Essays in Market Design

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Roadmap

Two papers:

- Chapter 2: Many-to-One matchings with wages
- Chapter 3: Walrasian equilibria of convex economies

and two preliminary ideas:

- Learning in college admissions
- Sequential college admissions

Chapter 2: Contribution and Literature

In the context of hospital-doctors matchings with wages, I provide

- Necessary and sufficient conditions for existence of a stable matching.
  - Existing work assumes all doctors are either gross substitutes or gross complements for each hospital (Kelso, Crawford (1982); Hatfield, Milgrom (2005); Rostek, Yoder (2020))
  - Any requirement on the preferences of *each agent independently* is too strong to be necessary.
- A measure of instability: Subsidy needed to 'stabilize' the efficient matching.
  - The least core (Kern and Paulusma, 2003); approximate core (Vazirani, 2022).

Some preferences are irrelevant

Two doctors: Alice (A) and Bob (B).

Two hospitals: UHS (U), Mount Nittany (M) with revenues:

	{A, B}	A	B
U	1000	500	700
M	7	2	2

Suppose doctors are indifferent between hospitals and only care about the wages.

It is irrelevant whether M's preferences satisfy GS or not.

A stable matching: $U - \{A, B\}$ with wages $w_A = w_B = 5$.

Example with no stable matching with some complementarity

Two doctors: Alice (A) and Bob (B); Two hospitals: UHS (U), Mount Nittany (M) with revenues:

	{A, B}	A	B
U	3	0	0
M	2	2	2

A and B are **complements** for U! (They are not gross substitutes.)

No stable matching.

(Result:) Stable matchings are efficient.

The efficient matching is $U - \{A, B\}$

but there is no solution to $3 \geq w_A + w_B$, $w_A \geq 2$, $w_B \geq 2$.

Stable matching with 'more' complementarity

Same agents, different revenues:

	{A, B}	A	B
U	**4**	0	0
M	2	2	2

A and B are still complements for U.

Now there is a stable matching: $U - \{A, B\}$ with wages $w_A = w_B = 2$.

Measuring Instability

Consider a 1 Dollar subsidy:

	{A, B}	A	B
U	3**+1**	0	0
M	2	2	2

Part-time Contracts

Same environment as before but suppose doctors can split their times.

Result: Every matching environment has a stable matching.

	{A, B}	A	B
U	3	0	0
M	2	2	2

Stable part-time matching: A and B works for U together for half of their times; then they work for M separately for the rest of their times.

Primitives

- A set of hospitals, H and a set of doctors, D.
- Hospitals maximize profit: $\pi_h(D') = r_h(D') - \sum_{d \in D'} w_d$, where $r_h(\cdot)$ is the revenue.
- Doctors maximize $u_d(h) + w$ (when d works for h at wage w).
  - 'Peer Effect': $u_d(h, D') + w$.

Definition

(μ, w) is a **Matching with Wages** if μ assigns each doctor to at most one hospital with wages for each doctor given by w.

Blocking a Matching

Definition

An outcome (μ, w) is **blocked** if there exists a hospital $h \in H$, a set of doctors $D' \subset D \setminus \mu(h)$, and some wages for the doctors in D' such that

- New profit of the hospital h is at least as high as before,
- New utility of each doctor in D' is at least as high as before,
- For at least one agent in $D' \cup \{h\}$, new utility is strictly higher than before.

Stable Matchings

Definition

An outcome (μ, w) is **stable** if

- it is individually rational for each hospital and doctor,
  - I.e., they are not worse off than being unmatched.
- it is not blocked.

Efficiency

Definition

An outcome (μ, w) is **efficient** if it maximizes the total welfare (among all feasible outcomes).

Lemma

If an outcome (μ, w) *is stable, then it is efficient.*

Associated Cooperative Game

An associated TU cooperative game: For any $N' \subset H \cup D$,

$$v(N') = \max_{\text{`Admissible' partitions of } N'} \begin{array}{c}\text{Total revenue and utility} \\ \text{of the partition}\end{array}$$

A partition is *admissible* if each coalition has at most one hospital.

Stability and the Core

Proposition

An economy has a stable outcome if and only if the associated transferable utility game v has a non-empty core.

Applying Bondareva-Shapley result to the game we defined:

Corollary

An economy has a stable outcome if and only if the associated transferable utility game satisfies balancedness.

Corollary

Stable outcome with partial contracts always exists.

Balancedness is Computationally ‘inefficient’

Balancedness of a TU game requires some constraints related to every coalition.

Due to the definition of blocking in this environment, some of those can never bind.

In the paper, I provide a way to check for the existence of a stable matching using less constraints than balancedness.

This method decreases the number of constraints from $2^{|H|+|D|}$ to $(|H|+1)2^{|D|}$.

So the problem grows linearly in the number of hospitals.

