

Coexistence of Centralized and Decentralized Markets

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Abstract

In this paper, I introduce a profit-maximizing centralized marketplace into a decentralized market with search frictions. Agents choose between the centralized marketplace and decentralized bilateral trade. I characterize the optimal marketplace in this market choice game using a mechanism design approach. In an equilibrium in undominated strategies, the centralized marketplace and the decentralized trade coexist. I find that the thickness of the centralized marketplace in the coexistence equilibrium does not depend on the search frictions. The profit of the marketplace is always higher than the half of the profit in case of monopoly.

Centralized marketplaces that bring buyers and sellers together have seen massive growth in the last decade. For instance, it has been estimated that Amazon generates half of all e-commerce sales in the US.¹ Another analysis estimates that usage of ride-share apps surpassed that of taxis in NYC as early as 2017.² Airbnb and Vrbo's market shares in vacation rentals reached half of the market.³

These centralized marketplaces are successful because they reduce search and information frictions present in decentralized markets. In a decentralized market, an agent may not meet with a trading partner, and even when he meets with a partner, it may not be the right one. Centralized marketplaces reduce these frictions by attracting agents, collecting

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information from them, and making sure the realized matches are efficient enough to allow the marketplace to make some profit. In turn, the near certainty of trade on these platforms attracts many agents and gives the platforms significant market shares.

But the reduction in frictions is not without cost. Centralized marketplaces take commissions from the participants –buyers or sellers, or both. It is up to the individual traders to decide whether it is worthwhile to buy/sell in the centralized marketplace or to do so in a decentralized manner. The two institutions compete. Given the rising market shares of platforms like Amazon, Uber, etc., there are concerns about the possible monopolization of trade by such platforms (?). Indeed, a congressional panel investigating competition in the digital markets has asserted that Amazon has monopoly power as an intermediary in the US e-commerce market (?).

In this paper I ask: Would a centralized marketplace monopolize all trade or is there room for some forms of decentralized trade to coexist? Moreover, if coexistence is possible, what are the welfare and profit consequences of multiple trading modes? How is the centralized marketplace affected by search frictions?

To begin, consider the example of someone who wants to buy or sell a used car. She can check the prices offered by Carvana, a platform specializing in the used car market. If the price offered by Carvana is acceptable, she can simply take this deal. However, if she thinks she can get a better deal by searching privately via newspaper ads, she can choose to do so. Even though Carvana may have a large market share in this sector, there are still endless possibilities for trading privately. When Carvana chooses what price to offer for each car, ignoring these possibilities would harm its profit. Moreover, it is hard to guess the impact of Carvana’s response to these other options.

Financial markets provide another example. Many assets can be traded at the stock exchanges as well as over-the-counter. In stock exchanges, there is essentially no uncertainty; agents can buy or sell at the posted prices. However, in over-the-counter markets, trade is not as transparent; dealers often do not post prices in a public manner. Instead, they provide quotes when someone is interested in trading with them. The agents who trade with these dealers only observe the prices offered by a limited number of dealers before they trade. This paper provides a framework to think about the problems faced by a stock exchange that competes with over-the-counter trades.

In this paper, I develop a model of a centralized marketplace that competes for agents who also have the option to trade in a decentralized manner. I consider a setup with a single, indivisible good where each of a continuum of agents can buy or sell one unit of the good. The endowments are common knowledge whereas the valuations are private information. A designer chooses an individually rational, incentive compatible mechanism to maximize

revenue of the marketplace – say, the commissions charged for intermediation.

The decentralized market is modeled a la Diamond-Mortensen-Pissarides (???); agents are randomly matched among those who choose to participate in decentralized trade and then they engage in Nash Bargaining in each realized match.⁴ I consider a market choice game where (i) the marketplace designer announces a mechanism, (ii) agents choose whether to join the mechanism or to search for a trading partner, (iii) outcomes are realized in both markets.

Informally stated the main findings of this paper are the following:

- *There is an equilibrium in undominated strategies in which both centralized and decentralized trade coexist. In such an equilibrium, the volume of trade in the centralized market is independent of search frictions.*⁵
- *There is no equilibrium in undominated strategies in which the marketplace monopolizes trade.*⁶

Equilibrium Description In the coexistence equilibrium, the agents with low and high values join the marketplace while the agents with intermediate values choose to search. Thus, high surplus trades take place in the marketplace while low surplus trades happen privately.

Segmentation The thickness of the centralized marketplace is *independent* of the search frictions. Even at the extremes, where the search frictions are absent or prohibitively high, the centralized marketplace targets and successfully attracts exactly the *same* agents to trade there. As the frictions decrease, the centralized marketplace has to “sweeten the deal” for traders to join there. Thus, with lower frictions in the decentralized market, the profit of the centralized marketplace from each trade is lower. It would be reasonable to expect the centralized marketplace to become more exclusive as a response. However, it is in fact optimal for the centralized marketplace to attract exactly the same agents.

Beyond the structure of the equilibrium and the independence of the segmentation from the search frictions, I also describe the profit of the marketplace and, both the total and consumer welfares in the equilibrium.

Profits One might expect that competition from the decentralized market will significantly decrease the profits of the marketplace. This is not the case. I demonstrate that the profit of the marketplace in the coexistence equilibrium is at least half of the profit that

⁴I establish robustness by showing that the results extend to trading with a double auction, instead of Nash Bargaining under some distributions in the online appendix.

⁵This is an informal statement of the Theorem 1 and Corollary 3.

⁶This is an informal statement of the Proposition 8.

the marketplace would make if there had been no decentralized trade. Moreover, I show that the ratio of the reduction in profit of the marketplace as result of competition from the decentralized market is independent of the distribution of agents valuations. In fact, this ratio is *only* a function of the search friction – the probability of finding a trading partner – in the decentralized market. A decrease in the search frictions in the decentralized market decreases the profits of the marketplace. However, even if these frictions were absent, the profit of the marketplace is half of its profit when it operates on its own.

Welfare I provide two types of welfare comparisons. First, I focus on traders’ welfare in a Pareto sense. I show that decreasing the frictions in the decentralized market increases the payoff of each trader, no matter which market they choose to trade in. Then, I compare the total welfare—measured as gains from trade—created in this equilibrium to the welfare from either modes of trade operating on their own. Coexistence always improves the welfare over the centralized marketplace alone. Furthermore, I provide conditions under which the coexistence generates higher total welfare than the search market alone. Essentially, the decentralized market extends the extensive margin of trade (more agents trades in the coexistence equilibrium than in the baseline, single marketplace) while the marketplace extends the intensive margin (some agents trade with a higher probability in the coexistence equilibrium than when the decentralized market operates alone). Thus, the combination of these leads to increased efficiency.

