Coexistence of Centralized and Decentralized Markets

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Types of Markets

- Many decentralized markets are characterized by search frictions:
  - Agents may not meet trade partners,
  - They may meet unsuited partners.
- Centralized marketplaces can solve these problems:
  - They can alleviate both search and matching frictions.
  - But they maximize their own profits.
  - E.g., Uber, Airbnb, Amazon, stock exchanges.

Efficiency losses due to profit-maximizing incentives?

THE WALL STREET JOURNAL.

TECH

Amazon Accused of Using Monopoly Power as E-Commerce 'Gatekeeper'

Online retailer says it welcomes regulatory scrutiny; congressional report mirrors findings from Wall Street Journal investigations

This Paper

Considers:

- Centralized marketplace introduced in a search market with frictions (as in Diamond-Mortensen-Pissarides).

- Would a revenue-maximizing centralized marketplace take over all transactions?

- How is it affected by search frictions?

This Paper

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- Centralized marketplace introduced in a search market with frictions (as in Diamond-Mortensen-Pissarides).
- Would a revenue-maximizing centralized marketplace take over all transactions?
- How is it affected by search frictions?

Preview:

- Centralized marketplace always coexists with a decentralized market.

Elementary Analysis

MODEL:

- Agents vary in valuations (distributed according to CDF F);
- Can buy or sell one unit of the good, each.
- Agents can join the centralized marketplace or trade in a decentralized search market.

No Frictions

"Competitive Equilibrium"

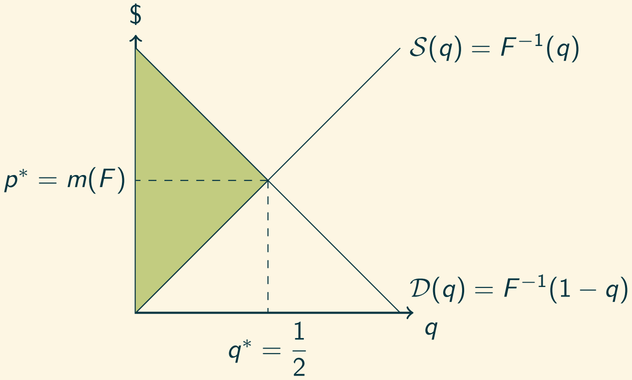

Figure: **Agents' Surplus**.

$q^* = 0.5$ and $p^* = m(F)$; the median of the distribution is the price that clears the market.

First-Best for the Designer

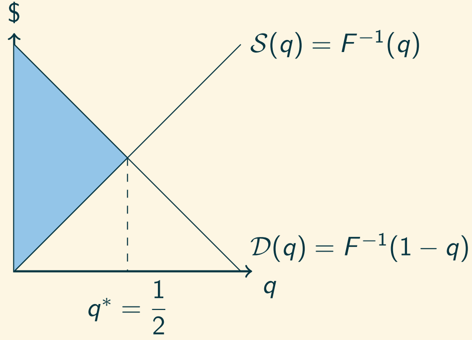

Figure: **First-Best Profit of the Marketplace**.

If the **designer has full info and there is no decentralized trade**, the designer implements the same allocation but gets the all surplus. ('**first degree price discrimination**')

Bid-Ask Mechanisms are Without Loss

Suppose the designer doesn't know agents' valuations.

Lemma (Informal)

Any deterministic mechanism is outcome equivalent to a mechanism that sets bid and ask prices.

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Lemma (Informal)

Any deterministic mechanism is outcome equivalent to a mechanism that sets bid and ask prices.

The competitive equilibrium is a special case of the lemma:

- Price for buying $= m(F) =$ price for selling.
- $\Pi = 0$.

Positive profit requires a positive ‘bid-ask spread’.

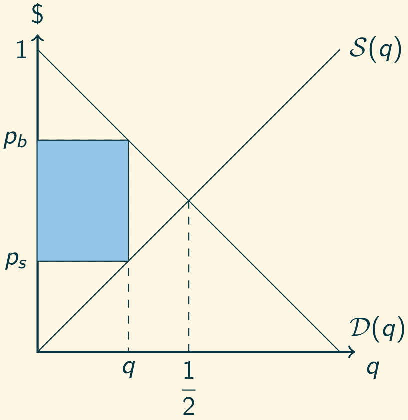


Figure: **Profit of the Marketplace**.

