         0.04536298           59         89.70         00000001010101110010         1111111111111111111110011         0.99998617         000000001110010110010         0.00523596           60         89.80         00000000111001001100         111111111111111111111111111111111111	54	83.257			0.99308276		0.11741607
56         85.00         100         11         0.99619469         100         0.08715574           57         86.25         00010000110000010101 001 11 1111111111	55	84.542			0.99542125		0.09511606
57         86.25         001         11         0.99783892         010         0.00540298           58         87.40         00001011100111011101         1111111111111001100101         0.99897062         00001011100111001110         0.04536298           59         89.70         00000001010101110010         111111111111111111110011         0.99998617         00000001010101110010         0.00523596           60         89.80         00000000111001001100         111111111111111111110011         0.99999300         00000000111001001100         0.00349065	56	85.00			0.99619469		0.08715574
58         87.40         110         01         0.99897062         100         0.04536298           59         89.70         00000001010101110010         1111111111111111110011         0.99998617         00000001010101110010         0.00523596           60         89.80         00000000111001001100         111111111111111111110011         0.99999300         00000000111001001100         0.00349065	57	86.25	001	11	0.99785892	010	0.06540298
59 89.70 010 10 0.00323390 60 89.80 00000000111001001100 111111111111111	58	87.40			0.99897062		0.04536298
60   X9 X0	59	89.70	00000001010101110010	111111111111111100011 10	0.99998617	00000001010101110010	0.00523596
	60	89.80			0.99999390		0.00349065

<sup>\*</sup>Decimal point exists in the very left of the binary string

# VII. CONCLUSION

The present work focuses mainly to reduce the number of clock cycles required to calculate the value of sine and cosine. The proposed system computes the sine and cosine values correctly up to 21 bits for all the inputs or more in only 6 clock cycles where as CORDIC architecture will require 21 clock cycles for the same. But the system resource utilization by the proposed methodology is more or less same compared to that of the

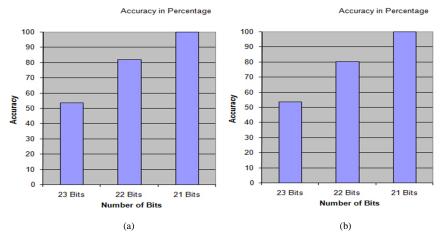


Figure 5. Shows accuracy achieved 23, 22 and 21 bits a) Sine values b) Cosine values

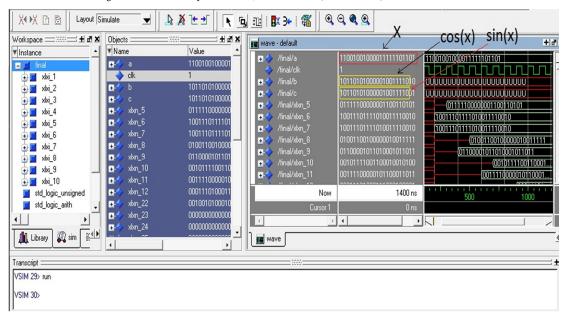


Figure 6. Snapshot of the ModelSim Simulation

CORDIC architecture. This work may be extended to design a complete module having memory unit for storing the data and a control unit to interface the computation unit and the memory unit.

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## REFERENCES

- J.E Volder, "The CORDIC Trigonometric Computing Technique", IRE Transactions on Electronic Computers, EC-8:330-334, September 1959.
- [2] Esteban O. Garcia, Rene Cumplido, Miguel Arias, "Pipelined CORDIC Design on FPGA for a Digital Sine and Cosine Waves Generator", 3rd International Conference on Electrical and Electronics Engineering (ICEEE), 2006.

- [3] P. K. Meher, J. Valls, T. B. Juang, K.Sridhan and K. Maharatna, "50 Years of CORDIC: Algorithms, Architectures, and Applications", IEEE Transactions on Circuits and Systems, Vol. 56, No.9, pp.1893-1907, 2009.
  [4] Javier Valls, Martin Kuhlmann, and Keshar K. Parhi, "Evaluation of CORDIC algorithms for FPGA design", Journal of VLSI signal Processing. Vol. 32, no. 3, pp. 207-222, 2002.
  [5] Shenk Al, Calculus and Analytic Geometry Scott Foresman & Co; 4<sup>th</sup> edition, July 2000.
  [6] Milton Abramowitz and Irene A. Stegun, Handbook of Mathematical Functions Dover Publications Inc., New York, 1965.

- [7] Frank W.J. Olver, Daniel W. Lozier, Ronald F. Boisvert and Charles W. Clark, The NIST Handbook of Mathematical Functions, Cambridge University Press, Cambridge, UK, 2010.