

Part 1)

First I have run the example system and property test to understand how tests and systems work in the MATLAB environment.

```
System 1 does not pass the property test for a = 0+5i and b = 0+10i
System 1 passes the property test for a = 0+5i and b = 1
fx >>
```

(Figure 1: Example Code's Output)

In this system, when b is not a real number, changing the order of multiplication and addition does not give the correct result. After understanding this example implementation, I have implemented five systems that I will experiment on.

I have implemented the systems in MATLAB, the code can be found at the end of this report. Before moving on to results, I believe it would be better to go through theoretical results for each system in terms of linearity, time-invariance, and causality.

Given Systems are,

a) $y[n] = nx[n + 2]$, **b)** $y[n] = x[2n]$

c) $y[n] = 3x[n + 4] + 5$, **d)** $y[n] = |x[n]|$

e) $y[n] = \sum_{k=1}^n x[k]$

For linearity,

If sequence 1 is $x_1[n]$, and sequence 2 is $x_2[n]$, the outputs of the system are $y_1[n]$ and $y_2[n]$ respectively. Let α and β be integers, if

$$\alpha y_1[n] + \beta y_2[n] = y[n] = \alpha x_1[n] + \beta x_2[n]$$

the system is said to be linear where $y[n]$ is output for the left-hand side sum of input sequences.

By applying this test, it can be concluded that systems a, b and e are linear since both sides of the equation give the same result. Again by using the same tests it can be said system d is not linear since for $\alpha = 1$, $\beta = -1$,

$$1 * |x_1[n]| - 1|x_2[n]| \neq |1 * x_1[n]| + |-1 * x_2[n]|$$

Similarly, system c is not linear since for $\alpha = 2$, $\beta = 2$

$$2 * (3x_1[n + 4] + 5) + 2 * (3x_2[n + 4] + 5) \neq 2 * 3x_1[n + 4] + 5 + 2 * 3x_2[n + 4] + 5$$

For time invariance

If sequence 1 is $x_1[n]$, and sequence 2 is $x_2[n]$, the outputs of the system are $y_1[n]$ and $y_2[n]$ respectively. Let n_0 be an integer delay amount, if $y[n - n_0] = w[n]$ where $w[n]$ is the output of the system for the input $x[n - n_0]$, system is said to be time-invariant. As a result of this test, systems c, d and e are time-invariant. However, systems a and b are time-variant.

For causality, a system is said to be causal if the system does not use any future values. By looking at the systems, it can be concluded that systems a, b and c are not causal. Since for $n = 2$, system a needs the value of $n = 4$. Similarly, for $n = 2$, system b needs the value of $n = 4$, and system c needs $n=6$ for $n = 2$. Similarly, it can be said that systems d and e are causal since do not use any future values.

Part 2)

For this part, I have run the test for 100 combinations of α and β for each system and each input set.

System a passed the linearity test for every combination with each of two input sets.

```

----- TESTS WITH INPUT SET #1 -----
----- Test for System a: -----
System a passes linearity test for alpha= -5, and beta= -5
System a passes linearity test for alpha= -5, and beta= -4
System a passes linearity test for alpha= -5, and beta= -3
System a passes linearity test for alpha= -5, and beta= -2
System a passes linearity test for alpha= -5, and beta= -1
System a passes linearity test for alpha= -5, and beta= 1
System a passes linearity test for alpha= -5, and beta= 2
System a passes linearity test for alpha= -5, and beta= 3
System a passes linearity test for alpha= -5, and beta= 4
System a passes linearity test for alpha= -5, and beta= 5

```

(Figure 2: Linearity test results for System a with input set 1)

```

----- TESTS WITH INPUT SET #2 -----
----- Test for System a: -----
System a passes linearity test for alpha= -5, and beta= -5
System a passes linearity test for alpha= -5, and beta= -4
System a passes linearity test for alpha= -5, and beta= -3
System a passes linearity test for alpha= -5, and beta= -2
System a passes linearity test for alpha= -5, and beta= -1
System a passes linearity test for alpha= -5, and beta= 1
System a passes linearity test for alpha= -5, and beta= 2
System a passes linearity test for alpha= -5, and beta= 3
System a passes linearity test for alpha= -5, and beta= 4
System a passes linearity test for alpha= -5, and beta= 5

```

(Figure 3: Linearity test results for System a with input set 2)

Since system a passed the tests for every value, it can be concluded that System a is linear. This also complies with the theoretical analysis in Part 1 of this report.

