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Answer 1

a) Sum of degrees of all nodes = 14

- deg(a) = 3
- deg(b) = 3
- deg(c) = 3
- deg(d) = 2
- deg(e) = 3

b) The number of non-zero entries in the adjacency matrix is 14.

$$\begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

c) The number of non-zero entries in the incidence matrix is 21.

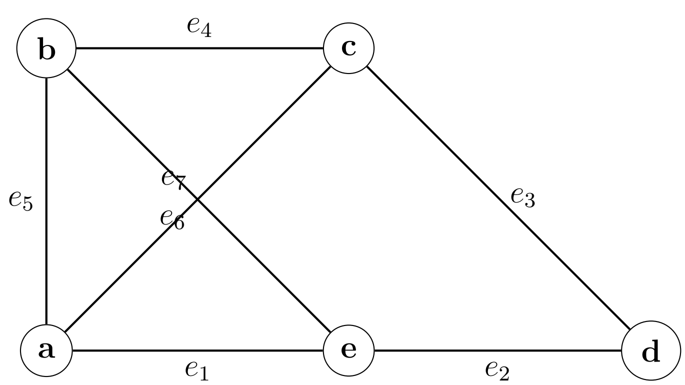


Figure 1: Graph G in Q1.

$$
\begin{array}{c}
 \\ a \\ b \\ c \\ d \\ e
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

d) No!

- A complete graph with 4 vertices has $C(4, 2) = 6$ edges. And a complete graph with 5 vertices has $C(5, 2) = 10$ edges.
- Initially G has 5 vertices and 7 edges. 4 of the vertices has degree 3 and other vertex has degree 2.
- G's itself is not a complete graph since it does not have enough edges.
- We can obtain subgraphs of G removing a vertex.
- If we remove one of the degree 3 vertex. The remaining subgraph has 4 vertices and 4 edges. It cannot be complete graph since it doesn't have 6 edges.
- If we remove degree 2 vertex. The remaining subgraph has 4 vertices and 5 edges. Similarly it cannot be complete graph since it doesn't have 6 edges.
- So we can conclude that the graph G does not have a subgraph with at least four vertices.

e) G is not bipartite since it does not satisfy graph color test

- Pick vertex b color it red and color adjacent vertices blue.
- Since a and c are adjacent vertices and same color. There is no need to continue. The graph G is not bipartite.

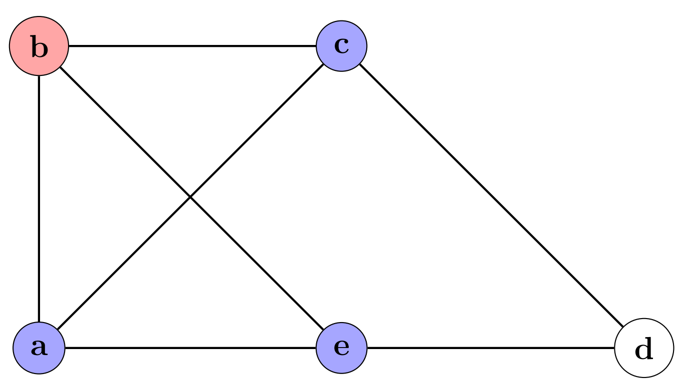


Figure 2: Graph G in Q1.

f) For each edge in the undirected graph G, there are two possible directions that the edge can be oriented in a directed graph. Thus, for the graph G with 7 edges, there are $2^7 = 128$ possible directed graphs that have G as their underlying undirected graph.

g) The length of the simple longest path in G is 6. One of the possible path is (a, b, e, a, c, d, e)

h) Number of connected components of G is 1.

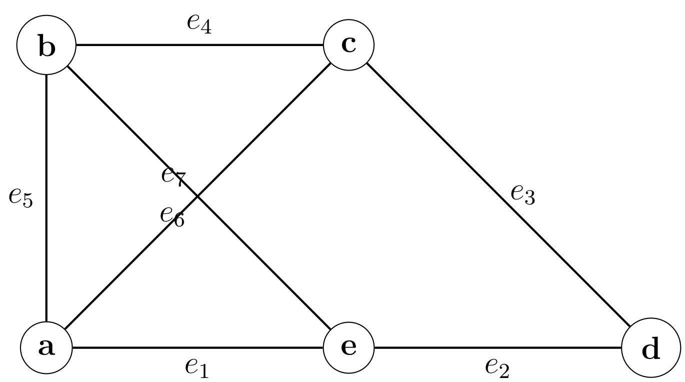


Figure 3: Graph G in Q1.

- Let each vertex in G be a disjoint set. $\{a\}, \{b\}, \{c\}, \{d\}, \{e\}$. For each edge (u, v), if the vertices u and v are in different sets, merge the sets containing u and v.
  - for e_1: merge $\{a\}$ and $\{e\}$.
  $$\{a, e\}, \{b\}, \{c\}, \{d\}$$
  - for e_2: merge $\{a, e\}$ and $\{d\}$.
  $$\{a, d, e\}, \{b\}, \{c\}$$
  - for e_3: merge $\{a, d, e\}$ and $\{c\}$.
  $$\{a, c, d, e\}, \{b\}$$
  - for e_4: merge $\{a, c, d, e\}$ and $\{b\}$.
  $$\{a, b, c, d, e\}$$
  - since there is only one set no need to continue.
- Thus, there is one connected component which is $\{a, b, c, d, e\}$

i) No there is not an Euler circuit in G.