Monopoly Equilibrium When every agent joins the centralized market, the outside option for each agent is not trading. Thus, of course the deviations are not profitable, given that the centralized market promises nonnegative payoffs. However, this is a very fragile equilibrium; I show that this equilibrium is always in dominated strategies. For agents with intermediate valuations, joining the centralized market is weakly dominated by joining the decentralized market. Therefore, I focus on equilibria in undominated strategies.

Multiple Designers Finally, I discuss an alternative setup where there are multiple profit-maximizing marketplace designers competing with each other. First, I study the case where they are restricted to choose direct mechanisms. This environment is akin to Bertrand Competition and the insight from the Bertrand Equilibrium carries over: In equilibrium, the designers make zero profit and the outcome is equivalent to a Walrasian Equilibrium.

Next, I allow the designers to choose more complex mechanisms and construct one particular equilibrium using strategies that include “price-matching guarantees.” In standard Bertrand Competition, price-matching guarantees are known to allow monopoly pricing to be a Nash equilibrium. Here, each marketplace posts prices that are equivalent to the baseline marketplace where there is no decentralized market and the agents uniformly randomize over the marketplaces. In this case, the marketplaces share the baseline profit. This discon-

tinuity between zero profit with direct mechanism and the collective baseline profit (similar to a cartel’s monopoly profit) reflects the inadequacy of direct mechanisms when there are multiple designers. If agents could also join the decentralized market, then the marketplaces would each post prices equivalent to the coexistence equilibrium mechanism of the main model with price-matching guarantees, and the agents again randomize.

0.1 Literature Review

There are many studies that consider the problem of incentivizing participation to centralized marketplaces. However, many of them do not have a decentralized trade option, as in the literature on competing platforms. In the papers where there is a decentralized market as well, the centralized marketplace is often a benevolent one.

Following the financial crisis of 2007-2008, a series of papers initiated by ?? focused on the ability of the public interventions to increase the efficiency of the investments in the financial markets. In these papers, the designers of the centralized markets are concerned with social welfare. Specifically, they focus on markets with adverse selection (and moral hazard in the case of ?) in terms of the quality of the investments. Without the public intervention, the level of investments is below the socially optimal level. To reduce the adverse selection and increase the level of investment, the government introduces a program by overpaying for some assets and removing the weakest assets from the market. ? studies a similar problem but considers the effects of the dynamic nature of the markets. By contrast, I focus on the profit-maximization problem of a centralized marketplace. Moreover, the nature of the frictions in the decentralized markets are different from the ones considered in this literature.

In another strand, some papers in matching theory (???) study the problem faced by a benevolent marketplace designer when the agents can choose between multiple venues. In kidney-exchange and school-choice settings, they show that it might be infeasible or undesirable to make sure everyone joins the centralized market, even if it aims to maximize the social welfare. This paper provides a natural counterpart where the centralized market is only concerned about its own profit.

? study a model where agents are allowed to deviate from a market mechanism to trade among themselves according to any feasible trading protocol. Their main result states that almost every market mechanism is inherently unstable in the sense that there is always a positive measure of agents who would like to deviate from it. My findings provide a partial counterpart to their result: By restricting the possible deviations from the market mechanism, I am able to find a stable market structure where both the centralized marketplace

and the outside trade are active.

The literature on the efficient dissolution of partnerships has important parallels with this study. Starting with ? and with contributions by many others (??), the setup in this literature includes a divisible good that is owned by many agents and a designer who wants to allocate the whole supply to one agent (or in some cases, to at least reduce the number of owners), thus dissolving the partnership. All of these papers have endogenous roles as buyers and sellers; each agent has some endowment which is less than the total endowment in the economy. Thus, like here, they obtain intermediate types who that are excluded from the trade and U-shaped utilities as a functions of agents types. Moreover, in extending the Nash bargaining, I use the efficient double auction which was introduced by ?.

? studies the problem of choosing a trade mechanism to maximize the total welfare in the economy. Their main result shows that it is generically impossible to have an efficient trade mechanism –that allocates the good always to the agent who values it the most– without outside resources to finance it. In my model without a decentralized market, unsurprisingly, the welfare achieved is even less than what ? provides. However, introducing the option to search improves the efficiency of the market as a whole.

In the literature on competing platforms (??), the questions mainly focus on the competition among platforms under numerous configurations of fee and price structures that could be employed by the platforms. The important distinction between this literature and my study is that in the context of competing platforms, the competition is between two profit-maximizing entities; they each react (or best respond) to the other’s actions. However, here, the centralized marketplace is in competition with a fixed set of rules that cannot react to the marketplace’s actions. This paper complements these studies by providing a insights into the nature of the competition in a decentralized market with a platform. More recently ? also bridges the gap between mechanism design and two-sided markets with a model where sellers can choose to join a platform that sets a menu of selling procedures or to develop their own selling venue.

? also studies a similar environment with centralized and decentralized markets. When the search technology in the decentralized market is improved so that it can support the Walrasian Equilibrium, he shows that the centralized market serves a vanishingly small part of the population.

1 The Model

Setup I consider a market with a single, indivisible good. There is a continuum of agents on $[0, 1]$. Each agent has 1 unit of endowment of the good and has a demand for 2 units

of it. As the good is indivisible, each agent can sell 1 unit, buy 1 unit, or neither buy nor sell any. Each agent has some valuation $\theta \in [0, 1]$ for a unit of the good. The valuations are drawn from some continuous distribution F with support $[0, 1]$, and they are agents' private information.

There is also a profit-maximizing mechanism designer, and they play the following game.

The Extensive-Form

1. Mechanism designer announces a mechanism and commits to it.
2. Agents observe the designer's marketplace and decide whether to join it or search for a trade partner in the decentralized market.
3. Trade happens in both markets simultaneously.

The Decentralized Market Following Diamond-Mortensen-Pissarides (???), the decentralized trade is modeled as a search market. Agents who choose the decentralized trade are randomly matched to each other and then use Nash bargaining to divide the surplus created by their trade.

Formally, suppose Θ^d is the set of agents who join the search market in a strategy profile and μ is the measure with respect to the distribution F . Then, the measure of meetings in the search market will be given by a matching function $M(\mu(\Theta^d))$ as a function of the measure of agents in the search market, $\mu(\Theta^d)$.