System b also passed each test in this part for each of the input sets.

```
----- Test for System b: -----  
System b passes linearity test for alpha= -5, and beta= -5  
System b passes linearity test for alpha= -5, and beta= -4  
System b passes linearity test for alpha= -5, and beta= -3  
System b passes linearity test for alpha= -5, and beta= -2  
System b passes linearity test for alpha= -5, and beta= -1  
System b passes linearity test for alpha= -5, and beta= 1  
System b passes linearity test for alpha= -5, and beta= 2  
System b passes linearity test for alpha= -5, and beta= 3  
System b passes linearity test for alpha= -5, and beta= 4  
System b passes linearity test for alpha= -5, and beta= 5  
System b passes linearity test for alpha= -4, and beta= -5  
System b passes linearity test for alpha= -4, and beta= -4  
System b passes linearity test for alpha= -4, and beta= -3
```

(Figure 4: Linearity test results for System b with input set 1)

```
----- Test for System b: -----  
System b passes linearity test for alpha= -5, and beta= -5  
System b passes linearity test for alpha= -5, and beta= -4  
System b passes linearity test for alpha= -5, and beta= -3  
System b passes linearity test for alpha= -5, and beta= -2  
System b passes linearity test for alpha= -5, and beta= -1  
System b passes linearity test for alpha= -5, and beta= 1  
System b passes linearity test for alpha= -5, and beta= 2  
System b passes linearity test for alpha= -5, and beta= 3  
System b passes linearity test for alpha= -5, and beta= 4  
System b passes linearity test for alpha= -5, and beta= 5
```

(Figure 5: Linearity test results for System b with input set 2)

Again for system b, it can be concluded that the system is linear since both experimental results support the theoretical analysis.

Unlike first two systems, system c could not pass all the tests but for each input set it only passed the test for alpha-beta value pairs of $\{-4, 5\}$, $\{-3, 4\}$, $\{-2, 3\}$, $\{-1, 2\}$, $\{2, -1\}$, $\{3, -2\}$, $\{4, -3\}$ and $\{5, -4\}$.

```

----- Test for System c: -----
System c does not pass linearity test for alpha= -5, and beta= -5
System c does not pass linearity test for alpha= -5, and beta= -4
System c does not pass linearity test for alpha= -5, and beta= -3
System c does not pass linearity test for alpha= -5, and beta= -2
System c does not pass linearity test for alpha= -5, and beta= -1
System c does not pass linearity test for alpha= -5, and beta= 1
System c does not pass linearity test for alpha= -5, and beta= 2
System c does not pass linearity test for alpha= -5, and beta= 3
System c does not pass linearity test for alpha= -5, and beta= 4
System c does not pass linearity test for alpha= -5, and beta= 5
System c does not pass linearity test for alpha= -4, and beta= -5
System c does not pass linearity test for alpha= -4, and beta= -4
System c does not pass linearity test for alpha= -4, and beta= -3
System c does not pass linearity test for alpha= -4, and beta= -2
System c does not pass linearity test for alpha= -4, and beta= -1
System c does not pass linearity test for alpha= -4, and beta= 1
System c does not pass linearity test for alpha= -4, and beta= 2
System c does not pass linearity test for alpha= -4, and beta= 3
System c does not pass linearity test for alpha= -4, and beta= 4
System c passes linearity test for alpha= -4, and beta= 5

```

(Figure 6: Linearity test results for System c with input set 1)

```

----- Test for System c: -----
System c does not pass linearity test for alpha= -5, and beta= -5
System c does not pass linearity test for alpha= -5, and beta= -4
System c does not pass linearity test for alpha= -5, and beta= -3
System c does not pass linearity test for alpha= -5, and beta= -2
System c does not pass linearity test for alpha= -5, and beta= -1
System c does not pass linearity test for alpha= -5, and beta= 1
System c does not pass linearity test for alpha= -5, and beta= 2
System c does not pass linearity test for alpha= -5, and beta= 3
System c does not pass linearity test for alpha= -5, and beta= 4
System c does not pass linearity test for alpha= -5, and beta= 5
System c does not pass linearity test for alpha= -4, and beta= -5
System c does not pass linearity test for alpha= -4, and beta= -4
System c does not pass linearity test for alpha= -4, and beta= -3
System c does not pass linearity test for alpha= -4, and beta= -2
System c does not pass linearity test for alpha= -4, and beta= -1
System c does not pass linearity test for alpha= -4, and beta= 1
System c does not pass linearity test for alpha= -4, and beta= 2
System c does not pass linearity test for alpha= -4, and beta= 3
System c does not pass linearity test for alpha= -4, and beta= 4
System c passes linearity test for alpha= -4, and beta= 5

```

(Figure 7: Linearity test results for System c with input set 2)

Since this system could not pass the test for each combination, it can be said that experimental results support theoretical analysis and system c is not a linear system.

System d does not pass any of the tests for each input set, hence it can be concluded that system d is not linear as it was found in the theory part.

```
----- Test for System d: -----  
System d does not pass linearity test for alpha= -5, and beta= -5  
System d does not pass linearity test for alpha= -5, and beta= -4  
System d does not pass linearity test for alpha= -5, and beta= -3  
System d does not pass linearity test for alpha= -5, and beta= -2  
System d does not pass linearity test for alpha= -5, and beta= -1
```

(Figure 8: Linearity test results for System d with input set 1)

As it was found in the theoretical part, system e passes the linearity test for each combination.

```
----- Test for System e: -----  
System e passes linearity test for alpha= -5, and beta= -5  
System e passes linearity test for alpha= -5, and beta= -4  
System e passes linearity test for alpha= -5, and beta= -3  
System e passes linearity test for alpha= -5, and beta= -2  
System e passes linearity test for alpha= -5, and beta= -1  
System e passes linearity test for alpha= -5, and beta= 1  
System e passes linearity test for alpha= -5, and beta= 2
```

(Figure 9: Linearity test results for System e with input set 1)

To conclude, as in discussed in part 1, systems a, b and e are linear whereas c and d not. The complete output for this part can be found in the output of tests part of this report.

