Dissertation Final Exam

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Roadmap

Two papers and two papers-to-be.

- Many-to-One matchings with wages.
- Competitive equilibria of convex economics.
- Sequential college choice.
- Learning in college choice.

Outline

Many-to-One Matchings with Transfers

- Consider the hospital-doctor matchings with wages.
- $\exists$ a stable matching if doctors are *gross substitutes* (GS) for hospitals.
- Any condition that is required for each preference individually is too strong.

Example-0

Two doctors: Alice (A) and Bob (B).

Two Hospitals: UHS (U), Mount Nittany (M) with revenues:

	{A, B}	A	B
U	1000	500	700
M	7	2	2

Suppose doctors are indifferent between hospitals and only care about the wages.

It is irrelevant whether M's preferences satisfy GS or not.

A stable matching: $U - \{A, B\}$ with wages $w_A = w_B = 5$.

Example-1

Two doctors: Alice (A) and Bob (B); Two Hospitals: UHS (U), Mount Nittany (M) with revenues:

	{A, B}	A	B
U	3	0	0
M	2	2	2

A and B are complements for U! (They are not gross substitutes.)

No stable matching: The efficient matching is $U - \{A, B\}$ but

there is no solution to $3 \geq w_A + w_B$, $w_A \geq 2$, $w_B \geq 2$.

Example-2

Same agents, different revenues:

	{A, B}	A	B
U	4	0	0
M	2	2	2

A and B are still complements for U.

Now there is a stable matching: $U - \{A, B\}$ with wages $w_A = w_B = 2$.

Example-1: Measuring Instability

Consider a 1 Dollar subsidy:

	{A, B}	A	B
U	3	0	0
M	2	2	2

Example-1: Part-time Contracts

	{A, B}	A	B
U	3	0	0
M	2	2	2

Stable partial matching: A and B works for U together for half of their times; then they work for M separately for the rest of their times.

Literature and Contribution

- Many-to-One Matchings with Wages.
  - Stable matching exists under **gross substitutability** (GS) (Kelso, Crawford (1982); Hatfield, Milgrom (2005))
  - Also exists under some complementarity conditions (Rostek, Yoder (2020))
- Measuring instability.
  - The least core (Kern and Paulusma, 2003); approximate core (Vazirani, 2022).

I provide

- Necessary and sufficient conditions for existence of a stable matching.
- A measure of instability: Subsidy needed to 'stabilize' the efficient matching.

Primitives

- A set of hospitals, H and a set of doctors, D.
- Hospitals maximize profit: $\pi_h(D') = r_h(D') - \sum_{d \in D'} w_d$, where $r_h(\cdot)$ is the revenue.
- Doctors maximize $u_d(h) + w$ (when d works for h at wage w).
  - 'Peer Effect': $u_d(h, D') + w$.

Matchings with Wages

Definition

(μ, w) is a **Matching with Wages** if μ assigns each doctor to at most one hospital with wages for each doctor given by w.

Abusing the notation, w will also denote the payoff vector for all agents. Then, we call (μ, w) an outcome.

Blocking a Matching

Definition

An outcome (μ, w) is **blocked** if there exists a hospital $h \in H$, a set of doctors $D' \subset D \setminus \mu(h)$, and some wages for the doctors in D' such that

- New profit of the hospital h is at least as high as before,
- New utility of each doctor in D' is at least as high as before,
- For at least one agent in $D' \cup \{h\}$, new utility is strictly higher than before.

Stable Matchings

Definition

An outcome (μ, w) is **stable** if

- it is individually rational for each hospital and doctor,
  - I.e., they are not worse off than being unmatched.
- it is not blocked.

Efficiency

Definition

An outcome (μ, w) is efficient if it maximizes total welfare (among all feasible outcomes).

Lemma

If an outcome (μ, w) is stable, then it is efficient.

