CSE4088 Introduction to Machine Learning
Homework 3

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Part 1 - Gradient Descent

For this section, Gradient_Descent.py is implemented. Script can be run to observe results. Plotting is included.
Results after running Gradient_Descent.py:

```
$ py Gradient_Descent.py
Gradient Descent:
Number of iterations: 10
Final(u,v): (0.04473629039778207, 0.023958714099141746)
Error rate after completion: 1.2086833944220747e-15
Coordinate Descent:
Number of iterations: 15
Final(u,v): (6.29707589930517, -2.852306954077811)
Error rate after completion: 0.13981379199615315
```

4.

The partial derivative of E(u,v) with respect to $\frac{\partial E}{\partial u}$ is

$$\frac{\partial E}{\partial u} = 2 \cdot (u \cdot e^{v} - 2 \cdot v \cdot e^{-u}) \cdot (e^{v} + 2 \cdot v \cdot e^{-u})$$

The answer is E.

5. and 6.

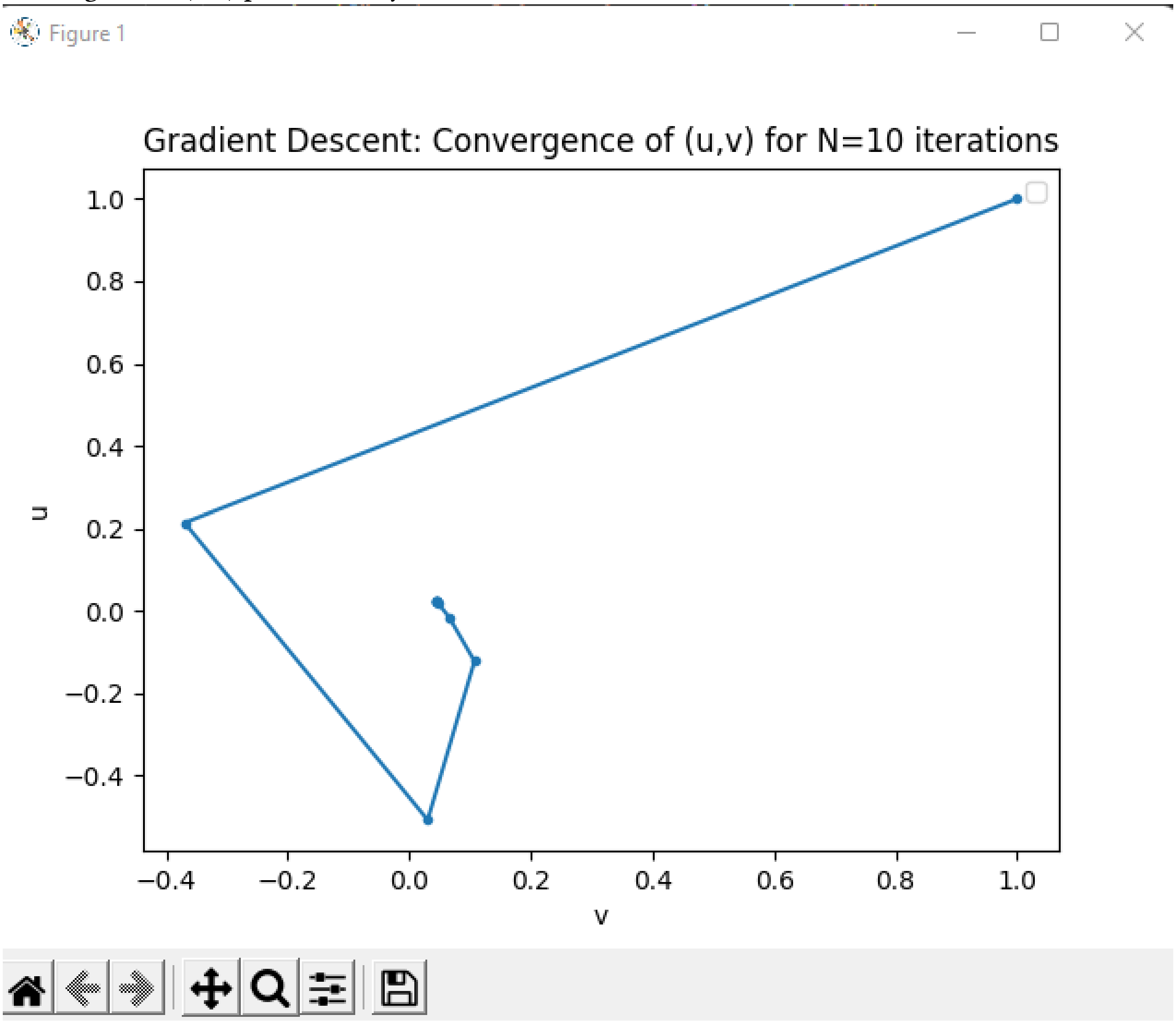
In this section we apply gradient descent algorithm to the nonlinear error space function by calculating the derivatives of squared residuals until error becomes less than $10 - 14$. Derivatives to apply gradient descent:

$$\frac{\partial E}{\partial u} = 2 \cdot (u \cdot e^{v} - 2 \cdot v \cdot e^{-u}) \cdot (e^{v} + 2 \cdot v \cdot e^{-u})$$

$$\frac{\partial E}{\partial v} = 2 \cdot (u \cdot e^{v} - 2 \cdot v \cdot e^{-u}) \cdot (u \cdot e^{v} - 2 \cdot e^{-u})$$

Number of iterations until error becomes 1.2086833944220747e-15 is 10.
The latest (u,v) pair is (0.04473629039778207, 0.023958714099141746)

Plotting of the (u,v) pair on every iteration:



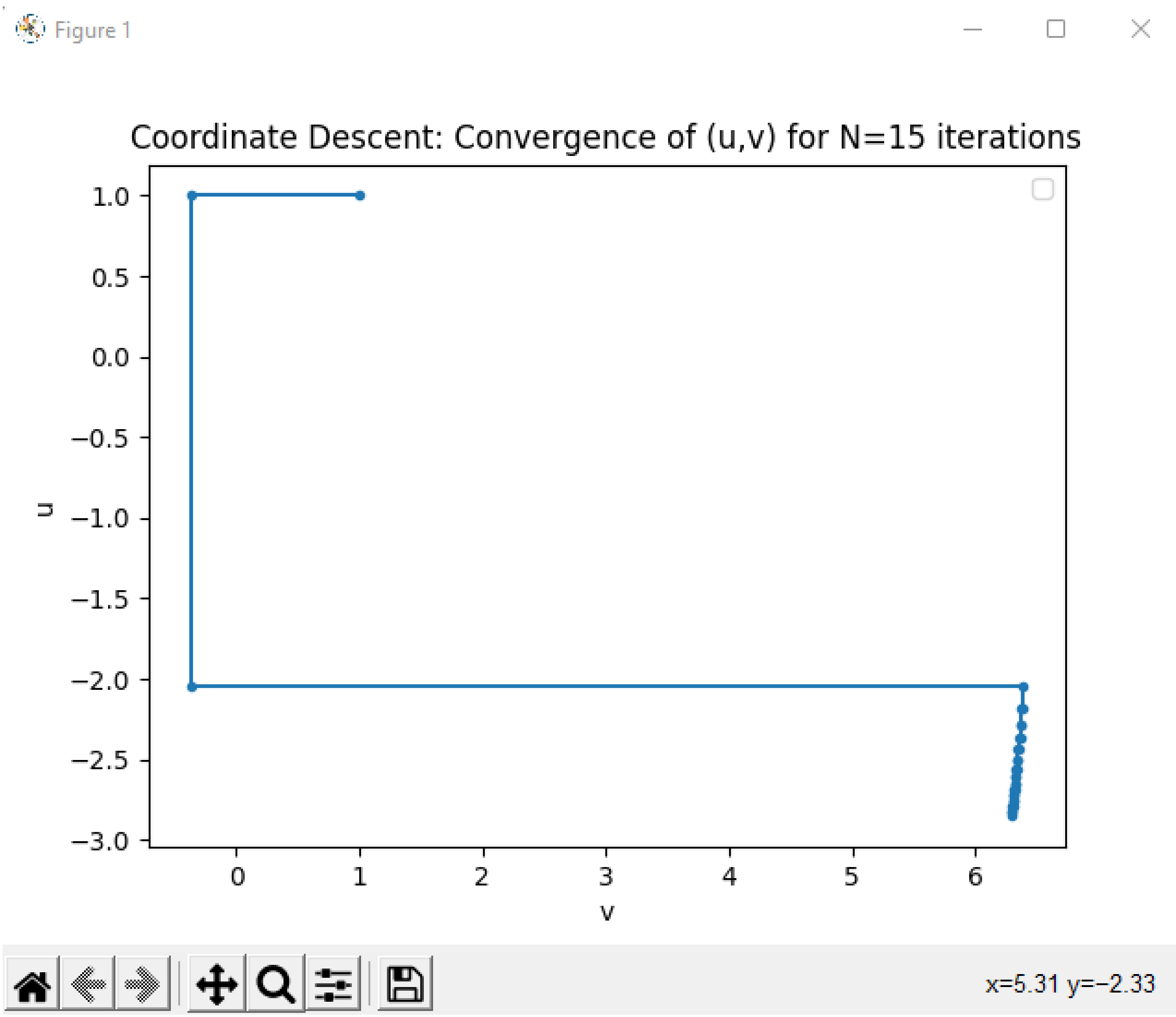
The answers are 5: D, and 6: E

7.

The error does not converge and stops at 0.13981379199615315 which is close to 10^-1

