Coexistence of Centralized and Decentralized Markets

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October 26, 2021

Types of Markets

- Many decentralized markets are characterized by search frictions:
  - Agents may not meet trade partners,
  - They may meet 'inefficient' partners.
- Centralized marketplaces can solve these problems:
  - They can alleviate both search and matching frictions.
  - But they maximize their own profits.

Efficiency losses due to profit-maximizing incentives?

THE WALL STREET JOURNAL.

TECH

Amazon Accused of Using Monopoly Power as E-Commerce 'Gatekeeper'

Online retailer says it welcomes regulatory scrutiny; congressional report mirrors findings from Wall Street Journal investigations

This Paper

Considers:

- Centralized marketplace introduced in a frictional search market (as in Diamond-Mortensen-Pissarides).
- Would a revenue-maximizing centralized marketplace take over all transactions?
- How is it affected by search frictions?

This Paper

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- Would a revenue-maximizing centralized marketplace take over all transactions?
- How is it affected by search frictions?

Preview:

- Centralized marketplace always coexists with a decentralized market.

Elementary Analysis

MODEL:

- Agents vary in valuations (distributed according to CDF F);
- Can buy or sell one unit of the good, each.
- Agents can join the centralized marketplace or trade in a decentralized market (as in Diamond-Mortensen-Pissarides).

"Competitive Equilibrium"

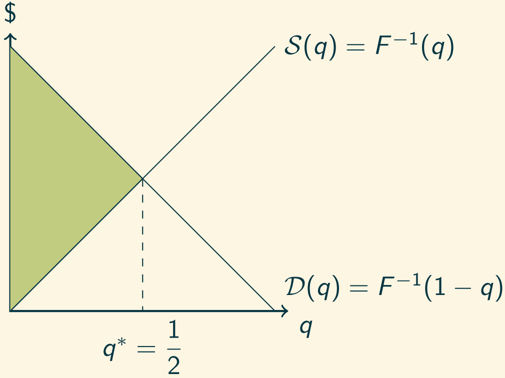

Figure: **Agents' Surplus**.

$q^* = 0.5$ and $p^* = m(F)$; the median of the distribution is the price that clears the market.

First-Best for the Designer

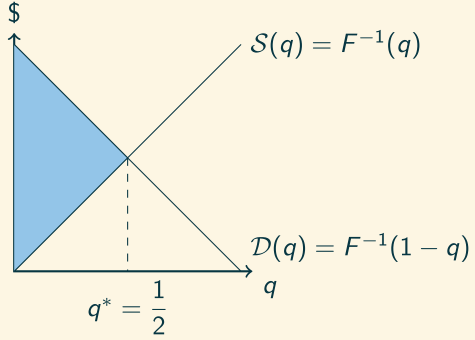

Figure: **First-Best Profit of the Marketplace**.

If the **designer has full info and there is no decentralized trade**, the designer implements the same allocation but gets the all surplus. ('**first degree price discrimination**')

Bid-Ask Mechanisms are Without Loss

Suppose the designer doesn't know agents' valuations.

Lemma (Informal)

Any deterministic mechanism is outcome equivalent to a mechanisms that sets bid and ask prices.

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Lemma (Informal)

Any deterministic mechanism is outcome equivalent to a mechanisms that sets bid and ask prices.

The competitive equilibrium is a special case of the lemma:

- Price for buying $= m(F) =$ price for selling.
- $\Pi = 0$.

Positive profit requires a positive 'bid-ask spread'.

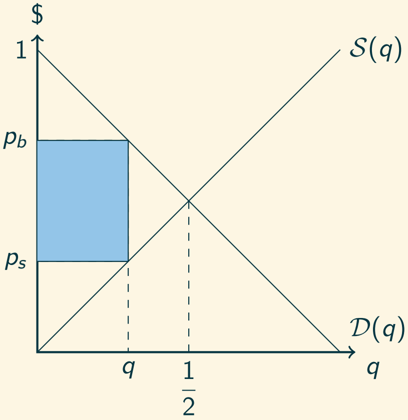


Figure: **Profit of the Marketplace**.