Part 3)

For this part I have extended input sequences as much as the range of delay interval, that is, I have appended 50 zeros to both sides of the input sequences. After that I have run the test procedure 100 times for each system.

System a could not pass the tests for any of the delay values. As it was concluded in the theoretical part. Therefore System a is time variant.

```
----- Test for System a: -----  
System a does not pass time invariance test for n0= -50  
System a does not pass time invariance test for n0= -49  
System a does not pass time invariance test for n0= -48  
System a does not pass time invariance test for n0= -47  
System a does not pass time invariance test for n0= -46  
System a does not pass time invariance test for n0= -45  
System a does not pass time invariance test for n0= -44  
System a does not pass time invariance test for n0= -43
```

(Figure 10: Time invariance test results for System a with input set 1)

System b also could not pass the tests for any of the delay values and any of the input sequences. This result complies with the theoretical analysis, hence system b is time variant.

```
----- Test for System b: -----  
System b does not pass time invariance test for n0= -50  
System b does not pass time invariance test for n0= -49  
System b does not pass time invariance test for n0= -48  
System b does not pass time invariance test for n0= -47  
System b does not pass time invariance test for n0= -46  
System b does not pass time invariance test for n0= -45
```

(Figure 11: Time invariance test results for System b with input set 1)

For system c, while delaying I have shifted and added 5s instead of 0s since this system never gets 0 because of the system was shifted 5 in the y axis. Moreover, system c passed all of the tests. This result also complies with theoretical values and it can be concluded that system c is time invariant.

```
----- Test for System c: -----  
System c passes time invariance test for n0= -50  
System c passes time invariance test for n0= -49  
System c passes time invariance test for n0= -48  
System c passes time invariance test for n0= -47  
System c passes time invariance test for n0= -46  
System c passes time invariance test for n0= -45  
System c passes time invariance test for n0= -44  
System c passes time invariance test for n0= -43
```

(Figure 12: Time invariance test results for System c with input set 1)

System d also passed all of the tests and it can be concluded system d is time invariant and results also complies with theory.

```
----- Test for System d: -----  
System d passes time invariance test for n0= -50  
System d passes time invariance test for n0= -49  
System d passes time invariance test for n0= -48  
System d passes time invariance test for n0= -47  
System d passes time invariance test for n0= -46  
System d passes time invariance test for n0= -45  
System d passes time invariance test for n0= -44  
System d passes time invariance test for n0= -43
```

(Figure 13: Time invariance test results for System d with input set 1)

For system e when the delay values are negative, I have added the last element of the system while shifting instead of 0. Moreover, system e also passed all the tests and it is time invariant.


```

----- Test for System e: -----
System e passes time invariance test for n0= -50
System e passes time invariance test for n0= -49
System e passes time invariance test for n0= -48
System e passes time invariance test for n0= -47
System e passes time invariance test for n0= -46
System e passes time invariance test for n0= -45
System e passes time invariance test for n0= -44
System e passes time invariance test for n0= -43

```

(Figure 14: Time invariance test results for System e with input set 1)

Again full version of the output can be found in Output of Tests part of the report

Part 4)

For this part, I have tested each of the systems index by index. For one system I have kept the future values, and for the other I have made future values 0 to see if the future values have effect on the current value. Moreover, I have not tested the systems for final element since it would pass the system anyway since there is no future values left.

System a only passed the tests for index 2 and 9 but failed others. As I indicated in part 1, the system is not causal.

```

----- Test for System a: -----
System a does not pass causality test for index 1
System a passes causality test for index 2
System a does not pass causality test for index 3
System a does not pass causality test for index 4
System a does not pass causality test for index 5
System a does not pass causality test for index 6
System a does not pass causality test for index 7
System a does not pass causality test for index 8
System a passes causality test for index 9

```

(Figure 15: Causality test results for System a)

System b passed the tests for 4 indices but it failed 5 of them. As concluded in theoretical part, this system also is not causal.

```

----- Test for System b: -----
System b does not pass causality test for index 1
System b passes causality test for index 2
System b does not pass causality test for index 3
System b does not pass causality test for index 4
System b does not pass causality test for index 5
System b passes causality test for index 6
System b passes causality test for index 7
System b passes causality test for index 8
System b passes causality test for index 9
-----

```

(Figure 16: Causality test results for System b)

System c only passed the last 3 tests and failed first 6, hence this system is also not causal.

```
----- Test for System c: -----
System c does not pass causality test for index 1
System c does not pass causality test for index 2
System c does not pass causality test for index 3
System c does not pass causality test for index 4
System c does not pass causality test for index 5
System c does not pass causality test for index 6
System c passes causality test for index 7
System c passes causality test for index 8
System c passes causality test for index 9
-----
```

