IE 301 - Operations Research II
Summer 2022
Midterm Exam
28.07.2022
Time Limit: 110 minutes **Name:**

INSTRUCTIONS

- Write clearly and legibly.
- **You have 110 minutes.**
- This is a **closed-book**, **closed-notes** exam. You are not allowed to use and refer to the book, class notes, homework assignments, homework solutions, handouts, or any other material in the test. You may use your **non-programmable calculators**. Cell phones or any other programmable devices can not be used.
- This is a personal exam. You are not allowed to collaborate with or provide/receive help to/from any other person during the exam by any means.
- **You are not allowed to ask any questions during the exam.** If you think the details are not clear about a question, you can make your own assumptions as long as they do not contradict with the question. In such a case, you have to state your assumptions clearly.
- Provide your answers on the same page of the questions. If you need extra space, you can use the backpage of the same question.
- **Make sure your definitions are complete and clear. Show all your work. This will increase your chance to get partial credits.**
- Good luck!

Question	Points	Score
1	20	
2	20	
3	30	
4	15	
5	15	
Total:	100	

1. (20 points) Consider the function below:

   $f(x) = 15 - 12x - 25x^2 + 2x^3$

   a) Find all stationary points and classify them.

   b) Show that f(x) has neither a global maximum nor a global minimum, and clearly explain your reasoning.

   Show all your work.

2. (20 points) Consider the following unconstrained NLP:

$$\min \quad f(x) = x^4 + 4x^3 + 7x^2 + 3x$$

a) Apply two iterations of the bisection method to (approximately) solve this problem. Find appropriate initial bounds $(\underline{x}, \bar{x})$ by inspection. Explain why your choice of upper and lower bounds is appropriate. Write down the next trial solution at the end of the second iteration clearly.

b) Starting from the initial solution $x = -0.5$, perform two iterations of the Newton's method for this problem. Write down the next trial solution at the end of the second iteration clearly.

Show all your work.

3. (30 points) Consider the following NLP:

$$\begin{array}{ll} max & f(x_1, x_2) = 8x_1 - x_1^2 + 4x_2 - x_2^2 \\ s.t. & x_1 + x_2 \leq 5 \\ & x_1, x_2 \geq 0 \end{array}$$

a) For this quadratic model, provide the b and c vectors, and Q and A matrices that correspond to the closed form.

b) Perform one iteration of the modified simplex algorithm to solve this problem. Clearly provide the KKT conditions, show how you manipulate them to obtain the related linear programming formulation, and identify the additional complementarity constraints that are enforced by the algorithm. Write down the list of candidate entering variables, the selected entering variable, and the leaving variable clearly.

Show all your work.

4. (15 points) Assume that you will be running a new construction project, and you need to manage the workforce through hiring and laying-off workers. Your goal is to minimize the total cost that is related to workforce.

   The project duration is 5 years, and the minimum labor force required in each year is given on the table below. You can easily lay-off or hire workers at the beginning of each year, however they incur cost. The hiring cost for each worker in each year is provided in the table below. The cost of laying-off is \$1000 per worker in any year. Apart from these, an additional cost is incurred if a year's available workforce exceeds the minimum requirement. For each of the excess workers in a year, you pay a penalty of \$400.

Years (t)	Minimum Labor Req'd. (R_t)	Hiring Cost (H_t)
1	100	500
2	150	500
3	75	800
4	140	800
5	125	800

   Formulate an appropriate DP recursion and boundary conditions. Define the stages, decision variables, and states clearly.

   Make sure your definitions are complete and clear. Do **not** solve.

5. (15 points) A company must meet the demand shown in the table below for a special product. The setup cost, unit variable cost and holding costs during each of the next four months are also given in the table. Also, production is instantaneous and takes place at the beginning of the month. At the beginning of month 1, there is one item in the inventory. Assume the storage and production capacity are infinite, and backorders are not allowed.

Months (n)	Demand (D_n)	Setup cost	Variable cost	Holding cost
1	3	3	1	1
2	1	3	1	1
3	1	3	2	1
4	2	3	2	1

Solve this dynamic lot sizing model using an Wagner-Whitin algorithm. Clearly express the optimal production plan and the total cost.

Show all your work.