$\Pi = q(p_b(q) - p_s(q))$. We know how to solve this.

Inspiration from the Monopoly Problem

Prohibitive search frictions (no decentralized trade) and **uninformed designer**:

Agents get **information rents** for their private information (their valuations).

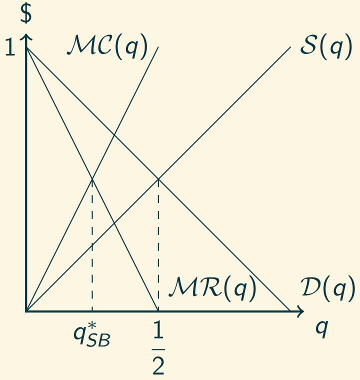


Figure: **Second-Best Profit of the Marketplace**, **Information Rents**.

Decentralized trade possible and **uninformed designer**:

Agents get information rents for their private information (their valuations) as well as '**Compensations**'.

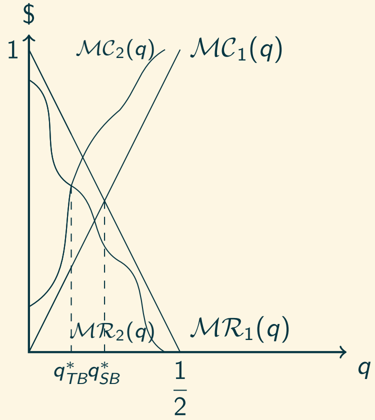
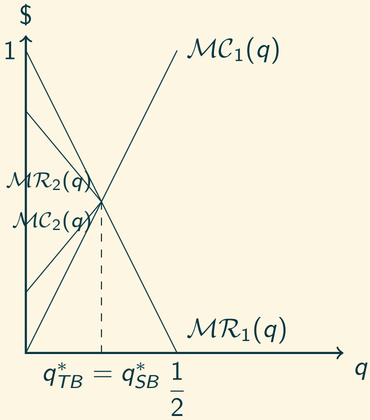


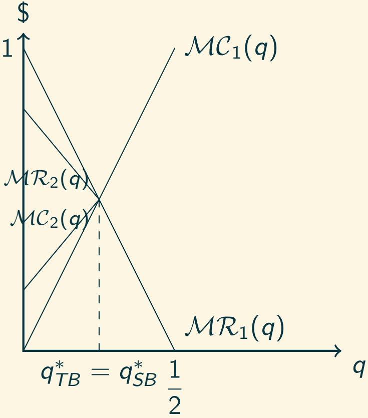
Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Hypothetical plots of $\mathcal{MC}_2(q)$ and $\mathcal{MR}_2(q)$.

Actual plots of $\mathcal{MR}_2(q)$, $\mathcal{MR}_2(q)$



Actual plots of $\mathcal{MR}_2(q)$, $\mathcal{MR}_2(q)$



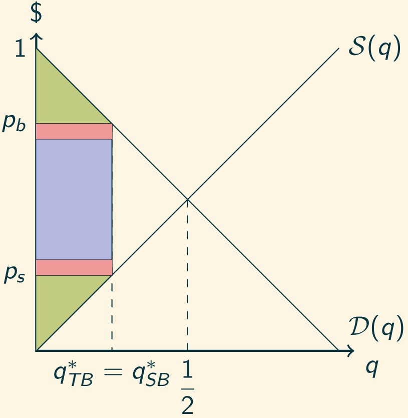


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Result 1

Coexistence (not all agents trade in the centralized marketplace.)

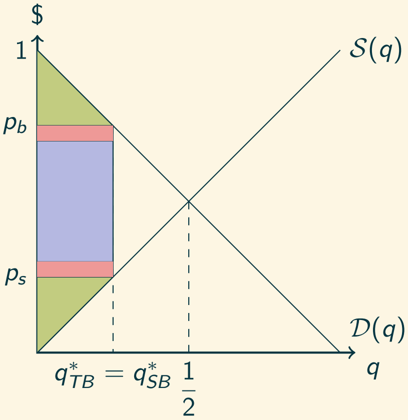


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Result 2

Thickness of the centralized marketplace is unaffected by the decentralized trade.