Measuring Instability

$$\min_{w} \quad \sum_{i \in H \cup D} w_i$$
$$s.t.$$
$$\sum_{i \in N'} w_i \geq v(N'), \text{ for each } N' \subset H \cup D.$$

If $\sum_{i \in H \cup D} w_i = v(N)$, there exists a stable matching.

If $\sum_{i \in H \cup D} w_i > v(N)$, there isn't. Then, $\sum_{i \in H \cup D} w_i - v(N)$ is the instability.

Question: Can we find an upper bound for $\dfrac{\sum_{i \in H \cup D} w_i - v(N)}{v(N)}$ in a 'large' economy?

Example-1 (Cont'd)

Why did we consider \$1 of subsidy?

	{A, B}	A	B
U	3	0	0
M	2	2	2

Outline

Famous Convex Sets

In economics, we assume/need the convexity of many sets:

- Upper contour sets
- Budget sets
- Strategy space

But what does convex mean?

Euclidean Convexity

A set C is **convex** if it includes its convex hull, $co(C)$.

A point x is in the **convex hull** of a set C if for every linear ordering $\succsim$, $\exists$ an element $y \in C$ such that $x \succsim y$.

Linear ordering: A preference represented by a linear utility function.

Illustration of Standard Convexity

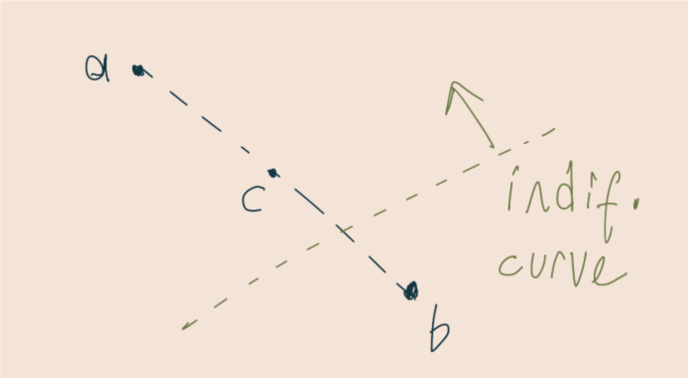

Figure: $c \in co(\{a, b\})$

An Example

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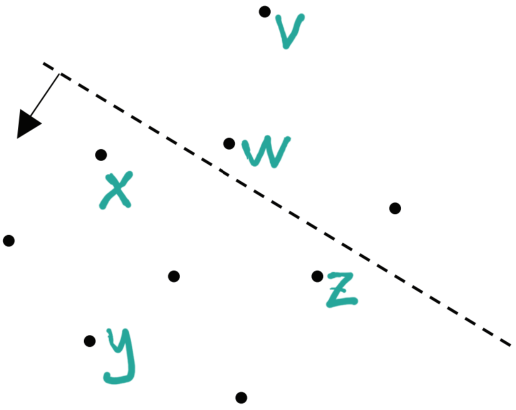


FIGURE 1

Figure: $w \notin K(\{x, y, z\})$ but $w \in K(\{x, y, z, v\})$

Another Convexity

A point x is in the convex hull of a set C if for every *increasing* linear ordering, $\succsim$, $\exists$ an element $y \in C$ such that $x \succsim y$.

$\forall$ increasing $\succsim$, $\exists\, x \in \{a,b\}$ s.t. $d \succsim x$

$$\Longrightarrow d \in co_{\uparrow}(\{a,b\}).$$

Abstract Convexities

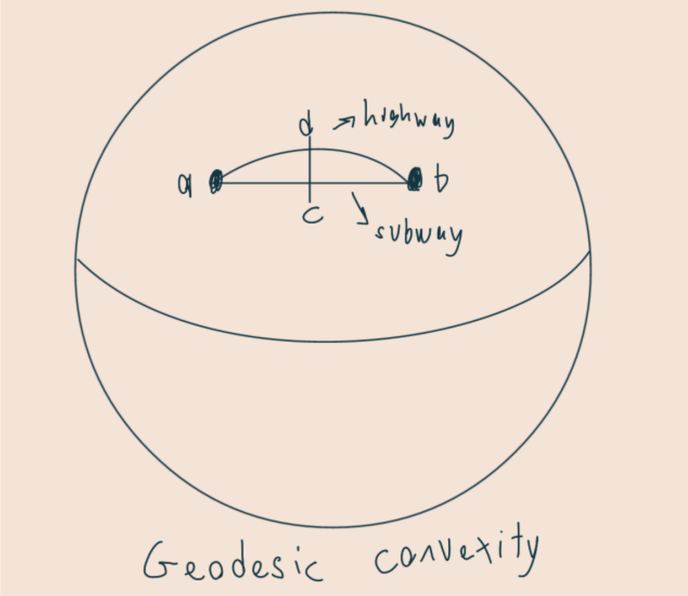
Definition

A family $\mathcal{C}$ of subsets of a set X is called a **convexity** on X if

1. The empty set $\emptyset$ and the universal set X are in $\mathcal{C}$;
2. $\mathcal{C}$ is closed under intersections;
3. $\mathcal{C}$ is closed under nested unions.