$\Pi = q(p_b(q) - p_s(q))$. We know how to solve this.

Inspiration from the Monopoly Problem

Prohibitive search frictions (no decentralized trade) and **uninformed designer**:

Agents get **Information Rents** for their private information (their valuations).

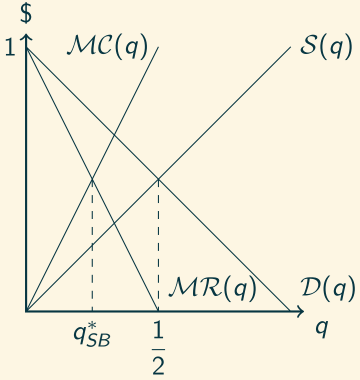


Figure: **Second-Best Profit of the Marketplace**, **Information Rents**.

Decentralized trade possible and **uninformed designer**:

Agents get '**Compensations**' as well as information rents for their private information (their valuations).

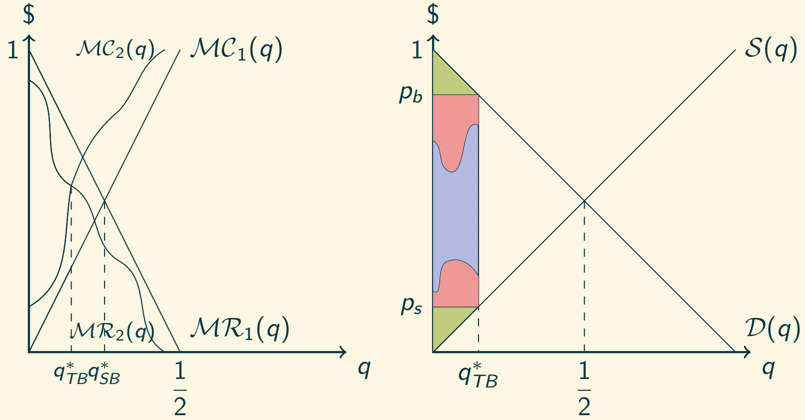


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Hypothetical plots of $\mathcal{MC}_2(q)$ and $\mathcal{MR}_2(q)$.

Result 0

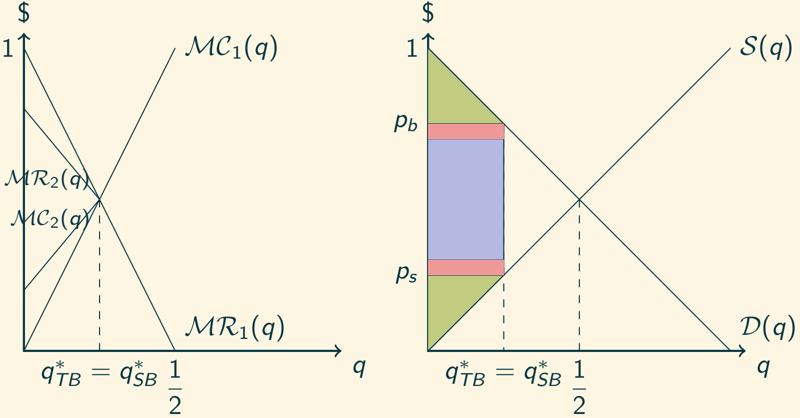


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Result 1

Coexistence (not all agents trade in the centralized marketplace.)

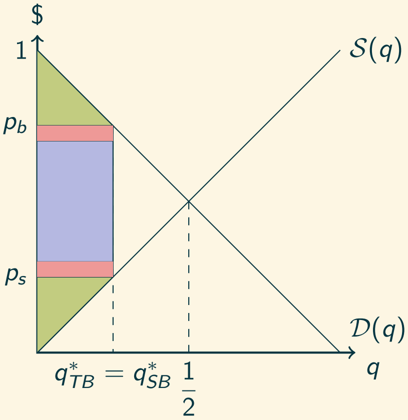
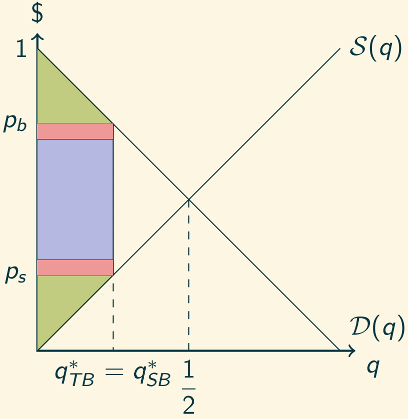


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Result 2

Compensations are a constant percentage of the baseline profit, independent of the distribution.

