Bilkent University

IE400

Principles of Engineering Management

Term Project Report

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Part 1

Problem Description

In the first part of the problem, our aim was to correctly assign fifteen trains with fixed paths to depots X and Y such that the total distance from depots to nodes was minimized. Those fifteen trains were traveling along subsets of eight stations (A to H). The trains need four hours of maintenance, and each train should complete its path as many times as possible. This assignment also required assigning at least five trains to each depot, and the paths from a depot to a starting node can be used by at most three trains.

Preprocessing

In our modeling, we assumed that the trains traverse round paths such that if a train has the path A→B→C and its maximum tour count is two, the train goes as A→B→C→B→A. First, we calculated each train's path length for one tour and how many tours it could traverse if this train were assigned to depot X or Y. As the starting and finishing nodes would be the same if the train traverses an even number of tours, we also computed a parameter that takes the value one when the tour number is even, zero otherwise. We also extracted each train's path's starting and finishing nodes for later use in our model. We also labeled nodes from 1 to 8, corresponding to A to H.

Our Model

Parameters

$$\mathrm{Even}_{ij} = \begin{cases} 1 & \text{if train i takes even number of tours when assigned to depot j} \\ 0 & \text{o.w.} \end{cases}$$

$$\forall i = 1, \ldots, 15$$
$$\forall j = X, Y$$

We need this parameter for our model to calculate the distance in the objective function correctly, as when the starting and finishing nodes are the same, the distance for this train is going to be two times the depot to the starting node. If the tour number is odd, the train's distance is the sum of the depot to the starting node and the depot to the finish node.

$$\mathrm{D}_{jk} = \text{Distance of depot j to node k}, \ \forall j = X, Y, \forall k = 1, \ldots, 8$$

$$\mathrm{W}_{ik} = \begin{cases} 1 & \text{if train i's path starts with node k} \\ 0 & \text{o.w.} \end{cases} \quad \forall i = 1, \ldots, 15 \quad \forall k = 1, \ldots, 8$$

Decision Variables

$$\mathrm{X}_{ij} = \begin{cases} 1 & \text{if train i assigned to depot j} \\ 0 & \text{o.w.} \end{cases} \qquad \forall i = 1, \ldots, 15 \quad \forall j = X, Y$$

Constraints

$$\sum_{j \in \{X,Y\}} X_{ij} = 1 \qquad \forall i = 1, \ldots, 15 \tag{1}$$

$$\sum_{i=1}^{15} W_{ik} \cdot X_{ij} \leq 3 \quad \forall k = 1, \ldots, 8 \quad \forall j = X, Y \tag{2}$$

$$\sum_{1}^{15} X_{ij} \geq 5 \quad \forall j = X, Y \tag{3}$$

Constraint (1) implies that a train can be assigned to at most one depot.

Constraint (2) implies that for each depot, paths to each node at the start can be used by at most three trains.

Finally, constraint (3) implies that at least five trains should assigned to each depot.

Objective Function

$$\min \sum_{i=1}^{15} \sum_{j \in \{X,Y\}} (1 + \mathrm{Even}_{ij}) \cdot X_{ij} \cdot D_{j,s_i} + (1 - \mathrm{Even}_{ij}) \cdot X_{ij} \cdot D_{j,f_i}$$

$$\text{where } \mathrm{s}_i \text{ is the starting node of train } \mathrm{s}_i$$
$$\text{and } \mathrm{f}_i \text{ the final node of train's path}$$

Results

Our results are as follows:

- Optimal total distance is 33.0

- Trains assigned to depot X: 1, 2, 6, 8, 9, 11, 12, 15
- Trains assigned to depot Y: 3, 4, 5, 7, 10, 13, 14

When we examine the parameters, we can see that all constraints held as we expected and assignment is made correctly.