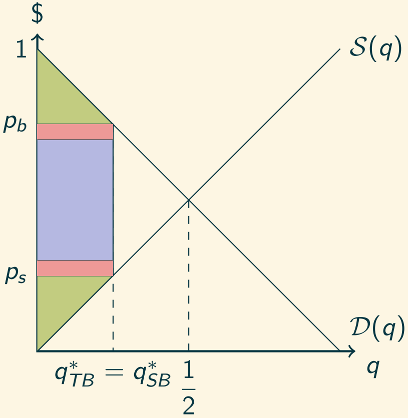
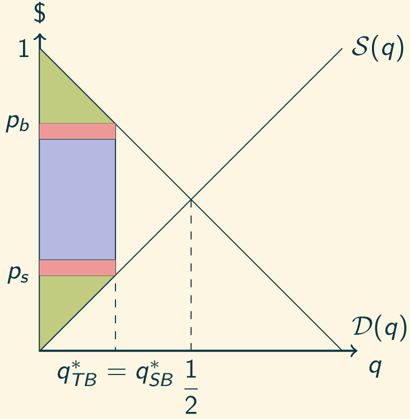


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Result 3

Compensations are a constant percentage of the baseline profit, independent of the distribution.

Let $p = \text{Prob(finding a match in decentralized market)}$.



$$\frac{\text{Second-Best Profit}}{\text{Third-Best Profit}} = 1 - \frac{p}{2}$$

Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Coming Up

- First: Centralized marketplace operates on its own.
  - Optimal allocation, simple economics of the marketplaces.
- Then: Centralized marketplace competes with a search market.
  - Coexistence result, segmentation, profit, and welfare.

Outline

Setup

- A single (indivisible) good.
- A continuum of agents over $[0, 1]$.
- Each agent has 1 unit of endowment and 2 units of demand.
  - The designer knows the endowments.
  - Agents have linear preferences.
    - θ is the private information of each agent.
    - $\theta \sim F(\cdot)$ and the designer knows $F(\cdot)$.
- Net utility: $u = \theta \times \min\{2, 1 + q\} - t$ where q is quantity, t is payment.

Marketplace

- For now, all trade goes through a marketplace.
- Marketplace designer
  - knows the endowments and distribution of agents' valuations;
  - wants to maximize her profit by choosing a strategy proof, deterministic mechanism for trade.
- By revelation principle, focus on direct, DS-IC mechanisms $(q(\theta), t(\theta))$.

Designer's Problem

Marginal Revenue: $\mathcal{V}(\theta) = \theta - \dfrac{1 - F(\theta)}{f(\theta)}$ (Virtual Value);

Marginal Cost: $\mathcal{C}(\theta) = \theta + \dfrac{F(\theta)}{f(\theta)}$ (Virtual Cost).

$$\begin{aligned}
\max_{q(\theta)} \Pi = \mathbb{E} \quad & [q(\theta)\,(\mathbb{1}\{\theta \text{ is a seller}\}\mathcal{C}(\theta) \qquad +\mathbb{1}\{\theta \text{ is a buyer}\}\mathcal{V}(\theta))] \\
& \text{s. t.} \\
& q(\theta) \text{ is increasing in } \theta \qquad && \text{(for Incentive Compatibility)} \\
& q(\theta) \geq -1 && \text{(Individual Feasibility)} \\
& \mathbb{E}[q(\theta)] \leq 0 && \text{(Market Clearing)}
\end{aligned}$$

Solving Designer's Problem

Monotonicity of the allocations $q(\theta)$ implies:

$$0 \ \underbrace{\hspace{3cm}}_{q(\theta) < 0} \ \underline{\theta} \ \underbrace{\hspace{4cm}}_{q(\theta) = 0} \ \overline{\theta} \ \underbrace{\hspace{3cm}}_{q(\theta) > 0} \ 1$$

Indivisible good + deterministic mechanism implies:

$$0 \ \underbrace{\hspace{3cm}}_{q(\theta) = -1} \ \underline{\theta} \ \underbrace{\hspace{4cm}}_{q(\theta) = 0} \ \overline{\theta} \ \underbrace{\hspace{3cm}}_{q(\theta) = 1} \ 1$$

Inspirations from the Monopolist

$\mathcal{MC}(q) = \mathcal{C}(F^{-1}(q))$ and $\mathcal{MR}(q) = \mathcal{V}(F^{-1}(1-q))$.

