# ENS 206: System Modelling and Control

# Project Report

*Project Summary:* **Designing a toy race car, its dynamics and simulate its performance along a given parkour**

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We have started the project by selecting a motor which is available on the given website. Then, we chose the one that has the following properties:

- Nominal Voltage: 48V
- Terminal Resistance: 4.68 Ω
- Terminal Inductance: 1.29 H
- Torque Constant: 75.8 mNm/A = 0.0758 Nm/A
- Velocity Constant: 126 rpm/V = 0.076 (sV/rad)
- Weight of the Motor: 340 g = 0.34 kg

After determining on the motor, we have solved 3 equations of motion to find a single equation in the form of (Input + Disturbance) * Transfer Function = Output.

## 1) Equation of Motion of the Race Car

$\sum F = m*\ddot{y}$

$F - M*g*\sin(\alpha) = M*\ddot{y}$

**$F = M*(\ddot{y} + g*\sin(\alpha))$** *[Equation 1]*

## 2) Equation of Motion of the Wheel

$\sum \tau = I_W*\ddot{\theta}_S$

$\tau_S - F*r = I_W * \ddot{\theta}_S$. Insert F from *Equation 1*:

$\tau_s - M*(\ddot{y} + g*\sin(\alpha))*r = I_W*\ddot{\theta}_S$

$5 * \tau_M - M*(\ddot{y} + g*\sin(\alpha))*r = I_W*(1/5)*\ddot{\theta}_M$ (since $\tau_S = 5 * \tau_M$ and $\ddot{\theta}_S = (1/5)*\ddot{\theta}_M$)

Let $M*(\ddot{y} + g*\sin(\alpha))*r = Z$ for the sake of simplicity. Then,

**$\tau_M = (I_W*(1/5)*\ddot{\theta}_M + Z)*(1/5)$** *[Equation 2]*

## 3) Equation of Motion of the Motor along with the Circuitry

$\sum \tau = I_M*\ddot{\theta}_M$

$\tau_M = I_M*\ddot{\theta}_M$ (As stated in the project description, we can ignore the rotor inertia of the motor, $I_M$)

We know that

$e = K_V * \dot{\theta}_M$

$\tau_M = K_T * i$

By the help of Kirchhoff's Current Law: $V - i*R_M - e = 0 \rightarrow V - i*R_M - K_V * \dot{\theta}_M = 0$

$V - (\tau_M/K_T)*R_M - K_V*\dot{\theta}_M = 0$

$V = (\tau_M/K_T)*R_M + K_V*\dot{\theta}_M$. Insert $\tau_M$ from *Equation 2*:

$V = (I_W*(1/5)*\ddot{\theta}_M + Z)*(1/5)*(1/K_T)*R_M + K_V*\dot{\theta}_M$

At this point, we need to get rid of $\ddot{\theta}_M$ and $\dot{\theta}_M$. To do that, we need the following equations:

$y = r * \theta_S \rightarrow \ddot{y} = r * \ddot{\theta}_S \rightarrow \ddot{y} = r * (1/5) * \ddot{\theta}_M \rightarrow \ddot{\theta}_M = 5 * \ddot{y} * (1/r)$ *[Equation 3]*

$\rightarrow \dot{y} = r * \dot{\theta}_S \rightarrow \dot{y} = r * (1/5) * \dot{\theta}_M \rightarrow \dot{\theta}_M = 5 * \dot{y} * (1/r)$ *[Equation 4]*

Insert $\ddot{\theta}_M$ and $\dot{\theta}_M$ from *Equation 3* and *Equation 4*, respectively:

$V = [(5*\ddot{y}*I_W)/(5*r) + Z] * [R_M/(5*K_T)] + K_V * 5 * \dot{y} * (1/r)$

If we rearrange the equation, $\mathbf{V = \ddot{y} * A + \ddot{y} * B + C + \dot{y} * D}$ *[Equation 5]* where

$A = (I_W*R_M) / (5 * K_T * r)$

$B = (M * r * R_M) / (5 * K_T)$

$C = (M*g*r* \sin(\alpha)*R_M) / (5 * K_T)$

$D = (5 * K_V) / r$

The internal variables consist of the given parameters that are provided us beforehand or design parameters that we have chosen at the beginning of the project. The values and their units of the internal variables are as follows:

- We determined $r = 0.39$ where r is the radius of the wheel
- $m_w = (0.5) * (0.39) = 0.195$ kg
- $I_W = m_w * r^2 = 0.195 * (0.39)^2 \approx 0.0297$
- $R = 4.68\ \Omega$
- $K_T = 0.0758$ Nm/A
- $M = m_b + m_c + m_w + m_m = 400 + 200 + 195 + 340 = 1135$ g $= 1.135$ kg
- $g = 10\ m/s^2$
- $K_V = 0.076$ (sV/rad)

The Laplace Transformation of the Equation 5 can be found below while considering the initial conditions as 0.

$V_{(s)} = s^2 * Y_{(s)} * (A + B) + s * Y_{(s)} * D + C$

$V_{(s)} - C = Y_{(s)} * [(A+B) * s^2 + D * s]$ where $1/G_{(s)} = [(A+B) * s^2 + D * s]$

$\mathbf{(V_{(s)} - C) * G_{(s)} = Y_{(s)}}$**.** We have reached our final goal!

Figure 1 and Figure 2 show the open-loop and closed-loop diagrams, respectively.

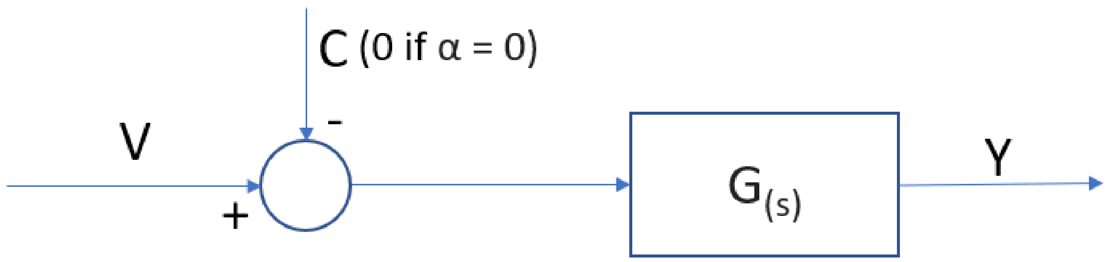


*Figure 1*

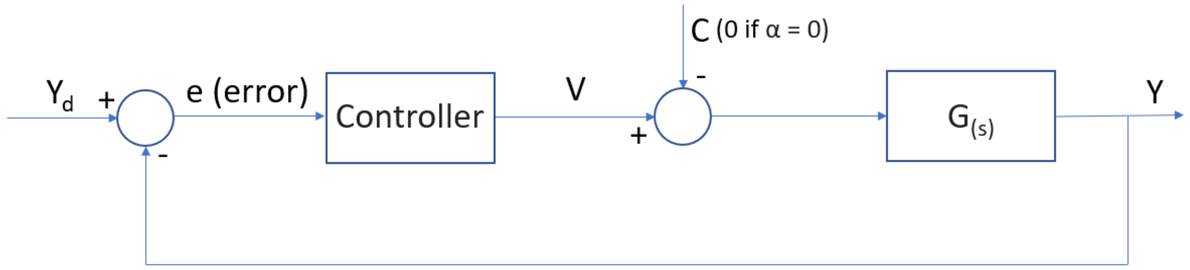


*Figure 2*

## TASK #1:

The simulink file (project_task1.slx) which simulates the task #1 can be found in the submission folder.

## TASK #2:

Changed Design Parameter: r (radius of the wheel)

For the task #2, we tested different r values to find out the best possible wheel size to finish the parkour. Our initial thought was to make the wheel as big as possible. At the end product, we used r = 0.39 which currently finishes the parkour in 17.414 seconds. We tested r = 0.1, which resulted in 41.607 seconds. We also tested r = 0.5 as it is the biggest wheel size we can choose for our car. But it was slower than r = 0.39. The reason for that is the change in $I_W$. While the car is moving, the energy is converted to its movement and the rotation of the wheels. Big wheels are resulting in a bigger $I_W$ value. Although big wheels are making us go further with the same rotation amount, the drawback coming from the increased $I_W$ value which makes r = 0.5 slower than r = 0.39.

## TASK #3:

The simulink file (project_task3.slx) which simulates the task #3 can be found in the submission folder.