- Theorem 1 from the section 10.5 of the textbook states that "A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree."

- From theorem 1 we can conclude that G has not an Euler circuit since it has vertices which has odd degree. Such as (a, b, c, e)

j) No there is not an Euler path in G.

- Theorem 2 from the section 10.5 of the textbook says "A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree."
- Since vertices a, b, c, e has odd degree and there are more than two vertices of odd degree we can conclude that the graph G has not an Euler path.

k) Yes. (d, c, b, a, e, d) is a Hamilton circuit.

l) Yes. (d, c, b, a, e) is a Hamilton path.

Answer 2

Let $G = (V_1, E_1)$ and $H = (V_2, E_2)$. Since G and H are simple graphs, if we can find a *one-to-one* and *onto* function f from V_1 to V_2 with property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1 we can show that G and H are isomorphic. The graph G has 5 vertices with degree 2 and 5 edges. Similarly the graph H has 5 vertices with degree 2 and 5 edges.

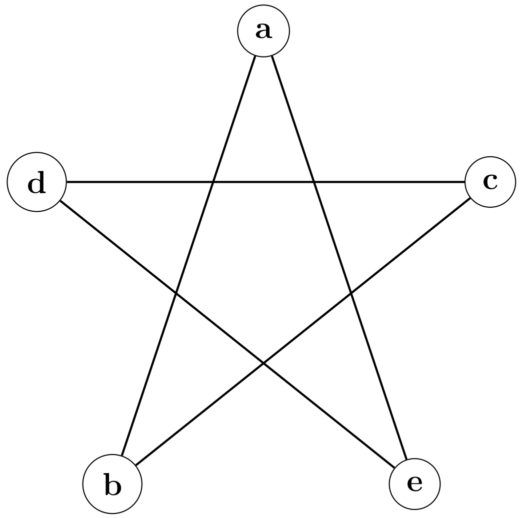
Let f be *one-to-one* and *onto* function from V_1 to V_2. Let the correspondence between graphs:

- $f(a) = a'$
- $f(b) = b'$
- $f(c) = c'$
- $f(d) = d'$
- $f(e) = e'$

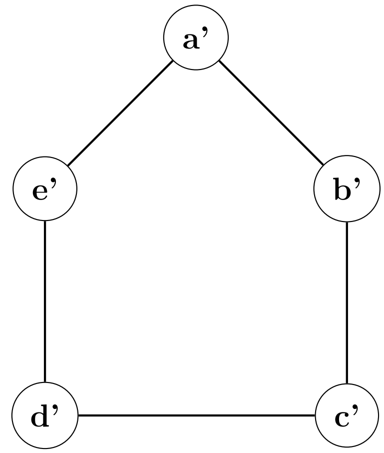
This correspondence preserves adjacency since:

- a is adjacent to b and e in G and $f(a) = a'$ is adjacent to $f(b) = b'$ and $f(e) = e'$ in H.
- b is adjacent to a and c in G and $f(b) = b'$ is adjacent to $f(a) = a'$ and $f(c) = c'$ in H.
- c is adjacent to d and b in G and $f(c) = c'$ is adjacent to $f(d) = d'$ and $f(b) = b'$ in H.
- d is adjacent to c and e in G and $f(d) = d'$ is adjacent to $f(c) = c'$ and $f(e) = e'$ in H.
- e is adjacent to a and d in G and $f(e) = e'$ is adjacent to $f(a) = a'$ and $f(d) = d'$ in H.

Hence G and H are isomorphic.



(a) G



(b) H

Figure 4: Graph G and H.

Answer 3

1) Set the distance of the neighbors of s to the length of the edge between s and them. Mark s visited since shortest path to s is found which is 0. For other vertices set the distance ∞

Vertex	**Distance**	**Prev.**	**Visited**
$\rightarrow$ s	0	-	**T**
***u**	**4**	**(s)**	F
***v**	**5**	**(s)**	F
***w**	**3**	**(s)**	F
x	∞	-	F
y	∞	-	F
z	∞	-	F
t	∞	-	F

2) The next vertex is w since it has minimum distance in unvisited vertices. Update distances and previous vertex for vertices x and z since new distances are smaller than previous distances. Mark w visited since shortest path to w is found which is 3.

Vertex	**Distance**	**Prev.**	**Visited**
s	0	-	T
u	4	(s)	F
v	5	(s)	F
→w	3	(s)	**T**
***x**	**11**	**(w)**	F
y	∞	-	F
***z**	**15**	**(w)**	F
t	∞	-	F

3) The next vertex is u since it has minimum distance in unvisited vertices. Update distance and previous vertex for vertex y since new distance is smaller than previous distance. Mark u visited since shortest path to u is found which is 4.