In search theory, matching functions are commonly assumed to have constant returns to scale (CRS). This means that doubling the size of the market also doubles the number of meetings. Since I focus on a market where every agent has the same endowment and demand, this is a one-sided market. In this setup, CRS matching functions are simply linear in the size of the market: $M(\mu(\Theta^d)) = m \times \mu(\Theta^d)$ where $m \in [0, 0.5]$ is the efficiency parameter of the search process. Then, probability that an agent finds a match in the search market, p is equal to $2m$ since the total measure of meetings is $M(\mu(\Theta^d))$, each agent is equally likely to be in any meeting, and there will be two agents in each meeting.

Notice that the probability of a match, p is independent of the set of agents who join the search market as well as the measure of the set. Since there is a one-to-one relationship between p and m , from now, an agents probability of finding a match is simply denoted by p . For the same reason, in this setup, p itself can be thought of as the primitive of the search market and the efficiency parameter of the matching process.

Given Θ^d and p , an agent with valuation θ computes his expected payoff from the decentralized market as follows: (i) He gets a match with the probability p . (ii) If he gets a

match, it is a random draw, θ' from the distribution of types restricted to Θ^d . (iii) If he is matched to θ' , his payoff from the Nash bargaining is half of the surplus created from their trade, $\frac{|\theta - \theta'|}{2}$. Thus, the expected payoff is given by

$$p\mathbb{E}\left[\frac{|\theta - \theta'|}{2} \middle| \theta' \in \Theta^d\right]. \quad (1)$$

Designer's Strategies The designer can choose any deterministic mechanism for each set of participants, and commit to implementing it. That is, a strategy for designer is a collection of mechanisms $\{\mathcal{M}_\Theta | \Theta \subset [0, 1]\}$ where \mathcal{M}_Θ represents the mechanism implemented when the set of agents who choose the marketplace is Θ . For each Θ , \mathcal{M}_Θ can be any direct or indirect mechanism with deterministic allocations and payments for each agent.

It is important to allow the designer to condition her marketplace on the set of participants for two reasons. First, without such conditioning, the mechanism can be infeasible for some sets of participants as market may fail to clear. Second, this provides a way to design the “off-the-equilibrium path” expected payoffs.

Revelation Principle and Individual Rationality Consider any equilibrium of the extensive-form game described above. Let Θ^* be the set of types that join the decentralized market in this equilibrium. By standard revelation principle arguments, the designer can instead offer a direct mechanism that implements the same outcome (allocation and payment) for each agent, given each set of possible participants. Since the agents with valuation in Θ^* joins the decentralized market in the original equilibrium, it must be the case that the centralized market is not strictly individually rational for them. Similarly, for other agents, the centralized market must be individually rational. Again, following the standard arguments, the direct mechanism given that the set of valuations of the participants is Θ^* will also be individually rational exactly for agents with valuations in Θ^* . Thus, from here on, I focus on direct mechanisms.

1.1 Bid-Ask Mechanisms are Without Loss

The mechanism implemented in the centralized marketplace has to be incentive compatible for every agents, whether their equilibrium behavior involves trading there or joining the decentralized market. Otherwise, there would be potential profitable deviations by changing the market and the report to the designer. Thus, in this subsection, I analyze the class of Bayesian Incentive Compatible mechanisms in this setting. The main finding of this section is that focusing on mechanisms with bid-ask prices is without loss of generality and profit,

i.e., the designer can simply set a price for buying and a price for selling in the marketplace and let the agents choose whether they want to buy, sell or not trade. This is in line with the findings of ? as it will become clear later.

As the mechanism has to be incentive compatible for every agent, for now, we can ignore the individual rationality constraints and simply study the consequences of incentive compatibility. Suppose a mechanism designer knows the distribution of valuations, F , and wants to design a Bayesian Incentive Compatible mechanism. Since there is a continuum of agents, there is no aggregate uncertainty. Thus, it is without loss to focus on direct, Ex-Post Incentive Compatible (or strategy-proof) mechanisms. Moreover, as agents are symmetric other than their valuations, I restrict attention to anonymous mechanisms; that is, the designer does not condition the mechanism on agents' 'names'.

In the Appendix A, I study the incentive compatibility of an allocation and solve the simpler case of designer's problem when there are prohibitive search frictions as a warm up exercise. There, the following result is obtained as a direct implication of incentive compatibility in this environment (independent of the individually rationality constraints and thus the decentralized market).

Proposition 1 *Bid-ask mechanisms are without loss of profit.*

Thus, any direct, incentive compatible mechanism is equivalent to a bid-ask mechanism.

1.2 The Simple Economics of Optimal Marketplaces

Before going into the complete analysis, here I provide a simple analysis of the optimal profit-maximizing marketplaces in the same spirit as ?. They have shown that the optimal auction design problem can be understood as a monopoly pricing problem. In my setting, we will see that the optimal marketplace design problem can be understood as simultaneously solving both a monopoly problem and a monopsony problem.

The objective function of the designer is the expected payments of the agents. This is made simpler by the observation that each mechanism the designer can implement is equivalent to a bid-ask mechanism (Proposition 1). In the Appendix A, we see that an equivalent way of thinking about the expected payments is the expected difference between the virtual values and costs, or expected *virtual surplus*. Thus, I will now use virtual values and costs to find the optimal level of trade graphically.

1.2.1 Prohibitive Search Frictions ($p = 0$)

To begin, suppose there are prohibitive search frictions. Thus, there is no decentralized trade and agents' only option to trade is the centralized marketplace.

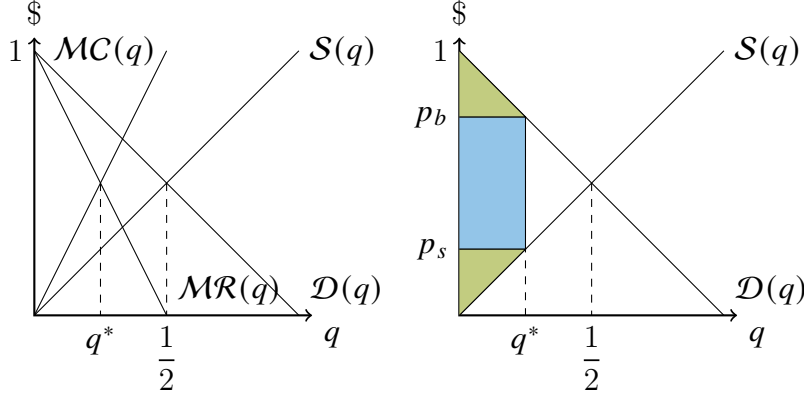


Figure 1: Profit of the Marketplace, Information Rents.