(Figure 17: Causality test results for System c)

As discussed in the theoretical analysis system d does not use any future values, and tests also proved this. System d passed all of the tests and it is causal.

```
----- Test for System d: -----
System d passes causality test for index 1
System d passes causality test for index 2
System d passes causality test for index 3
System d passes causality test for index 4
System d passes causality test for index 5
System d passes causality test for index 6
System d passes causality test for index 7
System d passes causality test for index 8
System d passes causality test for index 9
-----
```

(Figure 18: Causality test results for System d)

Similarly system e passed all of the tests and it is proven that system e is causal too.

```
----- Test for System e: -----
System e passes causality test for index 1
System e passes causality test for index 2
System e passes causality test for index 3
System e passes causality test for index 4
System e passes causality test for index 5
System e passes causality test for index 6
System e passes causality test for index 7
System e passes causality test for index 8
System e passes causality test for index 9
-----
```

(Figure 19: Causality test results for System e)

MATLAB Code:**Code for System (a) : $y[n] = nx[n + 2]$**

```
%% Author: Berk Saltuk Yılmaz
%%% System a

function [ output_args ] = system_a( input_args, mult )

%% Get an input signal and advance it by 2 samples
output_args = input_args;
advance = 2;

output_args(1 : end-advance) = input_args(advance + 1: end);
output_args(end - advance + 1: end) = zeros(1, advance);

output_args = mult.*output_args; %% Multiply the system with n

end
```

Code for System (b) : $y[n] = x[2n]$

```
%% Author: Berk Saltuk Yılmaz
%%% System b

function [ output_args ] = system_b( input_args, len)

output_args = input_args;

output_args(1 : end/2) = input_args(2:2:len);
output_args((end/2)+1:end) = zeros(1, len/2);

end
```

Code for System (c) : $y[n] = 3x[n + 4] + 5$

```
%% Author: Berk Saltuk Yılmaz
%%% System c

function [ output_args ] = system_c( input_args)

%% Get an input signal and advance it by 2 samples
output_args = input_args;
advance = 4;

output_args(1 : end-advance) = input_args(advance + 1: end);
output_args(end - advance + 1: end) = zeros(1, advance);

output_args = 3.*output_args + 5; %% Multiply the system with n

end
```

Code for System (d) : $y[n] = |x[n]|$

```
%% Author: Berk Saltuk Yılmaz
%%% System d

function [ output_args ] = system_d( input_args)

output_args = abs(input_args);
```

```
end
```

Code for System (e) : $y[n] = \sum_{k=1}^n x[k]$

```
%% Author: Berk Saltuk Yılmaz
%%% System e
function [ output_args ] = system_e( input_args, n)

output_args = input_args;

sum = 0;
for index = n
    for inner = 1:index
        sum = sum + input_args(inner);
    end
    output_args(index) = sum;
    sum = 0;
end

end
```

Code for Linearity Test (Part 2)

```
%% Author: Berk Saltuk Yılmaz
%%% Code for testing linearity
clc;
clear;

%%% Input index is same with the example test
n=1:10;

%%% Constants
alpha = -5:5;
beta = -5:5;

fprintf("----- TESTS WITH INPUT SET #1 ----- \n");

%%% Test first run
% Input set #1
x1 = [-3 -4 2 0 1 4 3 6 3 7];
x2 = [2 5 -1 7 -3 6 12 -9 8 -4];

fprintf("----- Test for System a: ----- \n");

% Test for System a
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_a = system_a(x1, n);
            y2_a = system_a(x2, n);
            w_a = ind.*y1_a + jind.*y2_a;

            x_a = ind.*x1 + jind.*x2;
            y_a = system_a(x_a, n);
            if(isequal(w_a,y_a))
                fprintf('System a passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System a does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end
```

```

        end
    end
end

fprintf("-----\n");

fprintf("----- Test for System b: -----\n");
% Test for System b
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_b = system_b(x1, 10);
            y2_b = system_b(x2, 10);
            w_b = ind.*y1_b + jind.*y2_b;

            x_b = ind.*x1 + jind.*x2;
            y_b = system_b(x_b,10);
            if(isequal(w_b,y_b))
                fprintf('System b passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System b does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end

fprintf("-----\n");

fprintf("----- Test for System c: -----\n");
% Test for System c
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_c = system_c(x1);
            y2_c = system_c(x2);
            w_c = ind.*y1_c + jind.*y2_c;

            x_c = ind.*x1 + jind.*x2;
            y_c = system_c(x_c);
            if(isequal(w_c,y_c))
                fprintf('System c passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System c does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end

fprintf("-----\n");

fprintf("----- Test for System d: -----\n");
% Test for System d
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_d = system_d(x1);
            y2_d = system_d(x2);
            w_d = ind.*y1_d + jind.*y2_d;

            x_d = ind.*x1 + jind.*x2;
            y_d = system_d(x_d);

