Student Information

Full Name : Berk Ulutaş
Id Number : 2522084

Answer 1

1)

- All states are reachable so no need to remove a state.
- Initially equivalence classes of $\equiv_0$ are: (namely $K - F$ and F)

$$\{q_0, q_1, q_3, q_4\} \ \{q_2, q_5\}$$

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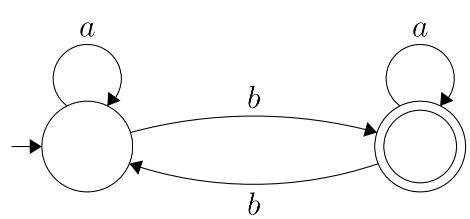
	a	b
q_0	q_1	q_2
q_1	q_1	q_2
q_3	q_4	q_5
q_4	q_0	q_2

	a	b
q_2	q_2	q_3
q_5	q_2	q_3

- $\delta(q_0, a) \equiv_0 \delta(q_1, a) \equiv_0 \delta(q_3, a) \equiv_0 \delta(q_4, a)$ and $\delta(q_0, b) \equiv_0 \delta(q_1, b) \equiv_0 \delta(q_3, b) \equiv_0 \delta(q_4, b)$
- $\delta(q_2, a) \equiv_0 \delta(q_5, a)$ and $\delta(q_2, b) \equiv_0 \delta(q_5, b)$
- After first iteration the classes of $\equiv_1$ are

$$\{q_0, q_1, q_3, q_4\} \ \{q_2, q_5\}$$

- There is no further splitting of classes, since $\equiv_0$ and $\equiv_1$ classes are same. The algorithm thus terminates.
- Minimum-state automaton is:



2)

There is 2 equivalence classes since there is 2 state in minimum-state automaton. Automaton recognizes languages which has odd number of b's. Let L to denote all strings recognized by the automaton. $L = (a^* + ba^*b)ba^*$. Equivalence are:

- $[b] = L$
- $[e] = Lba^*$

3)

According to Myhill-Nerode theorem a language is regular if and only if it has finitely many equivalence classes. To prove L' is not regular we need to show that L' has infinitely many equivalence classes.

- Let $S = \{a^n b^m | n, m \in N\}$. S is infinite since it contains one string for each natural number pair (n, m).
- Consider any two strings $a^{n_1}b^{m_1}, a^{n_2}b^{m_2} \in S$. Where $n_1 + m_1 \neq n_2 + m_2$
- Let $n_1 + m_1 = k_1 + 2u_1$ $(k_1, u_1 \in N)$
- Then $a^{n_1}b^{m_1}c^{k_1}d^{u_1} \in L'$ and $a^{n_2}b^{m_2}c^{k_1}d^{u_1} \notin L'$
- So $a^{n_1}b^{m_1}$ and $a^{n_2}b^{m_2}$ are distinguishable relative to L'. They should be in different equivalence class.
- Since S is infinite we can choose infinitely many such pairs. This means L' has infinitely many equivalence classes. Therefore, by Myhill-Nerode theorem, L' is not regular

Answer 2

1)

$G = (V, \Sigma, R, S)$ where;
$V = \{a, b, S, S_1\}$
$\Sigma = \{a, b\}$

$$\begin{aligned} R = \{&S \to S_1 b S_1, \\ &S_1 \to aS_1b \mid bS_1a \mid S_1S_1 \mid bS_1 \mid \epsilon\} \end{aligned}$$

2)

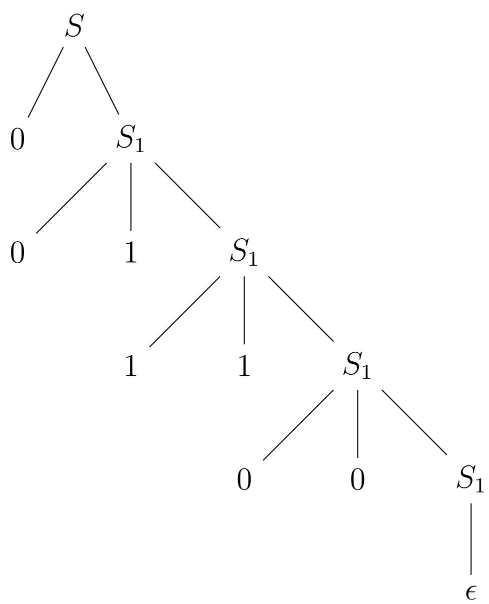
$G = (V, \Sigma, R, S)$ where;
$V = \{0, 1, 2, S, S_1, S_2\}$
$\Sigma = \{0, 1, 2\}$

$$\begin{aligned} R = \{S &\to S_1 S_2 \mid \epsilon, \\ S_1 &\to 0S_1 1 \mid e, \\ S_2 &\to 1S_2 2 \mid e\} \end{aligned}$$

3)

$G = (V, \Sigma, R, S)$ where;
$V = \{0, 1, S, S_1\}$
$\Sigma = \{0, 1\}$

$$\begin{aligned} R = \{S &\to 0S_1 \mid 1S_1, \\ S_1 &\to 00S_1 \mid 01S_1 \mid 10S_1 \mid 11S_1 \mid e\} \end{aligned}$$



Answer 3

1)

$$L_1 = \{e\} \cup \{w \in \{0,1\}^* \mid w \text{ starts and ends with same symbol}\}$$

2)

$$L_2 = \{w \in \{0,1\}^* \mid w \text{ has at least two 1's }\}$$