People get **information rents** for their private information (their valuations).

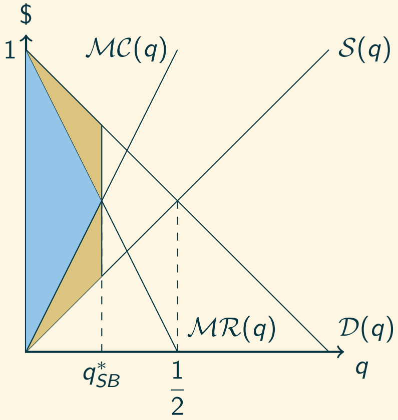


Figure: **Second-Best Profit of the Marketplace**, **Information Rents**.

Optimal Mechanism

Theorem

Suppose the distribution F is regular. Then, the optimal mechanism has the allocation rule

$$q(\theta) = \begin{cases} -1 & \text{if } \theta \leq \underline{\theta}^*, \\ 0 & \text{if } \underline{\theta}^* \leq \theta \leq \overline{\theta}^*, \\ 1 & \text{if } \theta \geq \overline{\theta}^*, \end{cases}$$

and the transfer rule

$$t(\theta) = \begin{cases} -\underline{\theta}^* & \text{if } \theta \leq \underline{\theta}^*, \\ 0 & \text{if } \underline{\theta}^* \leq \theta \leq \overline{\theta}^*, \\ \overline{\theta}^* & \text{if } \theta \geq \overline{\theta}^*. \end{cases}$$

Utilities

Utilities from the marketplace:

Figure: The utilities from the optimal marketplace (u^m).

Outline

Decentralized Market

Same primitives:

- Continuum of agents.
- Endowments: 1 unit; Demands: 2 units.
- Valuations: $\theta \sim F$ where F is regular.

Agents have a choice between the marketplace and a search market:

- In search market, they are **randomly matched** and they use **Nash bargaining**.

Market Choice Game

- Designer announces a mechanism and who should join it.
- Agents choose among the mechanism, a search market, and staying at home.
- Trade realizes in both markets.

Solution

- **Solution concept**: SPE with no bilateral deviations in the market choice stage.
  - Peivandi and Vohra (2021): No coalitional deviation to any mechanism.
- Deterministic mechanism.

Search Market

- Θ^d: agents who join the search market.
- $\mu(\cdot)$: the measure induced by F.
- A matching function, $M(\mu(\Theta^d))$ determines the measure of meetings.
- I assume M has constant returns to scale.
  - If the measure of agents is doubled, the measure of meetings also gets doubled.
- Then, $M(\mu(\Theta^d)) = m \times \mu(\Theta^d)$ for some $m \in [0, 0.5]$.
- Probability that an agent finds a match: $p = 2\frac{M(\mu(\Theta^d))}{\mu(\Theta^d)} = 2m$.

Search Market

Θ^d: agents who join the search market.

Expected utility of type θ from search:

$$\begin{aligned}\mathbb{E}[u^d(\theta)] &= \mathbb{P}[\text{match}]\mathbb{P}[\text{meet a higher value}]\mathbb{E}[\text{payoff from selling}] \\ &\quad + \mathbb{P}[\text{match}]\mathbb{P}[\text{meet a lower value}]\mathbb{E}[\text{payoff from buying}]\end{aligned}$$

$$\begin{aligned}&= p\mathbb{P}[x > \theta | x \in \Theta^d] \times \mathbb{E}\left[\frac{x-\theta}{2} | x > \theta, x \in \Theta^d\right] \\ &+ p\mathbb{P}[x < \theta | x \in \Theta^d] \times \mathbb{E}\left[\frac{x-\theta}{2} | x < \theta, x \in \Theta^d\right]\end{aligned}$$

Agents' outside options depend on everyone else's equilibrium behavior.