```

```

        if(isequal(w_d,y_d))
            fprintf('System d passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
        else
            fprintf('System d does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
        end
    end
end

fprintf("-----\n");

fprintf("----- Test for System e: -----\n");
% Test for System e
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_e = system_e(x1, n);
            y2_e = system_e(x2, n);
            w_e = ind.*y1_e + jind.*y2_e;

            x_e = ind.*x1 + jind.*x2;
            y_e = system_e(x_e, n);
            if(isequal(w_e,y_e))
                fprintf('System e passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System e does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end

fprintf("-----\n");

fprintf("----- TESTS WITH INPUT SET #2 -----\n");
% Tests second run
% Input set #2
x1 = [9 23 15 -10 18 14 5 -1 6 11];
x2 = [2 5 -1 7 -3 6 12 -9 8 -4];

fprintf("----- Test for System a: -----\n");

% Test for System a
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_a = system_a(x1, n);
            y2_a = system_a(x2, n);
            w_a = ind.*y1_a + jind.*y2_a;

            x_a = ind.*x1 + jind.*x2;
            y_a = system_a(x_a, n);
            if(isequal(w_a,y_a))
                fprintf('System a passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System a does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end
end

```

```
fprintf("-----\n");

fprintf("----- Test for System b: -----\n");
% Test for System b
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_b = system_b(x1, 10);
            y2_b = system_b(x2, 10);
            w_b = ind.*y1_b + jind.*y2_b;

            x_b = ind.*x1 + jind.*x2;
            y_b = system_b(x_b, 10);
            if(isequal(w_b,y_b))
                fprintf('System b passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System b does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end

fprintf("-----\n");

fprintf("----- Test for System c: -----\n");
% Test for System c
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_c = system_c(x1);
            y2_c = system_c(x2);
            w_c = ind.*y1_c + jind.*y2_c;

            x_c = ind.*x1 + jind.*x2;
            y_c = system_c(x_c);
            if(isequal(w_c,y_c))
                fprintf('System c passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System c does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end

fprintf("-----\n");

fprintf("----- Test for System d: -----\n");
% Test for System d
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_d = system_d(x1);
            y2_d = system_d(x2);
            w_d = ind.*y1_d + jind.*y2_d;

            x_d = ind.*x1 + jind.*x2;
            y_d = system_d(x_d);
            if(isequal(w_d,y_d))
                fprintf('System d passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System d does not pass linearity test for alpha= %d, and beta= %d\n', ind,
```

```

jind);
    end
    end
end

fprintf("-----\n");

fprintf("----- Test for System e: -----\n");
% Test for System e
for ind = alpha
    for jind = beta
        if( ind ~= 0 && jind ~= 0)
            y1_e = system_e(x1, n);
            y2_e = system_e(x2, n);
            w_e = ind.*y1_e + jind.*y2_e;

            x_e = ind.*x1 + jind.*x2;
            y_e = system_e(x_e, n);
            if(isequal(w_e,y_e))
                fprintf('System e passes linearity test for alpha= %d, and beta= %d\n', ind, jind);
            else
                fprintf('System e does not pass linearity test for alpha= %d, and beta= %d\n', ind,
jind);
            end
        end
    end
end
end

```

Code for Time Invariance Test (Part 3)

```

%% Author: Berk Saltuk Yılmaz
%%% Code for testing time invariance
clc;
clear;

n0 = -50:50; % Delay gets the values from -50 to 50
%%% Test sequence #1
fprintf("----- TESTS WITH INPUT #1 ----- \n");
x = [-3 -4 2 0 1 4 3 6 3 7];
x_extended(1:50) = zeros(1,50);
x_extended(51:60) = x;
x_extended(61:110) = zeros(1,50);
x = x_extended;
n = 1:110;
fprintf("----- Test for System a: ----- \n");
% Test for a
for ind = n0
    if(ind ~= 0)
        y_a = system_a(x, n);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_a_delayed = zeros(1, length(y_a));
            y_a_delayed(ind+1: length(y_a)) = y_a(1:length(y_a)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));

            y_a_delayed = zeros(1, length(y_a));
            y_a_delayed(1:length(y_a) + ind) = y_a(-ind+1:length(y_a));
        end
    end
end