The latest (u,v) pair is (6.29707589930517, -2.852306954077811))

Plotting of the (u,v) pair on every iteration:



The answer is A.

Part 2 - Logistic Regression

For this section, Logistic_Regression.py is implemented. Script can be run to observe results. Plotting is included.
Results after running Logistic_Regression.py:

```
$ py Logistic_Regression.py
Eout: 0.09444365341119473, epochs: 338.58
```

The used formulas for the logistic regression algorithm are:

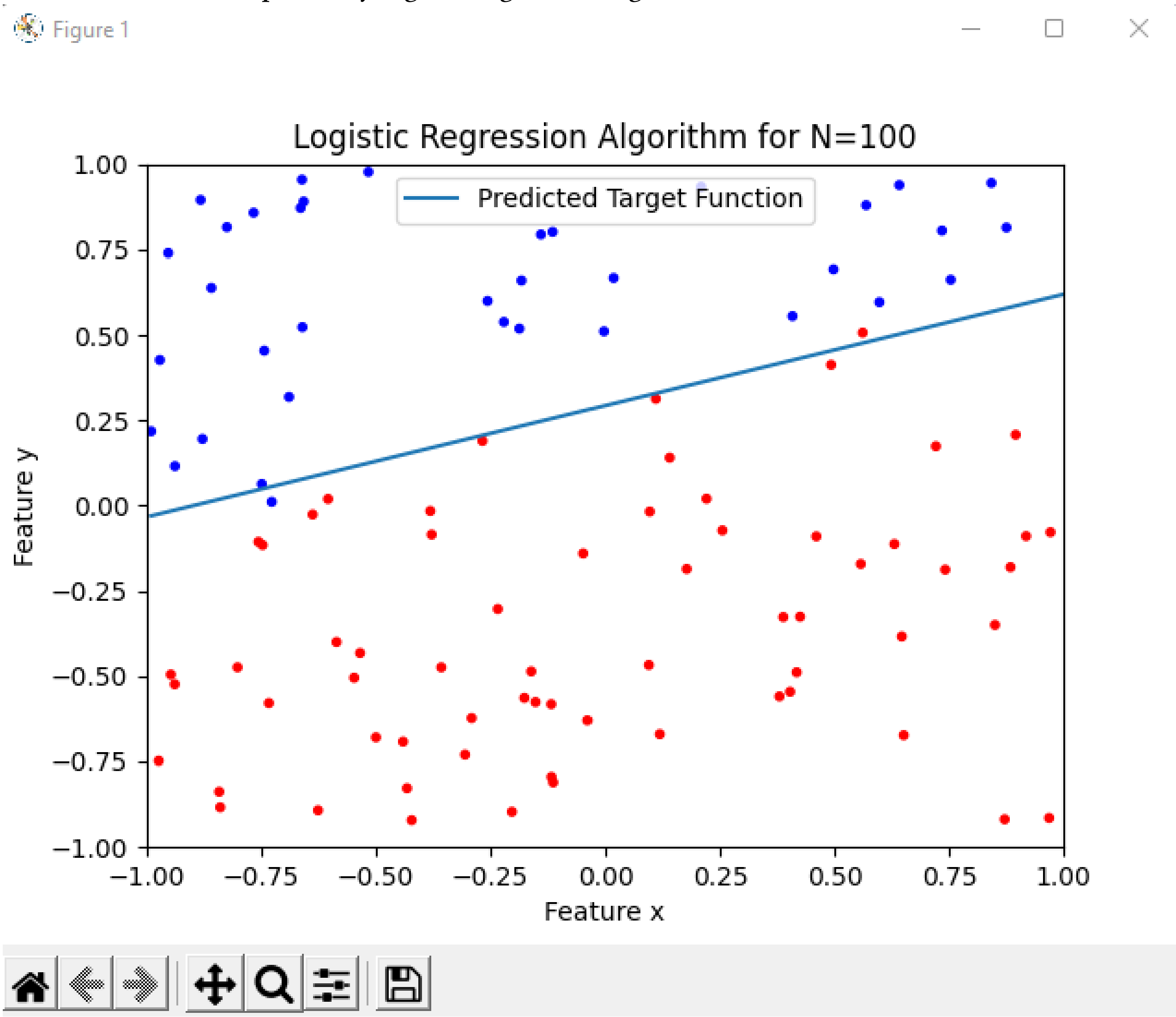
$$\text{Fixed step size: } w(1) = w(0) + n\nabla E_{in}$$