For each quantity level, q , if the marketplace wants measure q of sellers, the price for selling should be $F^{-1}(q)$. Similarly, to have measure q of buyers, the price for buying should be $F^{-1}(1-q)$. Then, the inverse supply and demand in the economy are given by $S = F^{-1}(q)$ and $D = F^{-1}(1-q)$

For each quantity level, q , let $MC(q) = C(F^{-1}(q))$ and $MR(q) = \mathcal{V}(F^{-1}(1 - F(\theta)))$ where C and \mathcal{V} are the *virtual cost* and *virtual value* functions, respectively.⁷ Here, the virtual cost is the marginal cost of the marketplace so it represents the ‘effective supply’ in the marketplace. Similarly, the virtual value gives us the marginal revenue curve so it represents the ‘effective demand’ for the marketplace. Since the virtual values and costs are the marginal revenue and cost curves for the profit-maximization problem, instead of agents’ willingness to pay (or willingness to get paid), I use their virtual values and costs to find the optimal quantity of trade.

This is very similar to using the marginal revenue instead of the demand in the monopoly problem. If the designer knew a buyer’s valuation, she would charge the buyer exactly his valuation. However, since the designer is uninformed about the valuations, she has to charge a buyer something less than his valuation. Similar reasoning holds for the sellers as well. So, each agent who trades, gets some *information rent* for his private information. This can be seen in the Figure 1.

Figure 1 shows D , S , MR , and MC together.⁸ As the marketplace is maximizing the profit, the optimal level of trade is given by q^* such that MR and MC are equal to each other. Moreover, the area of the rectangle given by q^* and the difference $p_b - p_s$ is equal to the profit of the marketplace. The triangles above p_b and below p_s are the buyers’ and

⁷They are defined formally in Section 1.4. However, the definition is not needed to follow the argument in this section.

⁸The figure is drawn for the uniform distribution over $[0, 1]$ for simplicity.

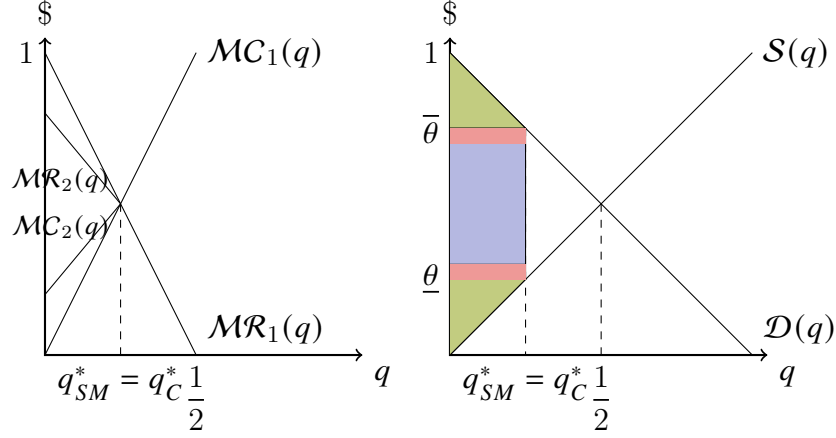


Figure 2: **Coexistence Profit of the Marketplace**, **Information Rents**, **“Compensations”**. q_{SM}^* is the optimal quantity when $p = 0$ (prohibitive search frictions) and q_C^* is the optimal quantity when $p > 0$ (coexistence).

sellers’ information rents. Finally, the triangle between p_b and p_s to the right of the profit is the deadweight loss created by (i) incomplete information and (ii) profit-maximization.

1.2.2 The Coexistence Equilibrium ($p > 0$)

Now I extend the analogy with the monopolist’s problem to the coexistence equilibrium as well.

With $p = 0$ (the case of prohibitive search frictions), the marketplace can ensure participation as long as the agents expect nonnegative payoffs from trade. Thus, the highest seller and the lowest buyer are offered 0 utility, in order to maximize the profit. However, when $p > 0$, the marketplace has to make sure that each agent it wants to attract receive a utility greater than what he expects from the decentralized market. This is accomplished by paying the agents the opportunity cost of joining the marketplace as compensations. Thus, with coexistence, the effective marginal revenue and cost curves will be different. Let MR_2 and MC_2 denote the new curves.

In Figure 2 (left panel), we have the plots of the single market marginal revenue and cost of the marketplace, MR_1 and MC_1 together with the plots of the coexistence marginal revenue and cost of the marketplace, MR_2 and MC_2 . This picture illustrates many of the results about the equilibrium structure.

MR_2 and MC_2 intersect at the same quantity level as MR_1 and MC_1 does. Thus, the optimal quantity under no decentralized trade, q_{SM}^* is equal to the optimal quantity under coexistence for any $p > 0$: q_C^* . Thus, the thickness of the centralized marketplace is unaffected by $p \in [0, 1]$ as stated earlier.

As q_C^* is below $\frac{1}{2}$ (that corresponds to the median of the distribution), some agents are excluded from the marketplace. But since these agents can always join the decentralized marketplace and get positive payoffs, it means there will be coexistence.

For a more detailed version of the figure with more results represented, see Appendix B.

1.3 Solution Concept

In this work, I focus on subgame perfect equilibrium in stage-undominated strategies. What this means is that given the mechanism announced by the designer, agents should play undominated actions in the following subgame. The reason for focusing on this selection is simple: Everyone joining the same market is always an equilibrium. However, these equilibria are very fragile. In particular, I will show that it is a dominated action for all agents to join the same market. Next, I show that coexistence is an undominated equilibrium. Finally, for technical reasons, I assume that the designer targets a closed set of agents as the participants of the marketplace in the equilibrium path.

Here, the solution concept is actually ‘stage-undominated subgame perfect equilibrium’⁹ because the domination depends on the designer choosing the optimal mechanisms for profit maximization. Of course, properly it should be called a ‘stage-undominated perfect Bayesian equilibrium’ as there is uncertainty about the agents’ valuations ex-ante. However, as there is a continuum of agents and no aggregate uncertainty, it is natural to think about it as a subgame perfect equilibrium as well.

For expository reasons, I first study these markets under the following assumption that restricts the set of mechanisms the designer can choose.

