How to Delegate the Choice of a Project

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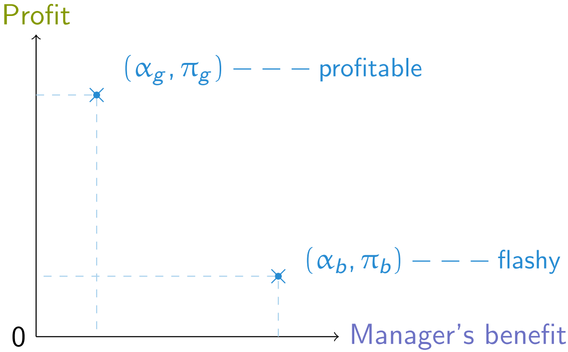
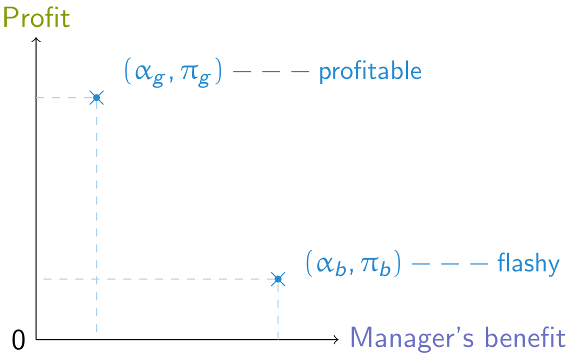
Manager and CEO choosing a project:

- New client; new division; switching software; acquisition; ...
- Potential projects (public info) vs. feasible projects (private info)
- Manager sequentially proposes feasible projects to CEO
- CEO can accept or reject proposal; cannot commit to future responses
- Conflicting preferences
  - CEO: profits/dividends
  - Manager: her influence ("empire building")

Problem


Profit
(α_g, π_g) — — — profitable
(α_b, π_b) — — — flashy
0
Manager's benefit

Problem



- Manager is able to "hide" the more profitable project
- CEO does not know which are feasible and if he should insist on the proposal of the more profitable project

Question

Instruments in dynamic interactions:

Information – Proposal power – Commitment power

- Manager has information + proposal power — as powerful as possible
- CEO has none of the instruments — taking away all the power

Question: Can the Manager exploit this imbalance to build her empire?

$\longrightarrow$ No! CEO benefits from lack of proposal power + can execute the "optimal" project selection

Preview of Results

- A commitment benchmark— best outcome if CEO could commit to his responses to proposals
- With two projects, CEO can always attain his commitment payoff in equilibrium
- With more than two projects, CEO can still attain his commitment payoff *under some conditions*

Two project example

- Two projects: $(3, 8)$ (good) and $(8, 3)$ (bad)
  - A subset is available
- Available projects represent Manager's *type* and it is privately observed
- Manager has 4 types:
  - $\emptyset$: empty type —— wp $\frac{1}{4}$
  - $\{(3, 8)\}$: good type —— wp $\frac{1}{4}$
  - $\{(8, 3)\}$: bad type —— wp $\frac{1}{4}$
  - $\{(3, 8), (8, 3)\}$: mixed type —— wp $\frac{1}{4}$
- Sequential recommendations: available or silence + accept or reject
- Perfect Bayesian Equilibrium: optimal strategies + consistent beliefs

Two project "best" outcome

What is the best outcome for the CEO if he could commit?

$\longrightarrow$ Collect reports and implement projects

- Good project whenever it's available
- Bad project with probability $\frac{3}{8}$ if it's the only available one

Best outcome:

$$\text{Good type: } \{(3,8)\} \text{ ——— } (3,8) \text{ wp } 1 \rightarrow 8$$

$$\text{Mixed type: } \{(3,8),(8,3)\} \text{ ——— } (3,8) \text{ wp } 1 \rightarrow 8$$

$$\text{Bad type: } \{(8,3)\} \text{ ——— } (8,3) \text{ wp } \tfrac{3}{8} \rightarrow \tfrac{3}{8}\cdot 3$$

Best outcome in the game?

How close can the CEO get to the best outcome through delegation?

Separating equilibrium outcome:

- Good & mixed types propose $(3,8)$ at $t=0$ + CEO accepts it
- Bad type stays silent for $t < t^*(\delta) := \min\{t : \frac{3}{8} \geqslant \delta^t\}$; proposes $(8,3)$ at $t = t^*(\delta)$ + CEO accepts it $\rightarrow$ signalling through delay
  - $(8,3)$ before $t^*(\delta)$ $\rightarrow$ CEO believes it's the mixed type + rejects it
  - Mixed type: $\underline{3 \text{ today}}$ vs. 8 at $t^*(\delta)$ $\rightarrow$ separated

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- 8 from good & mixed types
- $\delta^{t^*(\delta)} \cdot 3 \to \frac{3}{8} \cdot 3$ from bad type as $\delta \to 1$

How does the CEO still win?