```



```

end
w_a = system_a(x_delayed, n);
if(isequal(w_a,y_a_delayed))
    fprintf('System a passes time invariance test for n0= %d\n' ,ind);
else
    fprintf('System a does not pass time invariance test for n0= %d\n', ind);
end
end
end
fprintf("-----\n");

fprintf("----- Test for System b: -----\n");
% Test for b
for ind = n0
    if(ind ~= 0)
        y_b = system_b(x, 110);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_b_delayed = zeros(1, length(y_b));
            y_b_delayed(ind+1: length(y_b)) = y_b(1:length(y_b)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_b_delayed = zeros(1, length(y_b));
            y_b_delayed(1:length(y_b) + ind) = y_b(-ind+1:length(y_b));
        end
        w_b = system_b(x_delayed, 110);
        if(isequal(w_b,y_b_delayed))
            fprintf('System b passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System b does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
end
fprintf("-----\n");

fprintf("----- Test for System c: -----\n");
% Test for c
for ind = n0
    if(ind ~= 0)
        y_c = system_c(x);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_c_delayed = 5.*ones(1, length(y_c));
            y_c_delayed(ind+1: length(y_c)) = y_c(1:length(y_c)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_c_delayed = 5.*ones(1, length(y_c));
            y_c_delayed(1:length(y_c) + ind) = y_c(-ind+1:length(y_c));
        end
        w_c = system_c(x_delayed);
        if(isequal(w_c,y_c_delayed))
            fprintf('System c passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System c does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
end

```

```

fprintf("-----\n");

fprintf("----- Test for System d: -----\n");
% Test for d
for ind = n0
    if(ind ~= 0)
        y_d = system_d(x);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_d_delayed = zeros(1, length(y_d));
            y_d_delayed(ind+1: length(y_d)) = y_d(1:length(y_d)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_d_delayed = zeros(1, length(y_d));
            y_d_delayed(1:length(y_d) + ind) = y_d(-ind+1:length(y_d));
        end
        w_d = system_d(x_delayed);
        if(isequal(w_d,y_d_delayed))
            fprintf('System d passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System d does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
fprintf("-----\n");

fprintf("----- Test for System e: -----\n");
% Test for e
for ind = n0
    if(ind ~= 0)
        y_e = system_e(x, n);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_e_delayed = zeros(1, length(y_e));
            y_e_delayed(ind+1: length(y_e)) = y_e(1:length(y_e)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_e_delayed = y_e(end-1).*ones(1, length(y_e));
            y_e_delayed(1:length(y_e) + ind) = y_e(-ind+1:length(y_e));
        end
        w_e = system_e(x_delayed, n);
        if(isequal(w_e,y_e_delayed))
            fprintf('System e passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System e does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
fprintf("-----\n");

fprintf("----- TESTS WITH INPUT #2 -----\n");
%%% Test sequence #2
x = [5 -11 9 2 3 -6 0 -7 3 10];
x_extended(1:50) = zeros(1,50);
x_extended(51:60) = x;
x_extended(61:110) = zeros(1,50);
x = x_extended;

```

```

fprintf("----- Test for System a: -----\n");
for ind = n0
    if(ind ~= 0)
        y_a = system_a(x, n);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_a_delayed = zeros(1, length(y_a));
            y_a_delayed(ind+1: length(y_a)) = y_a(1:length(y_a)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_a_delayed = zeros(1, length(y_a));
            y_a_delayed(1:length(y_a) + ind) = y_a(-ind+1:length(y_a));
        end
        w_a = system_a(x_delayed, n);
        if(isequal(w_a,y_a_delayed))
            fprintf('System a passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System a does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
fprintf("-----\n");

fprintf("----- Test for System b: -----\n");
for ind = n0
    if(ind ~= 0)
        y_b = system_b(x, 110);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_b_delayed = zeros(1, length(y_b));
            y_b_delayed(ind+1: length(y_b)) = y_b(1:length(y_b)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_b_delayed = zeros(1, length(y_b));
            y_b_delayed(1:length(y_b) + ind) = y_b(-ind+1:length(y_b));
        end
        w_b = system_b(x_delayed, 110);
        if(isequal(w_b,y_b_delayed))
            fprintf('System b passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System b does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
fprintf("-----\n");

fprintf("----- Test for System c: -----\n");
for ind = n0
    if(ind ~= 0)
        y_c = system_c(x);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_c_delayed = 5.*ones(1, length(y_c));
            y_c_delayed(ind+1: length(y_c)) = y_c(1:length(y_c)-ind);
        else

```

```

        x_delayed = zeros(1,length(x));
        x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
        y_c_delayed = 5.*ones(1, length(y_c));
        y_c_delayed(1:length(y_c) + ind) = y_c(-ind+1:length(y_c));
    end
    w_c = system_c(x_delayed);
    if(isequal(w_c,y_c_delayed))
        fprintf('System c passes time invariance test for n0= %d\n' ,ind);
    else
        fprintf('System c does not pass time invariance test for n0= %d\n', ind);
    end
end
fprintf("-----\n");

fprintf("----- Test for System d: -----\n");
for ind = n0
    if(ind ~= 0)
        y_d = system_d(x);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_d_delayed = zeros(1, length(y_d));
            y_d_delayed(ind+1: length(y_d)) = y_d(1:length(y_d)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_d_delayed = zeros(1, length(y_d));
            y_d_delayed(1:length(y_d) + ind) = y_d(-ind+1:length(y_d));
        end
        w_d = system_d(x_delayed);
        if(isequal(w_d,y_d_delayed))
            fprintf('System d passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System d does not pass time invariance test for n0= %d\n', ind);
        end
    end
end
fprintf("-----\n");

fprintf("----- Test for System e: -----\n");
for ind = n0
    if(ind ~= 0)
        y_e = system_e(x, n);
        if(ind>0)
            x_delayed = zeros(1, length(x));
            x_delayed(ind+1:length(x)) = x(1:length(x)-ind);

            y_e_delayed = zeros(1, length(y_e));
            y_e_delayed(ind+1: length(y_e)) = y_e(1:length(y_e)-ind);
        else
            x_delayed = zeros(1,length(x));
            x_delayed(1:length(x)+ ind) = x(-ind+1:length(x));
            y_e_delayed = y_e(end-1).*ones(1, length(y_e));
            y_e_delayed(1:length(y_e) + ind) = y_e(-ind+1:length(y_e));
        end
        w_e = system_e(x_delayed, n);
        if(isequal(w_e,y_e_delayed))
            fprintf('System e passes time invariance test for n0= %d\n' ,ind);
        else
            fprintf('System e does not pass time invariance test for n0= %d\n', ind);
        end
    end
end