Assumption 1 *Suppose a convex set of types, $(\underline{\theta}, \bar{\theta})$ joins the search market in the equilibrium.*

The analysis is significantly more accessible in this case so the restricted set of mechanisms works well in explaining how the marketplace would work. Later, I consider the general case without this assumption. There, I show that the assumption is, in fact, without loss (see Theorem 3). Thus, the set of strategies that the Assumption 1 allows are exactly the type of strategies that could emerge in an equilibrium.

Now, I will study the problem where the designer chooses $\underline{\theta}$ and $\bar{\theta}$ optimally, anticipating the agents’ best responses to the announced mechanism. I call the equilibria that satisfy this assumption *simple equilibria* and the mechanisms that induce these equilibria *simple mechanisms*.

⁹See ? for more details on the notion.

1.4 Designer's Problem

Following Assumption 1, suppose the agents who join the mechanism will have valuations in $[0, \underline{\theta}] \cup [\bar{\theta}, 1]$ for some $\underline{\theta}$ and $\bar{\theta}$ with $0 \leq \underline{\theta} \leq \bar{\theta} \leq 1$. We can write their utilities as follows, using payoff equivalence.

$$u^m(\theta) = \begin{cases} u^m(\bar{\theta}) + \int_{\bar{\theta}}^{\theta} q(x) dx & \text{if } \theta \geq \bar{\theta} \\ u^m(0) + \int_0^{\theta} q(x) dx & \text{if } \theta \leq \underline{\theta}. \end{cases} \quad (2)$$

In this environment, there will be different binding individual rationality constraints for agents below $\underline{\theta}$ and for agents above $\bar{\theta}$. Thus, writing the utilities in this form is more convenient. Similarly, we can write the transfers:

$$t(\theta) = \begin{cases} \theta q(\theta) - u^m(\bar{\theta}) - \int_{\bar{\theta}}^{\theta} q(x) dx & \text{if } \theta \geq \bar{\theta}, \\ \theta q(\theta) - u^m(0) - \int_0^{\theta} q(x) dx. & \text{if } \theta \leq \underline{\theta}. \end{cases} \quad (3)$$

Now, we can study the profit from the optimal allocation given the cutoffs. The step-by-step derivation can be followed in the Appendix F but here is the end-result:

$$\begin{aligned} \Pi &= \underbrace{\mathbb{P}[\theta \in [0, \underline{\theta}]]}_{\text{Measure of sellers}} \underbrace{\mathbb{E}[t(\theta) | \theta \in [0, \underline{\theta}]]}_{\text{Exp. payment of a seller}} + \underbrace{\mathbb{P}[\theta \in [\bar{\theta}, 1]]}_{\text{Measure of buyers}} \underbrace{\mathbb{E}[t(\theta) | \theta \in [\bar{\theta}, 1]]}_{\text{Exp. payment of a buyer}} \\ &= \underbrace{-F(\underline{\theta})u^m(\underline{\theta}) - (1 - F(\bar{\theta}))u^m(\bar{\theta})}_{\text{Compensations for joining the mechanism}} \\ &\quad + \underbrace{\int_0^{\underline{\theta}} \left[\left(x + \frac{F(x)}{f(x)} \right) q(x) \right] f(x) dx + \int_{\bar{\theta}}^1 \left[\left(x - \frac{1 - F(x)}{f(x)} \right) q(x) \right] f(x) dx}_{\text{Profit if there were no search market}}. \end{aligned} \quad (4)$$

Here, the agents will get some compensations for joining the centralized marketplace in-

stead of the decentralized market. This is because the agents are giving up the opportunity to trade in the decentralized market when they join the centralized marketplace. Thus, the individual rationality constraints are endogenously determined by the equilibrium segmentation. The centralized marketplace has to pay the agents this lost ‘opportunity cost’, on top of the standard information rents created by the informational asymmetry. The exact form of the compensation will become clear below when we consider the individual rationality constraints.

Let the virtual cost, C and the virtual value \mathcal{V} be:

$$C(x) = x + \frac{F(x)}{f(x)} \text{ and } \mathcal{V}(x) = x - \frac{1 - F(x)}{f(x)}. \quad (5)$$

I assume both of them are increasing and call such distributions *regular*.

The Constraints The support of the distribution of valuations is $[0, 1]$. Thus, the first restriction is $0 \leq \underline{\theta} \leq \bar{\theta} \leq 1$.

Second, we have a market-clearing constraint in the form of

$$\int_0^{\underline{\theta}} q(x)f(x)dx + \int_{\bar{\theta}}^1 q(x)f(x)dx \leq 0. \quad (6)$$

This simply says that the designer cannot sell more than she buys.

Third, since each agent has one unit of endowment, they cannot sell more than that. Thus, we need $-1 \leq q(\theta)$. In fact, since the good is indivisible, each agent must have $q(\theta) \in \{-1, 0, 1\}$ as their allocation.

Fourth, we know that for incentive compatibility in the mechanism, we need the allocation to be increasing. The rest of the requirements of the incentive compatibility are already embedded in the transfers. So, as long as the allocation is increasing, the mechanism will be incentive compatible.

Finally, we need to consider the implications of the individual rationality.

Individual Rationality Notice that in any profit maximizing mechanism, individual rationality (IR) constraint for at least one type of agent in each segment who joins the mechanism ($[0, \underline{\theta}]$ and $[\bar{\theta}, 1]$) must bind. If not, then uniformly increasing the payment of all agents in the particular segment without a binding IR until there is a binding constraint increases the profit.

Moreover, the binding IR constraints in the optimal mechanism must be $\underline{\theta}$ and $\bar{\theta}$. To see this, notice that under the Assumption 1, we want to construct an equilibrium such that agents in $[0, \underline{\theta}]$ and $[\bar{\theta}, 1]$ join the mechanism while the rest of the agents join the

search market. In the equilibrium, each agent will choose the market that offer him a higher utility. Then, in the equilibrium, agents with valuations in $[0, \underline{\theta}]$ and $[\bar{\theta}, 1]$ must have a higher utility in the centralized marketplace and agents with valuations in $(\underline{\theta}, \bar{\theta})$ must have a higher expected utility in the decentralized market. Since the utilities from both markets are continuous in valuations, this means the utilities from the decentralized market, u^d , and the utilities from the centralized marketplaces, u^m , must cross each other at $\underline{\theta}$ and $\bar{\theta}$. Thus, agents with valuations $\underline{\theta}$ and $\bar{\theta}$ must be indifferent between the markets and these are the binding individual rationality constraints:

$$u^d(\underline{\theta}) = u^m(\underline{\theta}) = u^m(0) + \int_0^{\underline{\theta}} q(x)dx \text{ and } u^m(\bar{\theta}) = u^d(\bar{\theta}). \quad (7)$$

This allows us to make the following observation, proved in the Appendix C.