CEO mainly benefits from the lack of proposal power:

- $(3, 8)$ is proposed immediately when it's available
- At $t = 1$, CEO *knows* only $(8, 3)$ is available, not $(3, 8)$
  1. if he had proposal power, he would request $(8, 3)$ at $t = 1 \rightarrow$ mixed type would also wait $\rightarrow$ CEO would request $(8, 3)$ at $t = 0 \rightarrow 8 + 3 + 3$
  2. without proposal power, he **must** wait until $t = t^*(\delta)$ for $(8, 3)$ to be proposed despite his beliefs $\rightarrow 8 + 8 + \frac{3}{8} \cdot 3 > 8 + 3 + 3$

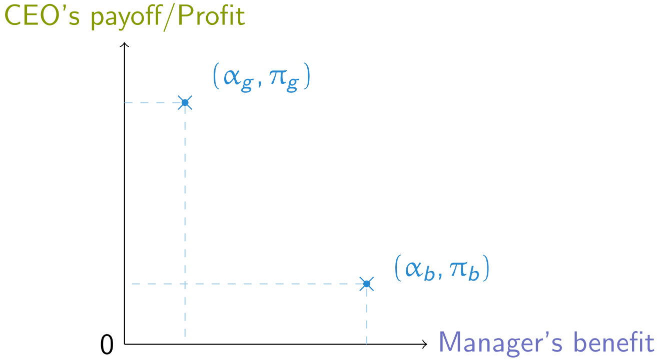
Threat of extremal off-path beliefs also helps: no $(8, 3)$ before $t^*(\delta)$

Overview

Projects

A project: pair of payoffs for Manager and CEO — $(\alpha, \pi) \in \mathbb{R}^2$

Nature: a subset of N potential projects available — focus on $N = 2$ today



Setup with $N = 2$

- Two projects: (α_g, π_g) (good) and (α_b, π_b) (bad) with

$$\pi_g > \pi_b > 0$$

$$\alpha_b > \alpha_g > 0$$

  Potential projects are common knowledge.

- A subset is *available*: drawn according to a distribution $\mu(\cdot)$
- Available projects $\equiv$ Manager's *type* + privately observed
- Manager has 4 types:
  - $\emptyset$: empty type —— $\mu_\emptyset$
  - $\{(\alpha_g, \pi_g)\}$: good type —— μ_g
  - $\{(\alpha_b, \pi_b)\}$: bad type —— μ_b
  - $\{(\alpha_g, \pi_g), (\alpha_b, \pi_b)\}$: mixed type —— μ_m

Sequential Delegation Game

Discrete time recommendation game with common discount factor δ

- Nature draws Manager's type before the game begins
- $t = 0, 1, \ldots, \infty$:
  - Manager proposes an available project or stays silent;
  - CEO accepts or rejects the proposal
    - if he accepts it, the game ends and players get their (discounted) payoffs
    - if he rejects it, the game proceeds to $t + 1$

Solution concept: Perfect Bayesian Equilibrium

Strategies

Commitment Benchmark

What if CEO could commit to his responses in the game?

- establishing a benchmark — his commitment payoff
- comparison to payoffs in equilibria

Proposition (Informal)

Optimal commitment in the game is equivalent to committing to a static stochastic mechanism with type-dependent message space.

Mechanism Design Problem

A mechanism is a mapping between types and probability of implementing for each available project:

- $g \mapsto q_{gg}$
- $b \mapsto q_{bb}$
- $m \mapsto q_{mg}, q_{mb}$ with $q_{mg} + q_{mb} \leqslant 1$

Type-dependent message space: a type can only report its subsets

$$g \rightarrow g$$

$$b \rightarrow b$$

$$m \rightarrow m, g, b$$

Optimal Mechanism

Two projects: (α_g, π_g), (α_b, π_b) with $\pi_g > \pi_b > 0$ and $\alpha_b > \alpha_g > 0$

CEO's objective:

$$\max_{q_{gg}, q_{bb}, q_{mg}, q_{mb}} \quad \text{CEO's expected payoff}$$

subject to

$$\text{Payoff of mixed} \geqslant \text{Payoff of good}$$

$$\text{Payoff of mixed} \geqslant \text{Payoff of bad}$$

$$q_{gg}, q_{bb}, q_{mg}, q_{mb} \quad \text{are probabilities}$$

$$q_{mg} + q_{mb} \leqslant 1$$

Notation

Solving for Optimal Mechanism

Observations about the problem:

1. We must have $q_{mg} + q_{mb} = 1$: otherwise, CEO gets a higher payoff while keeping the IC
2. First IC constraint is redundant: mixed type's expected payoff is always higher than the good type's

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2. First IC constraint is redundant: mixed type's expected payoff is always higher than the good type's $\Rightarrow q_{gg} = 1$

Problem becomes: whether or not CEO wants to extract the good project from the mixed type