```

```
end
end
```

Code for Causality Test (Part 4)

```
%% Author: Berk Saltuk Yılmaz
%% Code for testing causality
clc;
clear;

%% Input index is same with the example test
n=1:9;

x = [-3 -4 2 0 1 4 3 6 3 7];
z = x;
z1 = z;
z2 = z;
fprintf("----- Test for System a: -----\n");
for ind = n
    z2(1:ind) = z(1:ind);
    z2(ind+1:end) = zeros(1, length(z2)-ind);

    w_a = system_a(z1,10);
    y_a = system_a(z2, 10);
    if(w_a(ind) == y_a(ind))
        disp("System a passes causality test for index " + ind);
    else
        disp("System a does not pass causality test for index " + ind);
    end
end

fprintf("-----\n");

fprintf("----- Test for System b: -----\n");
z1 = z;
z2 = z;
for ind = n
    z2(1:ind) = z(1:ind);
    z2(ind+1:end) = zeros(1, length(z2)-ind);

    w_b = system_b(z1,10);
    y_b = system_b(z2, 10);
    if(w_b(ind) == y_b(ind))
        disp("System b passes causality test for index " + ind);
    else
        disp("System b does not pass causality test for index " + ind);
    end
end

fprintf("-----\n");

fprintf("----- Test for System c: -----\n");
z1 = z;
z2 = z;
for ind = n
    z2(1:ind) = z(1:ind);
    z2(ind+1:end) = zeros(1, length(z2)-ind);

    w_c = system_c(z1);
    y_c = system_c(z2);
    if(w_c(ind) == y_c(ind))
        disp("System c passes causality test for index " + ind);
```

```

    else
        disp("System c does not pass causality test for index " + ind);
    end
end
fprintf("-----\n");

fprintf("----- Test for System d: -----\n");
z1 = z;
z2 = z;
for ind = n
    z2(1:ind) = z(1:ind);
    z2(ind+1:end) = zeros(1, length(z2)-ind);

    w_d = system_d(z1);
    y_d = system_d(z2);
    if(w_d(ind) == y_d(ind))
        disp("System d passes causality test for index " + ind);
    else
        disp("System d does not pass causality test for index " + ind);
    end
end
fprintf("-----\n");

fprintf("----- Test for System e: -----\n");
z1 = z;
z2 = z;
for ind = n
    z2(1:ind) = z(1:ind);
    z2(ind+1:end) = zeros(1, length(z2)-ind);

    w_e = system_d(z1);
    y_e = system_d(z2);
    if(w_e(ind) == y_e(ind))
        disp("System e passes causality test for index " + ind);
    else
        disp("System e does not pass causality test for index " + ind);
    end
end
end

```

Outputs of Tests

Output of Linearity Test

```

----- TESTS WITH INPUT SET #1 -----
----- Test for System a: -----
System a passes linearity test for alpha= -5, and beta= -5
System a passes linearity test for alpha= -5, and beta= -4
System a passes linearity test for alpha= -5, and beta= -3
System a passes linearity test for alpha= -5, and beta= -2
System a passes linearity test for alpha= -5, and beta= -1
System a passes linearity test for alpha= -5, and beta= 1
System a passes linearity test for alpha= -5, and beta= 2
System a passes linearity test for alpha= -5, and beta= 3
System a passes linearity test for alpha= -5, and beta= 4
System a passes linearity test for alpha= -5, and beta= 5
System a passes linearity test for alpha= -4, and beta= -5
System a passes linearity test for alpha= -4, and beta= -4
System a passes linearity test for alpha= -4, and beta= -3
System a passes linearity test for alpha= -4, and beta= -2
System a passes linearity test for alpha= -4, and beta= -1
System a passes linearity test for alpha= -4, and beta= 1
System a passes linearity test for alpha= -4, and beta= 2
System a passes linearity test for alpha= -4, and beta= 3
System a passes linearity test for alpha= -4, and beta= 4
System a passes linearity test for alpha= -4, and beta= 5
System a passes linearity test for alpha= -3, and beta= -5
System a passes linearity test for alpha= -3, and beta= -4
System a passes linearity test for alpha= -3, and beta= -3
System a passes linearity test for alpha= -3, and beta= -2