Lemma 2 *In a coexistence equilibrium with cutoffs $\underline{\theta}$, $\bar{\theta}$, allocations must be such that:*

$$q(\theta) = \begin{cases} -1 & \text{if } \theta \leq \underline{\theta}, \\ 1 & \text{if } \theta \geq \bar{\theta}. \end{cases} \quad (8)$$

This is a useful observation. On intervals where the allocation is constant, transfer is constant as well due to payoff equivalence in this setup. We also know that the marketplace will be equivalent to a market with bid-ask prices. With this lemma and the equation above it, we can actually compute the optimal bid-ask prices for each possible segmentation. From here, it is also clear that the agents with valuations below $\underline{\theta}$ can only be sellers in any market and agents with valuations above $\bar{\theta}$ can only be buyers in any market.

1.5 Existence of a Coexistence Equilibrium

Here, I show how the designer can find the optimal pair of $\underline{\theta}$ and $\bar{\theta}$. The main result of this section shows that under Assumption 1, a coexistence equilibrium exists and the volume of trade in the centralized market is independent of search frictions. In this equilibrium, the marketplace designer attracts the agents with very low and very high valuations. The rest of the agents are left to search. Moreover, cutoffs are such that the marginal cost of the highest seller in the marketplace is equal to the marginal revenue of the lowest buyer in the marketplace. Finally, the measures of buyers and sellers in the marketplace are equal to each other.

Theorem 1 *Suppose F is regular. Then, there exists $\underline{\theta}, \bar{\theta}$ such that for any $p \in [0, 1]$, in the coexistence equilibrium,*

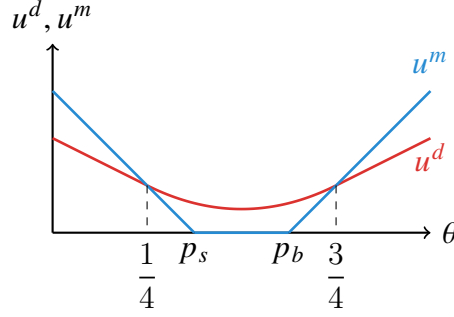


Figure 3: The utilities from the search market and the optimal mechanism under the simple equilibrium.

- agents in $[0, \underline{\theta}]$ and $[\bar{\theta}, 1]$ join the mechanism,
- agents in $(\underline{\theta}, \bar{\theta})$ join the search market,
- $C(\underline{\theta}) = \mathcal{V}(\bar{\theta})$ and $F(\underline{\theta}) = 1 - F(\bar{\theta})$.

The proof is in the Appendix H.

To illustrate the utilities agents are offered in the mechanism and expect from the search market, suppose everyone gets a meeting in the decentralized market ($p = 1$) and agents are uniformly distributed over the unit interval. The optimal cutoffs for the are $\underline{\theta} = \frac{1}{4}$ and $\bar{\theta} = \frac{3}{4}$. Figure 3 shows the utilities agents can expect from either market *in the equilibrium*.

As it can be observed from the Figure 3, agents with intermediate types receive a lower utility in this equilibrium in the centralized marketplace. So they are happy to join the decentralized market. Moreover, the agents with low and high values are offered higher utilities in the centralized marketplace than they expect from the decentralized market. Thus, no agent has any unilateral profitable deviation.

The preceding theorem describes the structure of the coexistence equilibrium. However, it does not completely describe the mechanism that induces this equilibrium. Lemma 2 pins down the allocations and transfers for the agents who join the mechanism. What remains to be determined is what should be offered to agents in $[\underline{\theta}, \bar{\theta}]$.

In fact there are many mechanisms that would induce the same equilibrium that only differ in the off-path payoffs. Proposition H.1 in the Appendix H.1 describes one such mechanism. As described before, it is equivalent to offering some bid-ask prices, i.e., prices for buying and selling. Essentially, I compute the transfers for the agents who join the mechanism and then extends the allocation and the transfer rules to the rest of the agents in a way that the allocations are increasing and we have $u^m(\theta) < u^d(\theta)$ for each $\theta \in (\underline{\theta}, \bar{\theta})$. The prices for buying and selling, p_b and p_s , implied by this proposition are the lowest and the highest valuations such that u^m is equal to 0, as can be seen in the Figure 3.

1.6 Profit of the Marketplace

Next, I establish the relationship between the profit in this equilibrium and the profit of the marketplace when there is no decentralized market. As the decentralized trade makes it costly to recruit agents to the centralized marketplace, it is not hard to guess that the decentralized trade decreases the profit of the centralized marketplace. Interestingly, the decrease in the profit only depends on the efficiency of the matching process in the decentralized market; it is independent of the distribution of the valuations.

Let Π^M denote the profit of the marketplace when there is no decentralized market.

Theorem 2 *Suppose F is regular. Then, the profit of the marketplace in coexistence is equal to $\left(1 - \frac{p}{2}\right) \Pi^M$.*

Moreover, the aggregate compensations agents receive in coexistence is equal to $\frac{p}{2} \Pi^M$.

It is surprising that the ratio of the profits is completely independent of the distribution of valuations. The ratio only depends on the probability that an agent finds a match in the search market, p . The intuition for this result is as follows. When there is no decentralized trade, the marketplace keeps all the surplus that remains after ‘paying’ the information rents. When agents can join the decentralized market, they find a match with probability p , and they get half of the surplus created by their meetings. I show that the expected surplus of a buyer and a seller is equal to the profit of the marketplace from each sale. Thus, in total, marketplace pays $\frac{p}{2} \Pi^M$ as compensation and the rest is kept as the profit of the marketplace.¹⁰

Since the ratio $1 - \frac{p}{2}$ is decreasing in p , the profit is also decreasing in p : As the matching process becomes more efficient, the decentralized market becomes more attractive. Then, the opportunity cost of each agent increases, meaning the compensations increase. Thus, the profit of the marketplace decreases.

Even if every agent finds a match in the decentralized market, meaning $p = 1$, the profit of the marketplace is half of the profit when it operates on its own. This provides a distribution-free lower bound for the profit as a function of Π^M . Here is why the marketplace can still make positive profit when $p = 1$. Due to the random matching process, even when each agent in the decentralized market gets a match with certainty, the matches may have small gains from trade. An agent might meet someone whose valuation is very close to her own. However, the marketplace solves this problem and makes profit by creating efficient matches. At the other extreme, $p = 0$, when there is no match in the decentralized market, agents cannot trade bilaterally. Then, the marketplace makes the profit Π^M . Thus, the baseline with the marketplace alone is obtained as a special case.