Associated Cooperative Game

An associated TU cooperative game: For any $N' \subset H \cup D$,

$$v(N') = \max_{\text{`Admissible' partitions of } N'} \begin{array}{c}\text{Total revenue and utility}\\ \text{of the partition}\end{array}$$

A partition is *admissible* if each coalition has at most one hospital.

Stability and the Core

Proposition

An economy has a stable outcome if and only if the associated transferable utility game v has a non-empty core.

Applying Bondareva-Shapley result to v:

Corollary

An economy has a stable outcome if and only if the associated transferable utility game v satisfies balancedness.

Corollary

Stable outcome with partial contracts always exists.

Balancedness is ‘inefficient’

Balancedness of a TU game requires some constraints related to every coalition.

Due to the definition of blocking in this environment, some of those can never bind.

In the paper, I provide a way to check for the existence of a stable matching using less constraints than balancedness.

Measuring Instability

$$\begin{aligned} \min_w \quad & \sum_{i \in H \cup D} w_i \\ s.t. \quad & \\ & \sum_{i \in N'} w_i \geq v(N'), \text{ for each } N' \subset H \cup D. \end{aligned}$$

If $\sum_{i \in H \cup D} w_i = v(N)$, there exists a stable matching.

If $\sum_{i \in H \cup D} w_i > v(N)$, there isn't. Then, $\sum_{i \in H \cup D} w_i - v(N)$ is the instability.

Can we find an upper bound for $\dfrac{\sum_{i \in H \cup D} w_i - v(N)}{v(N)}$ in a large economy?

Example-1 (Cont'd)

Why did we consider \$1 of subsidy?

	{A, B}	A	B
U	3	0	0
M	2	2	2

Outline

Famous Convex Sets

In economics, we assume/need the convexity of many sets:

- Upper contour sets,
- Budget sets,
- Strategy space.

But what does convex mean?

Euclidean Convexity

A set C is convex if it includes its convex hull, $K(C)$.

A point x is in the convex hull of a set C if for every linear ordering $\succsim$, $\exists$ an element $y \in C$ such that $x \succsim y$.

Linear ordering: A preference represented by a linear utility function.

An Example

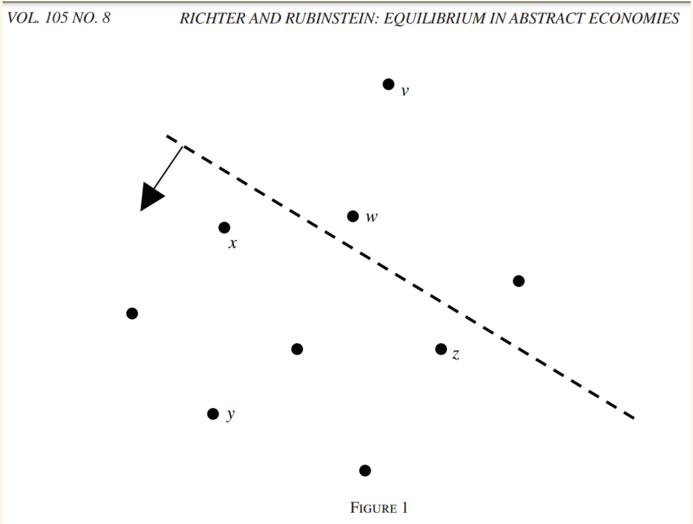

Figure: $w \notin K(\{x, y, z\})$ but $w \in K(\{x, y, z, v\})$

Another Convexity

A point x is in the convex hull of a set C if for every linear ordering that isn't decreasing in every dimension, $\succsim$, $\exists$ an element $y \in C$ such that $x \succsim y$.

This gets us the convexity of 'upward closed' sets.

[Drawings on the board, tikz if I can]

Abstract Convexities

Definition

A family $\mathcal{C}$ of subsets of a set X is called a **convexity** on X if

1. The empty set $\emptyset$ and the universal set X are in $\mathcal{C}$;
2. $\mathcal{C}$ is closed under intersections;
3. $\mathcal{C}$ is closed under nested unions.