$$\text{v is calculated by stochastic gradient descent: } \nabla E_{in} = -\frac{1}{N}\sum_{n=1}^{N}\frac{y_n \cdot x_n}{ln(1 + e^{y_n \cdot w^T(t) \cdot x_n}})$$

$$\text{cross entropy error: } E_{out}(w) = \frac{1}{N}\sum_{n=1}^{N} ln(1 + e^{-y_n \cdot w^T \cdot x_n})$$

More details can be found in the source-code.

Predicted line on 100 points by logistic regression algorithm:



8.

I used the cross entropy error formula to calculate E_out.

Average Eout after 100 runs: 0.09444365341119473.
The answer is D.

9.

Average epochs after 100 runs: 338.58.
The answer is A.

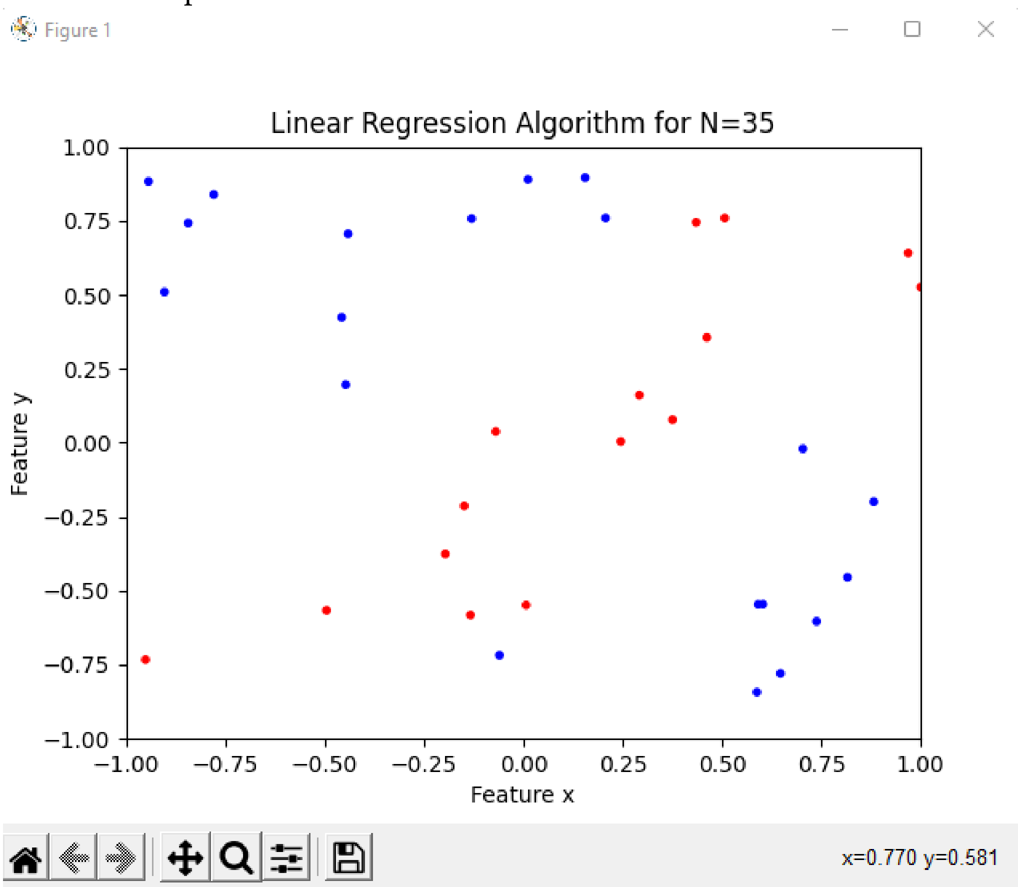
Part 3 - Regularization with weight decay