Part 2

Problem Description

In the second part, in addition to the assignment in the first part, we had to decide the type of each train to maximize energy efficiency. There are two types of trains: electric and diesel. Each train type requires a different type of energy source and, as a result, requires different types of energy infrastructure. As electric trains can only travel eight hours without recharging and as we also obtained in part 1 that each train travels more than eight hours, electric trains require both on-route and in-depot charging stations. On-route charging stations can be built at nodes A to H, and a train can only be charged at the time instance that it arrives at that node. On the other hand, diesel trains can go twenty hours once they have been fueled. Thus, they only require depot fueling stations. Both in-depot fueling and charging can be built at depots X and Y. Moreover, building stations, purchasing trains and operating those trains have the costs as follows:

- Cost of In-Depot Charging Station: 1,000,000$
- Cost of On-Route Charging Station: 350,000$
- Cost of In-Depot Fuel Station: 800,000$
- Cost of Purchasing an Electric Train: 750,000$
- Cost of Purchasing a Diesel Train: 250,000$
- Cost of Energy Spend by Diesel Train (by working hour): 100,000$
- Cost of Energy Spend by Electric Train (by working hour): 20,000$

In this part, we assumed that several stations can be built at nodes and depots and also each train starts the day with full charge/fuel.

Preprocessing

In the preprocessing of part two, first of all, we saved each train's assigned depot information. After that, by considering each train's actual path, we have derived the final path of each train by considering the maximum number of times that each train can complete its path; we have derived the "paths after tours." By incorporating distances to these final paths, we acquired a mapping from hour to train location. By filling the missing hours with placeholder values OW (on way) and MA (maintenance), we achieved a complete mapping for twenty-four hours of train locations (we again assumed round paths during this calculation). During this calculation, we also marked the hour that the train returned to its depot as the total operation time, and we used it as a parameter in our model. Moreover, using the mapping, we have acquired another parameter that indicates if a train is at a specific node at a given instance. We also indicate the train types as E: Electric and D: Diesel in our model. Moreover, since each train should be returned to its depot after hour twenty, we added our decision variables and constraints, considering twenty as the upper limit hour. Finally, for in-depot stations, as we knew that in four hours of maintenance, there will be five slots for charging or refueling, and one depot has seven trains assigned whereas the other has eight, we directly divided the arrived train count of each to depot by five and added it to our constraints.

Our Model

Parameters

$$\mathrm{X}_{ij} = \begin{cases} 1 & \text{if train i is assigned to depot j} \\ 0 & \text{o.w.} \end{cases} \quad \forall i = 1, \ldots, 15 \;\; \forall j = X, Y$$

$$\mathrm{T}_i = \text{ Total operation time for train i} \quad \forall i = 1, \ldots, 15$$

$$\mathrm{L}_{ijt} = \begin{cases} 1 & \text{if train i is at node j at time t} \\ 0 & \text{o.w.} \end{cases}$$
$$\forall i = 1, \ldots, 15$$
$$\forall j = 1, \ldots, 8$$
$$\forall t = 1, \ldots, 24$$

CIDF = Cost of In-Depot Fuel Station
CIDC = Cost of In-Depot Charging Station
CORC = Cost of On-Route Charging Station
CPE = Cost of Purchasing an Electric Train
CPD = Cost of Purchasing a Diesel Train
ESD = Cost of Energy Spend by Diesel Train (by working hour)
ESE = Cost of Energy Spend by Electric Train (by working hour)

Decision Variables

$$\mathrm{Y}_{ij} = \begin{cases} 1 & \text{if train i is of type j} \\ 0 & \text{o.w.} \end{cases} \qquad \forall i = 1, \ldots, 15 \quad \forall j = E, D$$

$$\mathrm{I}_i = \text{Number of in depot charging stations built at depot i} \quad \forall i = X, Y \quad \text{integer}$$

$$\mathrm{F}_i = \text{Number of fueling stations built at depot i} \quad \forall i = X, Y \quad \text{integer}$$

$$\mathrm{O}_i = \text{Number of on route charging stations built at node i} \quad \forall i = 1, \ldots, 8 \quad \text{integer}$$

$$\mathrm{D}_{it} = \text{Number of trains being charged at node i at time t}$$
$$\forall i = 1, \ldots, 8 \quad \forall t = 1, \ldots, 20 \quad \text{integer}$$

