

# Student Information

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## Answer 1

a) contradiction

$p$	$q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

b)

$$\begin{aligned} p \rightarrow ((q \vee \neg q) \rightarrow (p \wedge q)) &\equiv p \rightarrow (T \rightarrow (p \wedge q)) && \text{from table 6, Negation Law } q \vee \neg q \equiv T \\ &\equiv p \rightarrow (F \vee (p \wedge q)) && \text{from table 7, line 1} \\ &\equiv p \rightarrow (p \wedge q) && \text{from table 6, Identity Law for F} \\ &\equiv \neg p \vee (p \wedge q) && \text{from table 7, line 1} \\ &\equiv (\neg p \vee p) \wedge (\neg p \vee q) && \text{from table 6, first Distribution Law} \\ &\equiv T \wedge (\neg p \vee q) && \text{from table 6, Negation Law } \neg p \vee p \equiv T \\ &\equiv \neg p \vee q && \text{from table 6, Identity Law for T} \end{aligned}$$

## Answer 2

a)  $\forall x \exists y W(x, y)$

b)  $\exists y \forall x \neg F(x, y)$

c)  $\forall x (W(x, P) \rightarrow A(\text{Ali}, x))$

d)  $\exists x (W(\text{Büşra}, x) \wedge F(\text{TUBITAK}, x))$

e)  $\exists x \exists y \exists z (S(x, y) \wedge (S(x, z) \wedge (y \neq z)))$

f)  $\forall x \forall y \forall z ((W(x, z) \wedge W(y, z)) \rightarrow (x = y))$

g)  $\exists x \exists y \exists z ((x \neq y) \wedge W(x, z) \wedge W(y, z) \wedge \forall t (W(t, z) \rightarrow ((t = x) \vee (t = y))))$

### Answer 3

1		$p \rightarrow q$	
2		$(q \wedge \neg r) \rightarrow s$	
3		$\neg s$	
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4			$p$
5			$q$ $\Rightarrow$ E, 1, 4
6			
7			$\neg r$
8			$q \wedge \neg r$ $\wedge$ I, 5, 6
9			$s$ $\Rightarrow$ E, 2, 7
10			$\perp$ $\neg$ E, 3, 8
11			$\neg\neg r$ $\neg$ I, 6–9
12			$r$ $\neg\neg$ E, 10
13		$p \rightarrow r$	$\Rightarrow$ I, 4–11

## Answer 4

Ayşe :  $p$  , Barış :  $s \rightarrow \neg q$  , Can :  $p \rightarrow (q \wedge r)$  , Duygu :  $r \rightarrow s$   
 $p, p \rightarrow (q \wedge r), r \rightarrow s \vdash \neg(s \rightarrow \neg q)$

1	$p$	
2	$p \rightarrow (q \wedge r)$	
3	$r \rightarrow s$	
4	$q \wedge r$	$\Rightarrow$ E, 1, 2
5	$q$	$\wedge$ E, 4
6	$r$	$\wedge$ E, 4
7	$s$	$\Rightarrow$ E, 3
8	$s \rightarrow \neg q$	
9	$\neg q$	$\Rightarrow$ E, 7, 8
10	$\perp$	$\neg$ E, 5, 9
11	$\neg(s \rightarrow \neg q)$	$\neg$ I, 8–10

## Answer 5

1		$\forall x(P(x) \rightarrow (Q(x) \rightarrow R(x)))$	
2		$\exists(P(x))$	
3		$\forall x(\neg R(x))$	
4		$u$   $P(u)$	
5			$P(u) \rightarrow (Q(u) \rightarrow R(u)) \quad \forall E, 1$
6			$Q(u) \rightarrow R(u) \quad \Rightarrow E, 4, 5$
7			$\neg R(u) \quad \forall E, 3$
8			$Q(u)$
9			$R(u) \quad \Rightarrow E, 6, 8$
10			$\perp \quad \neg E, 7, 9$
11			$\neg Q(u) \quad \neg I, 8-10$
12		$\exists x \neg Q(x)$	$\exists I, 11$