Vertex	**Distance**	**Prev.**	**Visited**
s	0	-	T
→u	4	(s)	**T**
v	5	(s)	F
w	3	(s)	T
x	11	(w)	F
***y**	**15**	**(u)**	F
z	15	(w)	F
t	∞	-	F

4) The next vertex is v since it has minimum distance in unvisited vertices. Update distances and previous vertex for vertices x and y since new distances is smaller than previous distances. Mark v visited since shortest path to v is found which is 5.

Vertex	**Distance**	**Prev.**	**Visited**
s	0	-	T
u	4	(s)	T
→v	5	(s)	**T**
w	3	(s)	T
***x**	**7**	**(v)**	F
***y**	**11**	**(v)**	F
z	15	(w)	F
t	∞	-	F

5) The next vertex is x since it has minimum distance in unvisited vertices. Update distances and previous vertex for vertices y and z since new distance is smaller than previous distances. Mark x visited since shortest path to x is found which is 7.

Vertex	Distance	Prev.	Visited
s	0	-	T
u	4	(s)	T
v	5	(s)	T
w	3	(s)	T
→x	7	(v)	**T**
***y**	**8**	**(x)**	F
***z**	**13**	**(x)**	F
t	∞	-	F

6) The next vertex is y since it has minimum distance in unvisited vertices. Update distances and previous vertex for vertices z and t since new distance is smaller than previous distances. Mark y visited since shortest path to y is found which is 8.

Vertex	Distance	Prev.	Visited
s	0	-	T
u	4	(s)	T
v	5	(s)	T
w	3	(s)	T
x	7	(v)	T
→y	8	(x)	**T**
***z**	**12**	**(y)**	F
***t**	**17**	**(y)**	F

7) The next vertex is z since it has minimum distance in unvisited vertices. Update distances and previous vertex for vertex t since new distance is smaller than previous distance. Mark z visited since shortest path to z is found which is 12.

Vertex	Distance	Prev.	Visited
s	0	-	T
u	4	(s)	T
v	5	(s)	T
w	3	(s)	T
x	7	(v)	T
y	8	(x)	T
→z	12	(y)	**T**
***t**	**15**	**(z)**	F

8) The next vertex is t and there are no unvisited neighbor vertex. Mark t visited. So the algorithm finished.

Vertex	Distance	Prev.	Visited
s	0	-	T
u	4	(s)	T
v	5	(s)	T
w	3	(s)	T
x	7	(v)	T
y	8	(x)	T
z	12	(y)	T
t	15	(z)	T

To find shortest path vertex s to vertex t. We need to follow the previous vertices until we find s from t and reverse. The shortest path vertex s to vertex t is (s, v, x, y, z, t) with distance 15.

Answer 4

a)

Choosed Prim's Algorithm.

Order	Edge	Weight
1	(b,c)	2
2	(c,f)	2
3	(c,d)	3
4	(d,k)	2
5	(a,b)	3
6	(f,g)	4
7	(f,j)	3
8	(f,e)	4
9	(f,i)	4
10	(i,h)	2

b)

Minimum Spanning Tree with total weight 29.

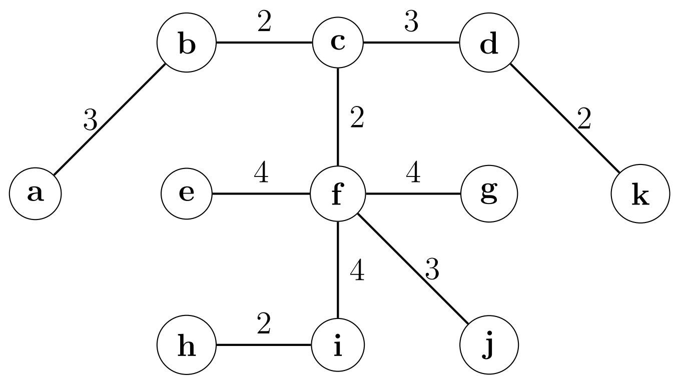


Figure 1: Minimum Spanning Tree of Graph G.

c)

No it is not unique. Since there is another Minimum Spanning Tree with total weight 29.

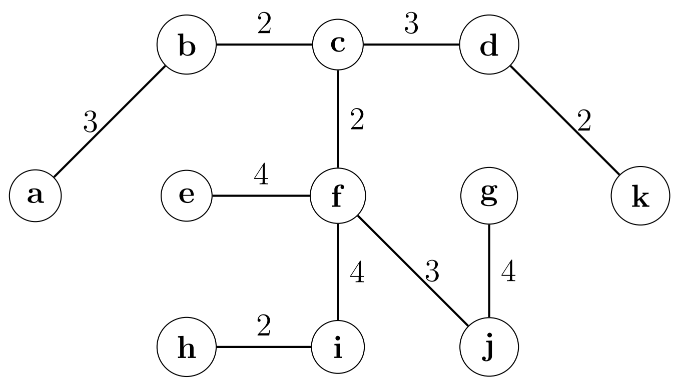


Figure 2: Another Minimum Spanning Tree of Graph G.