The pair $(X, \mathcal{C})$ is called a **convex structure**.

Geodesic Convexity - Nonlinear Betweenness



Nash Equilibrium

Proposition

Consider a game. If preferences of every player satisfies a 'well-behaved' convexity (together with the standard topological assumptions), then there exists a Nash Equilibrium.

- Existence of Nash Equilibrium with broad classes of 'convex' preferences.
- 'Expected Utility' without Independence Axiom.

Expected Utility with Nonlinear Betweenness

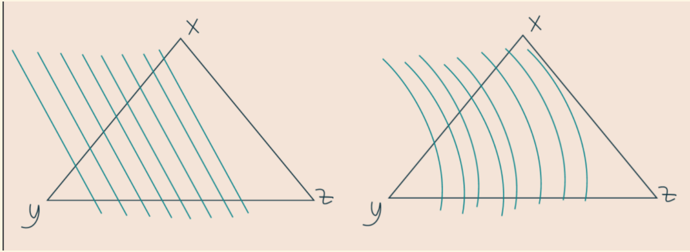


Figure: Indifference curves of preferences satisfying independence (left), violating independence (right)

Budget sets

Consider an exchange economy of n consumers with (u_i, e_i).

With endowment e_i and prices p, budget set is $\{x|px \leq pe_i\}$.

Budget sets generated by price vectors need not be convex in other convexities.

But I can define price orderings over bundles in an economy. (Piccione & Rubinstein 2007; Richter & Rubinstein 2015)

With endowment e_i and price ordering $\succsim_p$, budget set is $\{x|e_i \succsim_p x\}$.

Walrasian Equilibrium

Theorem

Consider an exchange economy. If the preferences of each consumer satisfies a well-behaved convexity (together with the standard topological assumptions), then there exists a Walrasian Equilibrium with a price ordering.

Outline

College Admission Design

- Students are misinformed about their rankings within their cohorts.
  - (Bobba & Frisancho, 2019; Bobba & Frisancho, 2020)
- Exposing them to more exams and information improves their outcomes.
  - (Bobba & Frisancho, 2019; Bobba & Frisancho, 2020; Doghonadze, 2022)
- Optimal feedback design in a contest setup?

Motivation

- In college admissions, most countries allow limited preference lists.
  - Can list 24 programs in Turkey, 10 in Chile, (in the past) 6 in Georgia...
- Students need to be strategic about the lists they submit.
- It leads to inefficiencies in realized matchings.
  - Luflade (2018), Ajayi & Sidibe (2017).
  - We can do better.

Toy Model

- Suppose all programs $(p_1, p_2, \ldots)$ have the same preferences $\succsim_p$ over students and one seat each.
- Say there are only two students: Alice and Bob, and Alice $\succsim_p$ Bob.
- Let Alice choose her top program, and then let Bob choose his top program, among the remaining ones.

Complications and Directions

- Can't go one by one with millions of students.
  - Use quantile cutoffs and let students know about the remaining seats.
    - Can it be done without hurting anyone compared to the status quo (deferred acceptance)?
- Programs may have different preferences over students.
  - Then, the deferred acceptance is generically not OSP.
  - Can we still make this an improvement over DA?

Moving Forward

- Chapter 2: Can I find an upper bound for the 'instability'?
- Chapter 3: Interesting, economically relevant examples of abstract convexities?
  - Iceberg costs in trade?
- College admissions ideas.

Appendix-Stable Matchings

Definition

An outcome (μ, w) is a **stable outcome** if

1. It is IR for each hospital and doctor (profits and utilities exceed 0).
2. There doesn't exist a hospital $h \in H$ and a set of doctors $D' \subset D$ such that they can deviate from this outcome in a way that makes everyone better off;

Appendix-Blocking A Matching

Definition

A matching with wages (μ, w) is **blocked** if there exists a hospital $h \in H$, a set of doctors $D' \subset D \setminus \mu(h)$ and wages for the doctors in D', w' such that there exists a set of doctors D'' with $D'' \subset D' \cup \mu(h)$ and:

- $r_h(D'') - \sum_{d \in D''} w_d \geq r_h(\mu(h)) - \sum_{d \in \mu(h)} w_d$;
- For each doctor $d \in D'$, $u_d(h, w_d) \geq u_d(\mu(d), w_d)$;
- For at least one agent in $D' \cup \{h\}$, the corresponding inequality above is strict.

Appendix-Existence of a Stable Outcome

We apply Bondareva-Shapley result to v:

Corollary

There exists a stable outcome (μ, w) if and only if for any set of balancing coefficients $(b_{N'})_{N' \subset N}$ (i.e., for any vector $(b_{N'})_{N' \subset N}$ such that $\sum_{N' \subset N} b_{N'} \delta_{N'} = 1$), we have that $v(N) \geq \sum_{N' \subset N} b_{N'} v(N')$.