- No $\longrightarrow$ Pooling
- Yes $\longrightarrow$ Separating

Optimal Mechanism: Pooling or Separating

The optimal mechanism can take two forms:

1. Pooling:
   - good project from the good type: $q_{gg} = 1$
   - bad project from the bad and mixed types: $q_{bb} = q_{mb} = 1$ (letting the mixed type **pool** with the bad)
2. Separating:
   - good project from the good and mixed types: $q_{gg} = q_{mg} = 1$ (**separating** the mixed type from the bad)
   - bad project from the bad type with some interior probability (for IC):

$$q_{bb} = \frac{\alpha_g}{\alpha_b} < 1$$

Pooling or Separating? → Gains vs. Losses

Gains from separation: higher payoff from the mixed type

$$(\pi_g - \pi_b)\mu_m$$

Losses from separation: lower payoff from the bad type for IC

$$(1 - \frac{\alpha_g}{\alpha_b})\pi_b\mu_b$$

Separate if

$$(\pi_g - \pi_b)\mu_m \geqslant (1 - \frac{\alpha_g}{\alpha_b})\pi_b\mu_b,$$

pool otherwise. Relative likelihood of types:

$$\lambda = \frac{\mu_m}{\mu_b} \geqslant \frac{(1 - \frac{\alpha_g}{\alpha_b})\pi_b}{(\pi_g - \pi_b)} = \lambda^*$$

Commitment Payoff in Equilibria

Theorem

There is an equilibrium of the sequential delegation game in which CEO's payoff approximates his commitment payoff as $\delta \to 1$.

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Commitment Payoff in Equilibria

Theorem

There is an equilibrium of the sequential delegation game in which CEO's payoff approximates his commitment payoff as $\delta \to 1$.

1. (Pooling) Each type of Manager proposes her favorite available project at $t = 0$. CEO accepts any proposal at $t = 0$. $[\lambda < \lambda^*]$
2. (Separating) Manager's good and mixed types propose the good project at $t = 0$ and bad type stays silent until $t = t^*(\delta) := \min\{t : \alpha_g \geqslant \delta^t \alpha_b\}$ at which point she proposes the bad project. CEO accepts the good project at $t = 0$ and the bad project at $t = t^*(\delta)$. $[\lambda \geqslant \lambda^*]$

Pooling Equilibrium

Equilibrium:

- Each type of Manager *always* proposes her favorite available project
- At any point, CEO believes that he will receive the same proposal if he rejects, so he accepts any proposal $\rightarrow$ accepts any project at $t = 0$

Attaining the commitment payoff:

- Easy to replicate the pooling outcome in the dynamic game
- Simply accepting any project in anticipation of Manager's 'stubborn' strategies
- Full discretion to Manager by always accepting

Separating Equilibrium

Equilibrium:

- Manager proposes the bad project only when there is no alternative and even then with a delay
- CEO expects the bad project after a certain delay and only then accepts it
- The delay is long enough that the mixed type does not wait
- Delay as costly signaling; supported by CEO's extremal off-path beliefs if the bad project is proposed earlier

Separating Equilibrium

Attaining the commitment payoff:

- CEO benefits in equilibrium from a combination of the *limited actions* available to him, the *extremal off-path beliefs*, and *delay as an instrument* for costly signaling of types
- With proposal power, he could give full discretion to Manager, in the spirit of Coasian bargaining
- *When* to accept a certain project is a way to incentivize Manager in combination with the threat of extremal beliefs

Conclusion

- We investigate how an uninformed CEO delegates the choice of a project to an informed Manager.
- It is possible to attain the commitment payoff without commitment power in the sequential delegation game.
- Even though the Manager has private information and proposal power, our analysis is in contrast with standard Coasian bargaining.
- Result extends to $N > 2$ projects under some conditions; currently working on providing a tight bound on the limit.

Strategies

A history consists of recommendations until t.

- Manager's strategy maps each history to a distribution over available projects at t
- CEO's strategy maps each history to a distribution over accepting and rejecting

Back

Optimal Mechanism

Two projects: (α_g, π_g), (α_b, π_b) with $\pi_g > \pi_b > 0$ and $\alpha_b > \alpha_g > 0$

CEO's objective:

$$\max_{q_{gg}, q_{bb}, q_{mg}, q_{mb}} \quad q_{gg}\mu_g\pi_g + q_{bb}\mu_b\pi_b + q_{mg}\mu_m\pi_g + q_{mb}\mu_m\pi_b$$

subject to

$$q_{mg}\alpha_g + q_{mb}\alpha_b \geqslant q_{gg}\alpha_g$$

$$q_{mg}\alpha_g + q_{mb}\alpha_b \geqslant q_{bb}\alpha_b$$

$$q_{gg}, q_{bb}, q_{mg}, q_{mb} \in [0, 1]$$

$$q_{mg} + q_{mb} \leqslant 1$$

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