¹⁰The result directly follows from the derivation of the coexistence profit in Appendix G.

Remark 1 *The independence of compensation:profit ratio holds for off-equilibrium as well. Given any (potentially sub-optimal) cutoffs $\underline{\theta}$ and $\bar{\theta}$ for segmentation, the total compensations is equal to $\frac{p}{2}$ times the virtual surplus generated by this segmentation.*

1.7 Thickness of the Marketplace

In this section, I compare the agents who trade on the centralized marketplace in the coexistence to those who get to trade in the marketplace when it operates on its own.

In the previous section, we have seen that the decrease in the profit of the marketplace is given by a ratio which is independent of the distribution. This means that the objective function of the marketplace, with or without the decentralized trade, is the same, up to a multiplication with a constant. Then, the solution -in terms of the allocation- is the same. The centralized marketplace wants to serve exactly the same types for each value of p including $p = 0$. Of course, for each value of p , the transfers are adjusted so that the cutoff types, $\underline{\theta}$ and $\bar{\theta}$, are always made indifferent between trading in the marketplace and the decentralized market.

Corollary 3 *The agents who trade on the marketplace are the same with or without the decentralized market. Thus, the thickness of the marketplace is unaffected by the decentralized trade.*

Although we can understand the corollary in light of the previous section, it is a counter-intuitive result on its own. When the agents have the option to trade in the decentralized market, they need to be compensated to join the centralized market. Thus, decentralized trade makes each trade in the marketplace costlier. Since the marketplace maximizes profit, it would be reasonable to expect the marketplace to become more exclusive to account for the higher costs. However, this is not optimal. Since the total compensations agents receive is a constant fraction of their virtual surplus, the optimal strategy is to maximize the virtual surplus. But Π^M is precisely the maximum attainable virtual surplus. Thus, marketplace keeps the same agents, no matter what happens in the decentralized market.

1.8 Welfare Comparisons

In this section, I first compare the consumers' welfare in a Pareto sense for different levels of p . Next, I compare the welfare generated in the coexistence to (i) welfare generated by the centralized market alone and (ii) welfare generated by the decentralized trade alone. For this analysis the welfare is measured as gains from trade. Thus, it includes the profit of the marketplace, when it exists. Omitted proofs are in the Appendix I.

1.8.1 Consumer Welfare

From Corollary 3, we know that the agents who trade in the decentralized market are the same. Then, conditional on meeting someone, agents' expected payoffs from the decentralized market are the same for each value of p . Thus, as p increases, agents' expected payoffs from the decentralized market increase. This directly leads to higher utilities for the intermediate types who join the decentralized market. It indirectly increases the payoff of each agent in the marketplace as well. When p increases, the cutoff types have a higher expected payoff from the decentralized market. Since the cutoff types remain the same, to make them indifferent under a higher p , centralized marketplace has to increase the compensations for each agent. Thus, a more efficient decentralized market increases the payoffs for all agents, regardless of their market choice.

Corollary 4 *Equilibrium payoff of each type is increasing in p .*

This provides a nice policy recommendation. Make the decentralized trade more efficient. It will make every agent strictly better off.

1.8.2 Total Welfare

Comparison between the coexistence and the centralized market alone is simple. Once again, by Corollary 3, the same agents trade on the marketplace, whether there is a decentralized market or not. Thus, the gains from trade generated on the marketplace is constant. However, in the coexistence, the agents with intermediate types also generate some trade. Thus, coexistence generates more gains under any regular distribution.

Corollary 5 *Coexistence leads to a higher welfare than the centralized market alone.*

For ease of exposition, I first focus on a concrete distribution for valuations to compare the welfare from the coexistence to the welfare from the decentralized trade alone. Later, I show that the result holds much more generally.

Proposition 6 *Suppose agents valuations are drawn from the uniform distribution over $[0,1]$. Then, the total welfare under the coexistence is greater than when either market operates on its own.*

Intuition behind this result is that the search market extends the extensive margin of trade by allowing more agents to trade while the centralized marketplace extends the intensive margin -by solving the search and matching frictions. Thus, when they operate together, everyone gets to trade with some probability and some agents trade for certain.

Although the proposition is stated for the uniform distribution, the result is true for many other distribution. Next, I provide a sufficient condition for distributions under which the coexistence is more efficient. This condition is satisfied by most commonly used distributions.

Assumption 2 *Under F , the following inequality holds:*

$$2\mathbb{E}[\theta F(\theta)|\theta \in [\underline{\theta}, \bar{\theta}]] + \mathbb{E}[\theta|\theta > \bar{\theta}] \geq \mathbb{E}[\theta|\theta \in [\underline{\theta}, \bar{\theta}]] + \mathbb{E}[\theta F(\theta)|\theta \leq \underline{\theta}] + \mathbb{E}[\theta F(\theta)|\theta \geq \bar{\theta}], \quad (9)$$

where $\underline{\theta}$ and $\bar{\theta}$ are the optimal cutoffs for a coexistence equilibrium, given the distribution F .

Standard Normal Distribution, Logistic Distribution, Exponential Distribution, Standard Beta Distribution are among the distributions that has this property¹¹.

Proposition 7 *If F satisfies Assumption 2, then the total welfare under the coexistence equilibrium is greater than when either market operates on its own.*

1.9 Simple Equilibrium is Without Loss

In this section, I remove Assumption 1 which restricted the designer to choose ‘simple’ market structures that required an interval of agents to join the search market. Here, I allow the designer to invite any closed set of agents. As I show in Theorem 3, the restriction to the simple equilibrium is without loss of generality. Thus, Assumption 1 was without loss of generality. Therefore, everything we have studied so far holds for all equilibria where both markets are active.

Theorem 3 *If both markets are active in an equilibrium, then the sets of agents in the search market must be an interval.*

This result is extremely helpful in reducing ‘the dimensionality’ of the problem. Without the structure provided by this result, searching for an equilibrium would require considering the consequences of every possible segmentation of a continuum of agents across markets. However, when we know that the agents in the search market will be an interval, the search is essentially reduced to finding a pair of cutoffs.