Let $p = \text{Prob(finding a match in decentralized market)}$.



$$\frac{\text{Third-Best Profit}}{\text{Second-Best Profit}} = 1 - \frac{p}{2}$$

Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Result 3

Thickness of the centralized marketplace is unaffected by decentralized trade.

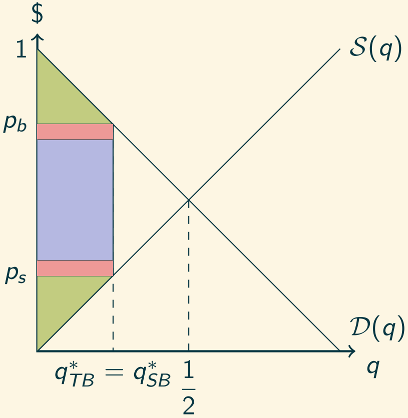


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Coming Up

- First: Centralized marketplace operates on its own.
  - Optimal allocation, simple economics of the marketplaces.
- Then: Centralized marketplace competes with a search market.
  - Coexistence result, segmentation, profit, and welfare.

Outline

Setup

- A single (indivisible) good.
- A continuum of agents over $[0, 1]$.
- Each agent has 1 unit of endowment and 2 units of demand.
  - The designer knows the endowments.
  - Agents have linear preferences.
    - θ is the private information of each agent.
    - $\theta \sim F(\cdot)$ and the designer knows $F(\cdot)$.
- Net utility: $u = \theta \times \min\{1, q\} - t$ where q is quantity, t is payment.

Marketplace

- For now, all trade goes through a marketplace.

- Marketplace designer
  - knows the endowments and distribution of agents' valuations;
  - wants to maximize her profit by choosing a deterministic mechanism for trade.

- By revelation principle, focus on direct, Bayesian IC mechanisms $(Q(\theta), T(\theta))$.
  - Continuum of agents $\Rightarrow$ without loss to focus on Dominant Strategy-IC mechanisms $(q(\theta), t(\theta))$.

Designer's Problem

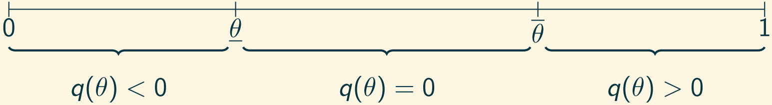
Marginal Revenue: $\mathcal{V}(\theta) = \theta - \dfrac{1 - F(\theta)}{f(\theta)}$ (Virtual Value);

Marginal Cost: $\mathcal{C}(\theta) = \theta + \dfrac{F(\theta)}{f(\theta)}$ (Virtual Cost).

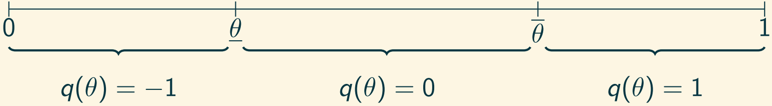
$$\begin{aligned}
\max_{q(\theta)} \Pi = \mathbb{E} \quad & \left[q(\theta) \left(\mathbb{1}\{\theta \text{ is a seller}\}\mathcal{C}(\theta) \quad + \mathbb{1}\{\theta \text{ is a buyer}\}\mathcal{V}(\theta) \right) \right] \\
& \text{s. t.} \\
& q(\theta) \text{ is increasing in } \theta \qquad && \text{(for Incentive Compatibility)} \\
& q(\theta) \geq -1 && \text{(Individual Feasibility)} \\
& \mathbb{E}[q(\theta)] \leq 0 && \text{(Market Clearing)}
\end{aligned}$$

Solving Designer's Problem

Monotonicity of the allocations $q(\theta)$ implies:



Indivisible good + deterministic mechanism implies:



Inspiration from the Monopoly Problem

$\mathcal{MC}(q) = \mathcal{C}(F^{-1}(q))$ and $\mathcal{MR}(q) = \mathcal{V}(F^{-1}(1-q))$:

Agents get **Information Rents** for their private information (their valuations).

