Student Information

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Answer 1

a)

We calculate the expected value $E(X) = \sum_x xP(x)$. Since the probabilities of all faces on a die are equal, we can write it as:

- $E(B) = \frac{1}{6} \cdot (1+2+3+4+5+6) = 3.5$
- $E(Y) = \frac{1}{8} \cdot (1+1+1+3+3+3+4+8) = 3$
- $E(R) = \frac{1}{10} \cdot (2+2+2+2+2+3+3+4+4+6) = 3$

b)

To maximize the total value on three dice we need to choose the option which gives higher expected value.

- Option 1: Rolling a single dice for each color.

$$E(X) = E(B) + E(Y) + E(R) = 3.5 + 3 + 3 = 9.5$$

- Option 2: Rolling three blue dice.

$$E(3B) = 3 \cdot E(B) = 3 \cdot 3.5 = 10.5$$

Option 2 (Rolling three blue dice.) should be chosen since the expected value is higher.

c)

If it is guaranteed that the yellow die's value will be 8 we can say that $E(Y) = 8$. Then

- Option 1: Rolling a single dice for each color.

$$E(X) = E(B) + E(Y) + E(R) = 3.5 + 8 + 3 = 14.5$$

- Option 2: Rolling three blue dice.

$$E(3B) = 3 \cdot E(B) = 3 \cdot 3.5 = 10.5$$

Option 1 (Rolling a single dice for each color.) should be chosen since the expected value is higher.

d)

R, B, Y denotes the events that red, blue, yellow die is picked respectively. The question asks for $P(R|3)$. From Bayes' Rule, we have

$$P(R|3) = \frac{P(3|R) \cdot P(R)}{P(3)}$$

- $P(R) = P(B) = P(Y) = \dfrac{1}{3}$ (each color has equal probability)
- $P(3|B) = \dfrac{1}{6}$, $P(3|Y) = \dfrac{3}{8}$, $P(3|R) = \dfrac{2}{10}$
- $P(3) = \dfrac{1}{3} \cdot P(3|B) + \dfrac{1}{3} \cdot P(3|Y) + \dfrac{1}{3} \cdot P(3|R) = \dfrac{89}{360}$ (using law of total probability, since events are mutually exclusive and exhaustive. Choosing any of dice is $\frac{1}{3}$)

$$P(R|3) = \frac{\frac{2}{10} \cdot \frac{1}{3}}{\frac{89}{360}} = \frac{24}{89} \approx 0.2697$$

e)

To get total value of 5, we have following combinations:

- Blue die: 1, Yellow die 4
- Blue die: 2, Yellow die 3
- Blue die: 4, Yellow die 1

Now, calculate the probability of each of these combinations. (Since events are independent we can simply multiply to find probability both events happening together):

- $P\{\text{Blue die: } 1\} \cap P\{\text{Yellow die: } 4\}$
  - $P\{\text{Blue die: } 1\} = \frac{1}{6}, P\{\text{Yellow die: } 4\} = \frac{1}{8}$
  - $\dfrac{1}{6} \cdot \dfrac{1}{8} = \dfrac{1}{48}$
- $P\{\text{Blue die: } 2\} \cap P\{\text{Yellow die: } 3\}$
  - $P\{\text{Blue die: } 2\} = \frac{1}{6}, P\{\text{Yellow die: } 3\} = \frac{3}{8}$
  - $\dfrac{1}{6} \cdot \dfrac{3}{8} = \dfrac{3}{48}$
- $P\{\text{Blue die: } 4\} \cap P\{\text{Yellow die: } 1\}$

- $P\{\text{Blue die: } 4\} = \frac{1}{6}, P\{\text{Yellow die: } 1\} = \frac{3}{8}$
- $\dfrac{1}{6} \cdot \dfrac{1}{8} = \dfrac{3}{48}$

Finally, we need to add up the probability of all events happening to get overall probability

$$\frac{1}{48} + \frac{3}{48} + \frac{3}{48} = \frac{7}{48}$$

Answer 2

a)

We can think this question as binomial distribution with parameters $n = 80$ and $p = 0.025$. Let X be denote the distributors of company A will offer a discount tomorrow.

- We try to find $P(X \geq 4)$
- $P(X \geq 4) = 1 - P(X < 4)$
- We can calculate $1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3))$
- $1 - (\binom{80}{0} \cdot 0.025^0 \cdot 0.975^{80} + \binom{80}{1} \cdot 0.025^1 \cdot 0.975^{79} + \binom{80}{2} \cdot 0.025^2 \cdot 0.975^{78} + \binom{80}{3} \cdot 0.025^3 \cdot 0.975^{77})$
- or we can use octave online to calculation with this query `1-binocdf(3,80,0.025)`

$$\begin{aligned} P(X \geq 4) &= 1 - P(X < 4) \\ &= \texttt{1-binocdf(3,80,0.025)} \\ &= 0.1406 \end{aligned}$$

b)

To calculate the probability of getting a discount on a phone within two days, we can use the binomial distribution. Let X and Y be denote, the distributors of company A and B will offer a discount tomorrow, respectively.