```


[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Test for System e:				
System e passes linearity test for alpha= -5, and beta= -5				
System e passes linearity test for alpha= -5, and beta= -4				
System e passes linearity test for alpha= -5, and beta= -3				
System e passes linearity test for alpha= -5, and beta= -2				
System e passes linearity test for alpha= -5, and beta= -1				
System e passes linearity test for alpha= -5, and beta= 1				
System e passes linearity test for alpha= -5, and beta= 2				
System e passes linearity test for alpha= -5, and beta= 3				
System e passes linearity test for alpha= -5, and beta= 4				
System e passes linearity test for alpha= -5, and beta= 5				
System e passes linearity test for alpha= -4, and beta= -5				
System e passes linearity test for alpha= -4, and beta= -4				
System e passes linearity test for alpha= -4, and beta= -3				
System e passes linearity test for alpha= -4, and beta= -2				
System e passes linearity test for alpha= -4, and beta= -1				
System e passes linearity test for alpha= -4, and beta= 1				
System e passes linearity test for alpha= -4, and beta= 2				
System e passes linearity test for alpha= -4, and beta= 3				
System e passes linearity test for alpha= -4, and beta= 4				
System e passes linearity test for alpha= -4, and beta= 5				
System e passes linearity test for alpha= -3, and beta= -5				
System e passes linearity test for alpha= -3, and beta= -4				
System e passes linearity test for alpha= -3, and beta= -3				
System e passes linearity test for alpha= -3, and beta= -2				
System e passes linearity test for alpha= -3, and beta= -1				
System e passes linearity test for alpha= -3, and beta= 1				
System e passes linearity test for alpha= -3, and beta= 2				
System e passes linearity test for alpha= -3, and beta= 3				
System e passes linearity test for alpha= -3, and beta= 4				

Output of Time Invariance Test

```
----- TESTS WITH INPUT #1 -----
Test for System a:
System a does not pass time invariance test for n0= -50
System a does not pass time invariance test for n0= -49
System a does not pass time invariance test for n0= -48
System a does not pass time invariance test for n0= -47
System a does not pass time invariance test for n0= -46
System a does not pass time invariance test for n0= -45
```


[illegible]

[illegible]

[illegible]

[illegible]

```
----- Test for System e: -----
System e passes time invariance test for n0= -50
System e passes time invariance test for n0= -49
System e passes time invariance test for n0= -48
System e passes time invariance test for n0= -47
System e passes time invariance test for n0= -46
System e passes time invariance test for n0= -45
System e passes time invariance test for n0= -44
System e passes time invariance test for n0= -43
System e passes time invariance test for n0= -42
System e passes time invariance test for n0= -41
System e passes time invariance test for n0= -40
System e passes time invariance test for n0= -39
System e passes time invariance test for n0= -38
System e passes time invariance test for n0= -37
System e passes time invariance test for n0= -36
System e passes time invariance test for n0= -35
System e passes time invariance test for n0= -34
System e passes time invariance test for n0= -33
```

```
----- TESTS WITH INPUT #2 -----
```

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

```
System e passes time invariance test for n0= 46
System e passes time invariance test for n0= 47
System e passes time invariance test for n0= 48
System e passes time invariance test for n0= 49
System e passes time invariance test for n0= 50
```

Output of Causality Test

```
----- Test for System a: -----
System a does not pass causality test for index 1
System a passes causality test for index 2
System a does not pass causality test for index 3
System a does not pass causality test for index 4
System a does not pass causality test for index 5
System a does not pass causality test for index 6
System a does not pass causality test for index 7
System a does not pass causality test for index 8
System a passes causality test for index 9
-----
----- Test for System b: -----
System b does not pass causality test for index 1
System b passes causality test for index 2
System b does not pass causality test for index 3
System b does not pass causality test for index 4
System b does not pass causality test for index 5
System b passes causality test for index 6
System b passes causality test for index 7
System b passes causality test for index 8
System b passes causality test for index 9
-----
----- Test for System c: -----
System c does not pass causality test for index 1
System c does not pass causality test for index 2
System c does not pass causality test for index 3
System c does not pass causality test for index 4
System c does not pass causality test for index 5
System c does not pass causality test for index 6
System c passes causality test for index 7
System c passes causality test for index 8
System c passes causality test for index 9
-----
----- Test for System d: -----
System d passes causality test for index 1
System d passes causality test for index 2
System d passes causality test for index 3
System d passes causality test for index 4
System d passes causality test for index 5
System d passes causality test for index 6
System d passes causality test for index 7
System d passes causality test for index 8
System d passes causality test for index 9
-----
----- Test for System e: -----
System e passes causality test for index 1
System e passes causality test for index 2
System e passes causality test for index 3
System e passes causality test for index 4
System e passes causality test for index 5
System e passes causality test for index 6
System e passes causality test for index 7
System e passes causality test for index 8
System e passes causality test for index 9
```