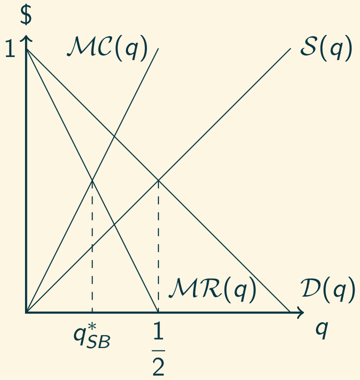


Figure: **Second-Best Profit of the Marketplace**, **Information Rents**.

Optimal Mechanism

Theorem

Suppose the distribution F is regular. Then, the optimal mechanism has the allocation rule

$$q(\theta) = \begin{cases} -1 & \text{if } \theta \leq \underline{\theta}^*, \\ 0 & \text{if } \underline{\theta}^* \leq \theta \leq \overline{\theta}^*, \\ 1 & \text{if } \theta \geq \overline{\theta}^*, \end{cases}$$

and the transfer rule

$$t(\theta) = \begin{cases} -\underline{\theta}^* & \text{if } \theta \leq \underline{\theta}^*, \\ 0 & \text{if } \underline{\theta}^* \leq \theta \leq \overline{\theta}^*, \\ \overline{\theta}^* & \text{if } \theta \geq \overline{\theta}^*. \end{cases}$$

Utilities

Utilities from the marketplace:

Figure: The utilities from the optimal marketplace (u^m).

Outline

Decentralized Market

Same primitives:

- Continuum of agents.
- Endowments: 1 unit; Demands: 2 units.
- Valuations: $\theta \sim F$ where F is regular.

Agents have a choice between the marketplace and a search market:

- In search market, they are **randomly matched** and they use **Nash bargaining**.

Market Choice Game

- Designer announces a mechanism and who should join it.
- Agents choose among the mechanism, a search market, and staying at home.
- Trade realizes in both markets.

Solution

- **Solution concept**: SPE with no bilateral deviations in the market choice stage.
  - Peivandi and Vohra (2021): No coalitional deviation to any mechanism.
- Deterministic mechanism.

Search Market

- Θ^d: agents who join the search market.
- $\mu(\cdot)$: the measure induced by F.
- A matching function, $M(\mu(\Theta^d))$ determines the measure of meetings.
- I assume M has constant returns to scale.
    - If the measure of agents is doubled, the measure of meetings also gets doubled.
- Then, $M(\mu(\Theta^d)) = m \times \mu(\Theta^d)$ for some $m \in [0, 0.5]$.
- Probability that an agent finds a match: $p = 2\dfrac{M(\mu(\Theta^d))}{\mu(\Theta^d)} = 2m$.

Search Market

Θ^d: agents who join the search market.

Expected utility of type θ from search:

$$\begin{aligned}\mathbb{E}[u^d(\theta)] &= \mathbb{P}[\text{match}]\mathbb{P}[\text{meet a higher value}]\mathbb{E}[\text{payoff from selling}] \\ &\quad + \mathbb{P}[\text{match}]\mathbb{P}[\text{meet a lower value}]\mathbb{E}[\text{payoff from buying}]\end{aligned}$$

$$\begin{aligned}&= p\mathbb{P}[x > \theta | x \in \Theta^d] \times \mathbb{E}\left[\frac{x-\theta}{2} | x > \theta, x \in \Theta^d\right] \\ &\quad + p\mathbb{P}[x < \theta | x \in \Theta^d] \times \mathbb{E}\left[\frac{x-\theta}{2} | x < \theta, x \in \Theta^d\right]\end{aligned}$$

Agents' outside options depend on everyone else's equilibrium behavior.