For this section, Linear_Regression.py is implemented. Script can be run to observe results. Some data scattering is included.
Results after running Linear_Regression.py:

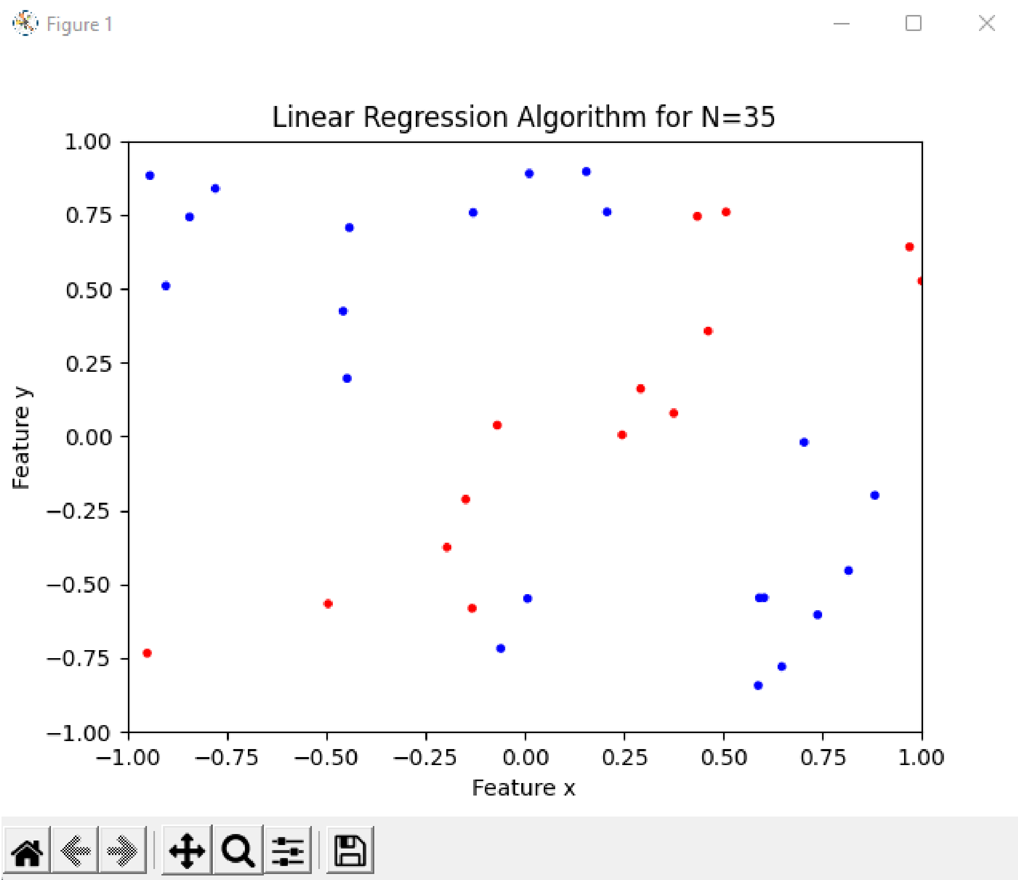
```
$ py Linear_Regression.py
Q2: Ein: 0.028571428571428571, Eout: 0.084
Q3: Ein: 0.028571428571428571, Eout: 0.08
Q4: Ein: 0.37142857142857144, Eout: 0.436
Q5- Q6: k: -1, Eout: 0.056
```

2.