Two Levels of Incentive Compatibility

Designer needs to announce a mechanism such that:

1. Agents the designer wants in the mechanism join the mechanism;
2. Agents the designer doesn't want in the mechanism join the search market or stay at home;
3. Every agent would reveal their type truthfully if they joined the mechanism.

(1)+(2) $\rightarrow$ Convexity of the utility function across markets.
(3) $\rightarrow$ Convexity of the utility function in the mechanism.

Equilibrium Segmentation

Myerson & Satterthwaite (1983): Informational frictions causes inefficiencies in trade.

Have seen: Marketplace excludes agents with intermediate values even without the decentralized market.

Conjecture: Marketplace again excludes an interval of agents, $(\underline{\theta}, \overline{\theta})$ with $0 < \underline{\theta} < \overline{\theta} < 1$.

Equilibrium Segmentation

Theorem

In an equilibrium, the set of agents in the decentralized market must be an interval $(\underline{\theta}, \overline{\theta})$ *such that* $0 < \underline{\theta} < \overline{\theta} < 1$.

Implication 1: Restriction to such segmentation is without loss of profit.

Implication 2: There is no equilibrium where everyone is in the same market.

Existence and Uniqueness

Theorem

There exists $\underline{\theta}, \overline{\theta}$ such that in the equilibrium,

- *agents in $(\underline{\theta}, \overline{\theta})$ join the search market,*
- *agents in $[0, \underline{\theta}]$ and $[\overline{\theta}, 1]$ join the mechanism,*
- $\mathcal{C}(\underline{\theta}) = \mathcal{V}(\overline{\theta})$ *and* $F(\underline{\theta}) = 1 - F(\overline{\theta})$.

Moreover, the equilibrium is unique.

Utilities

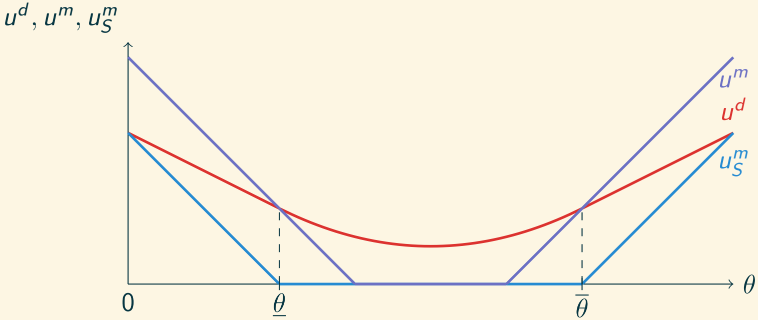


Figure: The utilities from the marketplace (u^m) and the decentralized market (u^d) in the coexistence, and utilities from the marketplace when it is the Single Market (u_S^m).

Marketplace Traders

Proposition

Same agents trade in the marketplace, with or without the decentralized market.

Profit of the Marketplace

$p \equiv \mathbb{P}[$ an agent finds a match in the search market.$]$

Π^M : Profit of the marketplace if there were no search market.

Theorem

The profit of the marketplace under coexistence is $\left[1 - \frac{p}{2}\right] \Pi^M$.

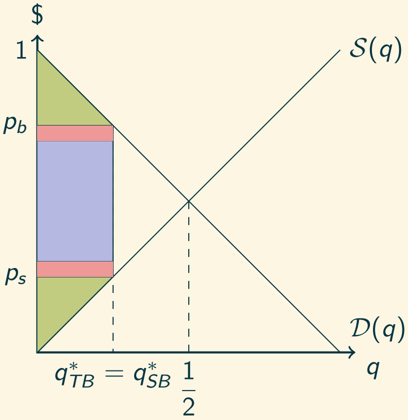


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Independent of the distribution, F,

$$\frac{\text{Compensations}}{\text{Second-Best Profit}} = \frac{p}{2} \text{ so } \frac{\text{Third-Best Profit}}{\text{Second-Best Profit}} = 1 - \frac{p}{2}.$$

Efficiency

Theorem

Suppose agents are uniformly distributed on $[0, 1]$*. Then, coexistence leads to a higher welfare than either market operating on its own.*

Remark

Also true for Standard Normal, Standard Beta, Exponential, Logistic distributions.