Two Levels of Incentive Compatibility

Designer needs to announce a mechanism such that:

1. Agents the designer wants in the mechanism join the mechanism;
2. Agents the designer doesn't want in the mechanism join the search market or stay at home;
3. Every agent would reveal their type truthfully if they joined the mechanism.

(1)+(2) $\rightarrow$ Convexity of the utility function across markets.
(3) $\rightarrow$ Convexity of the utility function in the mechanism.

Equilibrium Segmentation

Myerson & Satterthwaite (1983): Informational frictions causes inefficiencies in trade.

Have seen: Marketplace excludes agents with intermediate values even without the decentralized market.

Conjecture: Marketplace again excludes an interval of agents, $(\underline{\theta}, \overline{\theta})$ with $0 < \underline{\theta} < \overline{\theta} < 1$.

Equilibrium Segmentation

Theorem

In an equilibrium, the set of agents in the decentralized market must be an interval $(\underline{\theta}, \overline{\theta})$ *such that* $0 < \underline{\theta} < \overline{\theta} < 1$.

Implication 1: Restriction to such segmentation is without loss of profit.

Implication 2: There is no equilibrium where everyone is in the same market.

Existence and Uniqueness

Theorem

There exists $\underline{\theta}, \overline{\theta}$ such that in the equilibrium,

- *agents in $(\underline{\theta}, \overline{\theta})$ join the search market,*
- *agents in $[0, \underline{\theta}]$ and $[\overline{\theta}, 1]$ join the mechanism,*
- $\mathcal{C}(\underline{\theta}) = \mathcal{V}(\overline{\theta})$ *and* $F(\underline{\theta}) = 1 - F(\overline{\theta})$.

Moreover, the equilibrium is unique.

Utilities

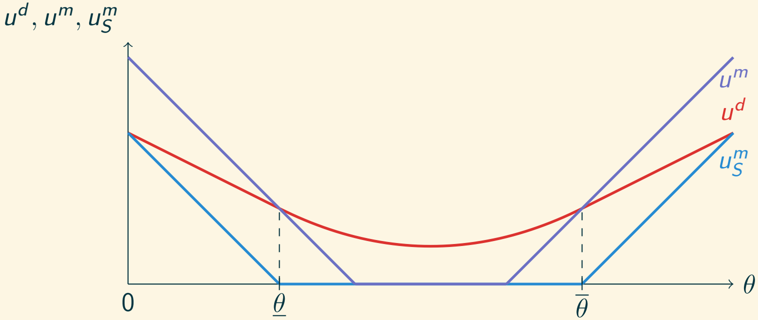


Figure: The utilities from the marketplace (u^m) and the decentralized market (u^d) in the coexistence, and utilities from the marketplace when it is the Single Market (u^m_S).

Marketplace Traders

Proposition

Same agents trade in the marketplace, with or without the decentralized market.

In other words, the thickness of the centralized marketplace is not affected by the decentralized trade.

Profit of the Marketplace

$p \equiv \mathbb{P}[$ an agent finds a match in the search market.$]$

Π^M : Profit of the marketplace if there were no search market.

Theorem

The profit of the marketplace under coexistence is $\left[1 - \frac{p}{2}\right] \Pi^M$.

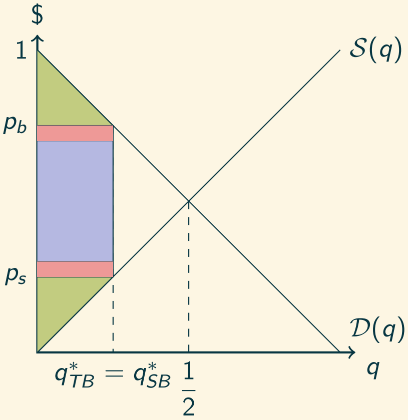


Figure: **Third-Best Profit of the Marketplace**, **Information Rents**, '**Compensations**'.

Independent of the distribution, F,

$$\frac{\text{Compensations}}{\text{Second-Best Profit}} = \frac{p}{2} \text{ so } \frac{\text{Third-Best Profit}}{\text{Second-Best Profit}} = 1 - \frac{p}{2}.$$

Efficiency

Theorem

Suppose agents are uniformly distributed on $[0, 1]$*. Then, coexistence leads to a higher welfare than either market operating on its own.*

Remark

Also true for Standard Normal, Standard Beta, Exponential, Logistic distributions.