- First we need to calculate not getting discount on a specific day for each company
- $P(X = 0) = \binom{80}{0}(0.025)^0(1 - 0.025)^{80} = 0.975^{80}$
- $P(Y = 0) = \binom{1}{0}(0.1)^0(1 - 0.1)^1 = 0.9^1$
- Now, we calculate the probability of not getting a discount from both companies on a specific day. Since companies act independently we can simply multiply what we found:

$$0.975^{80} \cdot 0.9^1 = 0.1187$$

- Next, we find the probability of not getting a discount from both companies in two days:

$$0.1187^2 = 0.0141$$

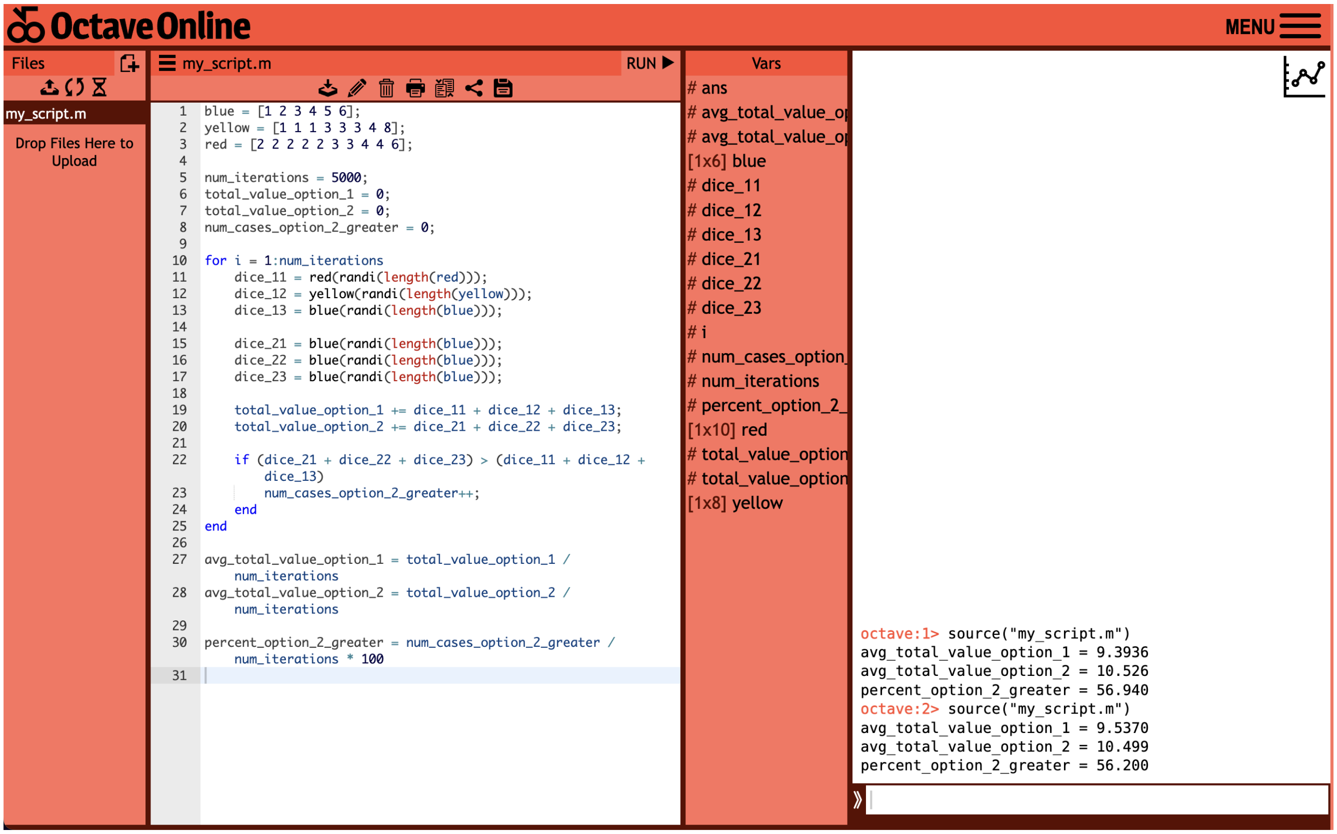
- Now, we calculate the probability of getting a discount within two days by subtracting the above probability from 1:

$$1 - 0.0141 = 0.9859$$

- So, the probability of being able to buy a phone within two days is ≈ 0.986

Answer 3

```
blue = [1 2 3 4 5 6];
yellow = [1 1 1 3 3 3 4 8];
red = [2 2 2 2 2 3 3 4 4 6];

num_iterations = 5000;
total_value_option_1 = 0;
total_value_option_2 = 0;
num_cases_option_2_greater = 0;

for i = 1:num_iterations
    dice_11 = red(randi(length(red)));
    dice_12 = yellow(randi(length(yellow)));
    dice_13 = blue(randi(length(blue)));

    dice_21 = blue(randi(length(blue)));
    dice_22 = blue(randi(length(blue)));
    dice_23 = blue(randi(length(blue)));

    total_value_option_1 += dice_11 + dice_12 + dice_13;
    total_value_option_2 += dice_21 + dice_22 + dice_23;

    if (dice_21 + dice_22 + dice_23) > (dice_11 + dice_12 + dice_13)
        num_cases_option_2_greater++;
    end
end

avg_total_value_option_1 = total_value_option_1 / num_iterations
avg_total_value_option_2 = total_value_option_2 / num_iterations
percent_option_2_greater = num_cases_option_2_greater / num_iterations * 100
```



Based on the given results of 5000 tries, we can see that the average total value for Option 2 (10.499) is indeed higher than the average total value for Option 1 (9.537). Analysis results match the expected values, it means we did the analysis correctly. Additionally, the percentage of times where Option 2 results in a higher total value (56.2%) supports the conclusion that Option 2 is the better strategy for maximizing the total value.