$$\mathrm{ET}_{it} = \text{Energy left in hours for train i at time t}$$
$$\forall i = 1, \ldots, 15 \;\; \forall t = 1, \ldots, 20 \;\; \text{integer}$$

$$\mathrm{E}_{ijt} = \begin{cases} 1 & \text{if train i is charged at node j at time t} \\ 0 & \text{o.w.} \end{cases}$$
$$\forall i = 1, \ldots, 15$$
$$\forall j = 1, \ldots, 8$$
$$\forall t = 1, \ldots, 20$$

Constraints

$$\mathrm{ET}_{i0} = 8 \;\; \forall i = 1, \ldots, 15 \quad (1)$$

$$\mathrm{D}_{it} \leq \; \mathrm{O}_i \cdot 1 \;\;\; \forall i = 1, \ldots, 8 \;\; \forall t = 1, \ldots, 20 \quad (2)$$

$$\left(\sum_{j=1}^{15} \mathrm{Y}_{jE} \cdot \; \mathrm{X}_{ji} \right) / 5 \; \leq 3 \cdot \; \mathrm{I}_i \quad \forall i = X, Y \quad (3)$$

$$\left(\sum_{j=1}^{15} \mathrm{Y}_{jD} \cdot \; \mathrm{X}_{ji} \right) / 5 \leq 2 \cdot \; \mathrm{F}_i \quad \forall i = X, Y \quad (4)$$

$$\sum_{i=1}^{15} \mathrm{E}_{ijt} = \mathrm{D}_{jt} \;\;\; \forall j = 1, \ldots, 8 \;\; \forall t = 1, \ldots, 20 \quad (5)$$

$$\sum_{j=1}^{8} \mathrm{E}_{ijt} = 1 \;\;\; \forall i = 1, \ldots, 15 \;\;\; \forall t = 1, .., 20 \quad (6)$$

$$\mathrm{ET}_{it} = (\mathrm{ET}_{i,t-1} - 1) \cdot \left(1 - \sum_{j=1}^{8} \mathrm{E}_{ijt} \right) + 8 \cdot \left(\sum_{j=1}^{8} \mathrm{E}_{ijt} \right)$$
$$\forall i = 1, \ldots, 15$$
$$\forall t = 1, \ldots, 20 \quad (7)$$

$$\mathrm{E}_{ijt} \leq \mathrm{L}_{ijt} \;\;\; \forall i = 1, \ldots, 15 \;\; \forall j = 1, \ldots, 8 \;\; \forall t = 1, \ldots, 20 \quad (8)$$

$$\mathrm{ET}_{it} \geq 0 \;\; \forall i = 1, \ldots, 15 \;\; \forall t = 1, \ldots, 20$$
$$\mathrm{ET}_{it} \leq 8 \;\; \forall i = 1, \ldots, 15 \;\; \forall t = 1, \ldots, 20 \quad (9)$$

$$\mathrm{E}_{ijt} \leq \mathrm{Y}_{iE} \;\;\; \forall i = 1, \ldots, 15 \;\; \forall j = 1, \ldots, 8 \;\; \forall t = 1, \ldots, 20 \quad (10)$$

$$\sum_{i=1}^{15} \sum_{j \in \{E,D\}} \mathrm{Y}_{ij} = 15 \quad (11)$$

$$\mathrm{Y}_{iE} + \mathrm{Y}_{iD} = 1 \quad \forall i = 1, \ldots, 15 \quad (12)$$

$$\mathrm{F}_i \geq 0 \quad \forall i = X, Y$$
$$\mathrm{I}_i \geq 0 \quad \forall i = X, Y$$
$$\mathrm{O}_i \geq 0 \quad \forall i = 1, \ldots, 8 \quad (13)$$

Constraint (1) implies that at time 0, all trains should have eight hours of charge, i.e., all trains start the day with a full battery.

Constraint (2) implies that the number of trains being charged at a time instance at an on-route charging station should be less than that node's charging capacity.