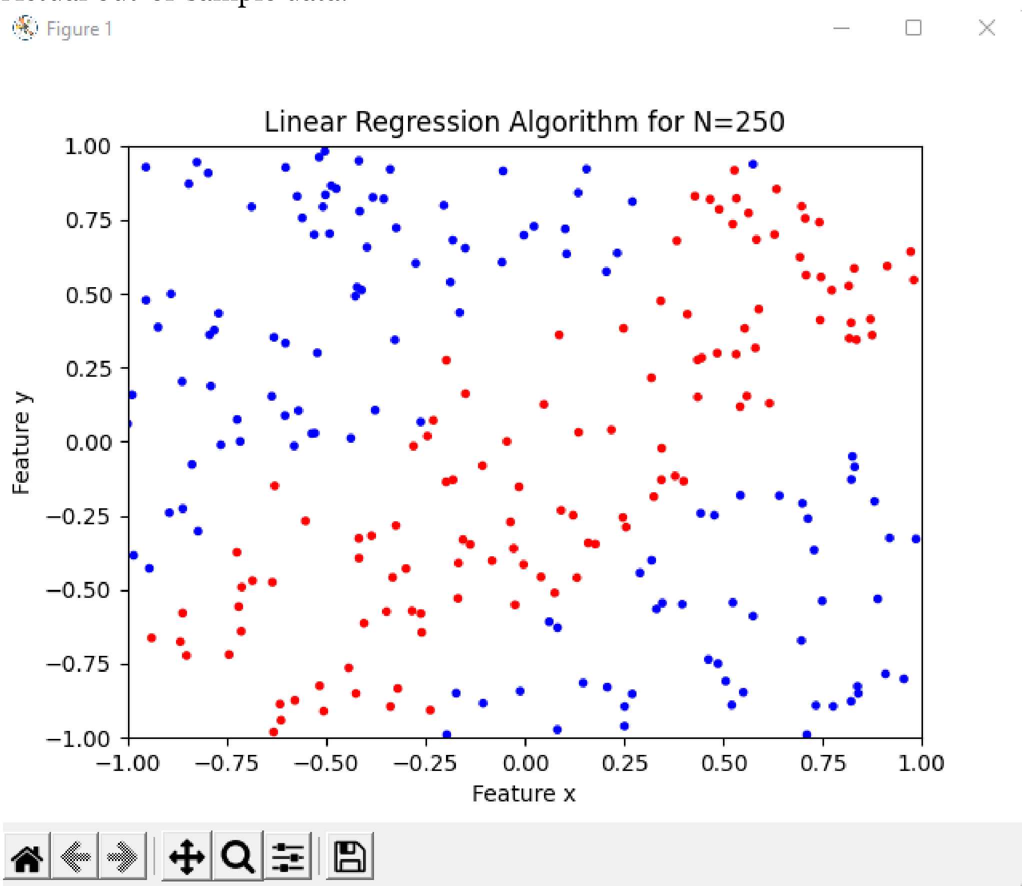
Actual in-sample data:



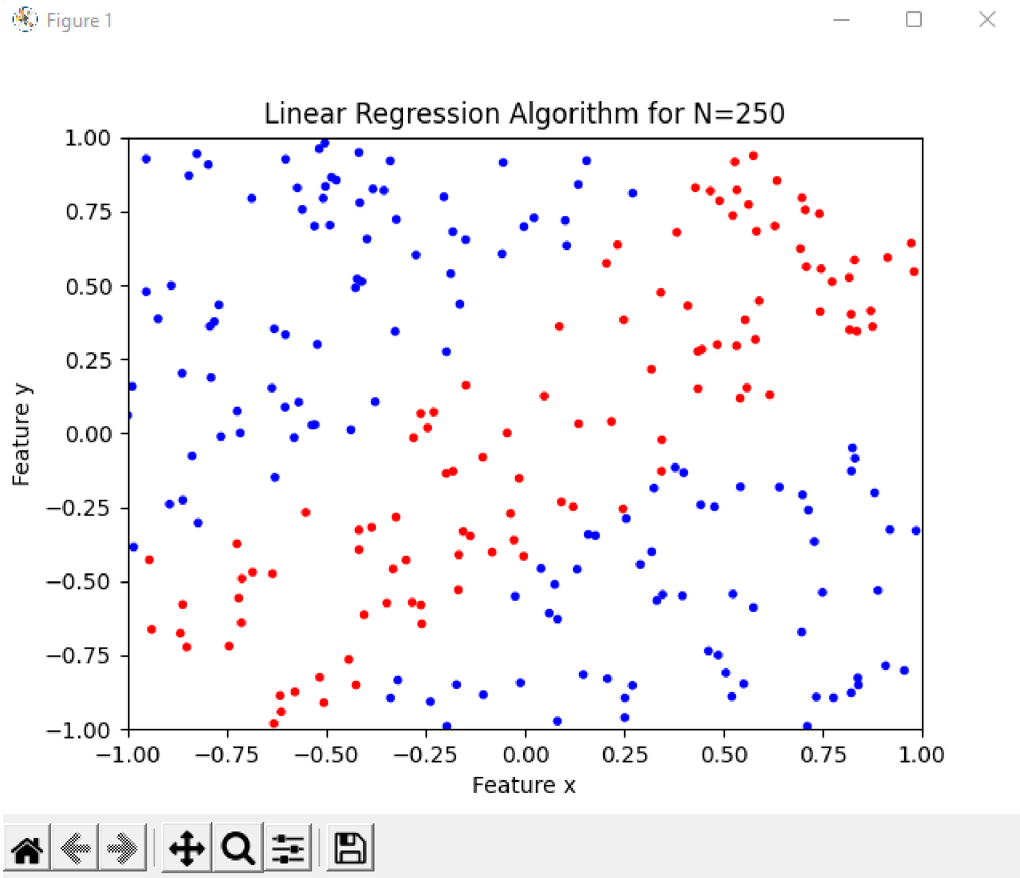
Predicted in-sample data:



Actual out-of-sample data:



Predicted out-of-sample data:



Ein: 0.028571428571428571, Eout: 0.084
The answer is A.

3.

0.028571428571428571, Eout: 0.08
The answer is D.

4.

Q4: Ein: 0.3714285714285714, Eout: 0.436
The answer is E.

5.

k = -1
The answer is D.

6.

Eout: 0.056
The answer is B.

Part 4 -Neural Networks

For this section, Neural_Network.py is implemented using the formulas below. Script can be run to observe results.
Neural Network operators:

$$w_{ij}^{(l)} = \left\{ \begin{array}{ll} 1 \leq l \leq L & \text{Layers} \\ 0 \leq i \leq d^{(l-1)} & \text{Inputs} \\ 1 \leq j \leq d^{(l)} & \text{Outputs} \end{array} \right\}$$

Activation function:

$$\frac{(e^s - e^{-s})}{(e^s + e^{-s})}$$

Forward propagation:

$$x_j^{(l)} = \theta(s_j^{(l)}) = \theta(\sum_{i=0}^{d^{(l-1)}} w_{ij}^{(l)} \cdot x_i^{(l-1)})$$

Backward propagation:

$$\text{For final layer: } \delta_1^{(L)} = 2 \cdot (x_1^{(L)} - y_n) \cdot (1 - \theta^2(s_1^{(L)}))$$

$$\text{For other layers: } \delta_i^{(l-1)} = (1 - (x_i^{(l-1)})^2) \cdot \sum_{j=1}^{d^{(l)}} w_{ij}^{(l)} \cdot \delta_j^{(l)}$$

$$\text{Updating perceptron weights: } w_{ij}^{(l)} \leftarrow w_{ij}^{(l)} - n \cdot x_i^{(l-1)} \cdot \delta_j^{(l)}$$

Results after running Neural_Network.py:

```
$ py Neural_Network.py
Q8: Total number of operations is: 28
Q9: Minimum number of weights is: 46
Q10: Maximum number of weights is: 522
```

8.

Total number of operations is: 28. More details can be observed from run_q8() function in the source-code
The answer is A.

9.

By setting 36 hidden layers with 1 weight we count all the weights in the neural network as 46. More details can be observed from run_q9() function in the source-code
The answer is A.

10.

By setting 2 hidden layers with weights of 18, 18 we count all the weights in the neural network as 522. More details can be observed from run_q10() function in the source-code.
The answer is E.