The pair $(X, \mathcal{C})$ is called a **convex structure**.

[Drawings on the board, tikz if I can]

Nash Equilibrium

Proposition

Consider a game. If preferences of every player satisfies a 'well-behaved' convexity (together with the standard topological assumptions), then there exists a Nash Equilibrium.

- Existence of Nash Equilibrium with broad classes of 'convex' preferences.
- 'Expected Utility' without Independence Axiom.

Budget sets

Consider an exchange economy of n consumers with (u_i, e_i).

With endowment e_i and prices p, budget set is $\{x | px \leq pe_i\}$.

Budget sets generated by price vectors need not be convex in other convexities.

But I can define price orderings over bundles in an economy. (Piccione & Rubinstein 2007; Richter & Rubinstein 2015)

With endowment e_i and price ordering $\succsim_p$, budget set is $\{x | e_i \succsim_p x\}$.

Walrasian Equilibrium

Theorem

Consider an exchange economy. If the preferences of each consumer satisfies a well-behaved convexity (together with the standard topological assumptions), then there exist a Walrasian Equilibrium with a price ordering.

Outline

College Choice Design

- Students are misinformed about their rankings within their cohorts.
  - (Bobba & Frisancho, 2019; Bobba & Frisancho, 2023)
- Exposing them to more exams and information improves their outcomes.
  - (Bobba & Frisancho, 2019; Bobba & Frisancho, 2023; Doghonadze, 2022)
- Optimal feedback design in a contest setup?

Outline

Motivation

- In college choice, most countries allow limited preference lists.
  - Can list 14 program in Georgia, 24 in Turkey...
- Students need to be strategic about the lists they submit.
- It leads to inefficiencies in realized matchings.
  - Luflade (2018), Ajayi & Sidibe (2017).
  - We can do better.

Toy Model

- Suppose all programs $(p_1, p_2, \ldots)$ have the same preferences $\succsim_p$ over students.
- Say there are only two students: Alice and Bob, and Alice $\succsim_p$ Bob.
- Let Alice choose her top program, and place Bob to either his top or second-top program.

Complications and Directions

- Can't go one by one with millions of students.
  - Use quantile cutoffs and let students know about the remaining seats.
    - Can it be done without hurting anyone compared to the status quo (deferred acceptance)?
- Programs may have different preferences over students.
  - Then, the deferred acceptance is generically not OSP.
  - Students can submit their preferences gradually.

Appendix-Stable Matchings

Definition

An outcome (μ, w) is a **stable outcome** if

1. It is IR for each hospital and doctor (profits and utilities exceed 0).
2. There doesn't exist a hospital $h \in H$ and a set of doctors $D' \subset D$ such that they can deviate from this outcome in a way that makes everyone better off;

Appendix-Blocking A Matching

Definition

A matching with wages (μ, w) is **blocked** if there exists a hospital $h \in H$, a set of doctors $D' \subset D \setminus \mu(h)$ and wages for the doctors in D', w' such that there exists a set of doctors D'' with $D'' \subset D' \cup \mu(h)$ and:

- $r_h(D'') - \sum_{d \in D''} w_d \geq r_h(\mu(h)) - \sum_{d \in \mu(h)} w_d$;
- For each doctor $d \in D'$, $u_d(h, w_d) \geq u_d(\mu(d), w_d)$;
- For at least one agent in $D' \cup \{h\}$, the corresponding inequality above is strict.

Appendix-Existence of a Stable Outcome

We apply Bondareva-Shapley result to v:

Corollary

There exists a stable outcome (μ, w) *if and only if for any set of balancing coefficients* $(b_{N'})_{N' \subset N}$ *(i.e., for any vector* $(b_{N'})_{N' \subset N}$ *such that* $\sum_{N' \subset N} b_{N'} \delta_{N'} = 1$*), we have that* $v(N) \geq \sum_{N' \subset N} b_{N'} v(N')$*.*