Beyond Nash Bargaining

Suppose the decentralized bilateral trades happen with double auction.

Two-sided Double Auction:

- Each agent makes a price bid;
- Agent with the higher bid buys the other's endowment;
- Pays the mid-point of the prices.

Equilibrium of the Double-Auction

Suppose the agent's distribution is given by a smooth distribution, G.

Lemma (Cramton, Gibbons, Klemperer (1987); Kittsteiner (2003))

In the unique equilibrium of this game, agent's bids are given by

$$b(\theta) = \theta - \frac{\int\limits_{G^{-1}(\frac{1}{2})}^{\theta} [G(x) - \frac{1}{2}]^2 dx}{[G(\theta) - \frac{1}{2}]^2}$$

Bids are monotone in valuations, so the equilibrium is efficient.

Equilibrium of the Double-Auction

We know the equilibrium only when G is smooth.

Assume an interval of agents $(\underline{\theta}, \overline{\theta})$ join the decentralized market.

Then, F restricted to $(\underline{\theta}, \overline{\theta})$ is a smooth distribution.

Suppose agents are uniformly distributed over $[0, 1]$.

Equilibrium of the Double-Auction

Suppose agents are uniformly distributed over $[0, 1]$.

Theorem

There exists $\underline{\theta}, \overline{\theta}$ such that in the equilibrium,

- *agents in $[0, \underline{\theta}]$ and $[\overline{\theta}, 1]$ join the mechanism, the rest searches;*
- *$\mathcal{C}(\underline{\theta}) = \mathcal{V}(\overline{\theta})$ and $F(\underline{\theta}) = 1 - F(\overline{\theta})$.*

Proposition

The market thickness is unaffected by the decentralized market.

Proposition

The profit of the marketplace is $1 - \frac{5p}{6}$ times the baseline profit.

Proposition

The total welfare under the simple equilibrium with the double auction is greater than when either market operates on its own.

Outline

Multiple Designers

$n \geq 2$ profit-maximizing marketplace designers in competition.

No (practical) revelation principle:

Each designer wants to condition her marketplace on the other marketplaces.

Infinite regress of conditions.

Approach 1

Restrict the designers to choose direct mechanisms.

(Not without loss anymore.)

Each direct, IC mechanism is equivalent to a pair of prices.

$\Rightarrow$ Bertrand competition.

Equilibrium

Suppose one designer makes positive profit.

Another can 'cut' the prices just enough to poach agents from her.

$\Rightarrow$ No equilibrium with positive profit.

Suppose each designer posts prices $p_b = p_s = m(F)$.

$\Rightarrow$ Bertrand Equilibrium.

Approach 2

Allow the designers to choose any mechanism.

One equilibrium:

- Each designer posts baseline marketplace prices;
- They also offer price-matching guarantees;
- agents randomize uniformly.

They collectively make the baseline 'monopoly' profit.

Compare with 0 profit under direct mechanisms.

Similar construction works when there is decentralized market as well!

Outline

Proof with the Uniform Distribution.

Total welfare when everyone searches:

$$\int_0^1 [p\theta\theta - p(1-\theta)\theta]d\theta = \cdots = \frac{p}{6}.$$

Cutoffs for the marketplace: $(\underline{\theta}, \overline{\theta}) = (0.25, 0.75)$. So, the welfare the marketplace creates:

$$\left[\int_{0.75}^1 \theta d\theta - \int_0^{0.25} \theta d\theta\right] = \cdots = \frac{3}{16}.$$

$\frac{3}{16} > \frac{p}{6}$ for every $p \in [0, 1]$. $\square$

Simplifying the Profit

Suppose the marketplace excludes $(\underline{\theta}, \overline{\theta})$ with $0 < \underline{\theta} < \overline{\theta} < 1$.