Beyond Nash Bargaining

Suppose the decentralized bilateral trades happen with double auction.

Two-sided Double Auction:

- Each agent makes a price bid;
- Agent with the higher bid buys the other's endowment;
- Pays the mid-point of the prices.

Equilibrium of the Double-Auction

Suppose the agent's distribution is given by a smooth distribution, G.

Lemma (Cramton, Gibbons, Klemperer (1987); Kittsteiner (2003))

In the unique equilibrium of this game, agent's bids are given by

$$b(\theta) = \theta - \frac{\int_{G^{-1}(\frac{1}{2})}^{\theta} [G(x) - \frac{1}{2}]^2 dx}{[G(\theta) - \frac{1}{2}]^2}$$

Bids are monotone in valuations, so the equilibrium is efficient.

Equilibrium of the Double-Auction

We know the equilibrium only when G is smooth.

Assume an interval of agents $(\underline{\theta}, \overline{\theta})$ join the decentralized market.

Then, F restricted to $(\underline{\theta}, \overline{\theta})$ is a smooth distribution.

Suppose agents are **uniformly** distributed over $[0, 1]$.

Equilibrium of the Double-Auction

Suppose agents are uniformly distributed over $[0, 1]$.

Theorem

There exists $\underline{\theta}, \overline{\theta}$ such that in the equilibrium,

- *agents in $[0, \underline{\theta}]$ and $[\overline{\theta}, 1]$ join the mechanism, the rest searches;*
- $\mathcal{C}(\underline{\theta}) = \mathcal{V}(\overline{\theta})$ *and* $F(\underline{\theta}) = 1 - F(\overline{\theta})$.

Proposition

The market thickness is unaffected by the decentralized market.

Proposition

The profit of the marketplace is $1 - \frac{5p}{6}$ times the baseline profit.

Proposition

The total welfare under the simple equilibrium with the double auction is greater than when either market operates on its own.

Outline

Multiple Centralized Marketplaces

$n \geq 2$ profit-maximizing marketplace designers in competition.

No (practical) revelation principle:

Each designer wants to condition her marketplace on the other marketplaces.

Infinite regress of conditions.

Approach 1

Restrict the designers to choose direct mechanisms.

(Not without loss anymore.)

Each direct, IC mechanism is equivalent to a pair of prices.

$\Rightarrow$ Bertrand competition.

Equilibrium

Suppose one designer makes positive profit.

Another can ‘cut’ the prices just enough to poach agents from her.

$\Rightarrow$ No equilibrium with positive profit.

Suppose each designer posts prices $p_b = p_s = m(F)$.

$\Rightarrow$ Bertrand Equilibrium.

Approach 2

Allow the designers to choose any mechanism.

One equilibrium:

- Each designer posts baseline marketplace prices;
- They also offer price-matching guarantees;
- agents randomize uniformly.

They collectively make the baseline ‘monopoly’ profit.

Compare with 0 profit under direct mechanisms.

Similar construction works when there is decentralized market as well!

Outline

Proof with the Uniform Distribution.

Total welfare when everyone searches:

$$\int_0^1 [p\theta\theta - p(1-\theta)\theta]d\theta = \cdots = \frac{p}{6}.$$

Cutoffs for the marketplace: $(\underline{\theta}, \overline{\theta}) = (0.25, 0.75)$. So, the welfare the marketplace creates:

$$\left[\int_{0.75}^1 \theta d\theta - \int_0^{0.25} \theta d\theta\right] = \cdots = \frac{3}{16}.$$

$\frac{3}{16} > \frac{p}{6}$ for every $p \in [0, 1]$. □

Simplifying the Profit

Suppose the marketplace excludes $(\underline{\theta}, \overline{\theta})$ with $0 < \underline{\theta} < \overline{\theta} < 1$.