The proof is simple but lengthy and therefore it is included in the online appendix (Appendix E). Here is the idea behind it. Given that the centralized market is characterized

¹¹Mathematica codes for computations that show these distributions satisfy this assumption are available upon request.

by a set of bid-ask prices, the utility function the centralized market offers has a ‘bucket shape’. On the other hand, the expected utility from the decentralized trade will always have a U-shape as long as there is a positive measure of agents there. Finally, for an outcome to be an equilibrium, each agent should choose the market that offers higher expected utility, given the valuation. Considering the ways in which these utility functions can intersect reduce the possible structures of segmentation. Among the potential segmentations that can be an equilibrium outcome, the interval structure described above is shown to be the only one that the designer would like to sustain in an equilibrium.

1.10 Monopoly Equilibrium is in Dominated Strategies

If all agents join the centralized market, of course, unilateral deviations to the decentralized market are not profitable; a deviating agent would end up alone in a market. Thus, it is always an equilibrium for all agents to join the centralized market. Below I argue that this *monopoly equilibrium* is very fragile.¹²

Proposition 8 *Monopoly equilibrium is in dominated strategies.*

It is easy to see why this is the case: The expected payoff from the decentralized market is strictly positive as long as there is a positive measure of agents in the decentralized market. Moreover, the designer chooses the mechanism so that the utilities offered to the agents with intermediate valuations are lower than what they expect from the decentralized market for each possible segmentation. Thus, either the decentralized market is empty and the intermediate agents are indifferent between two markets (as their IR constraints would bind in that case) or the decentralized market is active and thus the intermediate agents have a higher expected payoff in the decentralized market.

Corollary 9 *Coexistence equilibrium is in undominated strategies.*

The proofs for both results can be found in Appendix J.

1.11 Coexistence Equilibrium with Double Auction

So far, I assumed that when agents are matched to each other in the decentralized market, they engage in Nash bargaining. This provides a nice benchmark since Nash bargaining is

¹²In an earlier draft of this work, I allow agents to deviate bilaterally between markets. There, I showed that the coexistence is the unique equilibrium of this game. This provides another way in which the ‘monopoly equilibrium’ is fragile; it wouldn’t be an equilibrium if agents could form coalitions. Earlier draft with results on equilibrium under bilateral deviations is available on Arxiv: arxiv.org/pdf/2111.12767.pdf

efficient: Even with an efficient trading protocol, the marketplace can attract the agents it targets and make positive profit. However, in Nash bargaining, the assumption is that the agents observe each others' valuations upon meeting. This creates an information asymmetry between markets since the designer cannot observe agents' valuations. Thus, we might be concerned that the results I have presented may not be robust to other models of decentralized trade. In this section, I consider an alternative bargaining protocol and I show that under some conditions, all of the main results hold true more generally.

Suppose the agents who join the decentralized market are randomly matched to each other and then they participate in a double auction. The random matching process is unchanged from the main model. The double auction works as follows. Each agent submits a bid, b_i . Agent with the higher bid buys the other's endowment and pays $\frac{b_i + b_j}{2}$, when bids are given by b_i and b_j ; so they are trading a unit of the good at the midpoint of the bids.

This is a special case of the "simple trading rule" studied by ?. Assuming the valuations are drawn from the same, smooth distribution for each agent, they show that this game has a symmetric equilibrium where the bids are increasing in agents' valuations. Thus, the equilibrium is ex-post efficient in the sense that, the agent who values the good more ends up with the whole quantity. Moreover, ? has shown that the symmetric equilibrium characterized by ? is indeed the unique equilibrium.

Suppose the distribution of agents who participates in this double auction is given by some CDF, G . Suppose G is strictly increasing on its support, $[\underline{\theta}, \bar{\theta}]$, and is differentiable. Then, agents' bids in the unique equilibrium are given by:

$$b(\theta) = \theta - \frac{\int_{G^{-1}(\frac{1}{2})}^{\theta} [G(x) - \frac{1}{2}]^2 dx}{[G(\theta) - \frac{1}{2}]^2} \quad (10)$$

It follows from the Proposition 5 of ? that following this bidding strategy constitutes an equilibrium. Theorem 1 in ? further shows that there is no other equilibrium.

Here, I focus on simple mechanisms that exclude an interval of agents: Suppose agents in $(\underline{\theta}, \bar{\theta})$ join the decentralized market and the rest join the centralized marketplace. Then, the endogenous distribution of agents in the decentralized market is simply F truncated from both below and above. Thus, $G(\theta) = \frac{F(x) - F(\underline{\theta})}{F(\bar{\theta}) - F(\underline{\theta})}$ on its support $(\underline{\theta}, \bar{\theta})$.¹³

In the Appendix K, I develop an analysis parallel to that on Nash bargaining for the double auction. I summarize my results in this section. Suppose the agents' valuations are drawn from the uniform distribution over $[0, 1]$. Then, essentially every result I obtained

¹³The equilibrium of this double auction game when G has gaps in its support is not known.

under the Nash bargaining holds true for the decentralized market with double auction as well:

Theorem 4 *There exists $\underline{\theta}, \bar{\theta}$ such that in the coexistence equilibrium,*

- *agents in $[0, \underline{\theta}]$ and $[\bar{\theta}, 1]$ join the mechanism,*
- *agents in $(\underline{\theta}, \bar{\theta})$ join the decentralized market with double auction,*
- *$C(\underline{\theta}) = \mathcal{V}(\bar{\theta})$ and $F(\underline{\theta}) = 1 - F(\bar{\theta})$.*

Proposition 10 *The profit of the marketplace in coexistence is $1 - \frac{5p}{6} = 1 - \frac{5m}{3}$ times the profit the marketplace would make if there were no decentralized market with double auction.*

Proposition 11 *The total welfare under the coexistence equilibrium with the double auction is greater than when either market operates on its own.*

Proposition 12 *The agents who trade on the marketplace are the same with or without the decentralized market. Thus, the thickness of the marketplace is unaffected by the decentralized trade.*

Thus, exactly same agents trade in the centralized marketplace (i) for each level of friction in the decentralized market (including the extreme case of no decentralized trade with $p = 0$) and (ii) whether the decentralized trade happens according to Nash bargaining or a double auction.

2 Multiple Designers

A natural question to ask is how the equilibrium would change if instead of one centralized marketplace competing with a decentralized market, there were multiple centralized marketplaces competing with each other. This is closer to the models extensively studied by the competing platforms literature. However, it is still worthwhile to understand what the structure of the competition would look like within the framework of this paper.

Mechanism design problems with multiple designers are notoriously difficult to solve. In the appendix, I develop a toy model to provide some insight into this case. Main take away is that existence of multiple designers would not change the segmentation between the centralized markets and the decentralized market.