As we mentioned in the preprocessing part, constraints (3) and (4) imply that the total number of trains of type electric or diesel divided by five (indicating the total available charging slots during maintenance) should be less than the depots charging facilities' capacities.

Constraint (5) implies that the total number of trains charged at a node at a given time should be equal to the number of trains charged at a node at time t.

Constraint (6) implies that a train can be charged at most at one node at any timestep.

Constraint (7) gives us the energy level update formula. If a train is charged at a node at a given time, the train's energy level becomes eight. Otherwise, its energy level becomes one less than the previous timestep.

Constraint (8) is one of the most important constraints for our model. It implies that a train can be charged at a node at a timestep if and only if this train is present at that node at that exact time. This prevents our model from charging a train while it is on the way between two nodes.

Constraint (9) defines the bounds for energy levels. The energy level of a train is bounded between zero and eight.

Constraint (10) implies that a train can be charged if and only if it is electric.

Constraint (11) implies that the total number of trains (electric or diesel) must be fifteen.

Constraint (12) implies that each train can be either electric or diesel.

Constraint (13) is the nonnegativity constraint for station numbers.

Objective Function

$$\min \sum_{i\in\{X,Y\}} (\mathrm{CIDF}\cdot \mathrm{F}_i) + \sum_{i\in\{X,Y\}} (\mathrm{CIDC}\cdot \mathrm{I}_i) + \sum_{i=1}^{8} (\mathrm{CORC}\cdot \mathrm{O}_i) + \sum_{i=1}^{15} (\mathrm{CPE}\cdot \mathrm{Y}_{iE} + \mathrm{CPD}\cdot \mathrm{Y}_{iD})$$
$$\sum_{i=1}^{15} (\mathrm{ESE}\cdot \mathrm{T}_i \cdot \mathrm{Y}_{iE} + \mathrm{ESD}\cdot \mathrm{T}_i \cdot \mathrm{Y}_{iD})$$

We formulated the objective function as the sum of building in-depot fueling & charging station cost, on-route charging station cost, purchasing trains cost, and energy cost.

Results

Our results are as follows:

- Optimal cost is 20,230,000.0$

- Electric trains: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
- Diesel trains: None

- Number of in-depot charging stations at depot X: 1
- Number of in-depot charging stations at depot Y: 1

- Number of in-depot fueling stations at depot X: 0
- Number of in-depot fueling stations at depot Y: 0

- Number of on-route charging stations at node A: 0
- Number of on-route charging stations at node B: 1
- Number of on-route charging stations at node C: 1
- Number of on-route charging stations at node D: 0
- Number of on-route charging stations at node E: 0
- Number of on-route charging stations at node F: 1
- Number of on-route charging stations at node G: 1
- Number of on-route charging stations at node H: 0

We were not expecting to all trains to be electric trains and we were a little surprised but when we carefully examine the costs given as parameters, these results seem to be logical. Even if purchasing electric trains and building in-depot charging stations are more costly than purchasing diesel trains and building necessary facilities, the operating cost of an electric train is one-fifth of that of a diesel one. Moreover, building one in-depot charging station at each depot is enough for charging every train at the end of each day; these facilities have a larger capacity than diesel ones, and building four on-route charging stations can handle the need. As a result, the model favors purchasing electric trains as it would probably require more in-depot fueling stations if it chose to make all trains diesel, and the advantage of high traveling capacity and not needing on-route facilities would disappear due to the high operating cost. Moreover, it can be asked why the model does not choose to purchase both electric and diesel trains. The answer to that might be that fifteen trains (which is not a high number) can be charged with a few stations and in such a case building both types of facilities would be an unnecessary cost for us. Moreover, it would feel like we balanced the cost since diesel trains are cheaper, but again, there would not be balance as the operating costs would break it. As a result, our model chooses to make all trains electric, builds one in-depot charging station for each depot, and builds one on-route charging station for each of nodes B, C, F, and G. Since we do not have any diesel trains, we do not have in-depot fueling stations either. Reasonable.