Then, the profit from the pair $\underline{\theta}, \overline{\theta}$, $\Pi^C(\underline{\theta}, \overline{\theta})$:

$$\underbrace{-F(\underline{\theta})u^d(\underline{\theta}) - (1 - F(\overline{\theta}))u^d(\overline{\theta})}_{\text{Compensations for joining the mechanism}} + \underbrace{\int_0^{\underline{\theta}} \mathcal{C}(x)q(x)f(x)dx + \int_{\overline{\theta}}^1 \mathcal{V}(x)q(x)f(x)dx}_{\text{Profit if there were no search market.}}$$

Simplifying the Profit

$$-F(\underline{\theta})u^d(\underline{\theta}) - (1 - F(\overline{\theta}))u^d(\overline{\theta}) + (-1)\int_0^{\underline{\theta}} \mathcal{C}(x)f(x)dx + (1)\int_{\overline{\theta}}^{1} \mathcal{V}(x)f(x)dx$$

$$\int_0^{\underline{\theta}} \mathcal{C}(x)f(x)dx = \int_0^{\underline{\theta}} \left[x + \frac{F(x)}{f(x)}\right] f(x)dx = \int_0^{\underline{\theta}} xf(x)dx + \int_0^{\underline{\theta}} F(x)dx$$

$$= \left([xF(x)]_0^{\underline{\theta}} - \int_0^{\underline{\theta}} F(x)dx \right) + \int_0^{\underline{\theta}} F(x)dx = \underline{\theta}F(\underline{\theta}).$$

Similarly, it is possible to show that: $\int_{\overline{\theta}}^{1} \mathcal{V}(x)f(x)dx = \overline{\theta}(1 - F(\overline{\theta}))$.

Simplifying the Profit

Let $E[\theta^d] = E[\theta | \underline{\theta} \leq \theta \leq \overline{\theta}]$ and $p = \mathbb{P}[\text{match}]$. Then,

$$
\begin{aligned}
& F(\underline{\theta}) u^d(\underline{\theta}) + (1 - F(\overline{\theta})) u^d(\overline{\theta}) \\
&= F(\underline{\theta}) \frac{p}{2} \left[E[\theta^d] - \underline{\theta} \right] + (1 - F(\overline{\theta})) \frac{p}{2} \left[\overline{\theta} - E[\theta^d] \right] \\
&= \frac{p}{2} \left(F(\underline{\theta}) \left[\cancel{E[\theta^d]} - \underline{\theta} \right] + (1 - F(\overline{\theta})) \left[\overline{\theta} - \cancel{E[\theta^d]} \right] \right) \\
&= \frac{p}{2} \left(-\underline{\theta} F(\underline{\theta}) + \overline{\theta} (1 - F(\overline{\theta})) \right)
\end{aligned}
$$

Simplifying the Profit

$$-F(\underline{\theta})u^d(\underline{\theta}) - (1 - F(\overline{\theta}))u^d(\overline{\theta}) = -\frac{p}{2}\left(\overline{\theta}(1 - F(\overline{\theta})) - \underline{\theta}F(\underline{\theta})\right)$$

$$-\int_0^{\underline{\theta}} \mathcal{C}(x)f(x)dx + \int_{\overline{\theta}}^{1} \mathcal{V}(x)f(x)dx = (\overline{\theta}(1 - F(\overline{\theta})) - \underline{\theta}F(\underline{\theta}))$$

Then, simple algebra shows:

$$\Pi^C = \left[1 - \frac{p}{2}\right]\Pi^M,$$

where Π^M is the profit marketplace makes on its own.

Relation to the Literature

- Myerson and Satterthwaite (1983): Bilateral trade mechanisms for a buyer and a seller.
  - I find the profit-maximizing marketplace where the roles as buyers and sellers are endogenous.
- Peivandi and Vohra (2021) show that almost no market mechanism is stable.
  - I study stable market structures that allow trading outside the mechanism.
- Miao (2006) studies coexistence of centralized and decentralized markets in a different setup.
  - His results are driven by transaction and search costs; they are absent here.

Conclusions

- Marketplaces don't monopolize the markets.
  - Attract high value buyers and low cost sellers.
    - The same ones they get without the decentralized market!
  - The rest search.
- Decrease in the profit due to the decentralized market only depends on the efficiency of the decentralized market (p).
- Coexistence improves the welfare over either market alone.