Then, the profit from the pair $\underline{\theta}, \overline{\theta}$, $\Pi^C(\underline{\theta}, \overline{\theta})$:

$$\underbrace{-F(\underline{\theta})u^d(\underline{\theta}) - (1 - F(\overline{\theta}))u^d(\overline{\theta})}_{\text{Compensations for joining the mechanism}} + \underbrace{\int_0^{\underline{\theta}} \mathcal{C}(x)q(x)f(x)dx + \int_{\overline{\theta}}^{1} \mathcal{V}(x)q(x)f(x)dx}_{\text{Profit if there were no search market.}}$$

Simplifying the Profit

$$-F(\underline{\theta})u^d(\underline{\theta}) - (1 - F(\overline{\theta}))u^d(\overline{\theta}) + (-1)\int_0^{\underline{\theta}} \mathcal{C}(x)f(x)dx + (1)\int_{\overline{\theta}}^{1} \mathcal{V}(x)f(x)dx$$

$$\int_0^{\underline{\theta}} \mathcal{C}(x)f(x)dx = \int_0^{\underline{\theta}} \left[x + \frac{F(x)}{f(x)}\right] f(x)dx = \int_0^{\underline{\theta}} xf(x)dx + \int_0^{\underline{\theta}} F(x)dx$$

$$= \left([xF(x)]_0^{\underline{\theta}} - \int_0^{\underline{\theta}} F(x)dx \right) + \int_0^{\underline{\theta}} F(x)dx = \underline{\theta}F(\underline{\theta}).$$

Similarly, it is possible to show that: $\int_{\overline{\theta}}^{1} \mathcal{V}(x)f(x)dx = \overline{\theta}(1 - F(\overline{\theta}))$.

Simplifying the Profit

Let $E[\theta^d] = E[\theta|\underline{\theta} \leq \theta \leq \overline{\theta}]$ and $p = \mathbb{P}[\text{match}]$. Then,

$$
\begin{aligned}
&F(\underline{\theta})u^d(\underline{\theta}) + (1 - F(\overline{\theta}))u^d(\overline{\theta}) \\
&= F(\underline{\theta})\frac{p}{2}\left[E[\theta^d] - \underline{\theta}\right] + (1 - F(\overline{\theta}))\frac{p}{2}\left[\overline{\theta} - E[\theta^d]\right] \\
&= \frac{p}{2}\left(F(\underline{\theta})\left[\cancel{E[\theta^d]} - \underline{\theta}\right] + (1 - F(\overline{\theta}))\left[\overline{\theta} - \cancel{E[\theta^d]}\right]\right) \\
&= \frac{p}{2}\left(-\underline{\theta}F(\underline{\theta}) + \overline{\theta}(1 - F(\overline{\theta}))\right)
\end{aligned}
$$

Simplifying the Profit

$$-F(\underline{\theta})u^d(\underline{\theta}) - (1 - F(\overline{\theta}))u^d(\overline{\theta}) = -\frac{p}{2}\left(\overline{\theta}(1 - F(\overline{\theta})) - \underline{\theta}F(\underline{\theta})\right)$$

$$-\int_0^{\underline{\theta}} \mathcal{C}(x)f(x)dx + \int_{\overline{\theta}}^{1} \mathcal{V}(x)f(x)dx = (\overline{\theta}(1 - F(\overline{\theta})) - \underline{\theta}F(\underline{\theta}))$$

Then, simple algebra shows:

$$\Pi^C = \left[1 - \frac{p}{2}\right]\Pi^M,$$

where Π^M is the profit marketplace makes on its own.

Relation to the Literature

- Platform Competition (Rochet and Tirole (2003); Armstrong (2006)): Competing profit-maximizing platforms with explicit network effects.
  - I study how a profit-maximizing marketplace competes with decentralized trade.
- Peivandi and Vohra (2021) show that almost no market mechanism is stable.
  - I study stable market structures that allow trading outside the mechanism.
- Miao (2006) studies coexistence of centralized and decentralized markets in a different setup.
  - His results are driven by transaction and search costs; they are absent here.

Conclusions

- Marketplaces don't monopolize the markets.
  - Attract high value buyers and low cost sellers.
    - The same ones they get without the decentralized market!
  - The rest search.
- Decrease in the profit due to the decentralized market only depends on the efficiency of the decentralized market (p).
- Coexistence improves the welfare over either market alone.