

IE-231 In-Class Activity Solutions - Week 4

Due Date Mar 7, 2017, 14:00

This is a graded in-class assignment. Show all your work in R Markdown files. Submit compiled Word files only.

Question 1

The local coffee shop has three kinds of coffee, Turkish, espresso and filter coffee. A customer orders Turkish coffee with probability 0.4, espresso 0.25 and filter coffee 0.35.

- a. What is the probability that at least three customers among first 10 customers order espresso or filter coffee?

```
## [1] 0.9877054
```

- b. What is the probability that the first espresso is ordered by the fourth customer or before?

```
## [1] 0.6835938
```

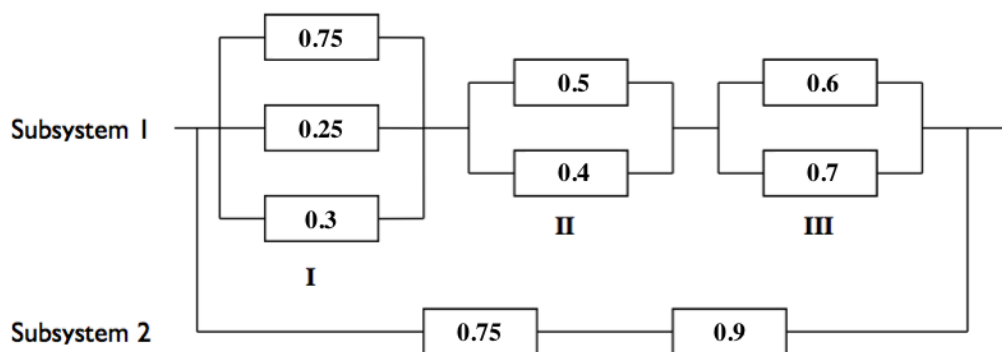
- c. The first 7 customers get a free cookie each day. What is the probability that at least three cookies are given to customers who order filter coffee?

```
## [1] 0.4677167
```

- d. If any type of coffee runs out, the remaining coffee types will be preferred proportionally (e.g. if espresso runs out Turkish coffee's probability will be $0.4/(0.75)$). Suppose, the coffee shop has only 1 cup of espresso left. What is the probability that 3 out of the first 5 customers will order filter coffee?

```
## [1] 0.2339599
```

Question 2



Consider the system above. Suppose the system works if either subsystem 1 or subsystem 2 works. Calculate the probability of the system working?

[1] 0.8489237

Question 3

A machine produces 15 items, 12 of which is non-defective. The items are randomly selected without replacement. The sixth selected item is found to be non-defective. What is the probability that this is the third non-defective one?

[1] 0.02197802

Question 4

A dice player rolls two dice.

- He wins if the sum is either 7 or 11.
- He loses if the sum is 2, 3 or 12.
- He repeats the roll if the sum is 4, 5, 6, 8, 9 or 10
 - Then repeats the roll until the initial sum is repeated
 - Loses if the sum is 7

What is $P(\text{Win})$? (Hint: $\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$ if $0 < a < 1$)

$$P(\text{Win}) = P(\text{Sum}_1 = 7) + P(\text{Sum}_1 = 11) + P(\text{Win}, \text{Sum}_1 = 4) + P(\text{Win}, \text{Sum}_1 = 5) + P(\text{Win}, \text{Sum}_1 = 6) + \\ P(\text{Win}, \text{Sum}_1 = 8) + P(\text{Win}, \text{Sum}_1 = 9) + P(\text{Win}, \text{Sum}_1 = 10)$$

$$P(\text{Sum} = 7) = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) = 6/36 = 1/6 \\ P(\text{Sum} = 11) = P(5, 6) + P(6, 5) = 2/36 = 1/18$$

$$P(\text{Win}, \text{Sum}_1 = 4) = P(\text{Sum}_1 = 4) * P(\text{Win} | \text{Sum}_1 = 4) \\ P(\text{Win} | \text{Sum}_1 = 4) = P(\text{Sum}_2 = 4) + P(\text{Sum}_2 \neq 4, 7) * P(\text{Win} | \text{Sum}_2 \neq 4, 7) \\ P(\text{Win} | \text{Sum}_2 \neq 4, 7) = P(\text{Sum}_3 = 4) + P(\text{Sum}_3 \neq 4, 7) * P(\text{Win} | \text{Sum}_3 \neq 4, 7) \\ P(\text{Win} | \text{Sum}_i \neq 4, 7) = P(\text{Sum}_{i+1} = 4) + P(\text{Sum}_{i+1} \neq 4, 7) * P(\text{Win} | \text{Sum}_{i+1} \neq 4, 7)$$

$$P(\text{Sum}_1 = 4) = P(1, 3) + P(2, 2) + P(3, 1) = 3/36 = 1/12 \\ P(\text{Sum}_1 \neq 4, 7) = 1 - 3/36 - 6/36 = 27/36 = 3/4 \\ P(\text{Win} | \text{Sum}_1 = 4) = 1/12 + 3/4 * (1/12 + 3/4 * (1/12 + \dots)) \\ P(\text{Win} | \text{Sum}_1 = 4) = 1/12 * (1 + 3/4 + (3/4)^2 + (3/4)^3 + \dots) \\ P(\text{Win} | \text{Sum}_1 = 4) = 1/12 * (1/(1 - 3/4)) = 1/3 \\ P(\text{Win}, \text{Sum}_1 = 4) = 1/12 * 1/3 = 1/36$$

Similarly for 5,6,8,9,10

$$P(\text{Win}) = 6/36 + 2/36 + 1/36 + 2/45 + 25/396 + 25/396 + 2/45 + 1/36 \\ = 0.4929293$$

```
##
##          2          3          4          5          6          7
## 0.02777778 0.05555556 0.08333333 0.11111111 0.13888889 0.16666667
##          8          9          10         11         12
## 0.13888889 0.11111111 0.08333333 0.05555556 0.02777778
##          7
## 0.4929293
```

Question 5

In a classroom of 22 students, what is the probability that none of them are born on the same day of the year? (ignore February 29)

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## [1] 0.5243047
```

```
## [1] 0.5243047
```